Structural Engineering and Mechanics, Vol. 14, No. 2 (2002) 135-150
DOI: http://dx.doi.org/10.12989/sem.2002.14.2.135

Ultimate response of bionics shells

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(Received January 29, 2001, Accepted June 26, 2002)

Abstract. Numerical analysis of ultimate behaviour of thin bionics shells is treated in present paper. Interactive conditions in resonance and stability ultimate response are considered. Numerical treatment of nonlinear problems appearing is made using the updated Lagrangian formulation of motion. Each step of the iteration approaches the solution of linear problem and the feasibility of parallel processing FETM-technique with adaptive mesh refinement and substructuring for the analysis of ultimate action of thin bionics shells is established. Some numerical results are submitted in order to demonstrate the efficiency of the procedures suggested.

Key words: adaptive mesh refinement; bionics shell; FETM-method; parallel processing; resonance; stability; substructuring; ultimate dynamics; updated Lagrangian formulation of motion; wave propagation.

1. Introduction

The bionics shells are approaching the shell configurations developed by nature and are nowadays often used in a number of applications in structural engineering.

When environment of bionics shells in structural engineering changes periodically with time, the question appears as to how an equilibrium path can lose its initial stability. The nonlinear bifurcation theory aims to answer the question of evolutionary stability by studying in both qualitative and quantitative terms the singularities that can arise in such an equilibrium path or surface. The merging of such theory with new and powerful topological techniques of catastrophe theory have laid the foundations for a unified theory which can act as a pilot study for a number of specialized formulations. Such approach reduces the original multi-dimensional problem by identifying a number of active internal coordinates and a corresponding number of external control parameters, allowing the 3-D simulations to be made for many of the most relevant singularities.

Illustrative examples are drawn from astrophysics of rotating planetary masses and gravitational collapse to a black hole, the buckling of plate girders, the resonance of elastic solids or the instability of an atomic lattice. Among such subjects, one of the most coherent and intensive developments has arisen recently to deal with the acute buckling problems of highly optimized engineering structures made of new composite materials. Such approach aims to delineate and classify the ways in which an evolving system can lose its stability by examining the singularities

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appearing in the equlibrium path. The item can be a highly pathological one that exhibits high-order singularities; an example being highly optimized systems which often seem to exhibit severe, sometimes compound instabilities.

2. Theoretical approach for stability phenomena

First of all a prototype general theory is sketched for a simple gradient system, with a view of delineating certain archetypal concepts and forms of instability. Considered is a gradient system governed by potential function $V(Q_i, \lambda^j)$, where Q_i is a set of n internal state variables or generalized coordinates and λ^j is a set of h external parameters. The necessary and sufficient conditions for equilibrium of the system is taken to be vanishing for all coordinate derivatives given by

$$\partial V/\partial Q_i = V_i = 0, \quad (1)$$

defining hereby h-dimensional equilibrium surfaces in the $(n + h)$-dimensional coordinate-parameter space. The necessary and sufficient condition for the stability of an equilibrium state is that the potential V, viewed as a function of the Q_i, should obtain a local relative minimum. Such familiar equilibrium and stability conditions can be accepted as two axioms forming the basis of the approach below, which is directly applicable to any bionics shell studied. The most obvious example is the conservative mechanical system of classical mechanics (the Hamiltonian or Lagrangian system), for which the addition of a little viscous damping allows a complete proof of above stability axiom adopting the conventional Liapunov definition. The elastic systems of structural engineering, simulated by classical models of modal or finite element analyses, are of such general type.

The external variables λ^j are often called control parameters, although they are here understood to incorporate all external influences into the system studied, including small, essentially uncontrollable perturbations or disturbances of the environment. They can be the lengths, modulae, geometric imperfections within the fabric of the system, loads, etc. They are assumed to vary slowly with time, in comparison with fast dynamic changes in the Q_i, generated by the minimum-seeking process.

Thus with varying λ^j, the system will follow a local minimum of $V(Q_i)$ until such minimum is destroyed at an unstable critical equilibrium state when a fast dynamic action carries the system rapidly to a new local minimum. Under such conditions there are analysed the ways in which an equilibrium solution evolving with the λ^j can lose its initial stability at some critical point which is invariably associated with a singularity of the equilibrium surface. The loss of stability can only occur at a critical equilibrium state implying the vanishing of the stability determinant $|\partial^2 V/(\partial Q_i \partial Q_j)|$. Critical surfaces are thus defined in coordinate-parameter space by the equations given by

$$V_i = |V_{ij}| = 0, \quad (2)$$

with particular interest in the projection of the stability boundary into the control subspace adopted.

In this situation it can be shown that the essential features of the developing instability can be adequately viewed in m-dimensional subspace of the full n-dimensional coordinate space adopted. Useful first step is to segregate the Q_i into two groups. Those participating in the instability will be m in number and will be called active coordinates, while those not participating in the instability, n-m

in number, are called passive coordinates. Thinking physically, the coordinates must be so segregated, that the placing of constraints on the active coordinates would be sufficient to inhibit the instability. The mathematical condition for this is that the subdeterminant of the passive coordinates should be non-zero, and it is a property of the full singular determinant of rank *n-m* that a valid segregation can always be found for a given initial set of generalized coordinates. Such elimination of passive coordinates must be done by solving their associated passive equilibrium equations. These give the passive coordinates as non-singular functions of the active coordinates and of the λ^j, and these are then substituted back into the original potential function. This submits finally a transformed energy function of the λ^j and the active coordinates only, containing all necessary and sufficient conditions for an examination of equilibrium and stability. Such elimination of passive coordinates is achieved so that Q_i submits the *m* active coordinates and the new transformed energy function *V* has a null determinant at the critical point with every $V_{ij} = 0$.

Having eliminated the passive coordinates, the attention turns now to the external parameter λ^j. Here the concepts of structural stability and of universal unfolding of singularity specify the number of control parameters needed to observe a phenomenon. Consider, for example, a system with a single active generalized coordinate Q_1 and the transformed energy function $V(Q_1)$, for which a critical equilibrium state is defined by the vanishing of V_1 (for equilibrium) and of V_{11} (for stability). The solution of two nonlinear equations $V_1 = 0$ nad $V_{11} = 0$ is required. In order to observe the critical point under the non-pathological conditions at least one λ parameter is to be introduced. Catastrophe theory (see, for example, Zeeman 1976) emphasizes that not only is this one λ necessary to observe the phenomenon, but that it is sufficient to fully understand the singularity provided that λ is contained in *V* in a prescribed manner. Generally, a singularity will only be observable if a scan is made through a sufficient number of external control parameters. In fact, the unfolding rules specify not only how many external parameters are needed, but show exactly how they should enter the energy function. In such a way, the original problem in $(n + h)$ dimensional space is reduced to a simpler problem in $(m + k)$ dimensional space where *m* is the number of active coordinates and *k* is the number of unfolding parameters. An excellent account of unfolding rules is given, for example, in Poston and Stewart (1976).

Special properties of elastic systems in structural engineering have led to a method of study that concentrates on bifurcations of equilibrium path rather than the more general topological study of an equilibrium surface. In the simplest conceptual terms an elastic structure there will be controlled by a gradient potential function and acted upon by a single control parameter, a load henceforth designated simply λ, although other parameters can play the role of external variables (Thompson and Hunt 1976).

3. Ultimate action of bionics shells

Numerical analysis of ultimate action, optimization and tuned vibration control of thin bionics shells has become the focus of intense efforts because of pressing problems of disaster prevention of shell bridges, planes, missiles, ships, etc. Required is sophisticated analysis in order to answer questions associated with ultimate behaviour of such structures acting in extreme environments and exploting conditions possibly occurring.

Technology, traffic and environmental forcing can produce dynamic forces with a predominant frequency concept around ultimate structural resonance and stability regions. In many cases slender shells are slightly damped and can undergo large vibrations when subjected to such forcing. Such vibrations create a serviceability problem because initiating of limit state response including large stress and strains as well as dynamic instabilities with large deformations appearing.

Considerable efforts have been directed towards improving the computational efficiency of numerical components of finite dynamic processes, such as temporal integration of incremental equations of motion, optimization of numerical techniques or development of parallel processing algorithms, that permit the analysis of complex thin bionics shells to whatever degree of modeling desired. Geometric and material nonlinearities and their interactions with time dependent ultimate structural behaviour are to be treated for an essentially unlimited range of shell structural configurations, geometries and exploitation conditions possibly appearing.

One way to manage the above problems efficiently is by adopting of parallel processing alorithms using adaptive mesh refinement and substructuring discretization approach.

Recently, the requirements for development of computational algorithms of discrete streamline type were formulated, exploiting the parallel processing facilities of modern computers. One of such algorithms is the FETM-method (combined finite element versus transfer matrix approach, see, for example, Tesár and Fillo 1988); the numerical technique adopting dynamic variable irregular mesh simulated by moving elements. The discrete simulation of bionics shell studied is modeled during the motion of basic element over the structure. Since a stiffness formulation is adopted for the development of such a basic element, the matrix inversion is required to eliminate the displacements of interior nodes before converting the governing equations for the periodic unit into transmission form.

The solution of nonlinear ultimate response of thin bionics shells requires the consideration of a number of physical phenomena appearing on a macroscopic level. As most significant there are to be mentioned the geometric imperfections and second order geometric effects, elastic-plastic material behaviour, local and total instability effects in critical and postcritical regions, nonlinear stiffness and damping parameters, etc. All above effects have to be taken into account in linear and nonlinear general interactions possibly appearing.

The solution of above problems below is concerned with the FETM-simulation of thin bionics shells consisting of an assemblage of substructures in space and time. The mixed multigrid schemes of discretization adopted in space and time, allow the problem oriented variability of substructure sizes and time steps in various space regions and time intervals of ultimate response of bionics shells studied. The mixed techniques for direct time integration of incremental equations of motion are used in combination with the FETM-method for mixed discretization when adopting the substructuring in space and time.

Perfect bionics shells under ultimate behaviour, governed by a purely membrane state of stress, there may be considered to be optimal in the sense that their load-bearing capacity is generally larger than that of shells which show more or less pronounced deviations in geometry. Such cases occur in ultimate response of thin bionics shells with linear and nonlinear general or local resonance modes of vibration acting as local linear imperfections. The behaviour of imperfect shell is closely related to that of the respective perfect one, because it strongly depends on the stability of the former's equilibrium state at the bifurcation point. Thin shells subjected to simultaneous stability and resonance limit states are often imperfection sensitive. The degree of imperfection sensitivity, that is the difference between the bifurcation load of the respective perfect structure and the failure load of its imperfect realization, is essentially a function of the initial slope of the perfect structure's bifurcation path as well as the magnitude and the shape of the imperfection mode occurring. The actual ultimate buckling loads are well below their classical critical values even though imperfection levels may only be a fraction of the shell thickness assumed.

In this paper the following is treated below:

1. Mathematical formulation of the problem including the treatment of governing incremental equations of motion for the analysis of ultimate response of thin bionics shells,
2. Description of the solution methodology adopted for numerical analysis of ultimate response of thin bionics shells studied,
3. Application of developed approach for numerical analysis of actual bionics shell problems treated.

4. Generalized analysis of motion

Displacements of a thin bionics shell are considered as a family of mappings from one region in space to another one. The current configuration of the shell structure is completely defined by the locations of displacements at given time point. The variations of configurations are assumed to be continuous and new boundaries will not arise during deformation. Each position is defined in relation to a reference position assumed.

Assuming the Cartesian coordinates x, y, z and corresponding displacements u, v, w, the Green strain tensor is defined by

$$E_{xx} = \partial u_x/\partial x + [(\partial u_x/\partial x)^2 + (\partial u_y/\partial x)^2 + (\partial u_z/\partial x)^2]/2, \tag{3}$$

$$E_{xy} = (\partial u_y/\partial x) + (\partial u_x/\partial y) + [(\partial u_x/\partial x)(\partial u_x/\partial y) + (\partial u_y/\partial x)(\partial u_y/\partial y) + (\partial u_z/\partial z)(\partial u_z/\partial z)]/2, \tag{4}$$

etc.

In order to establish the constitutive equations with Green strain tensor, a stress tensor with the same reference is needed. A symmetric one will be advantageous in present application. The 2nd Piola-Kirchhoff stress tensor denoted as S_{ij} has the desired properties. The general equilibrium equation for deformed configuration expressed by 2nd Piola-Kirchhoff stress tensor is given by

$$S_{ij} = g(E_{ij}), \tag{5}$$

where g is a single valued function of the Green strain tensor E_{ij}.

Consider a thin shell structure with volume, surface area and mass density in an initial configuration denoted by V, S, and ρ_0, respectively. The body forces per unit mass are denoted by $F_{0,i}$ and surface tractions are specified by force components T_i.

The shell structure in equilibrium is subjected to a virtual displacement δu_i which is kinematically consistent with boundary conditions assumed. The balance of work in the shell studied is given by

$$\int S_{ij}\ \delta E_{ij}\, dV - \int T_i\ \delta u_i\ dS - \int P_i \delta u_i\ dV = 0, \tag{6}$$

with

$$P_i = \rho_0\ F_{0,\,i}\,. \tag{7}$$

Expression (6) states that among all kinematically admissible displacement fields u_i the actual one renders the value of the total potential energy stationary.

The incremental form of the variational principle for two configurations of the shell analysed is given by

$$\int S_{ij}^{(1)}\delta E_{ij}^{(1)}dV - \int T_i^{(1)}\delta u_i^{(1)}dS - \int P_i^{(1)}\delta u_i^{(1)}dV = 0\,, \tag{8}$$

$$\int S_{ij}^{(2)}\delta E_{ij}^{(2)}dV - \int T_i^{(2)}\delta u_i^{(2)}dS - \int P_i^{(2)}\delta u_i^{(2)}dV = 0\,, \tag{9}$$

where superscripts (1) and (2) denote two neighbouring configurations studied.

The components of surface tractions and body forces refer to the same reference configuration and may therefore be subtracted directly to give

$$\Delta T_i = T_i^{(2)} - T_i^{(1)}\,, \tag{10}$$

$$\Delta P_i = P_i^{(2)} - P_i^{(1)}\,. \tag{11}$$

The variations of two displacement fields are chosen to be the same

$$\delta u_i = \delta u_i^{(1)} = \delta u_i^{(2)}\,. \tag{12}$$

An incremental form of the virtual work equations is then obtained by subtracting of Eqs. (8) and (9), giving

$$\int (S_{ij}^{(2)}\ \delta E_{ij}^{(2)} - S_{ij}^{(1)}\ \delta E_{ij}^{(1)})\,dV - \int \Delta T_i\ \delta u_i\ dS - \int \Delta P_i\ \delta u_i\ dV = 0 \tag{13}$$

and considering the virtual variations of both configurations analysed.

The neglection of higher order strain energy terms can be done only if two configurations are sufficiently close to each other. Eq. (13) gives configuration (2) from the known configuration (1) and known load increments.

5. Ultimate dynamics

The nonlinearity in ultimate response of thin bionics shell generally leads to a complex interactive resonance oscillation and buckling behaviour that often is joined by a snapping from one mode to another one, which is called the cascade buckling. The numerical treatment of this process is to be made adopting the parallel processing approach.

In this paper the motion is defined to be stable if the deformations remain in the pre-ultimate region without a transition to the post-ultimate range. The buckling process may be indicated by an increase of local or overall modes. As a proper measure for the state of deformation a norm of the deformation field is to be chosen. The procedure below is based on a comparison of the load induced strain energy with critical strain energy that is necessary to cause ultimate behaviour of the structure considering all deformation modes appearing.

Geometrical nonlinearities are to be taken into account as moderate rotations. The rate of total energy in ultimate range depends on displacements and slopes of the shell middle surface, as well as on corresponding velocities, integrated stress variables and forcing effects. Ultimate motion is effected by time dependent loads and describes a trajectory in the phase plane of displacements and velocities. The distance of the trajectory to the static equilibrium state is a measure of the energy induced by initial conditions. Ultimate state of motion is indicated by the critical trajectory called a separatrix. Such trajectory is the limit curve for all locally stable motions in the pre- and post-ultimate range of response. For the study of ultimate behaviour the energy level indicating the separatrix is of interest. Such energy level holds for all states of motion on the separatrix. The criterion of a stable motion is satisfied if the state of motion at the end of perturbation time is inside of separatrix since the energy level is lower than the critical one.

For computing of critical energy level the stable equilibrium state is given by the minimum of potential energy. A perturbation of this equilibrium leads to a locally stable motion of the structure with energy level ΔF. An increase form ΔF to ΔF_{crit} shifts the structure to a saddle point which denotes the critical deformation state of bionics shell studied, since two locally stable motions in ultimate range meet here. A further increase of the energy effects a motion in the whole ultimate range. The decisive factor for the stability of the equilibrium state is the energy level that is given by the maximum of strain energy occurring. Hence, the first variation of the strain energy has to vanish for the case of critical state for kinetic energy

$$\delta G = 0, \tag{14}$$

which leads to a system of nonlinear equations that may be solved iteratively as nonlinear eigenvalue problem.

6. Transient dynamics

The solution of nonlinear response in ultimate range of bionics shells subjected to dynamic loads is based on the application of updated Lagrangian formulation of motion, with reference state taken as the current configuration continuously updated throughout the entire deformation process (Tesár and Fillo 1988). A new reference frame is established at each stage along the deformation path. A major advantage of the formulation is the simplicity which provides an easy physical interpretation of the generalized nonlinear behaviour of bionics shell studied.

An incremental form of the equations of motion for the case of transient dynamics of bionics shells is obtained by considering the dynamic equilibrium at two configurations a time step Δt apart. The increments of external forcing then balance the dynamic equilibrium at time $t + \Delta t$ by

$$M_t\,\Delta a_t + C_t\,\Delta v_t + K_t\,\Delta u_t = R_{t+\Delta t} - (V_t^I + V_t^D + V_t^S), \tag{15}$$

with inertia forces $V_t^I = M_t\,\Delta a_t$, damping forces $V_t^D = C\,\Delta v_t$, elastic forces $V_t^S = K_t\,\Delta u_t$ and with corresponding accelerations, velocities and displacements a_t, v_t, and u_t, respectively. The vectors of nodal point accelerations, and velocities are given as time derivatives of the vector of nodal displacements u_t. The mass, damping and stiffness matrices M_t, C_t, and K_t, respectively, are constructed of element matrices established in incremental fashion directly for the shell simulation adopted. The subscript t denotes the current time and R is the vector of external forcing. If the structure is in equilibrium at time t, the right hand side of Eq. (15) will be identical with the increment of external forcing in time step Δt.

Increments in nodal displacements, velocities and accelerations are thus given by external load increments and known physical property matrices. If, however, these matrices change during time steps, as is the case of ultimate response of bionics shells studied, then Eq. (15) is only approximately true. The vector of local approximation error given by

$$\Delta V_{t+\Delta t} = R_{t+\Delta t} - (V^{I}_{t+\Delta t} + V^{D}_{t+\Delta t} + V^{S}_{t+\Delta t}), \tag{16}$$

is a measure of how close to equilibrium the solution has been increased by approximate Eq. (15).

Governing incremental equation of motion for nonlinear ultimate dynamics of the bionics shell studied is then given in modified form by

$$M_t\, \Delta a_t + C_t\, \Delta v_t + P_t\, \Delta u_t = \Delta R_t, \tag{17}$$

where $P_t\, \Delta u_t$ is the vector of internal, deformation dependent, nonlinear forces.

The pseudo-force method (Tesár and Fillo 1988) as applied here, is defined by

$$P_t\, \Delta u_t = K_t\, \Delta u_t + N_t\, \Delta u_t - \Delta V_{t+\Delta t}, \tag{18}$$

where $N_t\, \Delta u_t$ is the vector of nonlinear terms (pseudo-forces) and $\Delta V_{t+\Delta t}$ is the local approximation error defined above. In the application of the pseudo-force technique the term $P_t\, \Delta u_t$ is placed on the right-hand side of Eq. (17) and the vector of nonlinear terms is treated as pseudo-force vector. At each time step an estimate of $N_t\, \Delta u_t$ is computed and iterations are performed until $\Delta V_{t+\Delta t}$ becomes sufficiently small when compared to a prescribed tolerance norm. As an estimate of $N_t\, \Delta u_t$ for the first iteration at time step Δt an extrapolated value from previous solutions is to be used, i.e.,

$$N_t\, \Delta u_t = (1 + \alpha)\, N_{t-\Delta t}\, \Delta u_{t-\Delta t} - \alpha\, N_{t-2\Delta t}\, \Delta u_{t-2\Delta t}, \tag{19}$$

where α is an extrapolation parameter ranging from 0 to 1.

Because of the large computation effort required for the analysis of nonlinear ultimate response of thin bionics shells studied, it is desirable to seek a strategy for optimal numerical calculations which may be defined in the terms of a number of control parameters specifying the linearization techniques, the frequency of reformulation of effective stiffness matrix, convergence tolerances and limits on the maximum number of iterations and adaptively change of the time step size.

7. Ultimate wave dynamics

Simplified wave equation for the treatment of ultimate dynamics of bionics thin shells studied is given by

$$\mu\, \eta(u_t) + (\lambda + \mu)\, \mathrm{grad}(\mathrm{div}\, u_t) + f = \rho\, \partial^2 u_t/\partial t^2, \tag{20}$$

with λ and μ as Lame's constants, ρ as mass density, with Laplace operator η, with body force vector f and with the vector of displacements u_t.

In terms of derivatives of the displacement components u_t, the governing equation is given by

$$c_2\, u_t + (c_1^2 - c_2^2)\, u_t + f_i/\rho = a_t, \tag{21}$$

with propagation velocities for dilatational waves given by

$$c_1 = \sqrt{[(\lambda + 2\mu)/\rho]}\,, \tag{22}$$

and distortional or shear waves given by

$$c_2 = \sqrt{(\mu/\rho)}\,. \tag{23}$$

Strain and stress components are defined by

$$\varepsilon_{ij} = (u_{ij} + v_{ij})/2, \tag{24}$$

and

$$\sigma_{ij} = \lambda\, \varepsilon_{kk}\, \delta_{ij} + 2\, \mu\, \varepsilon_{ij}, \quad i, j = 1, 2, 3, \tag{25}$$

where δ_{ij} is the Kronecker delta function.

8. Parallel-processing approach

Mathematical and physical backgrounds of the approach are described below.

First of all, theoretical analysis of ultimate behaviour of single particle of microelement simulation mesh is dealt with.

Assumed is the Euclidean n-dimensional space B^n. The symbol (a, b) denotes an open interval in B^1. It holds $a < b$, $a, b \in B^1$ and $<a, b>$ in B^1. The symbol $G^{(k)}(a, b)$, with $k \in N$ is the set of real functions with continuous derivatives of order $s(0 \le s \le k)$ in (a, b). $C^{(k)}(<a, b>)$ is the set of functions from $C^{(k)}(a, b)$, with derivatives continuously extended into $<a, b>$. $L(B^n)$ is the set of real matrices $n \times n$.

Let in $<a, b>$ be given the system of n-linear differential equations of the first order given by

$$u_i'(t) = \sum a_{ij}(t) u_j, \quad i = 1, 2, \ldots, n, \tag{26}$$

where, for all i, j, $a_{ij}(t) \in C^{(0)}(<a, b>)$. In vector notation, the system (26) is given by

$$u' = Au\,. \tag{27}$$

Definition 1. An n-dimensional column vector $u(t) = [(u_j(t))_{j=1}^{n}]^T$ is said to be the solution of (27) if

$$\forall_j : u_j(t) \in G^{(1)}(<a, b>), \tag{28}$$

$$\forall t \in <a, b> : u' = Au. \tag{29}$$

Theorem 1. The solutions of the system (27) create the n-dimensional vector space over the field of real numbers. An arbitrary basis for such a space is called the fundamental system of solutions for the system (27). The matrix having n-columns, which creates the fundamental system of solutions for the system (27), is called the fundamental matrix of the system and is denoted below by $\Phi(t)$.

Theorem 2. The necessary and sufficient condition for the matrix $\Phi(t)$ of solutions $(\{\Phi_i(t)\}_1^n$ of the system (27) is det $\Phi(t)$ G, where G is the constant regular matrix of the same type.

Definition 2. An $n \times n$ square matrix of the type

$$U_A(a, t) = \Phi(t)\, \Phi^{-1}(a), \quad \forall t \in \langle a, b\rangle, \tag{30}$$

will be denoted below as the transfer matrix of the system (27) in the interval $\langle a, t\rangle$. If the fundamental system of solutions (27) is known, from (30) the transfer matrix in the interval $\langle a, b\rangle$ is directly to be specified.

For physical interpretation of above mathematical considerations let the internal and left-hand external displacements for a microelement of multigrid simulation mesh be denoted by u_i, u_a, and u_b, respectively. The u_a will match the u_b when the microelement is moved one bay to the right, so that u_a and u_b share the same dimension. The internal vector u_i can be eliminated beforehand, giving the stiffness matrix of the microelement given by

$$K(\omega) = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix}, \tag{31}$$

with

$$u = \begin{bmatrix} u_a \\ u_b \end{bmatrix}. \tag{32}$$

The submatrices K_{ii} are transcendental functions of frequency ω and $K(\omega)$ is symmetric for a non-dissipative system, i.e., $K_{ab} = K_{ba}$. The corresponding force vectors are derived as follows

$$n_a = K_{aa}\, u_a + K_{ab}\, u_b, \tag{33}$$

$$n_b = -K_{ba}\, u_a - K_{bb}\, u_b. \tag{34}$$

The state vector v is defined as combination of displacements and internal forces such that

$$v = [u, n]^T. \tag{35}$$

Thus Eqs. (33) and (34) are given in the transfer matrix shape, which relates the state vector at boundaries a and b by

$$v_b = S\, v_a, \tag{36}$$

where S is the corresponding transfer matrix. It holds

$$S(\omega) = \begin{bmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{bmatrix}, \tag{37}$$

where

$$S_{aa} = -K_{ab}^{-1}\, K_{aa}, \quad S_{ab} = K_{ab}^{-1}, \quad S_{ba} = -K_{ba} + K_{bb}\, K_{ab}^{-1}\, K_{aa}, \quad S_{bb} = -K_{bb}\, K_{ab}^{-1}. \tag{38}$$

The $S(\omega)$ is a symplectic matrix, i.e., its determinant equals 1 if τ is an eigenvalue, then so also is $1/\tau$, the symplectic orthonormality relationship exists between its eigenvectors and an arbitrary state vector can be expanded into the terms of corresponding eigenvectors.

The actual geometry of thin bionics shell studied is simulated by a multigrid space mesh of micro- and macro- shell elements. The model assumed allows there the simulation of general anisotropy of structural and material parameters as occurring in large deformation and elastic-plastic regions of nonlinear ultimate analysis of bionics shells studied. Some types and degrees of nonlinearities can be dealt with using various systems for discrete simulation adopted. In the updated Lagrangian formulation of motion the major rigid-body geometric nonlinearities are embodied into coordinate transformations of the microelement mesh used. The effects of physical nonlinearities (nonlinear damping, elastic-plastic material behaviour, viscoelasticity, etc.) are analysed on the level of the macroelement simulation mesh. Both systems are coupled and may be dynamically varied and combined. The proposed multigrid simulation mesh of the FETM-method uses there the variable size of micro- and macroelements in space and time as well as in the various regions of thin bionics shell studied and in various time and load steps of nonlinear ultimate analysis performed.

Generalized transfer hypermatrices of the FETM-method (Tesár and Fillo 1988) when applied for the analysis of the multigrid simulation model suggested are constructed by a diagonal set-up of transfer matrices of shell elements adopted. The transfer submatrices of macroelements consist of diagonal assembly of transfer submatrices for microelements adopted. Transfer submatrices are derived over inverse transformations of linear and nonlinear stiffness matrices of microelements adopted.

9. Adaptive mesh refinement

The adoption of a suitable refinement strategy for numerical analysis of ultimate response of bionics shells is required in addition. Such strategy takes into account both important aspects, efficiency and accuracy of computations made. Error indicators based on the weighted stress gradients yield good efficiency for singularity type solutions. An alternative indicator controls the linearization assumptions for the nonlinear formulation adopted, which is especially important for the analysis of finite rotations appearing. The output from the error estimation is a scalar measure of the error within each of the element of the simulation mesh adopted. Such information is used in advance to adapt the mesh in order to reduce the global error. In aposteriori for error estimation in nonlinear ultimate response computations the principal mechanisms for the control of the adaptive process are path control (selection of step length and local parameters or corrector adjustment) and approximation control (modification of the mesh and element type).

The mesh may be modified using so called *h*-methods and/or *p*-methods. When the *h*-methods are employed, the error information is converted into mesh refinement indicators giving the characteristic size of elements in the new mesh as a function of space. Alternatively, the mesh can be refined by the means of the *p*-methods where the polynomial order p of the elements are increased and the element size is fixed. The mesh can be redefined in different ways after the desired mesh size parameters have been found. A totally new mesh is generated in such a way that new elements are defined by succesive subdivision of old elements. In this case extra nodal points are inserted into the old mesh. All nodes of the old mesh are also present in the new mesh adopted.

From computational point of view it is necessary to organize the updated structural data in ultimate region efficiently in order to compute the best fitting meshes automatically for every important state and degree of the nonlinear ultimate analysis. The volume of the data set adopted must be flexible enough to establish suitable element meshes and to modify them efficiently.

10. Mapping of state variables

After the simulation model has been redefined according to a given error distribution at load step i, the values of state variables are to be mapped from the old mesh into the new one before the FETM solution procedure is continued. The mapping is performed at the previous load step ($i - 1$), which is the latest state with acceptable errors and where the internal stress is in equilibrium with external loads acting. This transformation process is referred to as rezoning and involve following steps:

1. The state variables in the old mesh are defined in terms of discrete variables located at the nodes. Smoothing is performed, if necessary, to obtain a continuous field.
2. The values of state variables are computed at location of the nodal points of the new FETM mesh adopted. Generally, this step involves a searching procedure and a parametric inversion, where an element of the old mesh that matches the location of a certain node of the new mesh is specified.
3. The distribution of the state variables is then defined within each new element using corresponding shape functions and necessary integration point data.
4. After the solution has been mapped into the new mesh, the equilibrium between internal forces and external loads may be violated. Additional equilibrium iterations may therefore be necessary before continuing with the next load or time steps.

The parametric inversion required for bionics shell configurations is there performed as a minimization of the distance between the new node and a parametric point on the element surface using, for example, a 2nd order Newton iteration procedure.

11. Numerical experiments with discussion

Example of the ultimate behaviour as well as of structural optimalization of thin bionics shell of the new wooden stadion designed in Slovakia is studied below. During design process the optimalization of geometric shape has been made by successive structural approximation adopting above theoretical approaches. Close cooperation of architects and structural engineers during design was emphasized.

Bionics shell with impressive architecture of the flower coming into blossom was adopted for the load bearing structural system of the stadion (Tesár and Minár 2001). The load-bearing system of the stadion is created of two bionics shells denoted as primary and secondary ones below. Primary bionics shell with two vertical curvatures was applied there as main supporting system of the roof. The wind bracing system of the stadion is implemented as secondary bionics shell having also the configuration of the flower coming into blossom.

The primary bionics shell roof structure is supported by laminated wood arch girders with two curva-tures in transverse direction. The arch girders together with suspended secondary bionics shell both dominated by architectural idea of developing flower create impressive configuration of the stadion (see Figs. 1, 2 and 3).

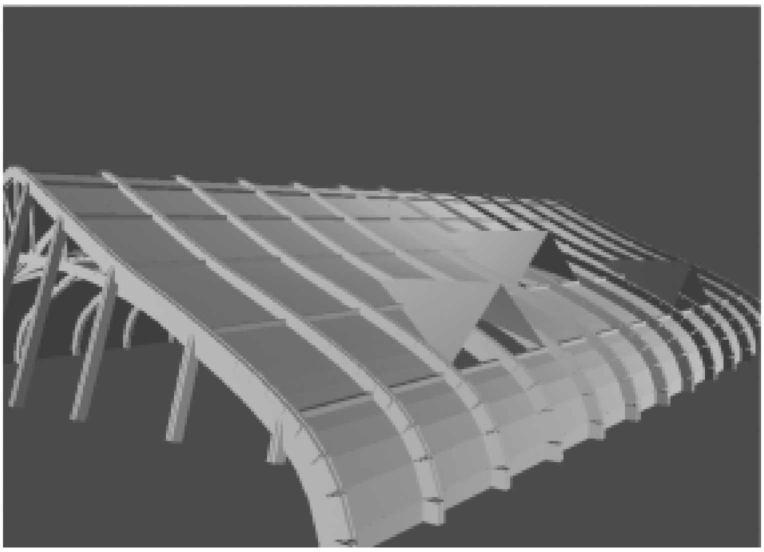

Fig. 1 Visualisation of the primary bionics shell of the structural system adopted

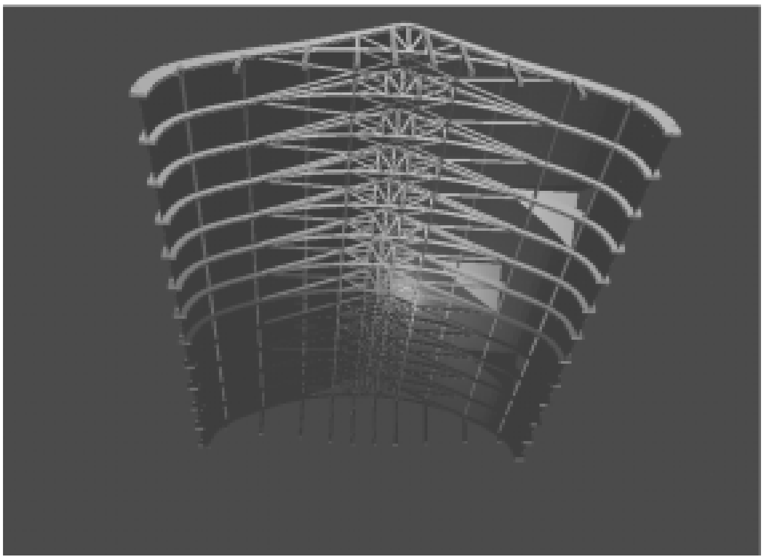

Fig. 2 Visualisation of the internal view on the roof structure

Main arches made of laminated wood are designed as twins of parallelly located reverse subarches having joints in end and midspan supporting points. The twins are connected by special pin joints.

Total span of the main arch girder is 55800 mm, the width is constant along the whole span and is 200 mm. The depth of the main arch girder is variable along the span from 1800 mm in the end supports until 780 mm in the middle of span.

The main arch girders are distanced á 6000 mm and behave in interaction with secondary bionics shell suspended below. The secondary shell is configurated of wood rod members in bionics

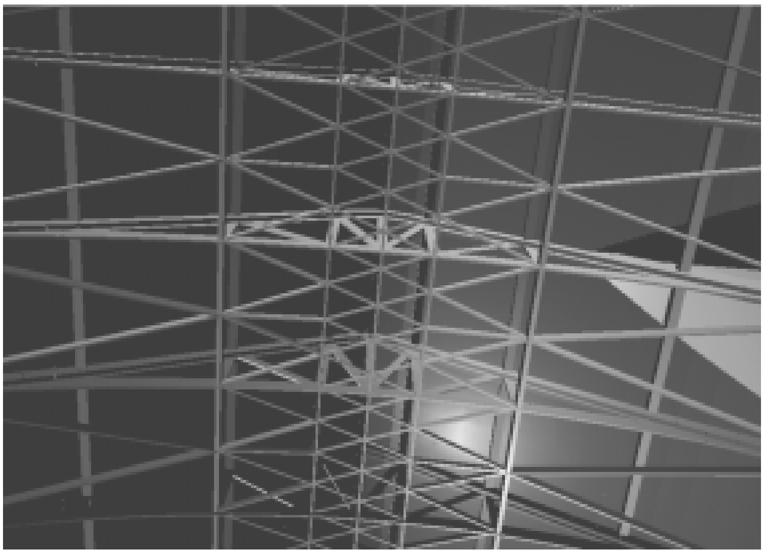

Fig. 3 Bracing system of secondary shell configurated as developing flower

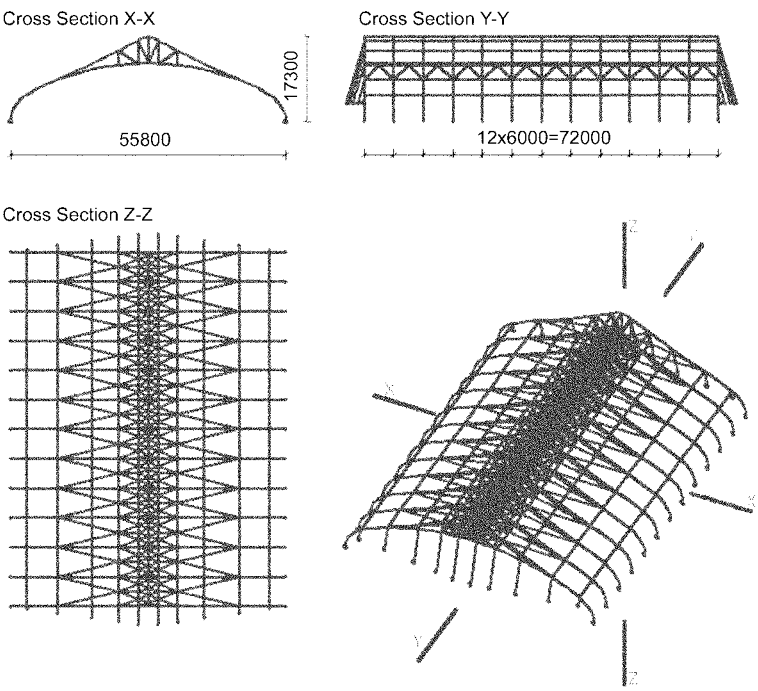

Fig. 4 Simulation model adopted

geometry and serves simultaneously as structural wind bracing.

For calculation there was adopted the simulation model as shown in Fig. 4 below. The optimization of structural configuration and dimensions of the load-bearing system of stadion has

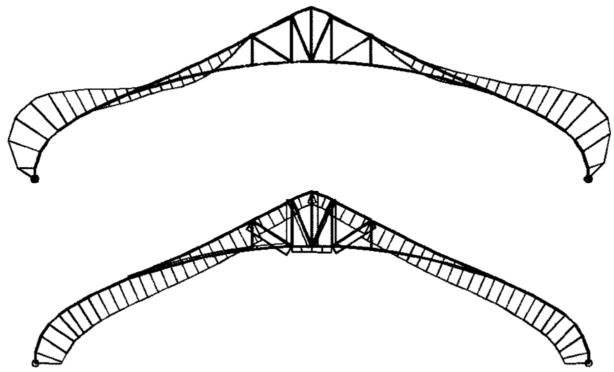

Fig. 5 Shapes of moments and lateral appearing

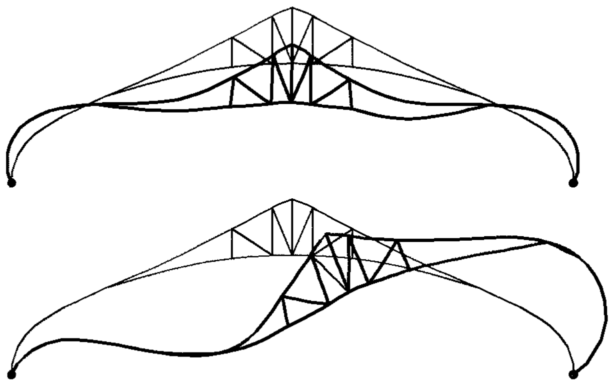

Fig. 6 Shapes of extremal symetric and antimetric deflections

been made using above theorethical approaches (see EC 5).

The plots of typical symetrical and antimetrical deformations of the structure subjected to standard loads assumed are submitted in Fig. 6.

12. Conclusions

Present parallel processing approach for numerical analysis of ultimate behaviour of thin bionics shells has been adopted for optimization, calculation and design of attractive configurations in structural engineering recently. Such trend of analysis is continued nowadays also in direction of more complicated bionics shell structurals configurations made of modern composite materials, for example, glass or carbon feabers and laminated wood. One of such structures, bionics shell of attractive wooden stadion in Slovakia, has been calculated and designed in accordance with scientific approaches submited in present paper. Some views of this optimized structure are shown in Figs. 1, 2 and 3. Simulation model adopted is shown in Fig. 4 and some results of calculation are submitted in Figs. 5 and 6.

The combination of bionics aesthetics with laminated wood material and shell load bearing systems permits design and erection of attractive structural configurations serving as monuments of present structural engineering. The aspect of economy, production and attractive appearance are significant points for succesful future design and erection of such structures inspirated by bionics configurations created by nature.

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