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Imperfection sensitivity to elastic buckling of wind loaded open cylindrical tanks

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Abstract. This paper considers the buckling and post-buckling behavior of empty metal storage tanks under wind load. The structures of such tanks may be idealized as cantilever cylindrical shells, and the structural response is investigated using a computational model. The modeling employs a doubly curved finite element based on a theory by Simo and coworkers, which is capable of handling large displacements and plasticity. Buckling results for tanks with four different geometric relations are presented to consider the influence of the ratios between the radius and the height of the shell (R/L), and between the radius and the thickness (R/t). The studies aim to clarify the differences in the shells regarding their imperfection-sensitivity. The results show that thin-walled short tanks, with $R/L = 3$, display high imperfection sensitivity, while tanks with $R/L = 0.5$ are almost insensitive to imperfections. Changes in the total potential energy of tanks that would buckle under the same high wind pressures are also considered.

Key words: buckling; cylindrical shells; finite elements; postbuckling; tanks; total potential energy; wind load.

1. Introduction

This paper considers the buckling and postbuckling response of wind loaded, thin-walled cylindrical tanks, and the sensitivity of such buckling response to the influence of small geometric deviations from the cylindrical shape. The research reported here focuses on the short tanks employed to store oil, water and petrochemical products in the Caribbean islands, in many parts of the United States, and in other geographical regions subject to high winds. The short cantilever cylinder is commonly employed in large capacity tanks to store oil, with ratio R/L of the order of 2 to 3, and R/t of the order of 1000-2000, where R is the radius of the cylinder, L is the height of the tank and t is the shell thickness.

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The failure of wind-loaded cantilever cylinders has been reported and investigated since the 1960s. For example, the collapse of oil-storage tanks in England in 1967 was reported by Kundurpi *et al*. (1975), but failures of this kind have also occurred in many other areas of the world that are subject to high wind conditions without being reported in the open literature.

The literature on the buckling of cantilever cylindrical shells under wind load has mainly concentrated on tanks that are not short. Pressure coefficients for tanks with dome roof (Maher 1966) or flat roof (Purdy *et al*. 1967) were reported from wind tunnel experiments in the 1960s. A fine set of wind tunnel experiments was done by Johns and co-workers during the 1970s (Kundurpi *et al*. 1975), who tested small scale cylinders with $1 < L/R < 5$ and $376 < R/t < 555$. Resinger and Greiner (1982), Uematsu and Uchiyama (1985), and Megson *et al*. (1987) reported other tests on cylinders carried out in wind tunnel facilities. Schmidt *et al*. (1998) published post-buckling results from tests on PVC and steel cylinders under internal suction, and also under a static simulation of wind by means of a pressure that varies over segments of the shell; all cylinders included ring stiffeners on top, except for one case with $L/R = 1$ and $R/t = 2500$, which was tested under internal pressure.

Recent computational research in this field includes the work of Schmidt *et al*. (1998), who developed design strategies based on the use of ring stiffeners as a way of preventing global buckling (i.e., buckling modes that have large displacements around the top edge). Their design recommendations are supported by computational results for perfect cylindrical shells. Greiner and Derler (1995) included the influence of imperfections using different patterns for the shape of the geometric deviation. For short shells it was found that imperfection sensitivity was highest for imperfections with the shape of the eigenmode associated to the lowest bifurcation load.

In all those studies the effects of wind were computed from a static analysis; however, one may question if the buckling process of tanks under wind is static or dynamic. The authors studied the nonlinear dynamic response of short tanks and found that inertia effects were not significant in this class of shells, so that static nonlinear studies could well be carried out to estimate instability under wind pressure (Flores and Godoy 1998).

The purpose of this work is not to formulate design specifications for tanks. In this point the authors agree with other researchers (Schmidt *et al*. 1998) about the need to provide a reinforcing ring as a way to prevent global buckling. But here the performance of already built tanks is considered, and many shells that failed during recent hurricanes do not have rings to stiffen the upper edge. The following sections describe the computer model and results for several geometries of cantilever shells, by assuming geometrical imperfections and geometrically nonlinear analysis. The lower edges of the shells were assumed to be clamped.

2. Computational model

The authors studied cantilever cylinders with different geometric relations, in order to highlight the differences in the buckling and post-buckling behavior depending on the geometric parameters, and to learn about their imperfection sensitivity. Of those, only four cases are discussed here, for reasons that will be clear in the following section.

The discretization of the shell is carried out using alternatively a four-node quadrilateral element with bilinear interpolation, and a six-node triangle (Flores *et al.* 1995) based on a shell theory developed by Simo and co-workers (1990, 1992). In all cases a full two-dimensional analysis of the shell is carried out, so that no uncoupling of the solution in terms of isolated modes is performed.

To avoid transverse shear locking phenomena, assumed strain models (derived using the Hu-Wazhizu variational principle) are considered according to the general methodology described by Oñate *et al.* (1993). For coarse meshes membrane locking can occur in initially curved quadratic triangles. To avoid this locking, an approximation similar to Oñate *et al.* (1993) was adopted, assuming linear variation of the mid-surface membrane strains (Flores *et al.* 1995). These elements have an excellent performance in geometric non-linear problems (Simo *et al.* 1990, Flores *et al.* 1995) and can also consider elastic-plastic behavior using the material model developed by Simo and Kennedy (Simo *et al.* 1992).

The elements were implemented in a computer code called ALPHA (Flores 1996) with capabilities for geometrically non-linear static and dynamic problems including large deformation plasticity. Using this model one can consider geometric imperfections and large displacements in post-critical paths. The code also has the possibility of computing limit and bifurcation points from a non-linear fundamental path using extended system (Wriggers and Simo 1990) and a numerical derivative of the tangent matrix.

The boundary conditions in all the problems studied in this work were clamped at the base and free at the upper edge, without any reinforcing ring. This configuration is representative of what has been observed in many tanks that buckled during hurricanes in the last decade in the Caribbean islands.

The pressure distribution on the walls of the tank was assumed as in other works (Flores and Godoy 1998), with a unit value (1 N/m^2) on the windward meridian, and this pressure is assumed to have a constant value in elevation at each meridian (see Fig. 1). The circumferential pressure distribution is assumed in the form (Rish 1967):

$$p = \lambda \sum_{i=0}^{6} c_i \cos(i\theta)$$

where the Fourier coefficients are $c_0 = 0.387$ $c_1 = -0.338$ $c_2 = -0.533$ $c_3 = -0.471$ $c_4 = -0.166$ $c_5 =$

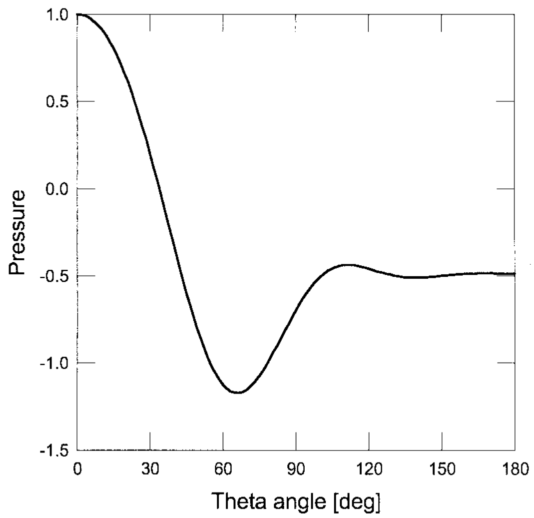

Fig. 1 Wind-pressure distribution assumed around the circumference

0.066 $c_6 = 0.055$. Previous results obtained by the authors (Flores and Godoy 1998) indicate that buckling is not so sensitive to the exact pressure distribution around the circumference, and is highly dependent on the pressure on the windward meridian. Such pressure is scaled by means of a load parameter λ.

To perform the computations, first a bifurcation buckling analysis was carried out to obtain the lowest bifurcation load parameter (eigenvalue) and the associated buckling mode (eigenvector). This provided an estimate of the values of the critical loads that can be expected in a nonlinear analysis, together with the possible modes; however, such values are not directly employed in the final nonlinear solution. Classical eigenvalue analysis provides unsafe values of buckling loads for some geometric shell configurations.

Second, a static nonlinear analysis is carried out to obtain the load-displacement response for several imperfection amplitudes. The present approach does not uncouple the solution using the classical eigenvalue analysis; instead, the response of the shell is followed with all possible deflection patterns being active and the results take multimode behavior into account. This has the advantage that there is no need to have a preferred basis to compute the nonlinear deflection of the shell; on the other hand, this analysis has the limitation that one does not obtain the details of how modes interact (Godoy 2000).

In the following, ξ denotes the amplitude of a geometric imperfection. The case with $\xi = 0$ is the perfect cylindrical shell, while cases with $\xi \neq 0$ represent imperfect geometries, with the imperfection shape given by the eigenmode associated to the lowest bifurcation load in a perfect analysis under wind pressure, and the maximum amplitude of the imperfection is given by ξ. The load factor λ^c is the maximum reached by a shell for $\xi = 0$, while for each imperfect shell $\lambda^{\max}$ is the maximum load reached for $\xi \neq 0$. The topic of the detailed shape of the geometric imperfection that is more dangerous in terms of load capacity reduction of the shell is not considered in this paper. However, other studies for pressure-loaded cylinders (see, for example, Yamada and Croll 1993) have shown that the dominant imperfection shape is that given by the lowest classical critical mode.

The computer model employed is capable of representing both elastic and plastic material conditions; however, in the present studies plasticity was only reached for very large post-buckling displacements, so that no further consideration will be given to it in the remainder of the paper.

3. Buckling behavior of thin walled cantilever shells under wind

Results from the analysis of four geometries of tanks, with $0.5 \leq R/L \leq 3$ and $1250 \leq R/t \leq 2000$, are reported in this section to highlight the main features of the buckling behavior of typical tanks. The shorter tank has $R/L = 3$, and results for $R/t = 2000$ are presented in Fig. 2. In Fig. 2a, the load factor λ employed to increase the unit pressure distribution (normalized with respect to λ^c) is plotted versus the out-of-plane displacement w at the top of the shell for a meridian located at 7^o from the direction of the wind. This is the point in the upper edge with maximum displacement in the eigenmode. The critical load parameter computed for $\xi = 0$ is $\lambda^c = 2282$ and the equilibrium path is linear up to a maximum value reached at $\lambda/\lambda^c = 1$. This indicates that in the short tanks the geometrically nonlinear behavior is well represented by a bifurcation analysis. The post-buckling equilibrium states occur along a descending, unstable path.

The imperfection-sensitivity diagram is shown in Fig. 2b, and out of the four cases considered,

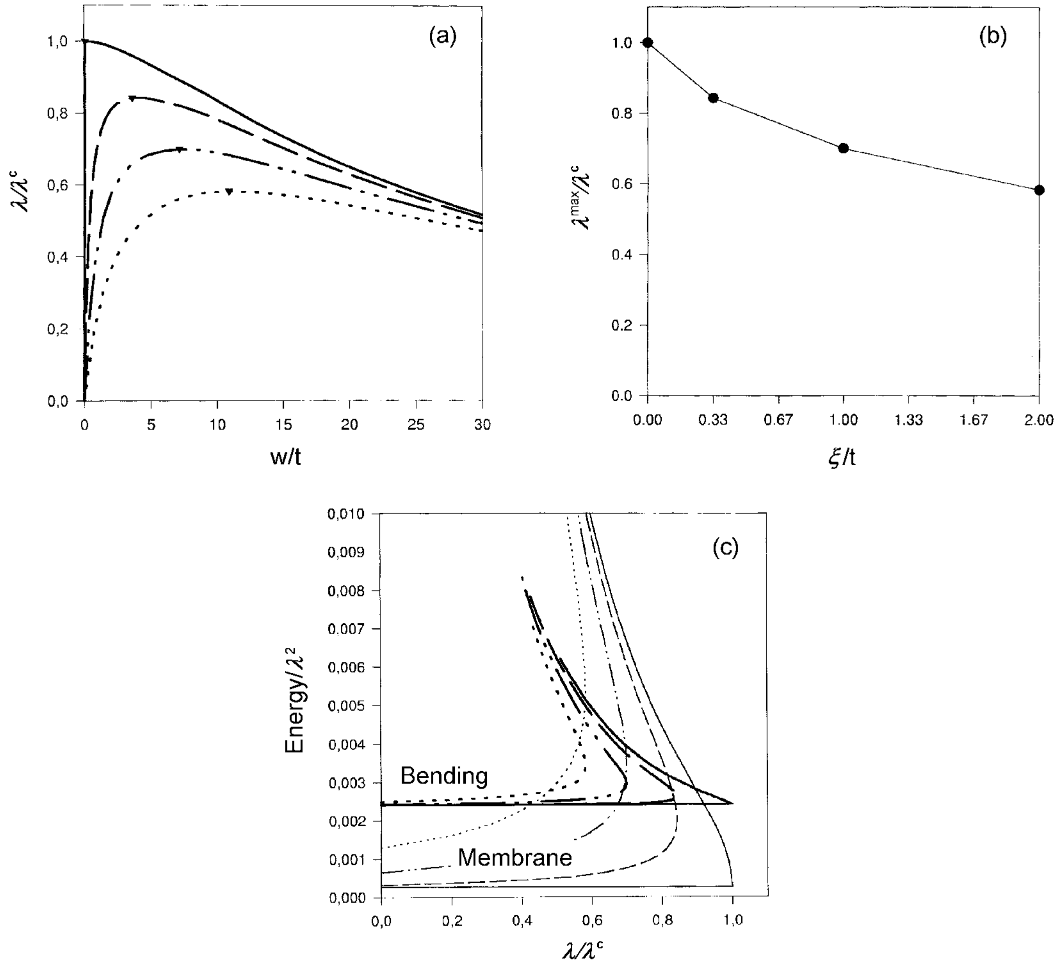

Fig. 2 Results for a tank with $R/L = 3$ and $R/t = 2000$. (a) Equilibrium paths; (b) Imperfection sensitivity; (c) Energy contributions. Thin lines are membrane contribution; thick lines are bending contributions. _____ $\xi/t = 0$; __ __ $\xi/t = 1/3$; __ . . __ $\xi/t = 1$; …….. $\xi/t = 2$

this one shows the largest drop in the maximum load. For an imperfection with amplitude $\xi/t = 2$ the maximum load obtained, λ^{max}, is only 60% of the maximum load in the perfect shell ($\lambda^{max} = 0.6\ \lambda^c$).

Next, the energy is plotted in Fig. 2c for values of increasing load λ/λ^c. The membrane energy contribution is seen to dominate the strain energy with an almost constant value until bifurcation is reached, and then it increases along the post-critical path. The bending contribution to the total energy has low levels along the fundamental path (lower than 0.0005 λ^2). In the post-critical path, for a value of $\lambda \approx 0.9\ \lambda^c$, the two energy components provide the same contributions to the total potential energy, and then bending dominates for larger postcritical displacements. The energy level at which the two contributions become the same tends to increase with the amplitude of the imperfection considered.

The results for two intermediate cases are shown in Fig. 3 ($R/L = 2$ and $R/t = 1750$) and Fig. 4 ($R/L = 1$ and $R/t = 1500$). The other extreme geometry among the cases considered in this section is

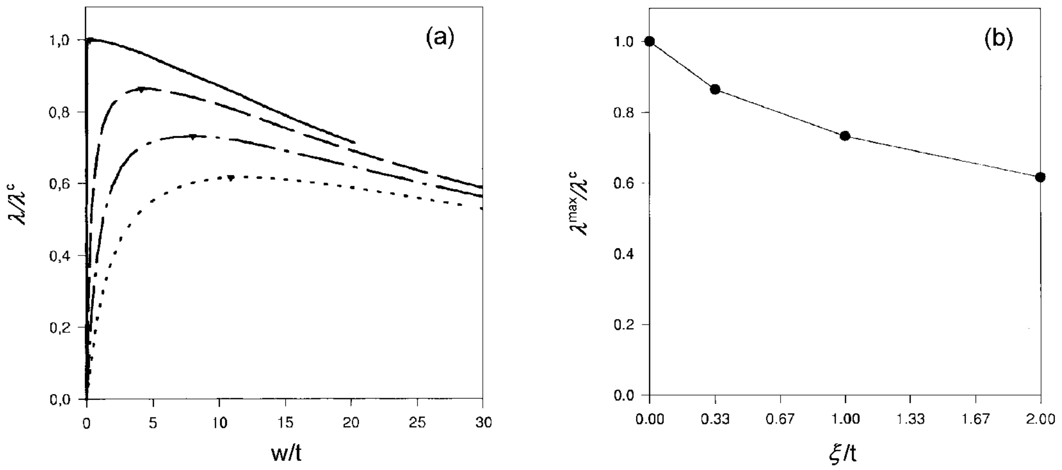

Fig. 3 Results for a tank with $R/L = 2$ and $R/t = 1750$. (a) Equilibrium paths; (b) Imperfection sensitivity. ______ $\xi/t = 0$; __ __ $\xi/t = 1/3$; __ . . __ $\xi/t = 1$; …….. $\xi/t = 2$

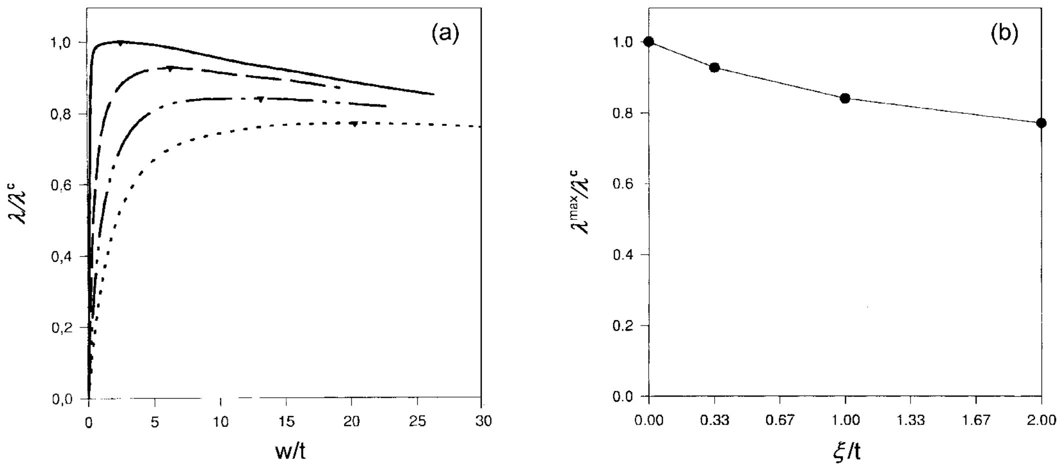

Fig. 4 Results for a tank with $R/L = 1$ and $R/t = 1500$. (a) Equilibrium paths; (b) Imperfection sensitivity. ______ $\xi/t = 0$; __ __ $\xi/t = 1/3$; __ . . __ $\xi/t = 1$; …….. $\xi/t = 2$

a cylindrical shell with $R = 0.5\ L$ and $R/t = 1250$. The results in Fig. 5a show curves for different imperfections that are much closer among them than in the shorter cylinder. Even for the perfect shell, $\xi = 0$, the behavior is clearly nonlinear and a limit point is reached at $\lambda = 1558$ for large deflections ($w/t = 17$), where the displacement w is measured at the top of the shell for a meridian located at 17^{o} from the wind direction. This is consistent with the limit point behavior found by Kundurpi *et al.* (1975) in wind tunnel tests. However, the results of those authors are not directly comparable with those of Fig. 5 because they investigated thicker shells (up to $R/t = 555$). The imperfection sensitivity in this case is much lower than in the short shell, with values of $\lambda^{max} \cong 0.95\ \lambda^c$

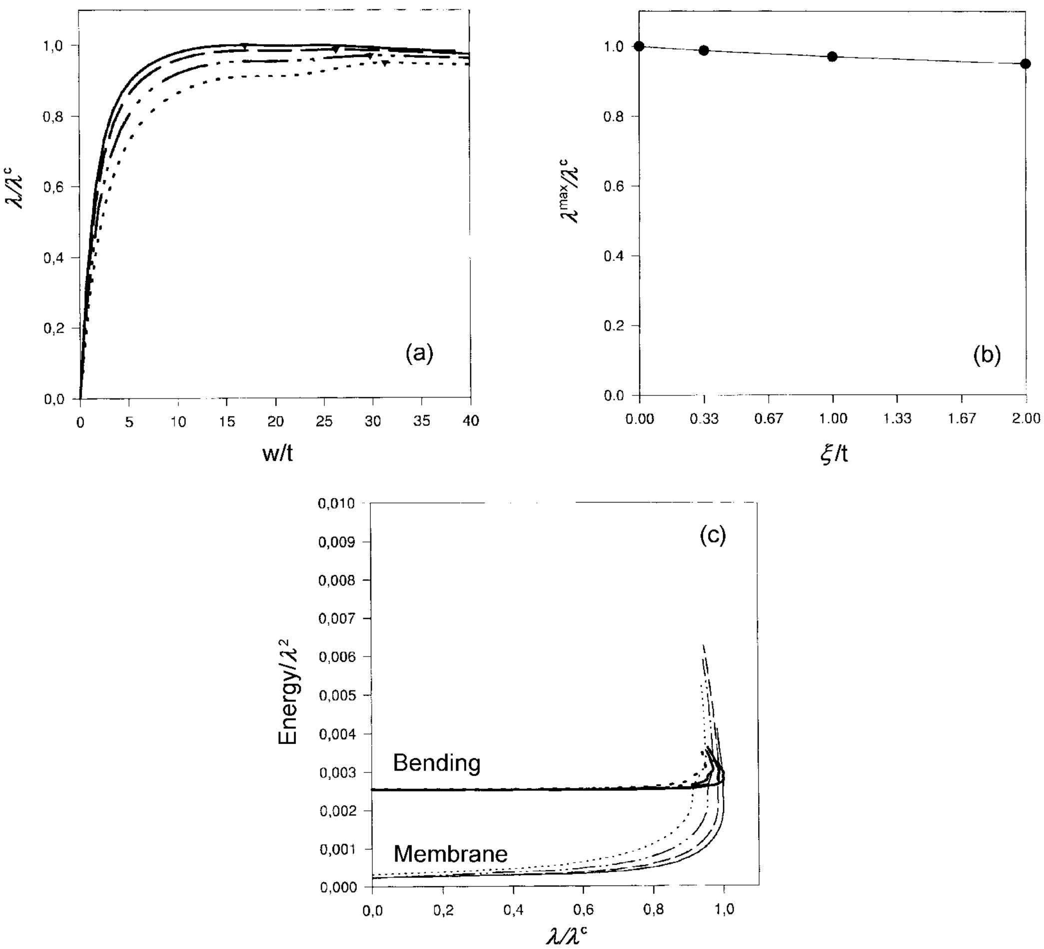

Fig. 5 Results for a tank with $R/L = 0.5$ and $R/t = 1250$. (a) Equilibrium paths; (b) Imperfection sensitivity; (c) Energy contributions. ______ $\xi/t = 0$; __ __ $\xi/t = 1/3$; __ . . __ $\xi/t = 1$; …….. $\xi/t = 2$

for $\xi/t = 2$. As in most limit-point problems, this cylinder is not significantly sensitive to initial imperfections. The energy components are given in Fig. 5c. For the perfect shell, the membrane energy does not significantly increase up to the limit point, but it is the bending contribution that increases for $\lambda > 0.9\ \lambda^c$. At the critical point the two components of the energy reach approximately the same values.

The results for the energy in the shell in Fig. 2, which is sensitive to imperfections, indicate that as the imperfection amplitude is increased, then there is a significant increase in the membrane energy required to maintain equilibrium prior to buckling. For example, for the largest imperfection amplitude computed, the membrane energy is initially (for $\lambda = 0$) about four times the value in the perfect case. On the other hand, the change in membrane energy in Fig. 5 (which is not sensitive to imperfections) is not significant.

According to Croll, "it is the membrane contributions to this initial resistance that are most likely to be lost in the buckling of highly optimized structures, like shells" (Yamada and Croll 1993, pp. 290).

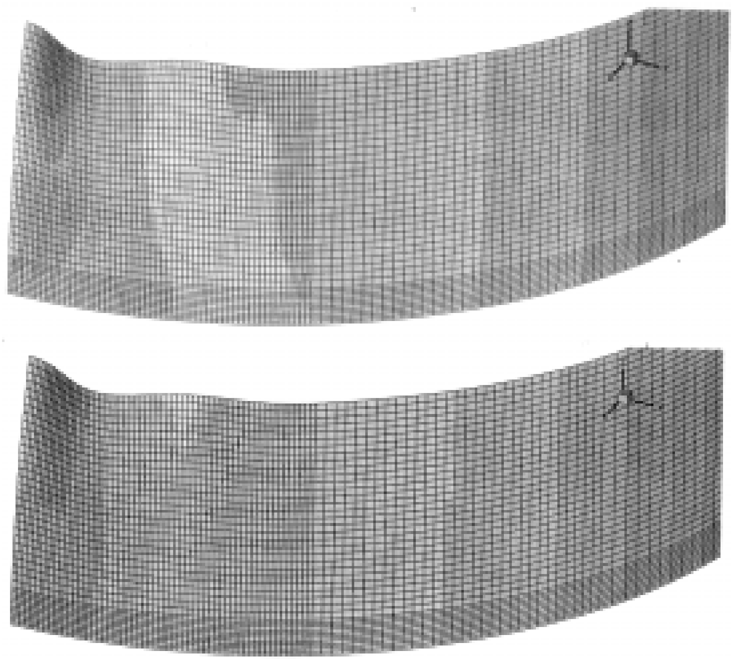

Fig. 6 Displacement pattern for a shell with R/L = 20 and R/t = 1750. (a) Perfect shell; (b) Imperfect shell, ξ/t = 2, for w/t = 10

This is shown in Fig. 2c in the postbuckling range: for example, for $\lambda = 0.8$ there is a drop in the membrane energy from the perfect case to the imperfect case. Such a loss must be compensated by an increase in the bending energy in the postbuckling range, which is also shown in Fig. 2c. This effect is small in the shell of Fig. 5c, which is not sensitive to imperfections.

The displacement pattern of the buckling mode for one case (the shell with R/L = 2.0 and R/t = 1750) is shown in Fig. 6a for $\xi = 0$. For $\xi = 2\ t$ and w/t = 10 the pattern of displacements (Fig. 6b) is essentially the same as in the perfect case (Fig. 6a). This means that the nonlinear postbuckling response of the shell under wind occurs without a change in the buckling mode.

4. Discussion

The present study shows that, from the mechanics point of view, "tanks" are not just one class of problem, and that according to their geometric features they may display very different buckling behavior and imperfection sensitivity under wind.

A common feature of the tanks studied in the previous section is that for the geometric parameters considered, the wind pressure required for buckling in the imperfect shell with an imperfection amplitude $\xi/t = 2$ is approximately the same, as shown in Table 1. This wind pressure is consistent with the wind velocity expected to occur in the Caribbean islands and in the east coast of the United States (v = 55.6 m/s or 125 miles/h). Notice that this wind velocity is much higher (twice) than what is considered a "normal" German wind load condition (v = 28.2 m/s), as stated by Schmidt *et al.* (1998).

Table 1 Maximum load factor λ^{max} reached by the shell under wind, for an imperfection $\xi = 2\,t$

R/L	R/t	ξ	$\lambda^{max} / \lambda^{c}$	λ^{max}
3.0	2000	2t	0.583	1331
2.0	1750	2t	0.617	1360
1.0	1500	2t	0.772	1391
0.5	1250	2t	0.951	1481

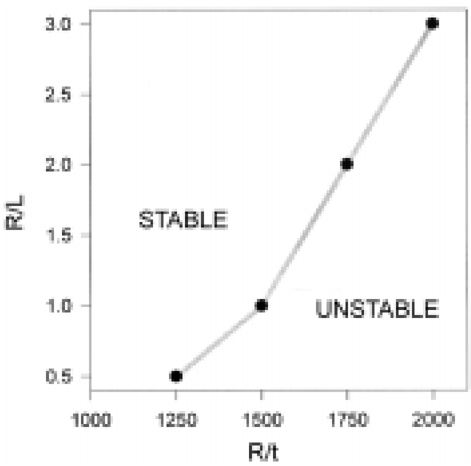

Fig. 7 Buckling constraints for imperfect tanks with imperfection amplitude $\xi = 2\,t$

The results are summarized in Fig. 7 in order to identify which geometric features of the shell lead to buckling. A buckling constraint for shells with $\xi/t = 2$ may be established in terms of the geometry of the shell as reflected by R/L and R/t. For example, a shell designed for winds of v = 55.6 m/s with $R/L = 1.5$ and $R/t = 1500$ would not be expected to buckle, while for the same R/L ratio but with R/t = 1800 the graph indicates that the shell would buckle.

The imperfection sensitivity of the short tanks considered in this paper is lower than in the axially loaded shells, but it is of the same order as in the cylinders under uniform lateral pressure. Thus, it would be very convenient to be able to estimate lower bounds to buckling loads based on the mechanics of behavior observed in the buckling process. Croll has developed a "reduced energy theory" to account for the imperfection-sensitivity of shells by means of a reduction in the membrane energy contributions (see Croll 1995, and the references cited there). It is in shorter tanks under wind that a reduced energy approach may be most helpful, as shown by the energy contributions in Fig. 2c. The development of such reduced energy approach for wind-loaded shells is seen as a topic for future research.

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