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Mechanics of missile penetration into geo-materials

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Abstract. The present study aims to improve an existing model for the prediction of deceleration time history, penetration depth and forces on ogive and conical nose shaped missiles under normal impact into geo-material targets. The actual ogive nose shaped missile has been considered in the analysis and the results thus obtained have been compared with the existing model and significant improvements are found. A close proximity in the results has also been observed with the experimental values. The results of ogive nose shaped missile have also been compared with equivalent conical nose shaped missile. Variation of radial stresses along nose length and radial direction has been studied. Effect of CRH on missile penetrating performance has been investigated.

Key words: missile penetration; axisymmetric impact; projectiles; deceleration; geo-material targets.

1. Introduction

The increasing fear of war has forced the countries for undertaking extensive study on the subject of missile impact upon different type of targets, particularly the geo-materials under which the strategic structures such as army bunkers, Nuclear Power Plants (NPP) etc. may be buried. The safety or destruction of these strategic structures require the correct estimation of penetration depth in overlying geo-materials by a missile. In the last two decades a number of attempts have been reported to understand the penetration phenomenon and to acquire reliable calculation tools for its estimation.

The study of the impact of projectiles on structural elements such as plates, shells etc. has long been of interest (Abbas 1995, 1996 and Goldsmith 1978) due to various reasons. Corbett *et al.* (1996) and Goldsmith (1999) presented a comprehensive survey of impact loading on plates and shells by free flying projectiles. Ansari (1998) studied various aspects of normal and oblique impact of projectiles on single and layered thin plates. Forrestal *et al.* (1994) developed an empirical equation for penetration depth of ogive nose projectiles penetrating concrete targets for normal impact. The equation was found in good agreement with experimental results. Forrestal *et al*. (1996) presented depth of penetration vs. striking velocity data for projectiles launched into grout and concrete targets. They determined experimentally the striking velocities corresponding to maximum depth of penetration. Frew *et al.* (1998) conducted some additional experiments to examine the accuracy of penetration equations proposed by Forrestal *et al.* (1994) and Forrestal *et al.* (1996) and

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found agreement with experiments. A detailed review of past investigators' work shows that considerable work has been done on impact of projectiles and missiles on plates, shells and concrete targets but studies available on impact of projectiles/missiles on geo-material targets are scanty.

It has been established that a high velocity missile when impacts and penetrates the geo-material target, it creates a cavity. This cavity expands under the action of stress waves generated into the target medium. To study the shape of the cavity thus formed and its expansion phenomenon three models have been reported in the literature viz. spherical cavity expansion model, cylindrical cavity expansion model and model of orthogonal layers(Goldsmith 1999). Using these models, past investigators have proposed various formulae for the prediction of possible deceleration time history, penetration depth, forces at the missile nose etc. These expressions, however, are usually analytical in nature and derived with number of idealizations. Numerical approach to study the penetration problem is, however, scanty. Moreover, analysis for both ogive and conical nose shaped missiles have not been reported widely by any single investigator. Most of the investigators have predicted deceleration time history only after the complete penetration of missile nose. Frictional forces acting on the missile nose and aft-body are also neglected by most of the researchers in their numerical modeling (Forrestal *et al*. 1981).

In the present study, a model proposed by Forrestal *et al*. (1981) for the prediction of deceleration time history, penetration depth and forces on equivalent conical nose shaped missiles under normal impact into geo-material targets has been improved. The actual ogive nose shape of the missile has been considered in the analysis and the results thus obtained have been compared with Forrestal *et al.* (1981) work and significant improvement has been observed. A close proximity in the results have also been observed with the experimental values. The results of ogive nose shaped missile have also been compared with an equivalent conical nose shaped missile. Variation of radial stresses along nose length and radial direction has been studied. Effect of caliber radius head (CRH) on missile penetrating performance has been investigated.

2. Problem formulation

The problem has been formulated under the following assumptions

· The missile is rigid i.e., deformation of missile is negligible and only soil deformation has been considered.
· Impact of missile is normal and axi-symmetric.
· Cavity expansion is cylindrical in nature.
· The target consists of thin independent layers normal to the direction of penetration (Fig. 1).
· Wave propagation is one dimensional and in the radial direction.
· Motion in individual target layers is one dimensional (i.e., radial) and independent of any other layer.
· The missile does not carry any warhead and no explosion has been considered.
· The loss of energy in the form of heat and sound has been neglected.

2.1 Material model

In the present work, the target medium is described by a linear hydrostat, assuming a linear shear failure stress and pressure relation as given below. Many rock materials with low water content can

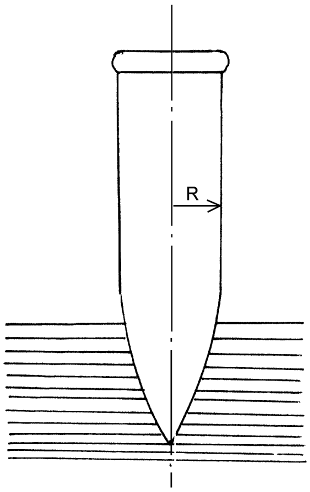

Fig. 1 Geometry of the problem

be modeled with these idealizations (Forrestal *et al.* 1981)

$$p = K\eta$$

and
$$\tau = \mu p \tag{1}$$

where, η = volumetric strain;

$$= \left(1 - \frac{\rho_0}{\rho}\right) \tag{2}$$

K = bulk modulus;
ρ_0 = initial mass density;
ρ = instantaneous mass density;
μ = shear strength parameter.

2.2 Formulation

When a rigid missile nose penetrates a uniform target medium with normal incidence, the idealised layer of target material expands. An element of such an expanded layer at a radial distance r from the axis of symmetry is shown in Fig. 1. This element is subjected to radial and circumferential components of Cauchy stress (taken postive in compression). The shear stress τ and hydrostatic pressure p in terms of these Cauchy stresses in the element has been taken as

$$\tau = (\sigma_r - \sigma_\theta) \tag{3}$$

$$p = \frac{1}{3}(\sigma_r + 2\sigma_\theta) \tag{4}$$

where, σ_r and σ_θ are the radial and circumferential components of Cauchy stress. The Eq. (4) has been obtained assuming the vertical stress σ_z to be equal to the circumferential stress σ_θ during the penetration event (Forrestal *et al*. 1981). Eqs. (1), (3) and (4) have been combined to obtain the radial stress σ_r as

$$\sigma_r = \left(1 + \frac{2\mu}{3}\right)p \tag{5}$$

The consideration of momentum and mass conservation in cylindrical Lagrangian coordinates for this elemental target layer results in the following equations:

Momentum conservation

$$\rho_0 r \frac{\partial^2 u}{\partial t^2} = -(r+u)\frac{\partial \sigma_r}{\partial r} - (\sigma_r - \sigma_\theta)\frac{\partial (r+u)}{\partial r} \tag{6}$$

Mass conservation

$$\rho_0 r = \rho(r+u)\left(1 + \frac{\partial u}{\partial r}\right) \tag{7}$$

The use of Eqs. (1), (2) and (5) of material model eliminates the stress components from Eqs. (6) and (7) and thus we have

$$\rho_0 r \frac{\partial^2 u}{\partial t^2} + \left(1 + \frac{2\mu}{3}\right)K(r+u)\frac{\partial \eta}{\partial r} + \mu K \eta\left(1 + \frac{\partial u}{\partial r}\right) = 0 \tag{8}$$

and,

$$\eta + \frac{\partial u}{\partial r} + \frac{u}{r}\left(1 + \frac{\partial u}{\partial r}\right) = 0 \tag{9}$$

when the missile penetrates at a velocity V_z, it begins to open a circular cavity in a given layer. The displacement at the cavity wall in Lagrangian coordinates (i.e., at $r = 0$) is given by

$$u(0, t) = Vt \tag{10}$$

where,

$$\begin{aligned} V &= \text{radial velocity of cavity interface} \\ &= V_z \tan\theta \end{aligned} \tag{11}$$

The other boundary condition requires that the radial displacement at the wave front is zero. This completes the cavity expansion problem. The solution of simultaneous partial differential equations (Eqs. (8) and (9)) with above two boundary conditions yields the radial stress component at the interface and in the target medium. In the present study, similarity transformation method, as described by Forrestal *et al*. (1981), has been employed for converting the above partial simultaneous differential equations into ordinary differential equations which have been solved numerically using Runge-Kutta method (Siddiqui 1994).

2.3 Forces on missile nose and deceleration

The penetration of missile into the target results in the radial movement of the target material at the cavity interface which produces radial stress in the target material. The incremental radial force

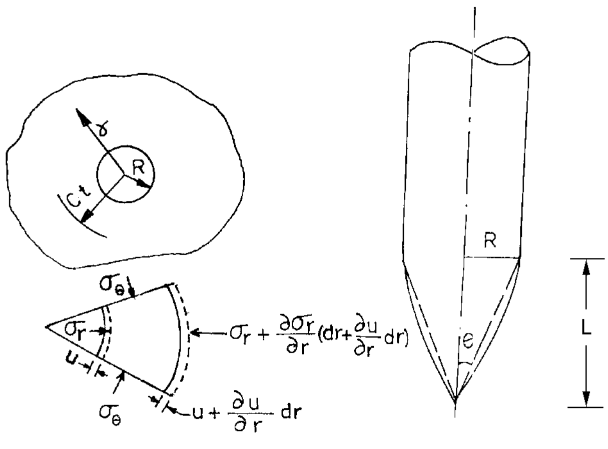

Fig. 2 An equivalent conical nose shaped missile

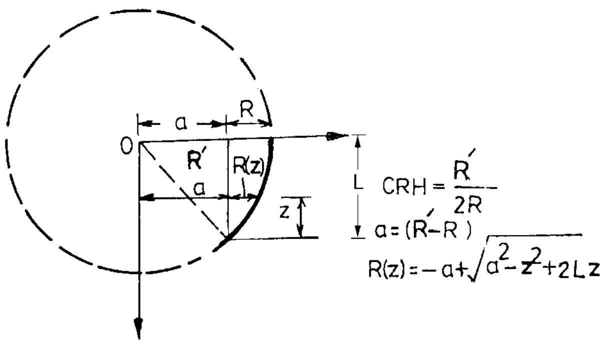

Fig. 3 Computation of $R(z)$

on the missile nose for a thin target thickness dz is given by

$$dF_r = 2\pi\sigma_r(0)R(z)dz \tag{12}$$

where $\sigma_r(0)$ is the radial stress in the target material at the cavity expansion and $R(z)$ is the radius of the missile nose at a distance z from its tip (Fig. 3). The force at the nose of the missile having a conical nose has already been obtained by Forrestal *et al.* (1981) and the same has been used by them for the ogive nose taking equivalent semi-vertex angle (Fig. 2). This approximation will obviously lead to lesser penetration depth, greater force and greater deceleration of the missile. Since, the missile nose is usually ogival, therefore, in the present study analysis has been carried out for actual ogive nose shaped missiles and the results thus obtained have been compared for an equivalent conical nose shaped missile. The expression of $R(z)$ for an ogive nose and a conical nose (Figs. 2 and 3) is given by

$$\begin{aligned} R(z) &= -a + \sqrt{a^2 - z^2 + 2Lz} \quad \text{for ogival nose} \\ &= z\tan\theta \qquad\qquad\qquad\quad \text{for conical nose} \end{aligned} \tag{13}$$

where,

$a = (R' - R)$;
R' = radius of the ogive nose;
R = radius of the aft body of missile; and
L = nose length of missile.

The vertical force at the nose of the missile due to the vertical stiffness of the target material of thickness dz is given by

$$dF_v = dF_r \tan\theta \tag{14}$$

Another force acting at the nose is the drag force which is tangential to the surface of the missile nose arising due to the friction between the target material and missile. The drag force has not been considered by Forrestal *et al.* (1981) in their penetration analysis. The magnitude of incremental drag force dF_d for the elemental target thickness dz is equal to the product of coefficient of dynamic friction between the missile surface and the target material (μ_d) and force normal to the missile nose (dF_n) i.e.,

$$dF_d = \mu_d dF_n \tag{15}$$

where, $$dF_n = dF_r \sec\theta$$

Therefore, $$dF_d = \mu_d dF_r \sec\theta \tag{16}$$

Hence, the total incremental vertical upward component (dF_z) of the target reaction will be

$$\begin{aligned} dF_z &= dF_v + dF_d \cos\theta \\ &= dF_r \tan\theta + \mu_d dF_n \cos\theta \\ &= dF_r(\mu_d + \tan\theta) \end{aligned} \tag{17}$$

where, the radial force dF_r and vertical force dF_v are in fact the radial and vertical components of the normal force dF_n. The total upward vertical target reaction on the missile nose has been obtained by integrating Eq. (17) from 0 to penetration depth z (where, $z \le L$) :

$$\begin{aligned} F_z &= \int_0^z (\mu_d + \tan\theta) dF_r \\ &= \int_0^z (\mu_d + \tan\theta) 2\pi\sigma_r R(z) dz \end{aligned} \tag{18}$$

If the depth of penetration of missile is greater than the nose length then the upper limit of integration will be up to L because we are getting reaction only on the nose. In the present work, it has been assumed that the size of cylindrical cavity due to the missile penetration is more than the diameter of the penetrator aft body.

Eq. (18) has been applied for the estimation of total vertical target reaction F_z for both conical and ogive nose shaped missiles. The force F_z for the conical nose shaped missiles will be simplified into

$$\begin{aligned} F_z &= \int_0^z (\mu_d + \tan\theta) 2\pi\sigma_r R(z) dz \\ &= \pi\sigma_r(\mu_d + \tan\theta) z^2 \tan\theta \end{aligned} \tag{19}$$

where, the angle θ is constant for conical nose shaped missiles. However, it is a variable for the ogive nose shaped missiles. The substitution of $R(z)$ for ogive nose from Eq. (13) and the value of $\tan\theta$ at a distance z from the tip of the nose in Eq. (18) leads to

$$\begin{aligned} F_z &= 2\pi\int_0^z \sigma_r R(z)\left(\frac{L-z}{R(z)+a} + \mu_d\right) dz \\ &= 2\pi\int_0^z \sigma_r\left(-a + \sqrt{a^2 - z^2 + 2Lz}\right)\left(\frac{L-z}{\sqrt{a^2 - z^2 + 2Lz}} + \mu_d\right) dz \end{aligned} \tag{20}$$

It is to be noted here that σ_r for ogive nose is a function of z whereas for conical nose it is independent of z. The force F_z given by Eq. (20) for the ogive nose can be integrated using any standard numerical integration scheme. The consideration of the dynamic equilibrium of missile of mass m results in the equation

$$m\frac{dV_z}{dt} = -F_z \tag{21}$$

The integration of Eq. (21) will yield time histories of velocity, penetration depth and the deceleration of missile. In the present study a forward finite difference approach has been employed for its integration. Using this approach the velocity V_z, deceleration a_z and the penetration depth z of missile at $(i + 1)^{th}$ time step can be obtained by the following relations

$$V_z^{i+1} = V_z^i - \frac{1}{m}(F_z^{i+1}\Delta t) \tag{22}$$

$$a_z^{i+1} = \frac{V_z^{i+1} - V_z^i}{\Delta t} \tag{23}$$

$$z^{i+1} = z^i + V_z^i \Delta t \tag{24}$$

The value of F_z^i which is the force F_z at the ith time step can be obtained from Eq. (19) or (20) depending upon the shape of the missile nose.

It is to be noted that Forrestal *et al.* (1981), however, were able to obtain closed form solutions for the velocity, acceleration and the depth of penetration of missile only after the instant of penetration of full nose of missile. It was possible by the assumption of a linear relationship between the two parameters σ_r/K and Y, where Y is given by

$$Y = \frac{V_z \tan\theta}{\sqrt{K/\rho_0}} \tag{25}$$

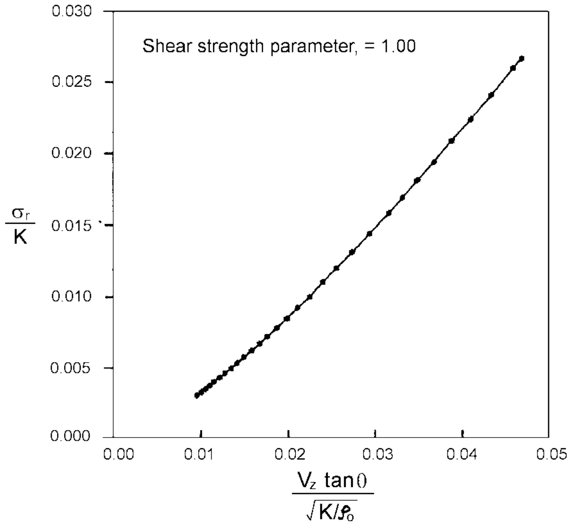

Fig. 4 Radial stress component of the conical nose

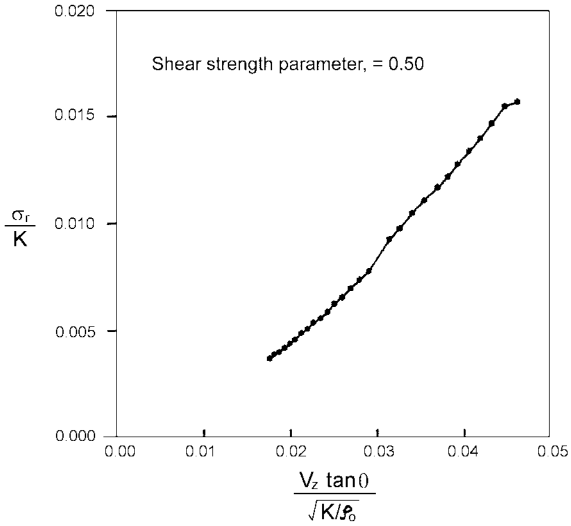

Fig. 5 Radial stress component on the ogive nose

But this assumption is not truly justified, as seen from Figs. 4 and 5, for conical and ogival nose. If this assumption is not to be considered then the only option left is to use the numerical methods as discussed above.

3. Discussion of results

3.1 Comparison with a field test

A missile penetration test (Forrestal *et al*. 1981) carried out in Sandia National Laboratories, Albuquerque has been taken for the purpose of validation of the present solution algorithm. In this test, a layer of welded tuff located at the Sandia Tonopah Test Range, Nevada was impacted by a missile of total length 1.52 m, outer diameter 0.165 m, an ogival nose profile with 9.25 CRH (Caliber Radius Head), nose length 0.495 m, and mass 182 kg. The missile was accelerated to a vertical velocity of 411 m/s with a Davis Gun. A 53.2 kg pusher plate which fits the internal diameter of the gun barrel was attached to the end of the penetrator until the pusher plate impacts the rock surface. The accelerometer was screwed into the penetrator nose after applying a light coat of epoxy on the threads and thus provided the rapid connection. The missile contains a 0.76 mm pad of rubber between the nose and the accelerometer. The pad was designed to act as mechanical filter and reduce the amplitude of high frequency excitation delivered to the accelerometer. Two accelerometers (Endevco 255MZ piezoelectric and Kistler 805 A quartz) were packaged within the missile to record the deceleration time response (Forrestal *et al*. 1981). This experiment has been used for subsequent discussion in the paper.

The deceleration time history observed during the experiment and that predicted for ogive nose shaped missile are shown in Fig. 6. The material data required for the application of the theory are (Forrestal *et al*. 1981)

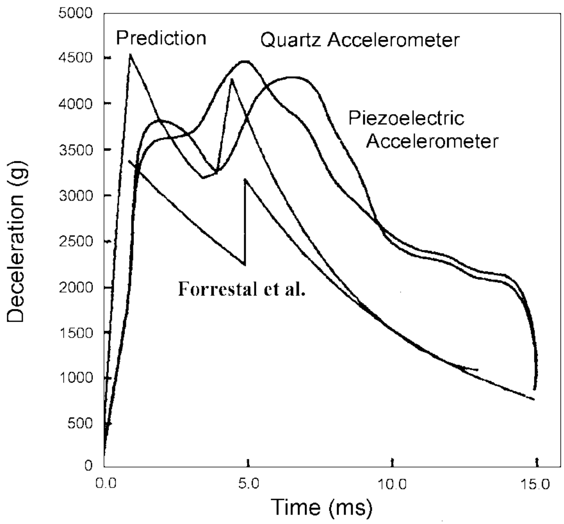

Fig. 6 Deceleration history of ogive nose shaped missile

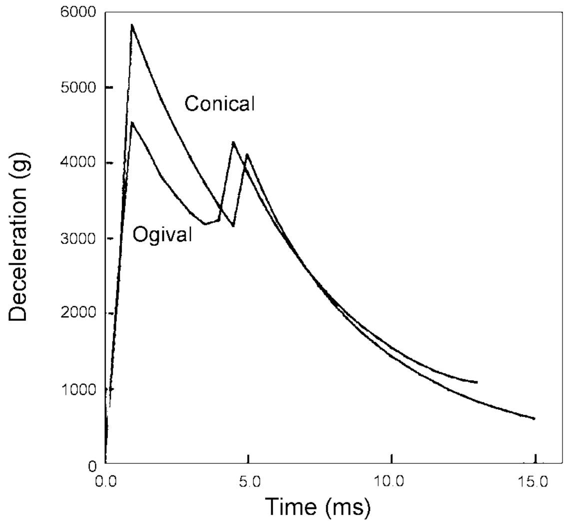

Fig. 7 Deceleration for ogive and conical nose shaped missile

Initial mass density , $\rho_0 = 1.97 \times 10^3$ kg/m^3
Bulk modulus, $K = 9.52 \times 10^3$ MPa
Shear strength parameter, $\mu = 4/3$
Coefficient of dynamic friction, $\mu_d = 0.13$ (assumed)

The Fig. 6 shows that the predicted deceleration has close proximity with experimental observation. There is a small difference in the period in which the pusher plate is trying to get detached. In the analysis sudden detachment of pusher plate has been assumed due to which there is discontinuity observed at this stage of penetration. Whereas, in the real experiment the removal of collar takes some time roughly about 2 ms due to which there is no abrupt change observed in the experimental observation. Another deviation from experimental time history is observed after the complete penetration of the missile aft body. At this stage the expanded target material comes in contact with the missile aft body and offers the drag on the aft body which is neglected in the present analysis. It has been assumed in the analysis that the effect of drag is limited to nose only. Owing to this assumption, the deceleration values are underestimated at later stage.

3.2 Comparison with Forrestal et al. results

The present study prediction and Forrestal *et al.* (1981) prediction for deceleration time history have been compared with the experimentally observed time history as shown in Fig. 6. This figure shows that the present study prediction has following major improvements over the Forrestal *et al.* (1981) prediction :

· The present prediction estimates the deceleration time response from zero time, whereas, Forrestal *et al.* (1981) theory predicts the deceleration time response only after the complete penetration of missile nose.
· Forrestal *et al.* (1981) theory neglects the effect of friction between the target material and the

missile nose because of which the predicted values are quite less than the experimental values. However, in the present analysis the effect of friction on the missile nose has been given due consideration which shifts the predicted values closer to the actual experimental values.

· Forrestal *et al*. (1981) have taken an equivalent conical nose shaped missile for actual ogive nose shaped missile (Fig. 2). However, the present work considers the actual shape of missile and then predicts the deceleration history.

3.3 Comparison with conical nose shaped missile

The deceleration time history for an equivalent conical nose shaped missile has been compared with an actual ogive nose shaped missile and shown in Fig. 7. This may be noted here that the nose length of the two missiles is same. The deceleration values for this equivalent conical nose shaped missile are significantly higher (about 20%) than that of similar ogive nose shaped missile for at least the duration of about 5 ms. Therefore, it is quite unjustified to assume an equivalent conical nose shaped missile for an actual ogive nose shaped missile.

3.4 Radial stress distribution along nose length

Fig. 8 shows the variation of radial stress for ogive nose shaped missile at the cavity interface, at a particular instant of time, along the nose length. This figure tells that the radial stress at the interface is maximum near the tip of the nose, where θ is maximum, and it is minimum near the end of the nose, where θ is minimum (i.e., $\theta = 0$). This indicates that the radial stress will be constant along its nose length if θ is constant, as for conical nose shaped missile. Therefore, it is quite unjustified to assume an equivalent conical nose shaped missile for an ogival nose shaped missile.

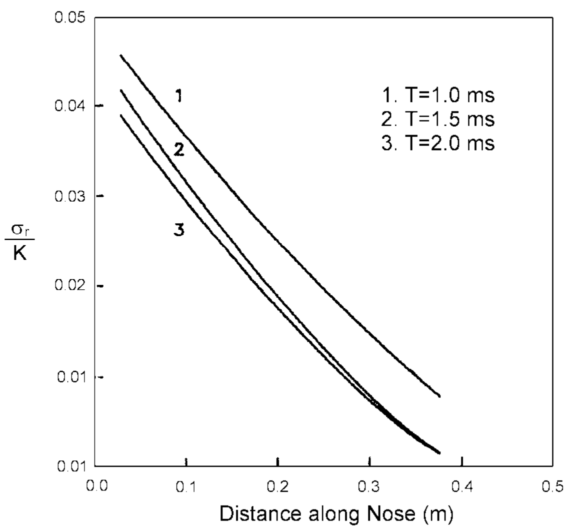

Fig. 8 Radial stress component along nose length (ogive nose)

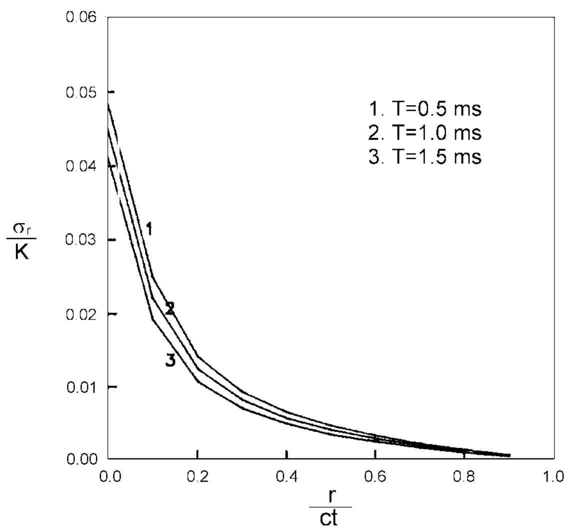

Fig. 9 Radial stress along radial direction (ogive nose) (Computed at the tip of the nose)

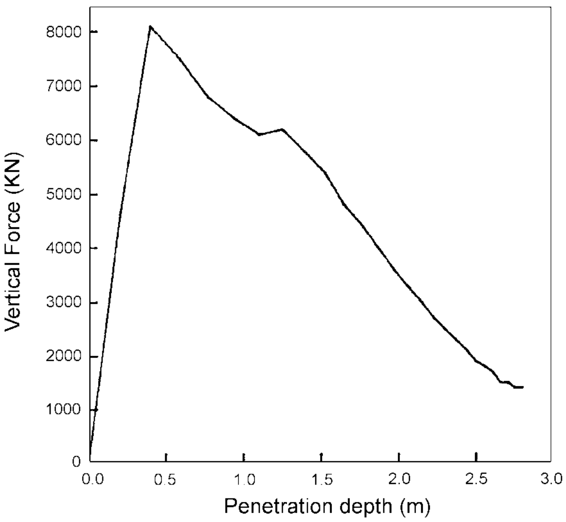

Fig. 10 Vertical force with penetration depth (ogive nose)

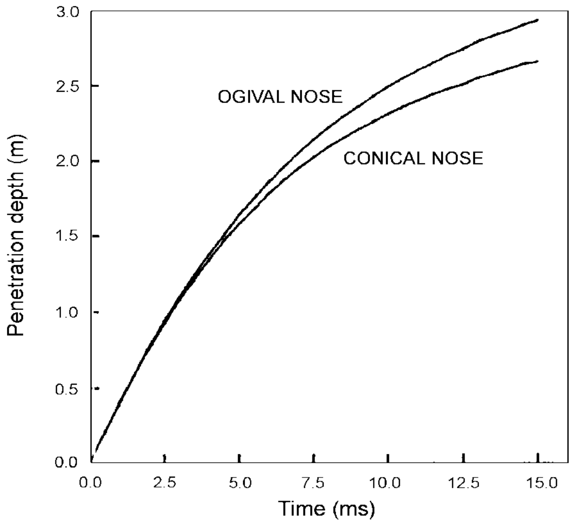

Fig. 11 Penetration depth with time

3.5 Radial stress distribution along radial direction

The variation of radial stress with non dimensional radial distance (r/ct), for different instant of time is shown in Fig. 9. This radial stress is observed to be reducing asymptotically as we move away from the nose of the missile. From this figure it can be concluded that the radial stress will be maximum just near the missile nose and at the instant when the missile strikes the target.

3.6 Vertical force at the nose

The variation of vertical force on the nose of the missile with the depth of penetration is shown in Fig. 10. This figure shows that the force is reducing non-linearly with the depth of penetration. The magnitude of the maximum force at the nose has been obtained as 8100 kN. Though there is sudden increase in deceleration because of the detachment of pusher plate from the missile but there is no corresponding discontinuity in the variation of force. It is due to the fact that the sudden increase in deceleration is associated with a sudden decrease in the mass.

3.7 Depth of penetration

The variation of depth of penetration with time for the ogive and equivalent conical nose shaped missile has been shown in Fig. 11. The figure shows that up to about 5 ms the depth of penetration is found to be the same for both types of missile. However, at later stage i.e., after 5 ms the depth of penetration at any instant is more for ogive nose shaped missile than that of similar equivalent conical nose shaped missile. In the present study analysis has been performed by considering drag on the nose, thus, these results are not the same as that of Forrestal *et al.* (1981) even for equivalent conical nose shaped missile. The final depth of penetration from the analysis has been found as 2.95 m

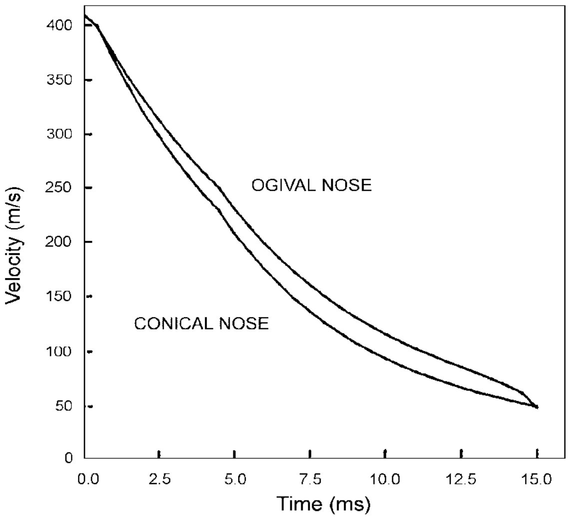

Fig. 12 Velocity with time

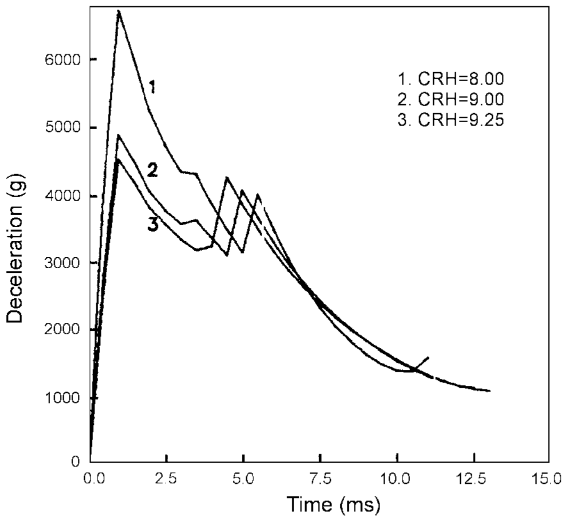

Fig. 13 Effect of CRH on deceleration

for ogive nose shaped missile, however, from the field it is 2.60 m. The predicted penetration depth is more than the experimental result because at later stage the expanded cavity recovers and comes in contact with aft body and thus frictional drag on the penetrator aft body becomes effective, which has been neglected in the present study. The frictional effect is assumed to be limited only to the missile nose.

3.8 Velocity

Fig. 12 shows the variation of velocity with time for ogive and conical nose shaped missiles. These curves are self explanatory. As the missile will go on penetrating the target material it will continuously lose its velocity. The fall in missile velocity is slower for ogive nose shaped missile than an equivalent conical nose shaped missile. At the velocity of about 30 m/s, (at time = 15 ms) the missile fails to further penetrate the target material and thus it suddenly comes to rest.

3.9 Effect of CRH

Fig. 13 shows the variation of deceleration with time for different CRH of ogival nose shaped missiles. This figure shows that as we increase the CRH of missile nose, the deceleration decreases. It is due to the fact that as CRH increases the nose length increases and shape of the nose becomes more pointed that makes the penetration easier and therefore deceleration decreases.

4. Conclusions

A model proposed by Forrestal *et al.* (1981) has been improved and forces on ogive and conical nose shaped missiles for normal impact into geo-material targets are predicted. The actual ogive nose shaped missile has been considered in the analysis and the results thus obtained have been compared with an experimental result. A close proximity has been observed with the experimental values. The deceleration time histories have been predicted and significant improvements have been observed over Forrestal *et al.* (1981) predictions. Consideration of friction; prediction of deceleration time history from the moment the missile strikes the target; and consideration of actual ogival nose in the analysis are the some of the significant improvements made over the Forrestal *et al.* (1981) theory. Radial stress variation along nose length and radial direction has been presented. The results of ogive nose shaped missile have been compared with an equivalent conical nose shaped missiles. Ogive nose shaped missiles are found more penetrating than equivalent conical nose shaped missiles. Effect of CRH on missile penetrating performance has been investigated.

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Notation

L	: nose length of missile
F_r	: radial force
F_z	: total vertical force
z	: depth of penetration ($z \le L$)
θ	: angle between tangent to missile nose with axis of missile
$R(z)$	: radius of the missile nose at a distance z from its tip
m	: mass of missile
CRH	: caliber radius head
ρ_0	: initial mass density
ρ	: instantaneous mass density
K	: bulk modulus
μ	: shear strength parameter
μ_d	: coefficient of dynamic friction
c	: speed of wave front
t, T	: Time
σ_θ	: circumferential stress

τ	: shear stress
p	: hydrostatic pressure
u	: radial displacement
R	: radius of aft body of missile
r	: radial distance
V_z	: vertical velocity
V	: radial velocity of cavity interface