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Probabilistic dynamic analysis of truss structures

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Abstract. The problem of dynamic analysis of truss structures based on probability is studied in this paper. Considering the randomness of both physical parameters (elastic module and mass density) of structural materials and geometric dimension of bars respectively or simultaneously, the stiffness and mass matrixes of the elements and structure have been built. The structure dynamic characteristic based on probability is analyzed, and the expressions of numeral characteristics of inherence frequency random variable are derived from the Rayleigh's quotient. The method of structural dynamic analysis based on probability is developed. Finally, two examples are given.

Key words: truss structure; physical parameter; geometric dimension; random variables; dynamic characteristic analysis.

1. Introduction

Dynamic analysis of structure is an important segment in the process of structural design. At present, the artery method of structural dynamic analysis have been developed from the equivalent predigest centralize parameter method and analytic method in forepart period of manual computation to the various numerical methods in nowadays computer period. It can be said that the methods have been considerably rich and perfect. So far, however, most of modeling on structural dynamic analysis basically belongs to the determinate models, that is, all structural parameters are regarded as determinate ones. Apparently, this kind of model can not reflect the influence of the randomness of structure on the structural dynamic characteristic. As a matter of fact, in some situations the randomness of structure must be considered, especially in design stage. Such as, for one kind numerous or batch producing structures, their values of physical parameter of material has dispersedness, there is error on the geometric dimension of structural number in process of manufacture and assemblage, the support boundary condition is indeterminate, and so on. Therefore, studying the problem of random structural dynamic analysis is of much realistic engineering background and important theoretical signification. Since 70 decade, some research works about the eigenvalue problem of structure with random parameters has been published successively (such as Astill & Shinozuka 1972, Vaicaitis 1974, Scheidt & Purkert 1983, Zhang & Chen 1989), in which the dynamic characteristics of structure with random parameter was analyzed. Their essential treatments are that the random factors in structure are described with small parameter, and the problem is solved by means of FEM or matrix perturbation method. Benaroya & Rehak reviewed

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on some problems of finite element methods in probabilistic structural analysis in 1988. Zhu & Wu analyzed the real eigenvalue problem by using the stochastic finite element method in 1992. Recent years, Wu & Yang (1996) took the randomness of structural parameter as the perturbation quantity nearby the mean value of random variables, and obtained the perturbing expression of structural dynamic characteristic by using the variation principle of Rayleigh's quotient. Li (1996) presented a kind of modeling method with two-phase iterations for random parameter system based on the extended order method in random structural analysis. Through the method of generalized sample subdomains, Wang & Mei (1997) set up the perturbation analyzing formula of random eigenvalue for the cooling tower with random parameter, and discussed the effect of statistical characteristic of random parameter of the boundary support in tower ground on structural eigenvalue.

In this paper, the truss structures are taken as analyzing objects and its structural dynamic modeling and analyzing based on probability are studied. Firstly, the stiffness and mass matrices of the element of structure and whole structure are constructed, in which the randomness of physical parameter of structural material and the geometric dimension of bars are respectively or simultaneously considered. Through introducing the random variable factor of bar length, the randomness of all bars' length is described with a united form. The situation of randomness of physical parameter of combination material is also considered in modeling. The computational expressions of numeral characteristics of inherence frequency random variable are derived from the constructed structural dynamic models and the Rayleigh's quotient in structural dynamics by means of the algebra synthesis method. Finally, through the examples the effect of randomness of physical and geometry parameters on dynamic characteristic is examined, and some significative conclusions are obtained. The results of examples show that the model and method presented in this paper are rational and feasible.

2. Probabilistic dynamic characteristic analysis of structures

2.1 Physical random variables

Considering that structural material will be affected inevitably by various random factors during the molding process, it makes the physical parameters, namely elastic module E and mass density ρ of one kind material present randomness in some degree. So the physical parameters, both E and ρ, are regarded as random variables. Apparently, the randomness of parameters E and ρ will lead the $[K]$ $[M]$ as well as the eigenvalues of structure which is determined by $[K]$ and $[M]$, to randomness.

Suppose that there are n_e elements in the analyzed truss structure and only one kind of material is used. By means of the finite element method, the stiffness matrix of eth element in local coordinate, $[K^{(e)}]$, can be expressed as:

$$[K^{(e)}] = E\frac{A_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = E \cdot [K^{(e)}]^{\#} \tag{1}$$

where l_e, A_e are bar length and cross section area of eth element respectively. $[K^{(e)}]^{\#}$ is the determinate part in stiffness matrix $[K^{(e)}]$. This expression shows that the element stiffness matrix can be divided into the product of two parts, i.e., the random variable E and the constant matrix $[K^{(e)}]^{\#}$, and that the randomness of $[K^{(e)}]$ is only dependent on the randomness of elastic module E.

Therefore, constructing the matrix $[K^{(e)}]^{\#}$ is same as constructing stiffness matrix of element before, just taking the parameter E as 1.

The stiffness matrix $[\hat{K}^{(e)}]$ of eth element and stiffness matrix $[K]$ of whole structure in global coordinate can be written as respectively:

$$[\hat{K}^{(e)}] = E[T^{(e)}]^T[K^{(e)}]^{\#}[T^{(e)}] = E \cdot [\hat{K}^{(e)}]^{\#} \tag{2}$$

$$[K] = \sum_{e}^{n_e} [\hat{K}^{(e)}] = E \cdot \sum_{e}^{n_e} [\hat{K}^{(e)}]^{\#} = E \cdot [K]^{\#} \tag{3}$$

where $[T^{(e)}]$ is the coordinate transform matrix of element e. $[K^{(e)}]^{\#}$, $[K]^{\#}$ are stiffness matrices of eth element and structure in global coordinate when parameter E=1.

Again by means of the finite element method, the mass matrix of eth element, $[m^{(e)}]$, can be expressed as:

$$[m^{(e)}] = \frac{\rho l_e A_e}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \rho \cdot [m^{(e)}]^{\#} \tag{4}$$

By concentrating, the whole mass matrix of structure $[M]$ can be expressed as:

$$[M] = \sum_{e}^{n_e} [m^{(e)}] = \rho \cdot \sum_{e}^{n_e} [m^{(e)}]^{\#} = \rho \cdot [M]^{\#} \tag{5}$$

where $[m^{(e)}]^{\#}$ and $[M]^{\#}$ are determinate matrices, i.e., the mass matrices of eth element and whole structure when the parameter $\rho = 1$.

If the materials of elements in the structure are different, but all variation coefficients of physical parameters $[E^{(e)}]$ and $\rho^{(e)}$ are the same, that is $\nu_E^{(e)} = \nu_E$, $\nu_\rho^{(e)} = \nu_\rho$. Then the ratios of numeral characteristic of all elements' parameters E and ρ are the same, i.e.

$$\frac{\mu_E^{(e)}}{\mu_E} = \frac{\sigma_E^{(e)}}{\sigma_E} = \alpha^{(e)}, \qquad \frac{\mu_\rho^{(e)}}{\mu_\rho} = \frac{\sigma_\rho^{(e)}}{\sigma_\rho} = \beta^{(e)} \tag{6}$$

where, μ_E, σ_E are the mean value and standard deviation of random variable E, respectively. μ_ρ, σ_ρ are the mean value and standard deviation of random variable ρ, respectively. $\mu_E^{(e)}$, $\sigma_E^{(e)}$ are the mean value and standard deviation of random variable $E^{(e)}$, respectively. $\mu_\rho^{(e)}$, $\sigma_\rho^{(e)}$ are the mean value and standard deviation of random variable $\rho^{(e)}$, respectively. $\alpha^{(e)}$, $\beta^{(e)}$ are the ratio factors of parameters $E^{(e)}$ and $\rho^{(e)}$, respectively.

When all ratio factors of element's physical parameter $\alpha^{(e)}$, $\beta^{(e)}$ are introduced into the total stiffness matrix and the total mass matrix of structure, they can be expressed respectively as:

$$[K] = \sum_{e}^{n_e} E^{(e)} \cdot [\hat{K}^{(e)}]^{\#} = E \cdot \sum_{e}^{n_e} \alpha^{(e)}[\hat{K}^{(e)}]^{\#} = E \cdot [K]^{\#} \tag{7}$$

$$[M] = \sum_{e}^{n_e} \rho^{(e)} \cdot [m^{(e)}]^{\#} = \rho \cdot \sum_{e}^{n_e} \beta^{(e)}[m^{(e)}]^{\#} = \rho \cdot [M]^{\#} \tag{8}$$

Where $[K]^{\#} = \sum_{e}^{n_e} \alpha^{(e)}[\hat{K}^{(e)}]^{\#}$, $[M]^{\#} = \sum_{e}^{n_e} \beta^{(e)}[m^{(e)}]^{\#}$.

Suppose that ith order inherence frequency and mode shape of structure are ω_i, $\{\phi_i\}$ respectively, by using the Rayliegh's quotient expression, the random variable of ith inherence frequency can be expressed as:

$$\omega_i^2 = \frac{\{\phi_i\}^T[K]\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}} \tag{9}$$

Substituting the stiffness and mass matrices of structure into above formula, then

$$\omega_i^2 = \frac{E \cdot \{\phi_i\}^T[K]^\#\{\phi_i\}}{\rho \cdot \{\phi_i\}^T[M]^\#\{\phi_i\}} = \frac{E \cdot K_i^\#}{\rho \cdot M_i^\#} = \frac{E}{\rho}(\omega_i^\#)^2 \tag{10}$$

where $K_i^\#$, $M_i^\#$, $\omega_i^\#$ are all determinate quantity, they are the ith order main stiffness, main mass and inherence frequency of structure when the parameters $E = 1$, $\rho = 1$.

According to previous formula, the computing expression of mean value μ_{ω_i} and variation coefficient ν_{ω_i} of ith order inherence frequency can be deduced by means of the algebra synthesis method (Chen *et al.* 2001).

$$\mu_{\omega_i} = \omega_i^\# \sqrt{\frac{1}{2}\frac{\mu_E}{\mu_\rho}\sqrt{4[1+\nu_\rho(\nu_\rho - c_{E\rho}\nu_E)]^2 - 2(\nu_E^2 + \nu_\rho^2 - 2c_{E\rho}\nu_E\nu_\rho)}} \tag{11}$$

$$\nu_{\omega_i} = \frac{\sigma_{\omega i}}{\mu_{\omega_i}} = \sqrt{\frac{2[1+\nu_\rho(\nu_\rho - c_{E\rho}\nu_E)] - \sqrt{4[1+\nu_\rho(\nu_\rho - c_{E\rho}\nu_E)]^2 - 2(\nu_E^2 + \nu_\rho^2 - 2c_{E\rho}\nu_E\nu_\rho)}}{\sqrt{4[1+\nu_\rho(\nu_\rho - c_{E\rho}\nu_E)]^2 - 2(\nu_E^2 + \nu_\rho^2 - 2c_{E\rho}\nu_E\nu_\rho)}}} \tag{12}$$

where μ_E, ν_E and μ_ρ, ν_ρ are mean value and variation coefficient of parameter E and ρ, respectively. $c_{E\rho}$ is the correlation coefficient of variables E and ρ.

If variables E and ρ are independent, then $c_{E\rho} = 0$. If variables E is completely correlative with ρ, then $c_{E\rho} = 1$. They are two kind of extreme situations. According to observation on the property of common metal material, it can be found that the elastic module E is usually positive correlative with mass density ρ, and that degree of correlation is rather higher. So in practical computation, it is suggested that the correlative coefficient $c_{E\rho} = 0.5$-0.9.

2.2 Geometric random variables

The geometric dimension of truss structure includes two kinds of dimension, the length $\vec{L}$ of bars and the cross section area $\vec{A}$ of bars. In structure optimum design where the structural topology form has been presented, the cross section area of bars $\vec{A}$ are usually taken as design variables. So here only the randomness of bars length $\vec{L}$ are considered. For the sake of easily analyzing, suppose that variation coefficient of all bars' length $l^{(e)}$ are equal, i.e., $\nu_l^{(e)} = \nu_l$ and that $l_e = \eta^{(e)} \cdot l$. Where $\eta^{(e)}$ is the determinate quantity that denotes the nominal length of eth bar. l is the random variable factor of all bars' length, its mean value is 1.0 and variation is ν_l^2, respectively. By means of the finite element method, the stiffness and mass matrices of every element and whole structure can be expressed respectively as:

$$[K^{(e)}] = \frac{EA_e}{l_e} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{l} \cdot \frac{EA_e}{\eta^{(e)}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{l} \cdot [K^{(e)}]^{\#} \tag{13}$$

$$[K] = \sum_{e}^{n_e} [\hat{K}^{(e)}] = \frac{1}{l} \sum_{e}^{n_e} \frac{1}{\eta^{(e)}} [\hat{K}^{(e)}] = \frac{1}{l} \cdot [K]^{\#} \tag{14}$$

$$[m^{(e)}] = \frac{l_e A_e \rho}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = l \cdot \frac{\eta^{(e)} A_e \rho}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = l \cdot [m^{(e)}]^{\#} \tag{15}$$

$$[M] = \sum_{e}^{n_e} [m^{(e)}] = l \cdot \sum_{e}^{n_e} \eta^{(e)} [m^{(e)}]^{\#} = l \cdot [M]^{\#} \tag{16}$$

Where $[K]^{\#} = \sum_{e}^{n_e} \frac{1}{\eta^{(e)}} [\hat{K}^{(e)}]$; $[M]^{\#} = \sum_{e}^{n_e} \eta^{(e)} [m^{(e)}]^{\#}$; $[K^{(e)}]^{\#}$, $[m^{(e)}]^{\#}$, $[K]^{\#}$, $[M]^{\#}$ are all determinate matrices, i.e., the stiffness and mass matrices of *e*th element and structure when the random variable factor l of bar's length is drawn from the original matrices.

Substituting the stiffness and mass matrices of structure, [*K*] and [*M*], into the Rayliegh's quotient expression (9), the random variable of *i*th inherence frequency ω_i can be expressed as:

$$\omega_i = \frac{1}{l} \omega_i^{\#} \tag{17}$$

where $\omega_i^{\#}$ is determinate quantity, i.e., the *i*th order inherence frequency of structure when each bar length is $l_e = \eta^{(e)}$ respectively.

From previous formula, the computing expression of mean value $\mu_{\omega i}$ and variation coefficient $\nu_{\omega i}$ of *i*th order inherence frequency can be deduced by means of the algebra synthesis method (Chen, J. J. 2001).

$$\mu_{\omega i} = \frac{1}{\mu_l} \omega_i^{\#} \tag{18}$$

$$\nu_{\omega i} = \frac{\sigma_{\omega i}}{\mu_{\omega i}} = \sqrt{\frac{2[1+\nu_z^2] - \sqrt{4[1+\nu_z^2]^2 - 2\nu_z^2}}{\sqrt{4[1+\nu_z^2]^2 - 2\nu_z^2}}} \tag{19}$$

$$\nu_z = \nu_{l^2}^2 = \frac{\sqrt{4\nu_l^2 + 2\nu_l^4}}{1+\nu_l^2}$$

where ν_l is the variation coefficient of random variable factor *l*.

2.3 Physical parameter random variables and geometric dimension random variables

Suppose that the physical parameters *E*, ρ and geometric dimension $\vec{L}$ are simultaneously random variables, and that the variables *E*, ρ and $\vec{L}$ are independent from each other. This is the most ecumenical situation in random structures. As preceding, suppose that the variation coefficients of physical parameter and bar's length of every element are same respectively, that is $\nu_E^{(e)} = \nu_E$, $\nu_\rho^{(e)} = \nu_\rho$, $\nu_l^{(e)} = \nu_l$, and that the all bars' length can be expressed as $l_e = \eta^{(e)} \cdot l$. Then the stiffness and mass matrices of eth element and whole structure can be expressed respectively as:

$$[K^{(e)}] = \frac{E^{(e)}A_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{E}{l} \cdot \frac{\alpha^{(e)}A_e}{\eta^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{E}{l} \cdot [K^{(e)}]^{\#} \tag{20}$$

$$[K] = \sum_{e}^{n_e} [\hat{K}^{(e)}] = \frac{E}{l} \cdot \sum_{e}^{n_e} [\hat{K}^{(e)}]^{\#} = \frac{E}{l} \cdot [K]^{\#} \tag{21}$$

$$[m^{(e)}] = \rho^{(e)} l_e \cdot \frac{A_e}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \rho l \cdot \frac{\beta^{(e)}\eta^{(e)}A_e}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \rho l \cdot [m^{(e)}]^{\#} \tag{22}$$

$$[M] = \sum_{e}^{n_e} [m^{(e)}] = \rho l \cdot \sum_{e}^{n_e} [m^{(e)}]^{\#} = \rho l \cdot [M]^{\#} \tag{23}$$

Substituting $[K]$ and $[M]$ into the formula (9), the ith order inherence frequency of structure, ω_i, can be derived as

$$\omega_i = \sqrt{\frac{E}{\rho l^2}\omega_i^{\#}} \tag{24}$$

where $\omega_i^{\#}$ is determinate quantity, that is the ith inherence frequency when the structural parameters $E = 1$, $\rho = 1$, and each bar's length are $l_e = \eta^{(e)}$ respectively.

From above formula, the mean value $\mu_{\omega i}$ and variation $\sigma_{\omega i}^2$ of random variable ω_i can be gained by means of the algebra synthesis method (Chen, J. J. 2001). They are

$$\mu_{\omega i} = \omega_i^{\#}\left(\frac{\mu_E}{\mu_\rho \mu_z}\right)^{\frac{1}{2}}\left\{[1+\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2 - c_{E\rho}\nu_E(\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2)^{1/2}]^2 - \frac{1}{2}[\nu_E^2+\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2 - 2c_{E\rho}\nu_E(\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2)^{1/2}]\right\}^{1/4} \tag{25}$$

$$\sigma_{\omega i}^2 = (\omega_i^{\#})^2\left(\frac{\mu_E}{\mu_\rho \mu_z}\right)\left\{[1+\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2 - c_{E\rho}\nu_E(\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2)^{1/2}] - ([1+\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2 - c_{E\rho}\nu_E(\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2)^{1/2}]^2 - \frac{1}{2}[\nu_E^2+\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2 - 2c_{E\rho}\nu_E(\nu_z^2+\nu_\rho^2+\nu_z^2\nu_\rho^2)^{1/2}])^{1/2}\right\} \tag{26}$$

$$\nu_{\omega i} = \frac{\sigma_{\omega i}}{\mu_{\omega i}} \tag{27}$$

From above formula, it is easily seen that the values of variation coefficient of every inherence frequency, $\nu_{\omega i}$, are equal to each other. They are only dependent on the physical parameters E, ρ

and the random variable factor of geometric dimension, l, as well as the correlative coefficient $c_{E\rho}$, but they are independent of the order number of structural vibration model. Therefore, in seeking the numeral characteristic of inherence frequency, the variation coefficient $\nu_{\omega i}$ can be obtained with formula (12), (19) or (27) by the given values of ν_E, ν_ρ, ν_l and $c_{E\rho}$. According to the determinate stiffness and mass matrices $[K]^{\#}[M]^{\#}$, the determinate values of every order inherence frequency $\omega_i^{\#}$ can be obtained by means of the conventional dynamic analysis method. Then substituting $\omega_i^{\#}$ into the formula (11), (18) or (25), $\mu_{\omega i}$, the mean value of random variable ω_i can be gained.

3. Examples

According to the preceding computing formula and the solving method, the corresponding computational program is designed. The sub-space iteration method is employed in structural dynamic analysis. For the sake of accelerating the convergence in computing process, the tactics is adopted as follows. Except that in first dynamic analyzing, the initial iterative eigenvector is created automatically by the program, in hereafter dynamic analyzing, the eigenvector obtained in former iteration is always taken as the current initial iterative vector. Two examples are given in the following as illustrations.

Example 1. 4-bar space truss structure (Fig. 1)

The structural material is aluminum alloy, its physical parameters E and ρ all are the normal random variables with mean value and variation coefficient $\mu_E = 6.894\times 10^4(\text{MPa})$, $\nu_E = 0.1$, $\mu_\rho = 2.714\times 10^{-2}$ (kg/cm^3), $\nu_\rho = 0.1$ respectively. The random factor of every bar's length, l, is also normal variables, its mean value is 1.0 and variation coefficient $\nu_l = 0.1$. The cross section area of all bars is 6.4516 (cm^2).

The computing results of first three order inherence frequencies of structure are given in Table 1. In order to compare, two kinds of models, the determinate model and random model, are adopted respectively in computational process. In the determinate model, the mean values of all random variables are regarded as determinate quantity, and their variation coefficients are taken as 0. In the random model, three situations are considered respectively. They are (1) The physical parameters of the structure are random variables; (2) The geometric dimensions of the structure are random variables; (3) Both the physical parameters and geometric dimensions are simultaneously random

Table 1 The computing results of inherence frequency of 4-bar space truss structure

Inherence Frequency	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)
Determinate Model	142.212	150.159	153.031
Random Model $\nu_E = \nu_\rho = 0.1$, $\nu_l = 0$	$\mu_{\omega 1} = 142.271$ $\sigma_{\omega 1} = 4.496$	$\mu_{\omega 2} = 150.275$ $\sigma_{\omega 2} = 4.749$	$\mu_{\omega 3} = 153.077$ $\sigma_{\omega 3} = 4.837$
Random Model $\nu_E = \nu_\rho = 0$, $\nu_l = 0.1$	$\mu_{\omega 1} = 142.212$ $\sigma_{\omega 1} = 13.674$	$\mu_{\omega 2} = 150.159$ $\sigma_{\omega 2} = 14.438$	$\mu_{\omega 3} = 153.031$ $\sigma_{\omega 3} = 14.714$
Random Model $\nu_E = \nu_\rho = 0.1$, $\nu_l = 0.1$	$\mu_{\omega 1} = 144.043$ $\sigma_{\omega 1} = 10.875$	$\mu_{\omega 2} = 152.147$ $\sigma_{\omega 2} = 11.487$	$\mu_{\omega 3} = 154.983$ $\sigma_{\omega 3} = 11.701$

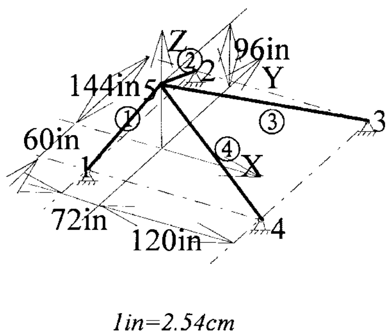

Fig. 1 4-bar space truss structure

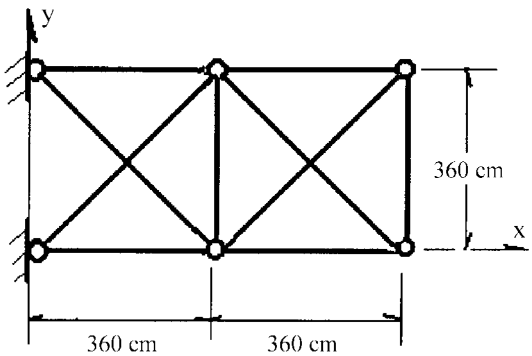

Fig. 2 10-bar plane truss structure

Table 2 The computing results of inherence frequency of 10-bar plane truss structure

Inherence Frequency	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)
Determinate Model	37.198	104.325	108.830
Random Model(I) $\nu_E = \nu_\rho = 0.01,\ \nu_l = 0.01$	$\mu_{\omega 1} = 37.204$ $\sigma_{\omega 1} = 0.289$	$\mu_{\omega 2} = 104.339$ $\sigma_{\omega 2} = 0.812$	$\mu_{\omega 3} = 108.844$ $\sigma_{\omega 3} = 0.847$
Random Model(II) $\nu_E = \nu_\rho = 0.1,\ \nu_l = 0.1$	$\mu_{\omega 1} = 37.681$ $\sigma_{\omega 1} = 2.845$	$\mu_{\omega 2} = 105.678$ $\sigma_{\omega 2} = 7.979$	$\mu_{\omega 3} = 110.240$ $\sigma_{\omega 3} = 8.323$

variables. It can be seen from Table 1 that, under the conditions that the variation coefficients of physical parameters and geometric dimensions are equal to each other, the randomness of geometric dimension will produce greater effect on the randomness of structural inherence frequencies than the one produced by the randomness of physical parameters.

Example 2. 10-bar plane truss structure (Fig. 2)

The structural material is the steel. Its parameters E and ρ are random variables with mean value $\mu_E = 2.058\times10^4$ (MPa), $\mu_\rho = 7.658\times10^{-2}$ (kg/cm^3). In order to investigate the effect of the dispersedness degree of random variables E, ρ and dimension random variable factor l on the structural dynamic characteristic, the values of variation coefficients of parameters E, ρ and l are respectively taken as two groups. Group I: $\nu_E = \nu_\rho = \nu_l = 0.01$. Group II: $\nu_E = \nu_\rho = \nu_l = 0.1$. The cross section area of all bars is taken as 1.0 (cm^2). The computing results of first three order inherence frequencies corresponding to two kinds of models, determinate one and random one, are given in Table 2. It is shown that changing the variation coefficients of physical parameter and geometric dimension will produce considerable effect on the computing results of structural inherence frequency. Along with the increase of the variation coefficients of physical parameter and geometric dimension, the dispersal degree of structure's eigenvalue will notably increase.

4. Conclusions

(1) The examples show that the analyzing results of structural dynamic characteristic of determinate

model are different from the one of random model. So that when one of both physical and geometric parameters of structure is random variable, the conventional determinate analysis method of structural dynamic will not reflect the effect of the randomness. It is only dependent on the structural dynamic analysis method based on probability.

(2) The results of example show that along with the increase of the variation coefficients of physical parameters and geometric parameters, the dispersal degree of structural dynamic characteristic will increase. This conclusion is completely coincident with the one that has been obtained in the structural reliability. Moreover, under the condition that the variation coefficients of physical parameters and geometric dimensions are equal to each other, the randomness of geometric dimension will produce more remarkable effect on the randomness of computing results of structural inherence frequencies than the one produced by the randomness of physical parameters. Therefore, in structural dynamic analysis the randomness (dispersal degree) of geometric dimension of structure should not be neglected.

(3) The examples show that the model and solving method of dynamic characteristic analysis of truss structure based on probability presented in this paper are rational and feasible, in addition they are comparatively simple.

References

Astill, C.J., and Shinozuka, M. (1972), "Random eigenvalue problems in structural analysis", *AIAA. J.*, **10**(4), 456-562.

Benaroya, H., and Rehak, M. (1988), "Finite element methods in probabilistic structural analysis: A selective review", *Appl. Mech. Rev.* ASME, **41**(5), 201-213.

Chen, J.J., Che, J.W., Ma, H.B., and Dai, J. (2001), "Dynamic optimum design based on reliability for truss structures", *Acta Mechanica Solida Sinica*, **22**(1), 54-60.

Li, J. (1996), "A research on modeling stochastic structures", *J. Vibration Engineering*, **9**(1), 46-53.

Scheidt, J.V., and Purkert, W. (1983), *Random Eigenvalue Problem*, Elsevier Science, New York.

Vaicaitis, R. (1974), "Free vibrations of beams with random characteristics", *J. Sound & Vibration*, **35**(1), 13-21.

Wang, X.T., and Mei, Z.X. (1997), "Random eigenvalue analysis of cooling tower on the spline subdomain random perturbation methods", *Comput. Struct. Mech. & Application*, **14**(2), 174-181.

Wu, X.F., and Yang, G.S. (1996), "Statistical analysis of the dynamic characteristics of structures including random parameters", *J. Beijing Science and Technical University*, **16**(1), 43-47.

Zhang, Z.F., and Chen, S.H. (1989), "Random eigenvalue problems in structural dynamics", *Proceeding of ICAM.*

Zhu, W.Q., and Wu, W.Q. (1992), "A stochastic finite element method for real eigenvalue problem", *Prob. Eng. Mech.*, **118**(3), 496-511.