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Structural dynamic optimization with probability constraints of frequency and mode*

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Abstract. The structural dynamic optimization problem based on probability is studied. Considering the randomness of structural physical parameters and the given constraint values, we develop a dynamic optimization mathematical model of engineering structures with the probability constraints of frequency, forbidden frequency domain and the vibration mode. The sensitivity of structural dynamic characteristics based on probability is derived. Two examples illustrate that the optimization model and the method applied are rational and efficient.

Key words: engineering structures; dynamic characteristics; frequency and mode; probability constraints; dynamic sensitivity; optimization design.

1. Introduction

The structural dynamic characteristic is a very important index of structural design. In structural design, making the structure's inherent frequency far away from the bandwidth of excitation frequency can avoid the resonance phenomena. Restraining or decreasing vibration by revising the structural dynamic characteristics is one of the effective techniques to control vibration. In the forepart of structural dynamic optimization, the main research works are concentrated on the structural optimization with constraints of inherent frequency or frequency forbidden domain. Such as Zarghamee (1968), Lin (1981), Wang (1982), Ramana (1993) are some representative literatures. Along with the development of structural dynamic optimization, the problem about structural optimization with constraints of frequency and vibration mode has attracted ones attention more and more in recent years, and has been discussed in Chen (1987), Xiang *et al.* (1995), Chen and Zhou (1996), Czyz and Lukasiewicz (1998). The background of this kind problem is that the dynamic design for flexibility carrier such as airplane, rocket, vehicle, ship and so on, not only requests their inherent frequency values but also has some restriction on the position of mode node (line) or mode abdomen point (line). In despite of the researching contents on optimum design of structural

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dynamic characteristic has been perfected more and more, but so far it is unluckily that the randomness of structures has been rarely considered in the optimization model of structural dynamic. As a matter of fact, in many instances the randomness exists impersonally in structural design, especially in those structures produced on a large scale or in batches. Such as the physical parameters of structural material has dispersivity, the geometric dimension of structural member has tolerance in process of manufacture and assemblage, the constraint values of frequency and vibration mode have indetermination, and so on. Thus studying the optimum design of structural dynamic characteristic based on probability is much significant theoretically and practically for perfecting the structural optimum design.

In this paper, the engineering structures (truss, beam, plate and shell) are taken as researching object, based on the author's forepart works (Chen and Cei 2000, Chen and Duan 1999), the problem of optimum design for structural dynamic characteristic is further studied. In constructed optimization model, the material physical parameters of structure and requested frequency and vibration mode are taken as random variables, at the same time, the frequency, forbidden frequency domain and position of vibration mode are taken as probability constraints.

2. The probability constraint of structural dynamic characteristic

The constraints of structural dynamic characteristic include two kinds; they are frequency constraints and vibration mode constraints respectively. The frequency constraints involve currently two forms, i.e., fundamental frequency and forbidden frequency domain. The constraint of fundamental frequency is namely requested that the lowest inherent frequency of structure should be greater than a given value Ω. The constraint of forbidden frequency domain is namely demanded the two borders upon inherent frequencies of structure should locate outside of given frequency range (Ω^l, Ω^u). Their mathematical expressions are respectively:

$$\omega_{\min} - \Omega \geq 0 \tag{1}$$

$$\Omega^l - \omega_j \geq 0, \qquad \omega_{j+1} - \Omega^u \geq 0 \tag{2}$$

where, $\omega_{\min}$, ω_j, ω_{j+1} are respectively the fundamental frequency, jth and $(j+1)$th inherent frequencies of structure. Ω is the given down bound value of the fundamental frequency. Ω^u, Ω^l are respectively the given upper and down bound values of forbidden frequency domain.

For the staff shape structures such as truss and beam, the vibration mode constraint is namely the constraint of the position of mode node or paunch point. While for the plate and shell structures, the vibration mode constraint is namely the constraint of the position of mode node line or paunch line. The positions of vibration mode node are commonly expressed with the distance from mode node to coordinate origin X_g. However, when we make the structural dynamic analysis by means of the finite element method, there is nearly no situation that the vibration mode nodes are superposed with the grid nodes of finite elements. In order to obtain the positions of vibration mode node, the interpolation method and the information of finite element nodes must be used. Suppose that through structural dynamic analyzing by means of finite element method, the ith order inherent frequency and corresponding vibration mode of structure are respectively: ω_i, $\{\varphi\}_i=\{\varphi_1, \theta_1, \varphi_2, \theta_2, \ldots, \varphi_n, \theta_n\}^T$, in where φ_j, θ_j are respectively the displacement and rotation angle of jth mode node. If the used interpolation points in vibration mode are $(X_v,\ \varphi_v)\sim(X_w,\ \varphi_w)$, then the position coordinates of vibration mode node X_g can be gained by using the Lagrange interpolation function $X(\varphi)$ (Feng 1978). That is:

$$X_g = X(0) = \sum_{j=v}^{w}\left(\prod_{\substack{i=v\\ i\neq j}}^{w} \varphi_i\right) X_j \Big/ \prod_{\substack{i=v\\ i\neq j}}^{w} (\varphi_i - \varphi_j) \tag{3}$$

In above formula, $X(0)$ express the position coordinates of vibration mode node. If the rotation angle θ stands for the displacement φ, then X_g is namely the position of vibration mode paunch point X_f.

Suppose that X_g^* and X_g are respectively the design-requested position of vibration mode node and the initial design ones. Then in order to the position of vibration mode node is satisfied with design demand, namely there is following condition:

$$d_i = |X_g^* - X_g| \leq \delta \tag{4}$$

where δ is the given allowable difference.

The expression of node line of vibration mode is more complex than the one of mode node. Its existence and description completely depends on the structural types. In Xiang *et al.* (1995), the area ΔS_i surrounded by the initial node line, the desired node line and the outline of structure is taken as the expression parameter of position of node line. Where the needed points on node line of vibration mode are obtained by means of interpolation approach, then the node line of vibration mode are produced by fitting with the thrice sample function. When the design demands of node position of vibration mode are satisfied, then the parameter $\Delta S_i \to 0$. Based on that the producing process of node line of vibration mode is first dispersing and then fitting treatment, therefore, the treating means for demand of node position of vibration mode is presented as follows. At first, the given node line of vibration mode is dispersed into multiple points, and then those points are taken as the demand design nodes. Thus the original position constraint to node line of vibration mode is dispersed into multiple position constraints to node of vibration mode. When all those position constraints to node of vibration mode are satisfied, then the node line of vibration mode is satisfied with the original position constraint. So the position constraints to node line of vibration mode can be changed into following forms:

$$d_i^{(j)} = |X_g^{(j)*} - X_g^{(j)}| \leq \delta \qquad (j=1,\ 2,\ \ldots,\ ng) \tag{5}$$

Where, $X_g^{(j)*}$, $X_g^{(j)}$ are respectively the given positions and currently position of jth discrete point on node line of vibration mode. ng is the number of discrete points on node line of mode.

If the node of vibration mode $X_g^{(j)}$ stands for the paunch node of vibration mode $X_f^{(j)}$, then the above formula will become the position constraint of paunch line of vibration mode.

The randomness of material physical parameters (elastic module E and mass density ρ) of structure leads to the randomness of the inherent frequency and vibration mode of structure. Further more, if the randomness of given frequency and corresponding position of vibration mode is taken into account, then the constraints of structural dynamic characteristic must be described with random variables and probability forms. According to this, the fundamental frequency constraint (1), the forbidden frequency domain constraint (2) and the position constraint of vibration mode (5) can all be expressed with the probability form as follows:

$$P^* - P_r\{\omega_{\min} - \Omega \geq \delta\} \leq 0 \tag{6}$$

$$P^* - P_r\{\Omega^l - \omega_j \geq \delta_l\} \leq 0 \tag{7}$$

$$P^* - P_r\{\omega_{j+1} - \Omega^u \geq \delta_u\} \leq 0 \tag{8}$$

$$P^* - P_r\{X_g^{(j)} - X_g^{(j)*} \geq \delta_j\} \leq 0 \qquad (\text{when } X_g^{(j)} \geq X_g^{(j)*}) \qquad (j=1, 2, \ldots, ng) \tag{9}$$

$$P^* - P_r\{X_g^{(j)*} - X_g^{(j)} \geq \delta_j\} \leq 0 \qquad (\text{when } X_g^{(j)} \leq X_g^{(j)*}) \qquad (j=1, 2, \ldots, ng) \tag{10}$$

where, the meanings of symbols $\omega_{\min}$, ω_j, ω_{j+1}, Ω, Ω^u, Ω^l are same as former ones, but they are regarded as random variables here. δ, δ_u, δ_l, δ_j are respectively allowable differences of the fundamental frequency, the upper and lower bound of forbidden frequency domain and the position of vibration mode. P^* is the allowable probability value (reliability). $P_r\{\cdot\}$ denotes the computational probability (reliability).

3. The mathematics model of optimization and equivalent treatment of probability constraints

3.1 The mathematical model of optimization

Considering the most common situation in structural dynamic characteristic design, the mathematical model of dynamic characteristic optimization of engineering structures is constructed here. In the model, the minimum mean value of structure weight is taken as the objective function, and it is subjected to the probability constraints of fundamental frequency, forbidden frequency domain and position of mode, as well as the upper and down bound of design variables. The mathematical model is expressed as follows :

$$\begin{aligned} find &: \{\bar{A}\} = (A_1, A_2, \ldots, A_{ne})^T \\ min &: \bar{W}(\bar{A}) = \sum_{i=1}^{ne} \bar{\rho}_i A_i S_i \end{aligned} \tag{11}$$

$$\begin{aligned} &s.\ t.: \text{Eqs. (6) ~ (10)} \\ &A^l \leq A_i \leq A^u \quad (i=1, 2, \ldots, ne) \end{aligned} \tag{12}$$

Where, $\{\bar{A}\}$ is the design vector. $\bar{\rho}_i$ is the mean value of mass density corresponding to the ith design variable. A_iS_i denotes the volume corresponding to the ith design variable. S_i is bar element's length or plate element's area corresponding to the ith design variable, which depends on the design variable A_i. $\bar{W}(\bar{A})$ is the mean value of structural weight. A^u, A^l are the upper and down bound of design variables respectively. A^l is depends on the condition of strength or stability. A^u is usually decided by the structural construction or manufacture techniques. ne is the dimension of design vector.

Above mathematical model of optimization is suited to many types of structures. For the plate and shell structures, the thickness of plate or shell are taken as design variables. For the staff shape structures (truss, beam), the cross section area of staff are taken as design variables. For the beams with different shape of cross sectional area, we suppose that there is following approximate relationship between the cross sectional area A and the section inertia moment J:

$$J = aA^b \tag{13}$$

where a and b are statistical constants, their values are decided by the section figure of beam.

3.2 The equivalent treatment of probability constraints

In above mathematical model of optimization, all probability constraints are implicit complex function of the design variables, and they are expressed in the probability forms. This makes the sensitivity analysis of constraint function difficult, and the conventional optimization approaches based on sensitivity information can not to be applied. Therefore, the probability constraints are equivalent treated as following:

Since all above probability constraints can be expressed with the uniform format as:

$$P^* - P_r\{R - S \geq \delta\} \leq 0 \tag{14}$$

where R, S denote the random variables of structural resistance and loading effect, they are usually independence from each other.

According to the first order and second moment method in structural reliability, the above probability constraints can be expressed with the reliability index β as:

$$\beta^* - \beta \leq 0 \tag{15}$$

$$\beta^* = \Phi^{-1}(P^*), \quad \beta = (\mu_R - \mu_S - \delta)(\sigma_R^2 + \sigma_S^2)^{-1/2} \tag{16}$$

where, $\Phi^{-1}(\cdot)$ denotes the inverse function of probability distribution function of the standard normal variable. μ, σ^2 denote respectively mean value and variance of random variable.

According to formulae (15) and (16), all original probability constraints in mathematical model of optimization (6)~(10) can be expressed with their equivalent form as:

$$\beta^* - (\mu_{\omega\min} - \mu_\Omega - \delta)(\sigma_{\omega\min}^2 + \sigma_\Omega^2)^{-1/2} \leq 0 \tag{6a}$$

$$\beta^* - (\mu_{\Omega^l} - \mu_{\omega_j} - \delta_l)(\sigma_{\Omega^l}^2 + \sigma_{\omega_j}^2)^{-1/2} \leq 0 \tag{7a}$$

$$\beta^* - (\mu_{\omega_{j+1}} - \mu_{\Omega^u} - \delta_u)(\sigma_{\omega_{j+1}}^2 + \sigma_{\Omega^u}^2)^{-1/2} \leq 0 \tag{8a}$$

$$\beta^* - (\mu_{X_g^{(j)}} - \mu_{X_g^{(j)*}} - \delta_j)(\sigma_{X_g^{(j)}}^2 + \sigma_{X_g^{(j)*}}^2)^{-1/2} \leq 0 \quad (\text{when } \mu_{X_g^{(j)}} \geq \mu_{X_g^{(j)*}}) \quad (j=1, 2, \ldots, ng) \tag{9a}$$

$$\beta^* - (\mu_{X_g^{(j)*}} - \mu_{X_g^{(j)}} - \delta_j)(\sigma_{X_g^{(j)*}}^2 + \sigma_{X_g^{(j)}}^2)^{-1/2} \leq 0 \quad (\text{when } \mu_{X_g^{(j)}} \leq \mu_{X_g^{(j)*}}) \quad (j=1, 2, \ldots, ng) \tag{10a}$$

4. The analysis of dynamic sensitivities

Here the semi-analytical method is employed for the sensitivity analysis of inherent frequency and vibration mode of structure. At first, the analytical expression of sensitivity is deduced out. The finite difference method is employed to solve those teams that are quite difficult to be solved, such as the sensitivities of stiffness and mass matrix with respect to the design variables, Based on this, the sensitivity expression of reliability index β can be deduced further more. Thereby, the sensitivities of probability constraint with respect to frequency and node position of vibration mode can be obtained.

Firstly, starting off from the Rayleigh's quotient expression in structural dynamics, differentiating it with respect to the design variables A_i, then the analytical expression of inherent frequency sensitivity of structure can be expressed as:

$$\frac{\partial\omega^2}{\partial A_i} = \{\phi\}^T\frac{\partial[K]}{\partial A_i}\{\phi\} - \omega^2\{\phi\}^T\frac{\partial[M]}{\partial A_i}\{\phi\} \tag{17}$$

where $[K]$, $[M]$ are the matrix of structural stiffness and mass, respectively. $\{\phi\}$ is the eigenvector of structural normal mode. $(\partial[K]/\partial A_i)$, $(\partial[M]/\partial A_i)$ are respectively the sensitivities of matrix of stiffness and mass with respect to design variables, they are solved by means of the finite difference method.

In order to solve the sensitivity of vibration mode, we start off from the expression of generalized eigenequation, that is:

$$([K]-\lambda[M])\{\phi\} = \{0\}$$

Where $\lambda=\omega^2$ is the eigenvalue of structure.

Differentiating above formula with respect to the design variables A_i, finally rearranging terms. There are :

$$[F]\frac{\partial\{\phi\}}{\partial A_i} = -\frac{\partial[F]}{\partial A_i}\{\phi\} \equiv \{W\} \tag{18}$$

$$[F] = [K]-\lambda[M] \tag{19}$$

where, $[F]$ and $\{W\}$ can be obtained from the frontal computational results, but the derivative of eigenvector $\partial\{\phi\}/\partial A_i$ can not be solved out from formula (18). Because the n-order matrix $[F]$ is a singularity matrix, its rank is only n-1 order. In order to solve out the $\partial\{\phi\}/\partial A_i$, the Nelson's algorithm (Nelson 1974) is used here. Firstly the $\partial\{\phi\}/\partial A_i$ is written as the sum of two terms. That is:

$$\frac{\partial\{\phi\}}{\partial A_i} = \{V\} - \left[\{\phi\}^T[M]\{V\} + \frac{1}{2}\{\phi\}^T\frac{\partial[M]}{\partial A_i}\{\phi\}\right]\{\phi\} \tag{20}$$

where $\{V\}$ should be satisfied with below formula :

$$[F]\{V\} = \{W\} \tag{21}$$

In order to remove the singularity of matrix $[F]$ in Eq. (21), we let kth component $V_k=0$ (k can be arbitrarily selected), and endow the element in $[F]$ corresponding to $V_k=0$ with a bigger number. Then the matrix $[F]$ becomes into non-singularity one. Consequently, the vector $\{V\}$ can be obtained from Eq. (21), and then the $\partial\{\phi\}/\partial A_i$ can be solved out from Eq. (20).

In order to solve the sensitivity of node position of vibration mode, we take derivation to formula (3) with respect to the design variables A_i, and then the sensitivity of node position of vibration mode can be obtained :

$$\frac{\partial X_g}{\partial A_i} = \sum_{j=v}^{w} \left\{ \left(\sum_{\substack{k=v \\ k\neq j}}^{w} \frac{\partial \varphi_k}{\partial A_i} \prod_{\substack{l=v \\ l\neq j,k}}^{w} \varphi_l \right) \left[\prod_{\substack{l=v \\ l\neq j}}^{w} (\varphi_l - \varphi_j) \right] - \prod_{\substack{l=v \\ l\neq j}}^{w} \varphi_l \cdot \left[\sum_{\substack{k=v \\ k\neq j}}^{w} \left(\frac{\partial \varphi_k}{\partial A_i} - \frac{\partial \varphi_j}{\partial A_i} \right) \prod_{\substack{l=v \\ l\neq j,k}}^{w} (\varphi_l - \varphi_j) \right] \right\} \frac{X_j}{\prod_{\substack{l=v \\ l\neq j}}^{w} (\varphi_l - \varphi_j)^2} \tag{22}$$

where $\partial\varphi_k/\partial A_i$ is the partial derivation of kth displacement component of vibration mode φ_k with respect to the design variable A_i. If the rotation angle component of vibration mode θ_k stands for φ_k in above formula, then the position sensitivity of paunch point can be obtained from formula (22).

After the probability constraints of the inherent frequency and the node position of vibration mode have been equivalently treated, their common expressions become into Eq. (15), where the reliability index β have two different expressions. They are respectively :

$$\beta = (\mu_R - \mu_S - \delta)[\sigma_R^2 + \sigma_S^2]^{-1/2} \quad (\text{when } R>S) \tag{23a}$$

$$\beta = (\mu_S - \mu_R - \delta)[\sigma_R^2 + \sigma_S^2]^{-1/2} \quad (\text{when } S>R) \tag{23b}$$

where μ_R, σ_R respectively express the mean value and variance of the allowable inherent frequency and node position of vibration mode, their values are given by design demand. μ_S, σ_S respectively express the computational result of mean value and variance of the inherent frequency and node position of vibration mode, their solving can be seen the author's forepart work (Chen and Cei 2000). δ is the given difference.

By differentiating the expression of reliability index, namely formulae (23a) and (23b), with respect to the design variable, then introducing the variation coefficient of random variable S, namely $v_S = \sigma_S/\mu_S$, and making some manipulations, the sensitivity expressions of reliability index can be written as :

$$\frac{\partial\beta}{\partial A_i} = \frac{-\sigma_R^2 - v_S^2\mu_S\mu_R + v_S^2\mu_S\delta}{[\sigma_R^2 + \sigma_S^2]^{3/2}} \cdot \frac{\partial\mu_S}{\partial A_i} \tag{24a}$$

$$\frac{\partial\beta}{\partial A_i} = \frac{\sigma_R^2 + v_S^2\mu_S\mu_R + v_S^2\mu_S\delta}{[\sigma_R^2 + \sigma_S^2]^{3/2}} \cdot \frac{\partial\mu_S}{\partial A_i} \tag{24b}$$

where, v_S is variation coefficient of random variable S; $\partial\mu_S/\partial A_i$ denotes the sensitivity of random variable of inherent frequency or node position of vibration mode with respect to the design variables.

From above formulae, it can be seen that the sensitivity of reliability index with respect to design variables is equal to the product of sensitivity of conventional constraint and a coefficient. This coefficient is decided by the given allowable difference δ and the first and second order moment of two random variables, which occur interference with each other. The formulae (24a) and (24b) also suit for solving the sensitivity of reliability constraint in similar problem.

5. The optimum approach and examples

5.1 The optimum approach

Since it is usually more difficult to ensure the initial design scheme to be a feasible point in the optimum design of structural dynamics, here we select the mixed penalty function and the DFP approach in optimum design, which has wider applicability. In order to promote the convergence and improve the initial design point, the extrapolation technique is introduced into the computational process. In structural dynamic analysis the sub-space iteration algorithm is employed, which suits for the system with multiple degree-of-freedom.

5.2 Examples

In order to demonstrate that the optimization model and approach presented in this paper are rational and effective, two engineering structural examples are given below.

5.2.1 The cantilever beam with the reliability constraint of node position of vibration mode

The initial geometric dimension of beam and plotting of element's are shown in Fig. 1. Due to that this beam is a deep one, its shearing deformation effect must be considered in computation. Here both elastic module E and mass density ρ of the beam are random variables, their mean values and variation coefficients are respectively $\mu_E=1\times 10^6$ psi, $v_E=0.01$ and $\mu_\rho=0.386\ lb/in^3$, $v_\rho=0.01$. The request for the node position of vibration mode is that the node of second order vibration mode should drop into the neighborhood of given point X_g. The mean value and variance of this point are respectively $\mu_{X_g}=7.5\ in$ and $\sigma_{X_g}=0.05\ in$, its allowable difference is $\delta=0.2\ in$. The given reliability of

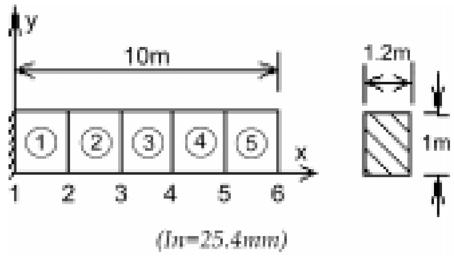

Fig. 1 The cantilever beam structure

Table 1 The initial design and the optimum design results of cantilever beam

Design variables	①	②	③	④	⑤	$\overline{W}(lb)$	$X_g^{(in)}$	P_r
Initial design(in^2)	1.2	1.2	1.2	1.2	1.2	4.636	8.069	<0.01
Optimum design(in^2)	0.1026	0.1005	0.1930	0.2262	0.1101	0.5583	7.526	0.972

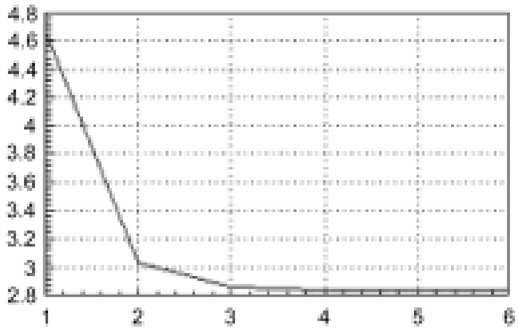

Fig. 2 The iterative history of cantilever beam's weight

probability constraints is 0.97. The cross section areas of each beam element are taken as design variables. Their allowable values should be located between 0.1 in^2 to 5 in^2.

Through six times optimization iterating, the computation converged. The initial and optimum design results of the beam are shown in Table 1. The iterative history of the mean value of beam's weight is drawn out in Fig. 2. From them it can be seen that the astringency and stability of optimization approach are satisfied considerably.

5.2.2 The single border clamped plate with reliability constraints of fundamental frequency and nodal line position of vibration mode

The length, width and thickness of the plate are respectively l=96 mm, b=48 mm and t=1 mm. The plate is dispersed into 12×8 triangle plate elements. There are three degrees of freedom in each node of element; they are one moving and two rotating degrees of freedom. In Xiang *et al*. (1995) the dynamic optimum design with conventional constraints of this plate is carried out. Where the constraints of inherent frequency are $8.5 \le \omega_1 \le 9.5$, $40 \le \omega_2 \le 41$, $58 \le \omega_3 \le 59(\mathrm{Hz})$, and the constraints of vibration mode are that the position of node line in first order wring mode and second order bend mode (they are respectively corresponding to second and third order mode of the plate) should be satisfied with the following design demands: The node line of first order wring mode must pass 19*th* and 53*th* finite element points, and the node line of second order bend mode must pass 36*th*, 46*th* and 56*th* finite element points. The constraint conditions of this example are different from (Xiang *et al*. 1995) ones somewhat (Xiang 1995). The lower limit of minimum inherent frequency of structure must be satisfied with reliability constraint. Furthermore, the position of node line in the first order wring mode and the second order bend mode must be satisfied with the given reliability constraints which are similar to above mode constraints. The mean value and variance of lower bound of given minimum inherent frequency are respectively μ_Ω=7.8 Hz and σ_Ω=0.8 Hz. Restricting the corresponding discrete points in the node line carries out the request for node line of vibration mode. The variances of all discrete points are 1 mm. The given reliability values of frequency constraint and mode constraint are 0.95. The mean value and variation coefficients of elastic module of the plate are respectively $\mu_E=2\times10^5$ kg/mm^2, $v_E=0.01$. The mean value and variation coefficients of mass density of the plate are respectively $\mu_\rho=7.8\times10^{-6}$ kg/mm^3, $v_\rho=0.01$. In designing process, the 96 plate elements are merged into 16 groups (see Fig. 3), there are 6 elements in each group. The plate's thickness of every group $t_i(i=1\sim16)$ are taken as design variables. The upper and lower bound of design variables are respectively 0.2 mm and 2 mm.

The positions of node line of the plate structure before and after optimum design are shown in Fig. 4. The results of initial design and optimum design that satisfied with the reliability constraint are given in Table 2.

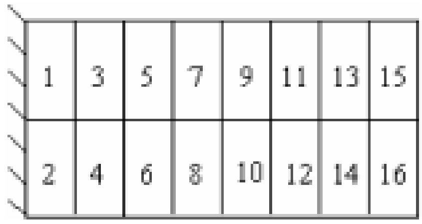

Fig. 3 The plate structure

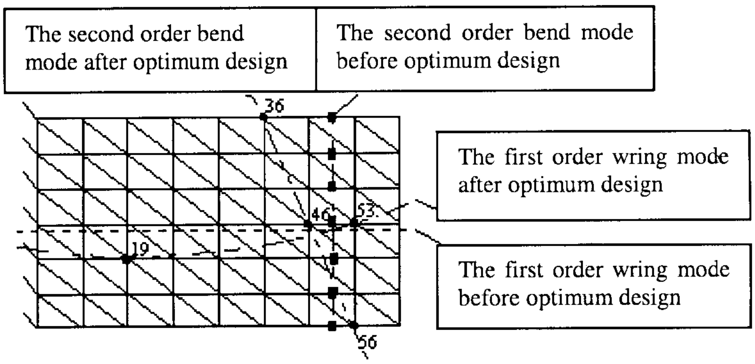

Fig. 4 The position of node line of plate structure before and after optimum design

Table 2 The optimum design results of the plate structure

Variables (mm)	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
Initial design	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
(Xiang *et al*. 1995)'s design	0.71	0.89	1.08	1.12	1.06	1.10	1.04	1.02	1.22
Reliability design	0.856	0.765	0.790	1.005	0.841	0.891	0.688	0.588	1.084
Variables (mm)	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	$\overline{W}$ (kg)	
Initial design	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0359	
(Xiang *et al*. 1995)'s design	0.99	1.10	0.79	0.74	0.94	0.36	1.14	0.0344	
Reliability design	0.473	0.687	0.553	0.311	0.559	0.200	0.488	0.0242	

Table 3 The inherent frequencies of plate structure after optimum design

Inherent frequency	ω_1(Hz)	ω_2(Hz)	ω_3(Hz)
Initial values	9.184	40.173	58.956
(Xiang *et al*. 1995)'s results	8.886	40.853	58.366
Author's results	9.581	33.649	44.124

The first three orders inherent frequencies of the plate structure before and after optimum design be given in Table 3.

From the optimum results it can be seen that since the requests to the node line of vibration mode are ultimately same as (Xiang *et al.* 1995)'s ones, the corresponding position of node line of vibration mode in the optimum design of this paper is considerably coincident with the one of Xiang *et al.* (1995). This shows that the treatment of constraint of node line of vibration mode is right and feasible. Due to that the number and request of frequency constraints here are different from (Xiang *et al.* 1995)'s once, the optimum design results of the structural weight and inherent frequency are different from (Xiang *et al.* 1995)'s ones.

6. Conclusions

The optimum examples shown that:

1. The mathematical model and solving approach of dynamic characteristic optimization of engineering structures based on probability, which presented in this paper, are rational and feasible.
2. The treatment of position constraint of mode node line in this paper is right and feasible.
3. The results of structural dynamic characteristic optimum design for the conventional model are different from the one for the model based on probability. From the viewpoint of probability, the optimum result of conventional design is usually an unfeasible solution for the optimum design based on probability. So that if the structural parameters have randomness, the conventional optimization model and approach can not give out rational design results.

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