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# A fracture criterion for high-strength steel cracked bars

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**Abstract.** In this paper a fracture criterion is proposed for cracked cylindrical samples of high-strength prestressing steels of different yield strength. The surface crack is assumed to be semi-elliptical, a geometry very adequate to model sharp defects produced by any subcritical mechanism of cracking: mechanical fatigue, stress-corrosion cracking, hydrogen embrittlement or corrosion fatigue. Two fracture criteria with different meanings are considered: a *global* (energetic) criterion based on the energy release rate G, and a *local* (stress) criterion based on the stress intensity factor  $K_I$ . The advantages and disadvantages of both criteria for engineering design are discussed in this paper on the basis of many experimental results of fracture tests on cracked wires of high-strength prestressing steels of different yield strength and with different degrees of strength anisotropy.

**Key words:** high-strength steel; cracked cylindrical bars; stress intensity factor; energy release rate; fracture criterion.

### 1. Introduction

In the framework of damage tolerance analyses in structural engineering, a problem of major concern is the formulation of a fracture criterion useful for engineering design against catastrophic failure (Barsom and Rolfe 1987). This is particularly important in the case of high-strength structural members in the form of bars, wires, strands, tendons and cables (e.g., high-strength steels for reinforcing and prestressing concrete) which are axially loaded under very severe loads (Elices 1985) and can suffer fatigue (Elices *et al.* 1994) or stress corrosion cracking (Parkins *et al.* 1982), thus increasing the risk of catastrophic failure and reducing the structural life (Valiente and Elices 1998).

The fracture problem of a cracked cylindrical bar of high-strength prestressing steel has been addressed in some pioneering works performed in the second half of the 70s decade. To the author's knowledge, the first fracture mechanics approach to the problem of catastrophic failure in prestressing steel wires was the Ph.D. Thesis defended by Astiz (1976) in the Civil Engineering School of the Polytechnic University of Madrid under the direction of Prof. Elices. Later, Athanassiadis defended his Doctoral Thesis on this topic at the end of 1978, the main results being published by Athanassiadis (1979) and Athanassiadis *et al.* (1981). Further research was performed in a new Ph.D. Thesis defended by Valiente (1980) and directed by Prof. Elices. After this date, it is

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Fig. 1 Geometry of the cracked bar

worth mentioning an interesting work by Astiz *et al.* (1986) to ascertain the validity of a generalized Irwin criterion for cracked cylindrical bars at cryogenic temperature (-196°C) at which the specimens exhibited brittle fracture and linear elastic fracture mechanics (LEFM) behaviour.

This paper focuses on the question of the most adequate fracture criterion for high-strength steel bars subjected to transverse cracking, i.e., when a surface mode I crack -probably of quasi-elliptic shape- appears in the plane perpendicular to the main axis of the bar as a consequence of the combined effect of mechanical and environmental actions. Although this issue has received attention in the past, the question is far from being fully understood, specially when the fracture process is not purely brittle. The present paper tries to clarify this important point on the basis of a broad experimental program on steels which have undergone different levels of cold drawing, so that the fracture criteria analyzed here account for the role of the yield strength of the material (strain hardening effect) and of the degree of anisotropy induced by cold drawing (microstructural orientation effect).

### 2. Problem statement

## 2.1 On the stress intensity factor $K_1$

In the estimation of the safe service life of cracked bars, a pre-requisite is the knowledge of the stress intensity factor  $K_I$  for the considered geometry and loading mode: a cylinder subjected to tension with a part-through crack (assumed to be semi-elliptical) perpendicular to the tensile loading direction, i.e., loaded in mode I, as is shown in Fig. 1. In this case, some difficulties arise because

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Fig. 2 Fracture modes in slightly drawn (a) and heavily drawn (b) steels; f: fatigue precrack; I: mode I cracking; II: propagation step with mixed mode cracking; F: final fracture

of the three-dimensional (3D) nature of the surface crack, causing the stress intensity factor to change along the crack front. This factor is a function of the crack depth, the crack aspect ratio and the position on the crack border, i.e.,

$$K_I = K_I(a/D, a/b, s) \tag{1}$$

where a is the crack depth (minor axis of the ellipse), b the other dimension of the crack (major axis of the ellipse), D the diameter of the bar and s the curvilinear coordinate marking the position of the specific point at the crack front (cf. Fig. 1).

A dimensionless stress intensity factor Y may also be defined as:

$$Y = K_I / \sigma \sqrt{\pi a} \tag{2}$$

 $\sigma$  being the remote axial stress on the cross section of the bar (uniform distribution of stress):

$$\sigma = 4F/\pi D^2 \tag{3}$$

where *F* is the tensile load applied on the cylinder.

Most of the K-solutions applicable to cracked cylindrical bars have been calculated by the finite element method. References published by Daoud *et al.* (1978) and Daoud and Cartwright (1984, 1985) present numerical computations of  $K_I$  by using a plane stress finite element model of variable width to model the cross section (pseudo three-dimensional analysis). Many papers (Blackburn 1976, Astiz and Elices 1980, Salah el din and Lovegrove 1981, Astiz 1986, Ng and Fenner 1988, Carpinteri 1992a, 1992b) deal with K-solutions for a cracked cylinder obtained by means of 3D finite element analyses (the most frequent method for computing stress intensity factors), whereas Athanassiadis (1979) and Athanassiadis *et al.* (1981) present results for the same body using the boundary integral equation method. Daoud *et al.* (1978) and Bush (1976, 1981) use a compliance experimental method to obtain  $K_I$ .

An important point is to know the variation of  $K_I$  along the crack line. Depending on the crack aspect ratio of the ellipse,  $K_I$  will be maximum or minimum at the center of the crack line (deepest point). This is a fact which has to be taken into account in fatigue crack propagation problems or in any problem of fracture mechanics where it is required to know the maximum value of  $K_I$  along the

crack front. For the semi-elliptical cracks obtained in the experimental programme (see next section of this paper) the aspect ratio was a/b < 1 in all cases and thus the maximum stress intensity value was achieved always at the crack center, according to the results given by Astiz (1986).

Another key point in this 3D problem is the stress-strain state in the neighbourhood of the crack border which may be between plane stress and plane strain considering the normal plane to the crack line at a specific point. With regard to this, the hypothesis of a plane strain situation in the vicinity of the crack front has been justified theoretically (Bui 1977) and numerically (Astiz 1976, Athanassiadis 1979, Athanassiadis *et al.* 1981). However, there is a loss of constraint and triaxiality at points of the crack near the free surface, as addressed by Sih and Lee (1989) in paper where the stress state and the strain energy density near the free surface are calculated. From the physical point of view, the loss of triaxiality can even change the microscopic mode of fracture form ductile (*micro-void coalescence*) to brittle (*cleavage-like*), as shown by Toribio (1997) in the case of fracture tests on axisymmetric notched samples of high strength pearlitic steel similar to those used in the present work.

#### 2.2 On the fracture criterion

The fracture criterion used in this work is that based on the stress intensity factor in the framework of LEFM principles, i.e.,  $K_I = K_{IC}$ . However, two formulations will be used, one of them global (energy-based) and the other one local (stress-based) to ascertain which describes with more accuracy the fracture process in a 3D complex geometry like that of the cracked bar. These two approaches have been proposed in previous research, although no definitive conclusion has been drawn as to which in more adequate for describing the fracture process in cracked cylindrical bars. The global formulation was used by Athanassiadis (1979), Valiente (1980), Athanassiadis *et al.* (1981) and Elices (1985) while the local one was proposed by D'Escatha and Labbens (1978), Bui and Dang Van (1979), Astiz *et al.* (1986) and Valiente and Elices (1998). In this paper both approaches are reviewed and applied to a broad range of fracture tests on prestressing steel bars.

Firstly, a *global* fracture criterion may be formulated on the basis of energetic considerations, using either the strain energy release rate concept or the specimen compliance (related to the former by a one-to-one relationship). According to this criterion, fracture will take place when the energy release rate reaches a critical value.

For a given geometry, it represents a single-parameter approach, since the energy release rate (or, accordingly, the compliance) for the considered geometry and loading mode (cf. Fig. 1) depends only on the crack depth a. Thus an average value of the energy release rate  $G^*$  along the crack front may be computed as follows (Athanassiadis 1979, Athanassiadis *et al.* 1981, Elices 1985).

$$G^{*} = \frac{1}{2s} \int_{-s}^{+s} \frac{K_{I}^{2}}{E'} ds$$
(4)

where E' is the generalized Young's modulus, i.e., E' = E in plane stress (bar surface) and  $E' = E/(1 - v^2)$  in plane strain (crack center). The single asterisk stands to emphasize that only one geometric parameter (the crack depth *a*) is required.

Another possibility is to evaluate the specimen compliance and calculate from it the energy release rate and thus the stress intensity factor. This method was used by Valiente (1980), leading to the following expression of the stress intensity factor:

$$K_I^* = Y^*(a/D)\sigma\sqrt{\pi a} \tag{5}$$

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where  $\sigma$  is the remote axial stress (far from the crack), a the crack depth, and  $Y^*(a/D)$  a dimensionless function given by:

$$Y^{*}(a/D) = [0.473 - 3.286(a/D) + 14.797(a/D)^{2}]^{1/2}[(a/D) - (a/D)^{2}]^{-1/4}$$
(6)

This function was obtained by a finite element computation of the compliance in a cracked cylinder with a straight-fronted edge crack. Again a single asterisk is used to indicate that only one geometric parameter (the crack depth a) is required, i.e., it is a simplified approach in which the aspect ratio a/b and the curvilinear coordinate s are not considered, the former because a representative geometry (the cylinder with a straight-fronted edge crack) is employed, and the latter due to the global character of the fracture criterion according to which failure takes place when a critical condition for the *whole crack* is achieved.

From the results presented by Valiente (1980) using precracked rods of different materials, it may be concluded that global criteria seem to be more adequate in fracture situations with a certain degree of plasticity, i.e., when not purely brittle materials are involved and the microscopic fracture process develops by a micro-void coalescence over a certain area.

Secondly, a *local* fracture criterion may be considered. This criterion was rigourously formulated by D'Escatha and Labbens (1978) and Bui and Dang Van (1979) in the form:

$$Sup_{\Gamma}K_{I}(s) = K_{IC} \tag{7}$$

where *s* is the curvilinear coordinate marking the position of the specific point on the crack front (cf. Fig. 1) and  $\Gamma$  is the domain, i.e., the crack line. Expression (7) indicates that fracture takes place when the maximum stress intensity factor along the crack curve reaches a critical value, i.e., a local instability at a certain point produces the catastrophic fracture of the overall cylinder.

To apply this local fracture criterion, a biparametric K-solution (depending not only on the crack depth but also on the crack aspect ratio) is required at any point of the crack. For the geometry under consideration, i.e., a cracked cylinder in tension with a part-through crack of semi-elliptical shape (Fig. 1), it was obtained by Astiz (1986) by the finite element method using singular elements and virtual crack extension technique to compute the stress intensity factor at any point of the crack front, and particularly at the crack center (i.e., at s = 0 where the maximum value is achieved for aspect ratio a/b < 1):

$$K_{I}^{**} = Y^{**}(a/D, a/b)\sigma\sqrt{\pi a}$$
(8)

A double asterisk is used to indicate that two parameters (the relative crack depth a/D and the crack aspect ratio a/b) are needed. In this case the dimensionless stress intensity factor is expressed by Astiz (1986):

$$Y^{**}(a/D, a/b) = \sum_{\substack{i=0\\i\neq 1}}^{4} \sum_{j=0}^{3} C_{ij}(a/D)^{i}(a/b)^{j}$$
(9)

and the coefficients  $C_{ii}$  are given in Table 1.

$C_{ii}$	j = 0	j = 1	<i>j</i> = 2	<i>j</i> = 3
i=0	1.118	-0.171	-0.339	0.130
i = 2	1.405	5.902	-9.057	3.032
<i>i</i> = 3	3.891	-20.370	23.217	-7.555
<i>i</i> = 4	8.328	21.895	-36.992	12.676

Table 1 Values of  $C_{ij}$  to compute the local stress intensity factor  $K_I^{**}$ 

This local criterion is formulated on the basis of pure stress considerations, and it seems to be adequate for brittle fracture (e.g., cleavage-like or low temperature). It has been successfully applied to the fracture of reinforcing steel displaying linear-elastic behaviour and brittle fracture at low temperature (Astiz et al. 1986) and of cracked cylindrical bars of prestressing steel exhibiting cleavage-like fracture at room temperature (Valiente and Elices 1998). In the former case (Astiz et al. 1986) the critical value of the stress intensity factor in the cracked bar was in agreement with the fracture toughness obtained from a standard fracture toughness test on a compact tension specimen.

# 3. Experimental programme

## 3.1 Experimental procedure

A high strength eutectoid steel supplied from commercial stock by EMESA TREFILERIA (La Coruña, Spain) was used in this work. The chemical composition is given in Table 2. Different degrees of cold drawing were analyzed, associated with intermediate steps of the manufacturing process from the hot rolled bar (steel A0 which is not cold drawn at all) to the prestressing steel

С	Mn	Si	Р	S	Cr	V	Al	
0.80	0.69	0.23	0.012	0.009	0.265	0.060	0.004	

Table 2 Chemical composition (wt %) of the steel

Table 5 Nome	ficiature, uia	meter reduction		uncar propert	ies of the siec	.15		
Steel	A0	A1	A2	A3	A4	A5	A6	
$D_i$ (mm)	12.00	10.80	9.75	8.90	8.15	7.50	7.00	
$D_i/D_0$	1	0.90	0.81	0.74	0.68	0.62	0.58	
E (GPa)	197.4	201.4	203.5	197.3	196.7	202.4	198.8	
$\sigma_{Y}$ (GPa)	0.686	1.100	1.157	1.212	1.239	1.271	1.506	
$\sigma_R$ (GPa)	1.175	1.294	1.347	1.509	1.521	1.526	1.762	
P(GPa)	1.98	2.26	2.33	2.49	2.50	2.74	2.34	
n	5.89	8.61	8.70	8.45	8.69	7.98	11.49	

Table 3 Nomenclature, diameter reduction and mechanical properties of the steels

*E*: Young's modulus,  $\sigma_{Y}$  yield strength,  $\sigma_{R}$ : ultimate tensile stress (UTS)

*P*, *n*: Ramberg-Osgood parameters:  $\varepsilon = \sigma/E + (\sigma/P)^n$ 

wire (steel A6 which has undergone six passes of cold drawing and represents the final commercial product). Table 3 gives the nomenclature and diameter of all the steel wires, so the number of drawing steps applied to each one is indicated by the digit in its own name.

Cylindrical samples of 30 cm were made by cutting the steel wires. A starter notch was produced by means of a jewellers' file in a plane perpendicular to the wire axis. Samples were subjected to axial fatigue (tensile loading/unloading in the direction of the wire axis) to produce a precrack before the fracture test. The precracking programme was designed so that the maximum stress intensity factor at the final stage (just before the fracture test) never exceeded 60% of the critical stress intensity factor reached at final failure. This condition was checked after each fracture test to reject those specimens precracked at higher loads.

After fatigue precracking, the cracked rods (Fig. 1) were subjected to monotonic tensile loading up to fracture at a crosshead speed of 3 mm/min. The load applied on the specimen and the relative displacement of two points symmetrically placed in relation to the crack plane were continuously monitored during the test, the first by means of load cell and the second using an extensometer (whose gage length was 12.5 mm) placed in front of the crack, so as to record the complete load-displacement plot.

A total number of 37 fracture tests were performed, more or less uniformly distributed among all wire diameters (i.e., among all degrees of cold drawing), although fewer tests were conducted on the intermediate levels of cold drawing due to the scarcity of material from these stages of the manufacturing route. Nevertheless, at least three replicate tests were made for each material condition (degree of drawing).

Since all the tests were performed on the same family of steels with identical chemical composition, no metallurgical variables are involved in this study and thus the main objective of the research, the establishment of a fracture criterion for cracked cylindrical wires of high-strenght steel, may be achieved, accounting for the role of the yield strength of the material (strain hardening effect) and the degree of anisotropy induced by cold drawing (microstructural orientation effect).

#### 3.2 Experimental results

As is shown in Fig. 2, while the fracture behaviour of slightly drawn steels (A0-A3) was isotropic, the most heavily drawn steels (A4 to A6) exhibited anisotropic fracture behaviour with crack deflection. Therefore, crack propagates in mode I over the whole fracture area in slightly drawn steels, whereas in heavily drawn steels a mixed mode appears with a propagation step oriented 90° in relation to the initial propagation direction by fatigue in mode I. This 90° step is thus parallel to the wire axis or cold drawing direction, a consequence of the marked microstructural orientation induced in the pearlitic colonies and lamellae of the steels by the manufacturing process in the form of progressive cold drawing (Ovejero 1998, Toribio and Ovejero 1998a, 1998b).

A fractographic analysis (Toledano 1998, Toribio and Toledano 1998, 2000) revealed that the fracture micromechanisms also depend on the cold drawing level, from predominant cleavage (brittle) in slightly drawn steels to predominant micro-void coalescence (ductile) in heavily drawn steels. This evolution from brittle to ductile microscopic mode of fracture is not sudden, but takes place progressively as the degree of cold drawing increases.

The load-displacement plots during the fracture tests exhibited a monotonously increasing aspect up to final fracture with no decrease in load, and this was independent of the degree of cold drawing. Nevertheless, the appearance of the load-displacement plot during the fracture tests was



Fig. 3 Double recording of load vs. displacement in a test on steel A6 with two extensioneters, one at the crack mouth (1) and the other on the opposite side ahead of the crack tip (2)

seen to depend on the strain hardening level, with a general evolution from a linear plot associated with elastic behaviour up to fracture in slightly drawn steels to non-linear (clearly curved) load-displacement plot in heavily drawn steels.

In heavily drawn steels the presence of the 90°-step can explain the non-linear part of the loaddisplacement curve, and this hypothesis was experimentally checked by performing a fracture test with continuous recording of two displacement signals by means of two extensometers placed symmetrically in relation to the crack plane, one of them in front of the crack mouth and the other in the opposite position (at the other side of the diameter). Fig. 3 shows the double plot of load versus displacement for the two extensometers in a fracture tests on steel A6 (heavily drawn). A clear bending effect -of geometrical nature- is observed in the specimen as a consequence of the 90°-step associated with *pop-in* phenomenon. Thus, while the displacement signal measured by the extensometer numer 1 (that located in front of the crark mouth) increases very much, the other signal (that of extensometer number 2 placed at the opposite position) remains quasi-constant or only slightly increasing, which indicates that the bending produced by the step (and the subsequent mixed mode propagation) is the reason for the non-linear behaviour of the samples.

## 4. Discussion

#### 4.1 Methods for evaluation of the critical stress intensity factor

In the load-displacement plots of heavily drawn steels (cf. Fig. 3), two characteristic load levels may be defined:

- (i) The level  $F_Y$  at which the pop-in takes place and the 90°-propagation step appears, i.e., the instant of fracture initiation.
- (ii) The maximum load  $F_{\text{max}}$  in the load-displacement plot, i.e., the instability point associated with final fracture.



Fig. 4 Critical values of the stress intensity factor

Fig. 5 Standard deviations  $\sigma_{n-1}$  associated with the results of Fig. 4

It should be noticed that in slightly drawn steels there is no pop-in, and thus the fracture initiation coincides with the instant of final fracture ( $F_Y = F_{max}$ ).

In addition, two expressions of the stress intensity factor may be used for the geometry and loading mode under consideration:

- (i) A uniparametric expression  $K_I^*$  (*a/D*) of the stress intensity factor (5) related to a global (energy-based) fracture criterion.
- (ii) A biparametric expression  $K_I^{**}(a/D, a/b)$  of the stress intensity factor (8) linked with a local (stress-based) fracture criterion.

Therefore, four characteristic stress intensity values -candidates to fracture toughness of the considered steels- may be evaluated from the test results:

- (i)  $K_{IY}^*$ : evaluated as the uniparametric  $K_I^*$  level at fracture initiation (pop-in) using the load  $F_Y$  and the fatigue crack depth *a*.
- (ii)  $K_{IY}^{**}$ : evaluated as the biparametric  $K_I^{**}$  level at fracture initiaion (pop-in) using the load  $F_Y$ , the fatigue crack depth *a* and the aspect ratio a/b.
- (iii)  $K_{I\max}^*$ : evaluated as the uniparametric  $K_I^*$  level at the final fracture instant using the maximum load  $F_{\max}$  and the fatigue crack depth *a*.
- (iv)  $K_{I\max}^{**}$ : evaluated as the biparametric  $K_I^{**}$  level at the final fracture instant using the maximum load  $F_{\max}$ , the fatigue crack depth *a* and the aspect ratio a/b.

Fig. 4 shows the four critical values of the stress intensity factor and Fig. 5 their corresponding statistical errors in the form of standard deviation  $\sigma_{n-1}$  defined as:

$$\sigma_{n-1} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}$$
(10)

where  $x_i$  is any measure,  $\overline{x}$  the average value and *n* the total number of measures.

# 4.2 Slightly drawn steels (from A0 to A3)

These are the steels with the most isotropic and brittle behaviour. In this case the initiation load  $F_Y$  coincides with the maximum one  $F_{\text{max}}$ , and accordingly, there is no difference between the stress intensity values  $K_{IY}$  and  $K_{Imax}$ . With regard to the distinction between the two fracture criteria (global and local), the following points should be emphasized:

- The measured fracture toughness is more reliable using the local stress intensity factor at the deepest point of the crack (central position) rather than the global stress intensity factor for the whole cracked geometry, since the standard deviation of the measures is lower in the first case (between 2 and 4 MPa m<sup>1/2</sup>) than in the second one (between 4 and 7 MPa m<sup>1/2</sup>).
- $\cdot$  The use of a global stress intensity factor based on a single crack geometry parameter (the crack depth *a*) overestimates slightly the evaluated fracture toughness value if compared to the evaluation through a biparametric expression of the local stress intensity factor accounting for both the depth and the aspect ratio of the crack.
- The reason for this may be the overestimation of the cracked area in the cross section of the cylinder when the global expression of stress intensity factor is used, since it assumes an edge crack with a straight front, obviously deeper than the real semi-elliptical cracks at any point other than that of the center position.

# 4.3 Heavily drawn steels (from A4 to A6)

These are the steels with the most anisotropic and ductile behaviour, and the situation is not so clear due to the mixed mode stress state caused by the 90° step. In this case the maximum load  $F_{\text{max}}$  associated with final catastrophic failure exceeds clearly the initiation load  $F_Y$  associated with the instant of pop-in and the formation of the 90°-step. In the matter of the fracture criteria, the following comments may be made:

- The measured toughness for fracture initiation can be evaluated using either of the expressions for the stress intensity factor: the global one (5) or the local one (8), since the standard deviation is really similar in both cases. Thus the choice of the fracture criterion is not relevant to the matter of fracture initiation.
- With regard to final fracture at maximum load, the standard deviation errors are in general lower when using a global fracture criterion with a single-parameter stress intensity factor than when using a local fracture criterion and a two-parameter  $K_I$ , which indicates that the former seems to be more adequate for fracture processes which are not purely brittle.
- As in the case of slightly drawn steels and for the same reason, again the global K-expression overestimates the characteristic value of the fracture toughness. However, in this case there is a ductile subcritical crack growth by micro-void coalescence (Toribio and Toledano 1998) which makes the crack front tend to a straight-fronted edge shape.

# 4.4 Global versus local fracture criterion

A local fracture criterion seems to be the more adequate for brittle failure (taking place in slightly drawn steels). This is consistent with a micromechanism of fracture of the *weakest link* type, according to which fracture occurs when a single point reaches the critical condition and the fracture of this unit promotes the failure of the overall specimen. This unit is the weakest link which

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plays the role of fracture initiator or promoter.

Thus, for slightly drawn steels breaking in purely brittle mode:

$$K_{IC} = K_{IY}^{**} = K_{Imax}^{**}$$
(11)

i.e., the critical stress intensity factor (fracture toughness) at the moment of failure may be evaluated using the expression (8). In this case there is no difference between fracture initiation and final fracture.

Conversely, a global fracture criterion appears to be more suitable for ductile failure, or brittle failure after a previous ductile development in heavily drawn steels, which is consistent with a micromechanism of fracture of the *process zone* type, according to which fracture occurs when the critical condition is reached over a certain area: the process zone, damaged area or fracture initiation region.

Thus, for heavily drawn steels breaking in a more ductile mode, the critical stress intensity factor for fracture *initiaton* is:

$$K_{IC}^{(i)} = K_{IY}^{*}$$
(12)

and for *final* fracture:

$$K_{IC}^{(f)} = K_{I\max}^{*}$$
 (13)

i.e., the characteristic stress intensity factors for fracture initiation and final fracture may be evaluated using the expression (5). It should be noticed that the characteristic value (13) is only an engineering estimation, but not a material property, since it is determined as if the crack propagated in mode I, in spite of the presence of the 90°-step (mixed mode propagation) which governs the non-linear portion of the load-displacement plot.

Apart from its adequacy for more ductile fracture processes, the global fracture criterion may be capable of accounting for the anisotropic fracture behaviour caused by the markedly oriented microstructure (in a direction parallel to the wire axis) of heavily drawn steels as a consequence of the drawing process (cf. Toribio and Ovejero 1998a, 1998b). Therefore, this global criterion could be useful for fracture processes in high-strength steel cracked wires with a certain degree of ductility and anisotropy, which is fully consistent with the results obtained by Valiente (1980).

In spite of the fact that a global criterion describes better the fracture process in heavily drawn steels, the use of a local one could be considered as a conservative approach to damage tolerance design of structural elements in wire form which can suffer fatigue or stress-corrosion cracking. Thus, for the sake of uniqueness, a common fracture criterion -valid for any degree of cold drawing in high-strength steel wires- could be formulated on the basis of a local stress intensity factor (8) in the following form:

Fracture will take place when the maximum stress intensity factor at any point of the crack front reaches a critical value which can be considered as a material property: the fracture toughness in slightly drawn steels or the critical stress intensity factor for fracture initiation in heavily drawn steels.

# 5. Conclusions

Two fracture criteria may be formulated for cracked samples of high-strength cold drawn steel: a *global* (energetic) criterion based on the energy release rate G, and a *local* (stress) criterion based on the stress intensity factor  $K_I$ . It is seen that the more adequate criterion depends on the degree of cold drawing.

In slightly drawn steels (which behave isotropically) the fracture process develops in a brittle manner, so that a *local* fracture criterion may be formulated for this case on the basis of the maximum value of the stress intensity factor at any point of the crack front. This criterion is consistent with a microscopic mode of fracture of the *weakest-link* type.

In heavily drawn steels (which exhibit anisotropic fracture behaviour) the fracture process develops in a more ductile manner, so that a *global* fracture criterion may be used in this case on the basis of the global value of the stress intensity factor (or, accordingly, the energy release rate). It is consistent with a microscopic fracture mode based on the *process zone* concept.

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# References

- Astiz, M.A. (1976), "Estudio de la estabilidad de una fisura superficial en un alambre de acero de alta resistencia", Ph.D. Thesis, Polytechnic University of Madrid, Spain.
- Astiz, M.A. (1986), "An incompatible singular elastic element for two- and three- dimensional crack problems", *Int. J. Fracture*, **31**, 105-124.
- Astiz, M.A. and Elices, M. (1980), "On the application of the stiffness derivative method to two and three dimensional fracture problems", *Proc. Second Int. Conf. Numerical Methods in Fracture Mechanics*, 93-106.
- Astiz, M.A., Elices, M. and Valiente, A. (1986), "Numerical and experimental analysis of cracked cylindrical bars", In: Van Elst, H.C., Bakker, A. (Eds.), Fracture Control of Engineering Structures-ECF 6. EMAS, West Midlands, 65-74.
- Athanassiadis, A. (1979), "Stabilité, ténacité, propagation des fissures dans les fils et barres en acier", Laboratoire Central des Ponts et Chaussées. Rapport de Recherche LPC n°89.
- Athanassiadis, A., Boissenot, J.M., Brevet, P., Francois, D. and Raharinaivo, A. (1981), "Linear elastic fracture mechanics computations of cracked cylindrical tensioned bodies", *Int. J. Fracture*, **17**, 553-566.
- Barsom, J.M. and Rolfe, S.T. (1987), *Fracture & Fatigue Control in Structures*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Blackburn, W.S. (1976), "Calculation of stress intensity factors for straight cracks in grooved and ungrooved shafts", *Engng. Fracture Mech.*, **8**, 731-736.
- Bui, H.D. (1977), "An integral equations method for solving the problem of a plane crack of arbitrary shape", J. *Mech. Physics Solids*, **25**, 29-39.
- Bui, H.D. and Dang Van, K. (1979), "Généralisation de la théorie de la rupture de Griffith", *Journal Mécanique Appliquée*, **3**(2), 205-225.
- Bush, A.J. (1976), "Experimentally determined stress-intensity factors for single-edge-crack round bars loaded in

bending", Expl. Mech., 16, 249-257.

- Bush, A.J. (1981), "Stress intensity factors for single-edge-crack solid and hollow round bars loaded in tension", *J. Test. Eval.*, **9**, 216-223.
- Carpinteri, A. (1992a), "Stress intensity factors for straight-fronted edge cracks in round bars", *Engng. Fracture Mech.*, **42**, 1035-1040.
- Carpinteri, A. (1992b), "Elliptical-arc surface cracks in round bars", *Fatigue Fracture Engng. Mater. Struct.*, 15, 1141-1153.
- D'Escatha, Y. and Labbens, R. (1978), "Remarque sur deux critères de rupture fragile pour les problèmes tridimensionnels en mode I", *Journal Mécanique Appliquée*, **2**(4), 541-552.
- Daoud, O.E.K. and Cartwright, D.J. (1984), "Strain energy release rates for a straight-fronted edge crack in a circular bar subject to bending", *Engng. Fracture Mech.*, **19**, 701-707.
- Daoud, O.E.K. and Cartwright, D.J. (1985), "Strain energy release rate for a circular-arc edge crack in a bar under tension or bending", *J. Strain Anal.*, **20**, 53-58.
- Daoud, O.E.K., Cartwright, D.J. and Carney, M. (1978), "Strain energy release rate for a single-edge-cracked circular bar in tension", J. Strain Anal, 13, 83-89.
- Elices, M. (1985), "Fracture of steels for reinforcing and prestressing concrete", In: Sih, G.C., DiTommaso, A. (Eds.), Fracture Mechanics of Concrete: Structural Application and Numerical Calculation, Martinus Nijhoff Publishers, Dordrecht, 226-271.
- Elices, M., Llorca, J. and Astiz, M.A. (1994), "Fatigue of steels for concrete reinforcement and cables", In: Carpinteri, A. (Ed.), Handbook of Fatigue Crack Propagation in Metallic Structures. Elsevier, Amsterdam, 191-220.
- Ng, C.K. and Fenner, D.N. (1988), "Stress intensity factors for an edge cracked circular bar in tension and bending", *Int. J. Fracture*, **36**, 291-303.
- Ovejero, E. (1998), "Fractura en ambiente agresivo de aceros perlíticos con distinto grado de trefilado", Ph.D. Thesis, University of La Coruña, Spain.
- Parkins, R.N., Elices, M., Sánchez-Gálvez, V. and Caballero, L. (1982), "Environment sensitive cracking of prestressing steels", *Corros. Sci.*, 22, 379-405.
- Salah el din, A.S. and Lovegrove, J.M. (1981), "Stress intensity factors for fatigue cracking of round bars", *Int. J. Fatigue*, **3**, 117-123.
- Sih, G.C. and Lee, Y.D. (1989), "Review of triaxial crack border stress and energy behaviour", *Theor. Appl. Fracture Mech.*, **12**, 1-17.
- Toledano, M. (1998), "Fatiga y fractura de aceros perlíticos con distinto grado de trefilado", Ph.D. Thesis, University of La Coruña, Spain.
- Toribio, J. (1997), "A fracture criterion for high-strength steel notched bars", *Engng. Fracture Mech.*, 57, 391-404.
- Toribio, J. and Ovejero, E. (1998a), "Effect of cold drawing on microstructure and corrosion performance of high-strength steel", *Mech. Time-Dependent Mater.*, **1**, 307-319.
- Toribio, J. and Ovejero, E. (1998b), "Microstructure orientation in a pearlitic steel subjected to progressive plastic deformation", J. Mater. Sci. Lett., 17, 1037-1040.
- Toribio, J. and Toledano, M. (1998), "Anisotropic fracture behaviour of eutectoid steels with different degrees of drawing", In: Brown, M.W., de los Ríos, E.R., Miller, K.J. (Eds.), Fracture from Defects-ECF12, EMAS, West Midlands, 685-690.
- Toribio, J. and Toledano, M. (2000), "Fatigue and fracture performance of cold drawn wires for prestressed concrete", *Constr. Building Mater.*, **14**, 47-53.
- Valiente, A. (1980), "Criterios de fractura para alambres", Ph.D. Thesis, Polytechnic University of Madrid, Spain.
- Valiente, A. and Elices, M. (1998), "Premature failure of prestressed steel bars", *Engng. Failure Analysis*, **5**, 219-227.