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Dynamic characteristics of elastic beams subjected to traffic loads

Chia-Chih Tang†

Department of Civil Engineering, Chinese Military Academy, 1 Wei-Wu Rd. Feng-Shan, 83000, Taiwan, R.O.C.

Yang-Cheng Wang‡

Academic Affairs Development Office, National Defense University, 156 Chung-Shin Rd. Lung-Tang, 32500, Tao-Yuan, Taiwan, R.O.C.

Abstract. The objective of this study is to investigate the dynamic behavior of elastic beams subjected to moving loads. Although analytical methods are available, they have limitations with respect to complicated structures. The use of computer technology in recent years is an effective way to solve the problem; thus using the latest technology this study establishes a finite-element solution procedure to investigate dynamic behaviors of a typical elastic beam having a set of constant geometric properties and various span lengths. Both the dead load of the beam and traffic load are applied in which the traffic load is considered a concentrated moving force with various traveling passage speeds on the beam. Dynamic behaviors including deflection, shear, and bending moment due to moving loads are obtained by both analytical and finite element methods; for simple structures, they have an excellent agreement. The numerical results show that based on analytical methods the fundamental mode is good enough to estimate the dynamic deflection along the beam, but is not sufficient to simulate the total response of the shear force or the bending moment. The linear dynamic behavior of the elastic beams subjected to multiple exciting loads can easily be found by linear superposition, and the geometric nonlinear results caused by large deformation and axial force of the beam are always underestimated with only a few exceptions which are indicated. In order to make the results useful, they have been nondimensionalized and presented in graphical form.

Key words: dynamic; characteristics; beams; bridges; moving loads; traffic loads.

1. Introduction

Engineers and researchers become more interested in a particular research problem on dynamic behavior of bridges subjected to moving loads (Biggs 1972, Chen 1978, Vlahinos and Wang 1994, Casas 1995, Yang and Yau 1997, Filho 1996, and Yang and Fonder 1998). The vibration of bridges due to moving loads is important in that excessive vibration, noticeable to the persons on the bridge, has the psychological effect of impairing public confidence in the structure and that the stresses and bending moments are increased as a result of the vibration (Agrawal 1997). In the 1970s, Biggs

† Assistant Professor
‡ Professor & Director

(1972) and Chen (1978) provided the analytically based structural response function of simply supported beams subjected to a moving load. The analytical methods are bound by the following case: a load travelling across an elastic beam to find the vertical deflection. However, most of researchers didn't investigate the bending moment and stress of the structure subjected to moving loads. (Chen 1978, Vlahinos and Wang 1994, Yang and Yau 1997, Lin *et al*. 1997, and Yang and Fonder 1998). The use of high strength material and computer technology increases the span length of bridges and the elastic girder becomes more flexible. This type of dynamic analysis becomes more important, especially for long span bridges.

Elastic beams subjected to dynamic excitations may create additional deflection along the span in vertical, longitudinal, torsional directions, and their coupling movements (Yang and Fonder 1998). Rieker and Trethewey (1999) stated that if only the fundamental mode of vibration was used in the solution, the maximum value of 1.732 times the static deflection was achieved. These values occur when the load is traveling at the rate of 0.81 times the structure's fundamental period of vibration.

Associated with the dynamic deflection of the beam caused by moving loads, the dynamic stresses, and bending moment, can be larger than that obtained by static analysis. For most girder-supported bridges, the bridge decks are designed for the function of passage traffic, dynamic behavior of bridge deck shouldn't be ignored (ASCE 1992).

This study investigates the nonlinear effects on dynamic whereas other published articles only related to linearly dynamic behaviors of this type of structure. Lin *et al.* (1997) numerically investigated the dynamic effects on continuous concrete bridges subjected to traffic load. Yang and Yau (1997) have addressed the dynamic interaction of vertical deflection between the moving vehicle and the simply supported beam. These articles have not taken the beam's dead load into account in their structural dynamic analysis. In order to investigate the dead load effect on dynamic behaviors, the realistic loading conditions including both the moving loads and the beam's dead load have been considered.

The theory of a structure subjected to a moving load has been developed sufficiently to simulate and to handle many engineering situations such as beams, highway structures, train structures, or bridges. However, effective methods to estimate the bending moment and the shear forces along the elastic beam simultaneously subjected to a moving load associated with the dead load of the beam itself have not been generally addressed. This work will present all the dynamic behaviors such as the deflection, bending moment, and shear force, of an elastic beam subjected to a moving load with various velocities. Two types of loading conditions are considered: (1) a massless beam subjected to a unit moving load; and (2) a realistic beam with its dead load subjected to a wheel load according to AASHTO (1992). Finally, a parametric evaluation of the effects of velocity, span length, and nonlinear analysis on dynamic response is performed.

2. Solution procedures

Analytical solutions for the governing partial differential equations of moving load problems are limited to rather simple cases, especially for those with known mode shape functions. On the other hand, closed-form solutions become very cumbersome for complicated structures; therefore, these types of structures may need the use of numerical methods such as finite difference or finite element methods to investigate their dynamic behaviors.

This study describes an analytical method, and develops a finite element procedure to solve this type of problem. Furthermore, the problem idealization and the procedure verification of the finite element methods are performed.

2.1 Analytical method

The dynamic governing equation of an elastic beam subjected to a moving load is briefly derived using Lagrange's equation. The elastic beam with a variety of spans is subjected to a concentrated load that traverses across the beam in a time-dependent manner. For forced vibration of the beams, the dynamic deflection may be represented by the summation of the model components for lumped-mass systems:

$$Y(x,t) = \sum_{n=1}^{N} G_n(t)\phi_n(x) \tag{1}$$

where $G_n(t)$ is the modal amplitude function, and $\phi_n(x)$ is the modal-shape function. Then, the velocity is given by

$$\dot{Y}(x,t) = \sum_{n=1}^{N} \dot{G}_n(t)\phi_n(x) \tag{2}$$

For use in Lagrange's equation the kinetic energy of the complete system is expressed as

$$T = \sum_{i=1}^{r} \frac{1}{2} m_i \left[\sum_{n=1}^{N} \dot{G}_{in}(t) \right]^2 \tag{3}$$

where r is the number of discretized mass; n is the number of mode, and N is the total number of modes; $\dot{G}_{in}(t)$ represents the velocity of the i_{th} discretized mass at n_{th} mode; m_i is the mass of discretized element i. The integration provides the summation of all kinetic energy along the span of the beam.

The potential energy of the whole system U is:

$$U = \sum_{p=1}^{P} \frac{1}{2} k_p \left[\sum_{n=1}^{N} \delta_{pn} \right]^2 \tag{4}$$

where δ_{pn} represents the relative displacement at the n_{th} mode, and the stiffness constant is k_p of element p.

W_d is the total energy taken by damper. It can be expressed by

$$W_d = \sum_{j=1}^{J} (-C_j) \sum_{n=1}^{N} \dot{\delta}_{jn} \sum_{n=1}^{N} \delta_{jn} \tag{5}$$

where $\dot{\delta}_{jn}$ represents the velocity of damper j at the n_{th} mode, C_j is j_{th} damping coefficient.

The work done W_e by external dynamic force $F_i(t)$ during an arbitrary disorder is

$$W_e = \sum_{i=1}^{r} F_i(t) \sum_{n=1}^{N} G_{in}(t) \tag{6}$$

In order to form a general equation of motion in the generalized coordinate system, the above equations can be rewritten into the well known Lagrange's equation.

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{G}_n} + \frac{\partial U}{\partial G_n} = \frac{\partial W_e}{\partial G_n} \tag{7}$$

Then one obtains,

$$M_e\ddot{Y}_n(x,t)+C_e\dot{Y}_n(x,t)+K_eY_n(x,t)=F_e(t) \tag{8}$$

Where the element mass matrix $M_e=\sum_{i=1}^{r} m_i\beta_{in}^2$, the element damping matrix $C_e=\sum_{j=1}^{r} C_j\beta_{jn}^2$, the element stiffness matrix $K_e=\sum_{p=1}^{r} m_p\beta_{pn}^2$, the element force vector $F_e(t)=\sum_{i=1}^{r} F_i(t)\beta_{in}$, in which β_{in}, β_{jn}, and β_{pn} are defined as $\beta_{in}=\frac{G_{in}}{Y_n(x,t)}=\frac{\dot{G}_{in}}{\dot{Y}_n(x,t)}$, $\beta_{jn}=\frac{G_{jn}}{Y_n(x,t)}=\frac{\dot{G}_{jn}}{\dot{Y}_n(x,t)}$, and, $\beta_{pn}=\frac{G_{pn}}{Y_n(x,t)}=\frac{\dot{G}_{pn}}{\dot{Y}_n(x,t)}$, respectively. In the structural system, any particle motion can be described by the generalized displacement function at the n_{th} node $Y_n(x, t)$. Thus, the dynamic governing equation can also be written in the following way

$$\ddot{Y}_n(x,t)+2\xi\omega_n\dot{Y}_n(x,t)+\omega_n^2Y_n(x,t) = \frac{F_e(t)}{M_e} \tag{9}$$

where $2\xi\omega_n = \frac{C_e}{M_e}$, $\omega_n^2 = \frac{K_e}{M_e}$, in which ω_n is the natural frequency of the n_{th} mode. ξ is the damping ratio of the structure. In order to form the equation of motion in a general form, Eq. (9) is rewritten as the following.

$$\ddot{Y}_n(x,t)+2\xi\omega_n\dot{Y}_n(x,t)+\omega_n^2Y_n(x,t) = g(t)\frac{\sum_{i=1}^{r} F_i(t)\beta_{in}}{\sum_{i=1}^{r} m_i\beta_{in}^2} \tag{10}$$

where $g(t)$ is the time function depending on the external force, i.e., $F_i(t)=g(t)F_i$. In order to determinate the deformational function of the beam, the shape function must be known. Eq. (11) is the general form of beam's shape function $\Phi_n(x)$ in which it may be applied to various spans with any type of end restraints. The constraints are determined by the boundary conditions of the particular problem.

$$\Phi_n(x) = A_n\sin Z_nx+B_n\cos Z_nx+C_n\sinh Z_nx+D_n\cosh Z_nx \tag{11}$$

where $Z_n = \sqrt[4]{\frac{m\omega_n^2}{EI}}$. If the dynamic stress is desired, the well known relationships, the bending moment, $M(x, t) = -EI\frac{\partial^2 y(x,t)}{\partial x^2}$, and the shear force, $V(x, t) = \frac{\partial M(x,t)}{\partial x}$, should be applied to the deformational function. Thus, the computation involves only the differentiation with respect to x of the expressions given above for dynamic deflection.

2.2 Finite element procedure

Analytical methods have trouble solving complicated structures at present, but the numerical methods such as the finite difference or the finite element methods seem to work. Based on finite element methods, this study proposes a finite-element procedure to solve those problems no matter what kind the model of the beam is or what kind the moving load is. The solution procedure is described in the following.

1. The first step is to set the initial condition at which the external load is zero.
2. Set up the time increment, dt. The time increment for each velocity considered is determined by $dt = \dfrac{dx}{v}$ in which dx is length between two subsequent nodes and v is constant velocity.
3. If $P_1(t)$ is the applied load at node i, at $t = 0$, $p_i = 0$ for all nodes, at time $t = \dfrac{m \cdot dx}{v}$ the load at node m is the weight of the vehicle and zero for all other nodes; i.e., $P_m\left(\dfrac{m \cdot dx}{v}\right) = W_m$ at $i = m$, $P_i\left(\dfrac{m \cdot dx}{v}\right) = 0$, for $i = 1, 2, \ldots, m-1, m+1, \ldots, n$.
4. Perform time history analysis for t_0 through $t = \dfrac{x}{v}$.
5. Record the deflection, shear force, and bending moment, of the bridge at the point, which has been specified by finite element methods.

For linear analysis, the direct time integration used in this procedure is an implicit, unconditionally stable scheme based on the Newmark method. The term implicit means that the displacement vector is a function of both previous (known) and current (unknown) displacements, velocities and accelerations. The term unconditionally stable means that the solution of a linear system will never diverge, no matter how large the time increment (dt) is. If nonlinearities exist, this procedure is solved iteratively at a single time point to provide any number of equilibrium iteration. Convergence criteria are identical to that used in a static analysis.

The above simulative procedure performs the dynamic influence lines of vertical displacement and dynamic stress for the structure subjected to a moving load with various vehicle velocities.

2.3 Problem idealization

In order to effectively model and solve the problem, the idealizations are in the following.

1. The material is originally straight and linear elastic.
2. The moving traffic is considered a moving concentrated load with a constant traveling passage velocity on the beam.
3. The mass of the traveling vehicle is not considered in the global system.
4. The roughness of the roadways is not taken into account.
5. Damping is not taken into account in numerical examples to simplify the procedure.

2.4 Procedure verification

To verify the finite-element solution procedure, a simply supported beam subjected to a moving load is investigated; the numerical results such as the vertical deflection, bending moment, shear force, and natural frequencies obtained by both the analytical and finite element methods, are compared to assess the method's validity. The shape function of a simply supported beam is $\Phi_n(x) = \sin\frac{n\pi x}{L}$ based on Eq. (11) in which the position function of the moving load is defined as $x = vt$; v is a constant vehicle velocity. In order to simplify this problem, the beam is considered an undamped structure. Substituting the shape function $\Phi_n(x)$ into Eq. (12) then, integrate the equation to find the generalized displacement function $Y_n(x, t)$ of a simply supported beam subjected to a moving load.

$$Y_n(x,t) = \frac{\frac{F'}{\omega_n^2}}{1-\alpha_n^2}(\sin\Omega_n t - \alpha_n \sin\omega_n t)\Phi_n(x) \tag{12}$$

where the parameters are defined: $\alpha_n = \frac{\Omega_n}{\omega_n}$, $\Omega_n = \frac{n\pi v}{L}$, and $F' = \frac{2F}{mL}$. The well known relationships, $M(x,t) = -EI\frac{\partial^2 y}{\partial x^2}$ and $V = \frac{\partial M(x,t)}{\partial x}$, are applied into the deformation function to obtain the dynamic stress functions, bending moment, and shear force, $M_n(x, t)$, $V_n(x, t)$, respectively.

$$M_n(x,t) = \left(\frac{n\pi}{L}\right)^2 \frac{EI\,F'}{\omega_n^2(1-\alpha_n^2)}(\sin\Omega_n t - \alpha_n \sin\omega_n t)\sin\left(\frac{n\pi x}{L}\right) \tag{13}$$

$$V_n(x,t) = -\left(\frac{n\pi}{L}\right)^3 \frac{EI\,F'}{\omega_n^2(1-\alpha_n^2)}(\sin\Omega_n t - \alpha_n \sin\omega_n t)\cos\left(\frac{n\pi x}{L}\right) \tag{14}$$

Note that, in going from the deflection to bending moment and shear force, the higher order modes become increasingly important, as indicated by the increasing power of n. However, the amplitudes of the dynamic stress decrease for the higher order modes because the natural frequency ω_n increases and ω_n is proportional to n^2.

If a beam has different spans and boundary conditions from the simply supported beam, its mode shape should be other than $\Phi_n(x) = \sin\frac{n\pi x}{L}$. The deformational, bending moment, and shear force functions can still be derived by the same procedure.

In order to verify the finite-element solution procedure and to study the dynamic characteristics, a typical simply supported beam proposed by Biggs (1972) with span length 40 ft (1ft = 0.3048 m) subjected to a moving force is investigated using both analytical and finite element methods. The parameters of the system are given as: $m = 0.1$ lb-sec^2/in^2 (2.2 lb = 1.0 kg, in = 2.54 cm), $EI = 2.0*10^{10}$ lb-in^2 , the total span $L = 40$ ft, and the travel speed $V = 50$ ft/sec. To verify this numerical procedure by the deflection and stress of the beam, Dynamic Load Factor (DLF) and dynamic stress including the bending moment and the shear force are obtained. The solutions obtained by both the finite element and the analytical methods are also compared. DLF is defined as the ratio of vertical dynamic deflection with respect to the maximum static deflection along the span caused by the same load.

Fig. 1 represents the geometry of the beam, the moving force, and the comparison of the DLFs. The figure also shows the dynamic stress obtained by analytical and finite element methods at the midpoint of the beam subjected to moving traffic with the speed of 50 ft/sec (1ft/sec = 0.3048 m/sec).

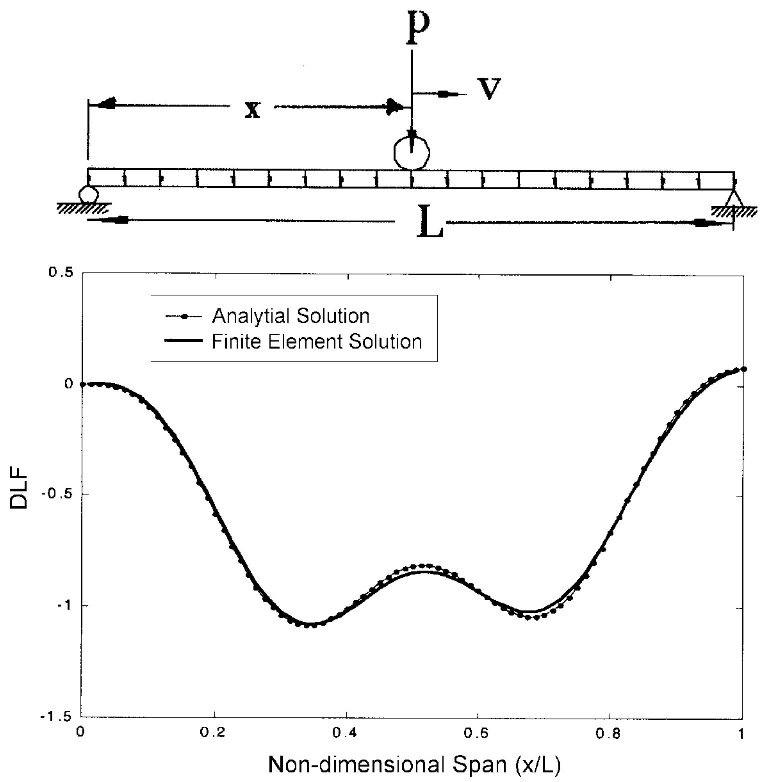


Fig. 1(a) DLF of the simply supported beam subjected to a moving load at the midspan

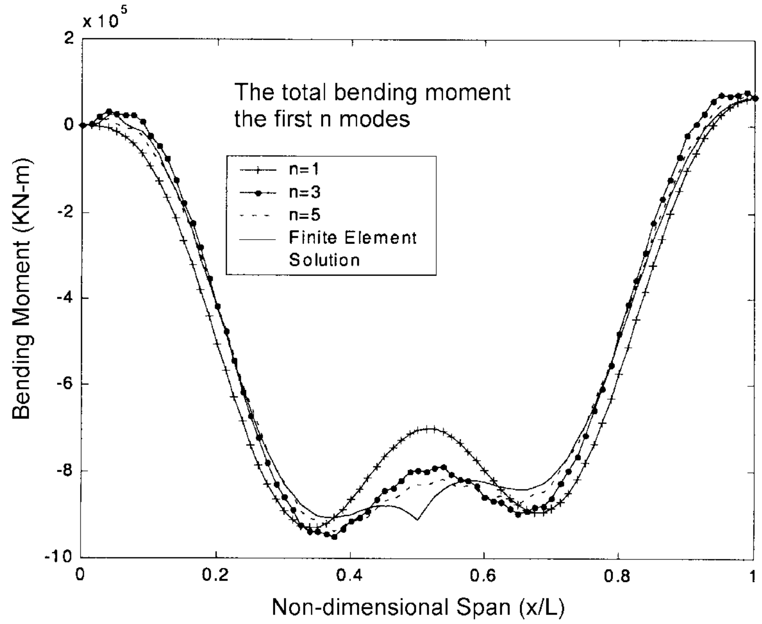


Fig. 1(b) Influence lines of bending moment of the simply supported beam at the midspan

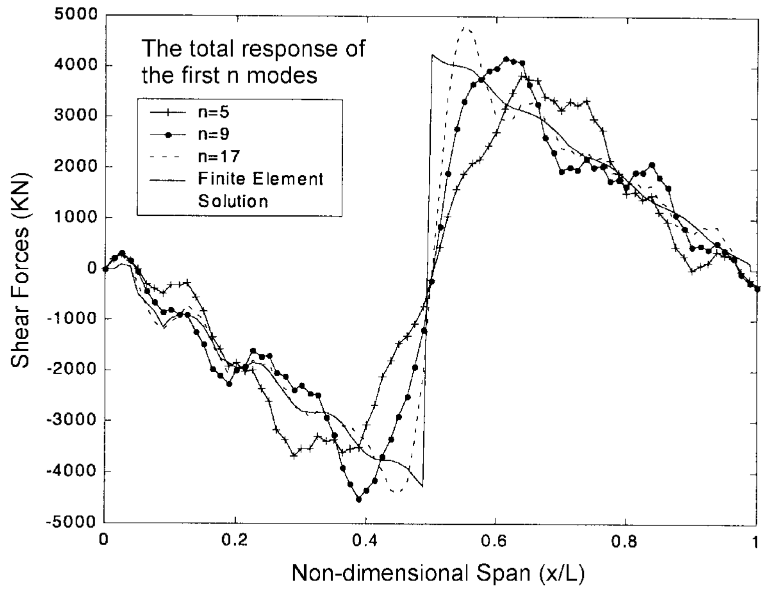


Fig. 1(c) Influence lines of shear force of the simply supported Beam at the midspan

The horizontal axis of Fig. 1 is the non-dimensional distance (x/L) in which x is the traveling distance from the initial position, and L is the total span length of the beam.

Fig. 1(a) represents DLF of the simply supported beam subjected to a moving force. The DLFs, obtained by both analytical and numerical methods, are almost identical in which the solid line indicates the DLF obtained by finite element method, and the dot line indicates the DLF obtained by analytical methods. The first dynamic mode is the primary contribution to the DLF, and the contribution of the rest modes to the DLF is less then 2% to the total response.

The bending moment and the shear force of the beam at the midspan are represented in Fig. 1(b) and 1(c), respectively. The non-dimensional bending moment $M^N(x, t)$ is defined as $M^N(x, t) = -EI\dfrac{\partial^2(DLF)}{\partial x^2}$, and the non-dimensional shear $V^N(x, t)$ is defined as $V^N(x, t) = \dfrac{\partial M^N(x, t)}{\partial x}$.

The characteristics of dynamic stresses, bending moment, and shear force, differ from those of DLF. The fundamental mode is only the primary contribution to the structural deflection; for the dynamic stresses, the higher order modes have more contributions to the total structural responses. Fig. 1(b) shows that the first 5 modes are the primary contribution to the total bending moment and the higher order modes are negligibly small. Fig. 1(c) indicates that the first 17 modes are the primary contribution to the shear forces, which are almost equal to that obtained by analytical methods. The fact is, quite simply, that the increasing power of n induces the bending moment and the shear force contributed by higher order modes because the amplitudes are proportional to the power of n. The total responses obtained by both the analytical and the finite element methods have a good agreement.

The finite element analysis is an effective method in solving this type of beam subjected to moving load. It is not only to obtain the total response of the structure, but also to find the natural frequencies and their corresponding mode shapes. To verify the dynamic analysis, eigen frequency

(ω_n) is important for the solution of this type of beam. For the beam demonstrated by Biggs, the natural frequencies obtained by the analytical and finite element methods also have an excellent agreement. The first six modes of the natural frequency obtained by both the analytical and finite element methods are almost identical. The natural frequency of the fundamental mode obtained by the analytical method is 3.05 Hz (19.16 rad/sec); and so is that obtained by the finite element methods. It clearly indicates that the finite element procedures are effective in solving this type of problem, especially for solving dynamic stress such as the bending moment and shear forces.

3. Numerical examples

Dynamic responses -- Dynamic Load Factor (DLF), bending moment, and shear force -- of an elastic beam subjected to moving load depend on its span length, the velocity and the magnitude of

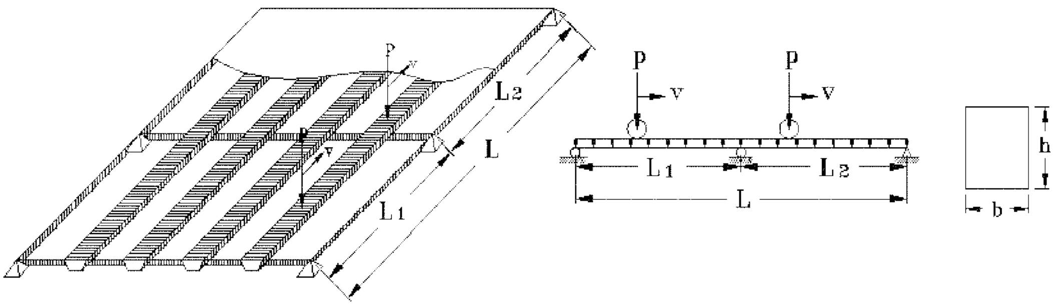
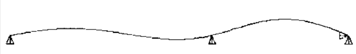
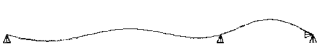


Fig. 2 Geometry of the elastic beam and the loading conditions

Table 1 The eigen-frequencies and their corresponding mode shapes of the two-span girder with total length 120 m

Mode #	Span Ratio (L_1/L)		
	0.3	0.4	0.5
1	 0.946 Hz	 1.202 Hz	 1.418 Hz
2	 3.092 Hz	 2.762 Hz	 2.216 Hz
3	 4.985 Hz	 4.598 Hz	 5.672 Hz

the moving load. In order to investigate the dynamic behavior, the finite-element procedure investigates the dynamic behaviors and the eigen frequencies of a typical two-span elastic beam having different span lengths subjected to moving loads with a variety of velocities. The geometry of the elastic beam and the moving loading conditions are presented in Fig. 2. The elastic beam is one of the two-span girders supporting the bridge deck. Its cross-section is rectangular represented by the width b and the depth h, respectively. The first span length of the beam is L_1, the second span length is L_2, and the total span length is L, which remains 120 m. The parameters of the steel girder are given as: $E = 300$ Gpa, $b = 0.5$ m and $h = 1.0$ m. The traveling vehicle is considered a concentrated force shown in Fig. 2 passage on the beam with a constant velocity v. Both the pseudo-excited (unit force) and the realistic loading conditions are used to examine the structural dynamic characteristics. For pseudo-excited loading conditions, the moving load is considered a unit concentrated force passage on the massless beam with a constant velocity. The realistic loading condition is the dead load of the beam itself and a variety of a wheel load according to AASHTO (1992). The numerical results are nondimensionalized and plotted as influence lines versus the span length for different loading conditions.

To investigate the importance of the natural frequencies to the dynamic behavior of the two-span girder.

Table 1 represents the first three modes of the eigen frequencies and their corresponding mode shapes. The ratios of the span length (L_1/L) vary from 0.3, 0.4 and 0.5. The first mode of the beam is usually dominated by the longer span, which creates lower eigen-frequency. In opposition, the frequency of the second mode is dominated by the shorter span. Generally speaking, eigen-frequencies are different when the girders have different internal supports along the span.

3.1. Characteristics of dynamic response

Using finite-element methods, in this section the authors study the characteristics of DLF of an elastic two-equal-span beam subjected to both pseudo and realistic loads. The numerical results are represented in the plot of DLF versus the non-dimensional span. The elastic beam has two 60 m-equal spans. DLF of the two-span beam is defined as the ratio of the deflection created by the moving load with respect to the maximum static deflection of the first span L_1. For static analysis, the maximum deflection occurs at the point of $0.480\,L_1$ from the end support of the first span, and the amplitude is $0.015PL_1^3/EI$ in which P is the amplitude of the moving force; EI is the rigidity of the beam.

Fig. 3(a) plots the non-dimensional span versus DLF at the midpoint of the first span. The figure indicates that the oscillation of DLF decreases but the amplitude increases when the velocity of the moving force increases. The maximum DLF reaches 1.53 when the velocity of the moving load is 120 Km/hr. It means that the dynamic loading creates the additional 53% of deflection than static loading does. If the velocity of the moving load increases, the maximum DLF increases and the location of the maximum DLF is varied along the span.

The characteristics of dynamic stress are presented in Figs. 3(b) and (c). Fig. 3(b) represents the normalized shear $V^*(x, t) = V(x, t)/Max.\ Static\ V(x, t)$ versus the non-dimensional span (x/L). If the velocity of the moving force is less than 80 km/h, the maximum shear force obtained by both dynamic and static analyses is almost identical; however, the influence lines will be different when the velocity is beyond 80 km/h. In other words, Fig. 3(b) shows that the maximum shear obtained by dynamic analysis is greater than that obtained by static analysis if the velocity of the moving force is greater than 80 km/h. The maximum shear obtained by dynamic analysis is 1.17 times as

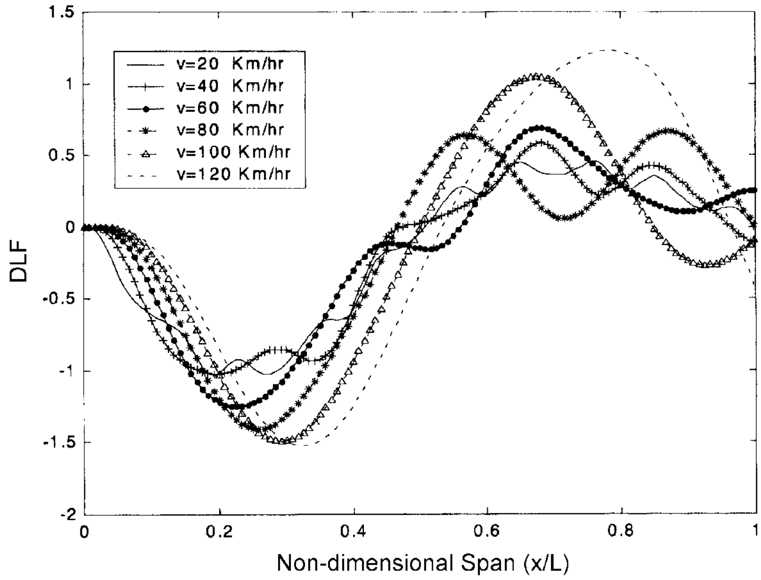


Fig. 3(a) Nondimensional span versus DLF at the midpoint of the first span for various velocities

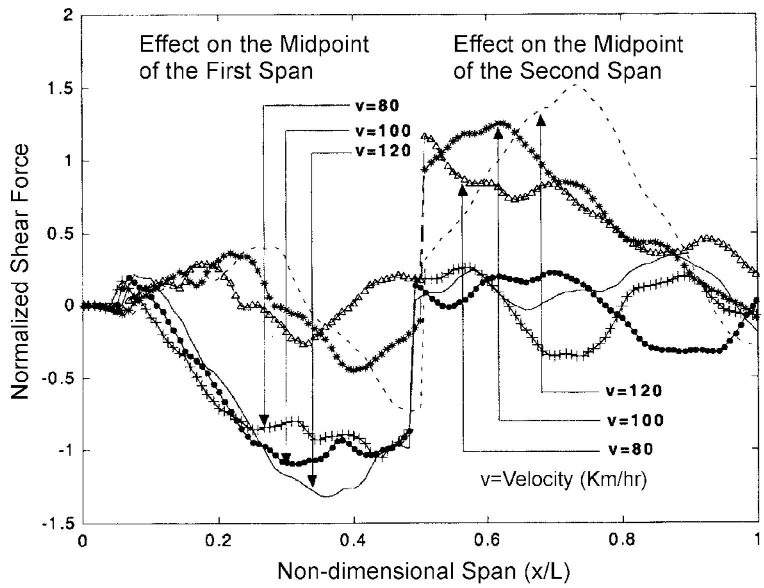


Fig. 3(b) Normalized shear V^* versus non-dimensional span (x/L) for various velocities at the midpoint of the first span

much as the shear force obtained by static analysis when the speed of moving load is 80 km/h. The ratio can reach 1.25 and 1.51 for the speed of 100 km/h and 120 km/h, respectively. Generally speaking, the maximum shear occurs near the support. Moreover, the location of the maximum dynamic shear occurs around internal supports if the velocity of the moving force is faster than 80 km/h. The characteristics of dynamic stress are different from those obtained by static analysis,

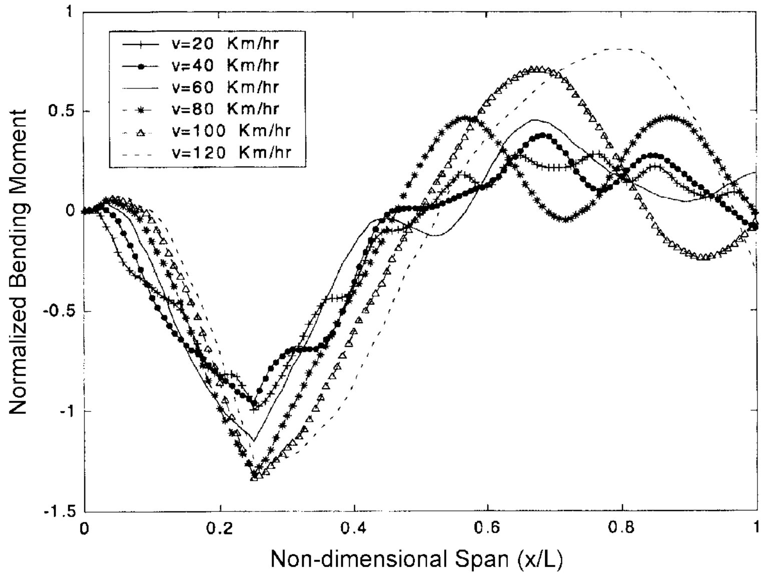


Fig. 3(c) Normalized bending moment M^* at the midpoint of the first span versus non-dimensional span (x/L) for various velocities

and it shall be noticed that the maximum shear may cause the maximum stress to exceed the designed limit.

Different from the characteristics of shear force, the maximum dynamic bending moment is always sited near the midpoint of the span. Fig. 3(c) represents the relationship between the normalized span and bending moment $M^*(x, t) = M(x, t)/Max.\ Static\ M(x, t)$. In general, the higher speed creates higher dynamic bending moments, especially when the speed is faster than 100 Km/hr in which the maximum $M^*(x, t)$ reaches 1.34. If the beam is subjected to low speed, less than or equal to 40 km/h, the moment influence line has only more oscillation but the amplitude is close to that obtained by static analysis.

Fig. 4 shows total structural response for the aforementioned structure in which both the dead load of the beam itself and the moving load are taken into account (Collins and Mitchell 1991). The dead load of the beam itself is equivalent to 123 kN. Based on AASHTO (1992) for truck loading, the moving load is considered ranging from 230.3 kN for HS15-44 to 320.2 kN for HS20-44. For lane loading, the uniform load is considered 7.0 kN/m for H15-44 or HS15-44 trucks and it is considered 9.3 kN/m for H20-44 of HS20-44; the concentrated load is ranging from 60.1 kN for moment of H15-44 or HS15-44 to 115.7 kN for shear of H20-44 or HS20-44.

Fig. 4(a) shows DLF, which is the deflection due to the loading condition with respect to the maximum deflection of the first span due to dead load, at the midpoint of the first span. The maximum DLF is 2.6, which occurs at the 0.37 of the first span from the end support due to the dead load associated with a 200 kN moving load with traveling velocity, 100 km/h. Although DLF increases as the amplitude of the moving load increases, the increment of DLF is not linear and depends on the locations of the moving loads. According to the numerical results, the deflection is significantly underestimated for the structural design based on static analysis.

Fig. 4(b) plots the normalized shear influences located at internal support versus the non-

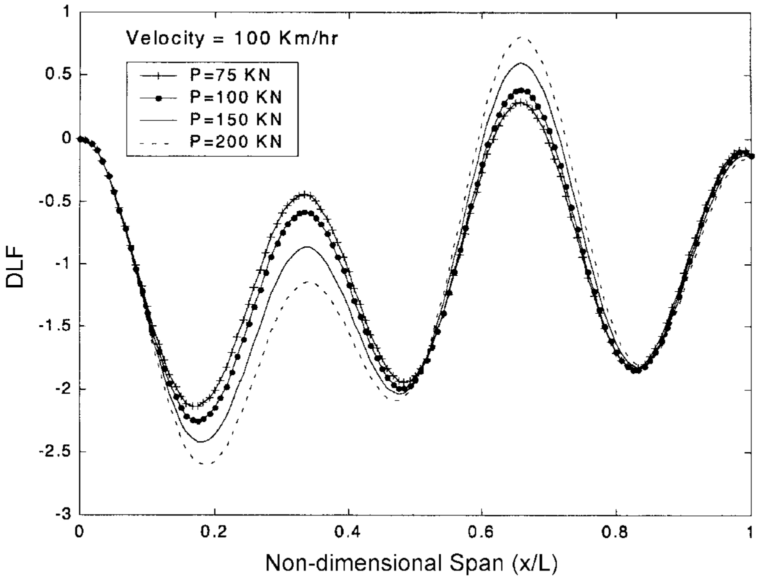


Fig. 4(a) Nondimensional span versus DLF of realistic loading conditions at the midpoint of the first span for various velocities

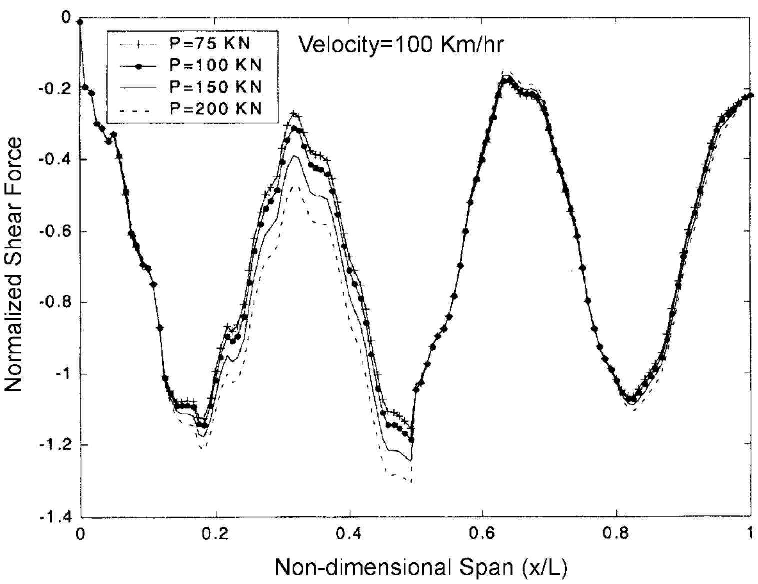


Fig. 4(b) Normalized shear V^* of realistic loading conditions at the internal support versus nondimensional span (x/L) for various velocities

dimensional span for realistic loading condition in which the traveling velocity of the moving load is 100 km/h. Under realistic loading conditions, dynamic response creates more than 30% of shear force than the static response does when the moving live load is 200 kN. The effect of shear forces on dynamic analysis depends on the live-dead load ratio. The maximum shear always occurs at the

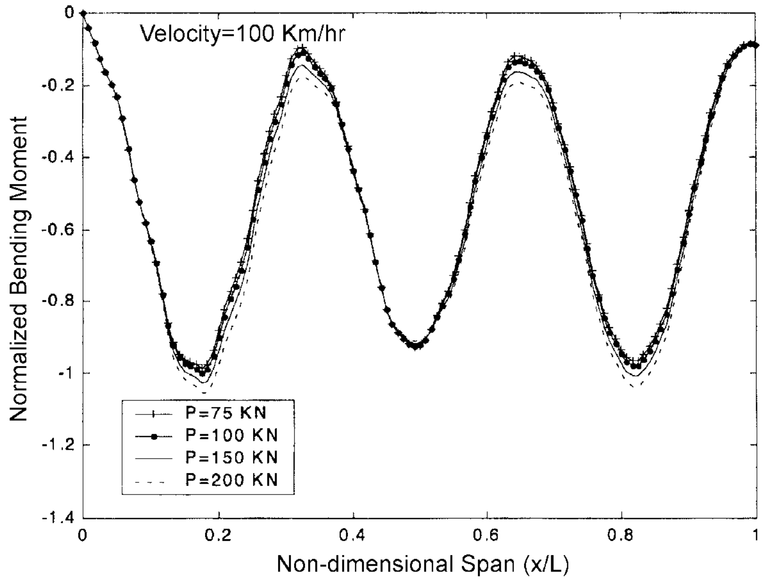


Fig. 4(c) Normalized bending moment M^* of realistic loading conditions at the midpoint of the first span versus nondimensional span (x/L) for various velocities

internal support for this type of structures.

The characteristics of the bending moment of the structure subjected to realistic dynamic loading are different from those of either DLF or shear force. The bending moment is not significantly different if the structure is subjected to the combination of its dead load and different amplitude of live load. Under the realistic loading condition, the maximum bending moment occurs when the moving load travels at the midpoint of each span, but the magnitude is only 5% more than that in static analysis if the live moving load is 200 kN as shown in Fig. 4(c). If the moving load is less than 100 kN, the bending moment created by dynamic loading is less than that created by static loading.

3.2. Effects of the span length

According to the designed function of a bridge, the use of high strength materials, and working methods, bridges usually have different span lengths. The dynamic characteristics become significant when the flexibility of bridges increases as the span length increases. The effects of the span length on dynamic analysis including the deflection, shear force, and bending moment along the whole span are investigated.

The span length effects of a typical two-span beam with total span length of 120 m, which remains constant, subjected to a unit moving force with traveling velocity of 100 Km/hr shows that if the span length increases the structural response becomes significant, and the effect increases nonlinearly. Fig. 5(a) shows that the maximum DLF increases nonlinearly when the length increases. The DLF is the same as that of Fig. 3(a), in which the maximum deflection obtained by static analysis of the two-equal span beam is considered as unity. When the ratio of L_1/L reaches 0.7, the maximum DLF is as high as 4.04 because the flexibility of the beam increases. The numerical results show that the two-equal span beam is the best design if the criteria are based on maximum deflection.

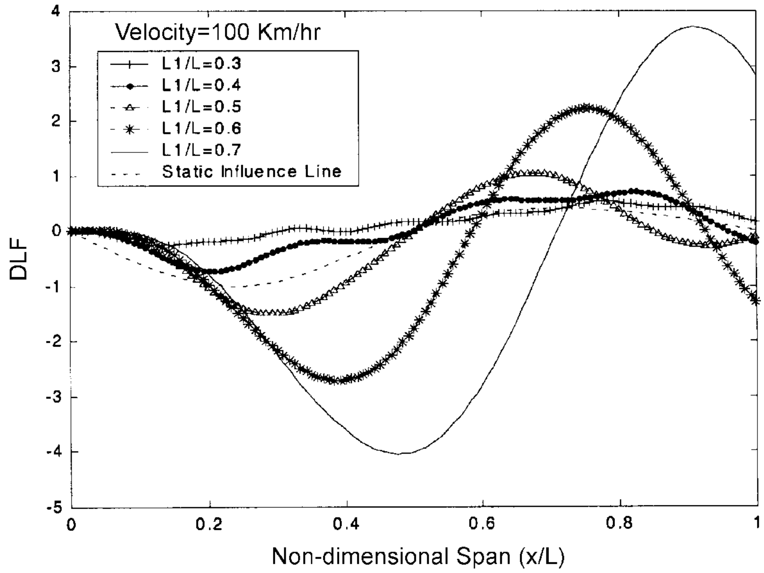


Fig. 5(a) DLF at the midpoint of the first span versus nondimensional span for various the first span lengths

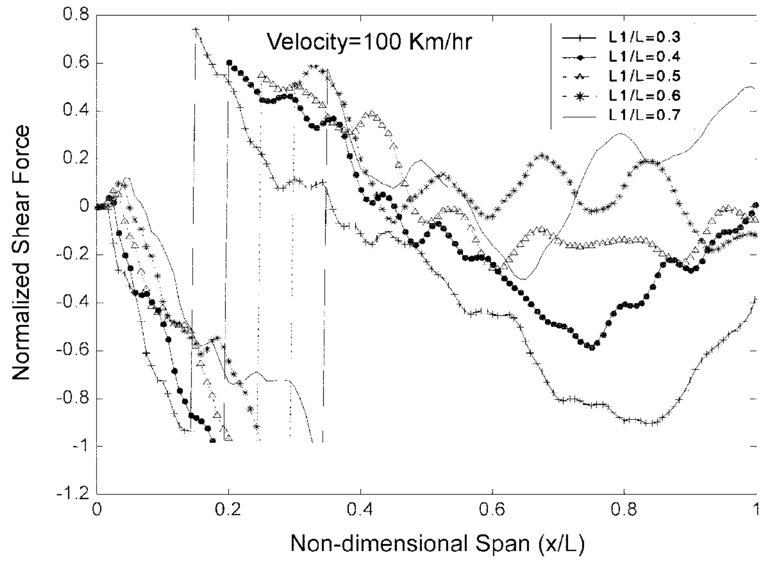


Fig. 5(b) Normalized shear force versus span for different span lengths

The span length effect of shear force on this type of dynamic analysis is not significant. Fig. 5(b) plots the normalized shear force at the midpoint of the first span versus the non-dimensional span for the beam with various span lengths of the first span. The non-dimensional shear force is the same as that of Fig. 3(b), in which the shear force at the mid-point obtained by static analysis of the two-equal span beam is considered as unity. Against the structural characteristics of DLF, however, the beam with longer span does not necessarily have the maximum shear force. Fig. 5(b) shows that

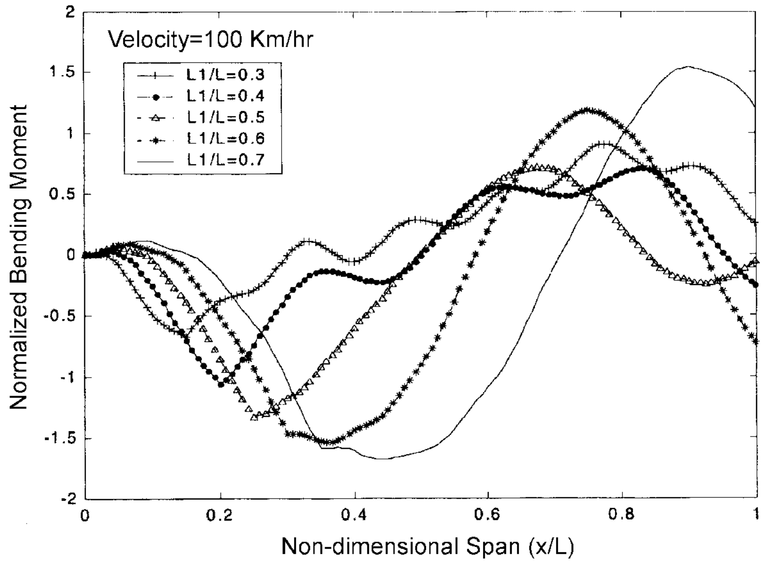


Fig. 5(c) Normalized bending moment at the midpoint of the first span versus span for various the first span lengths

the maximum shear force occurs when the ratio of L_1/L is 0.6, and its magnitude is 1.16.

The span length effect of bending moment on this type of dynamic analysis is less significant than that of DLF, but it is more significant than that of shear force. Fig. 5(c) plots the normalized bending moment versus the non-dimensional span for the beam with various spans. The normalized bending moment is the same as that of Fig. 3(c), in which the bending moment at the mid-point obtained by static analysis of the two-equal span beam is considered as unity. The increment of the bending moment is nonlinear for the beam with various spans. The maximum bending moment occurs when the ratio of L_1/L is 0.7; in other words, if the beam has longer span length, it has larger bending moment. The increment becomes not significant when the ratio of L_1/L is greater than 0.7.

3.3. Geometric nonlinear effect

Geometric nonlinear (large deformation and axial force) effect on the vertical displacement of the beam subjected to this type of dynamic loading is more significant than the effect dynamic on shear force and bending moment. This study investigates the nonlinear effect for a two-equal span beam with total span of 120 m subjected to a realistic loading associated with a moving load of 144 kN that is an axial load of HS-20 based on AASHTO (1992).

Usually, non-linearities are four primary categories: Geometric, boundary, force and material non-linearities. In this paper the geometric non-linearity is only taking into account. The finite element methods used in this paper handle the structures subject to transverse loading as the structures are preloaded in axial direction. The methods handle prestressed analysis by means of stress stiffening. Prestressing is handled by means of a stress stiffening matrix $[s]$ which is obtained from the stress state of the structure from the previous solution. Apply the stiffening matrix into the equation of motion. The displacement vector $\{u\}$ can be solved by the well known equation.

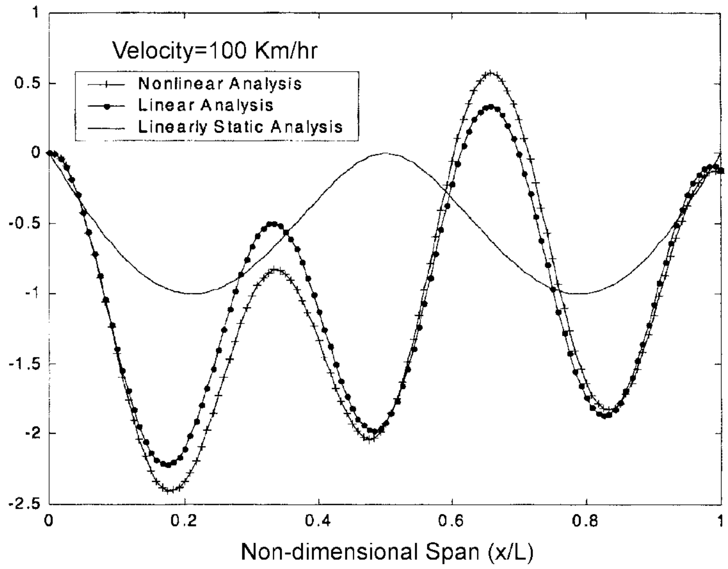


Fig. 6(a) Nonlinear effect on DLF

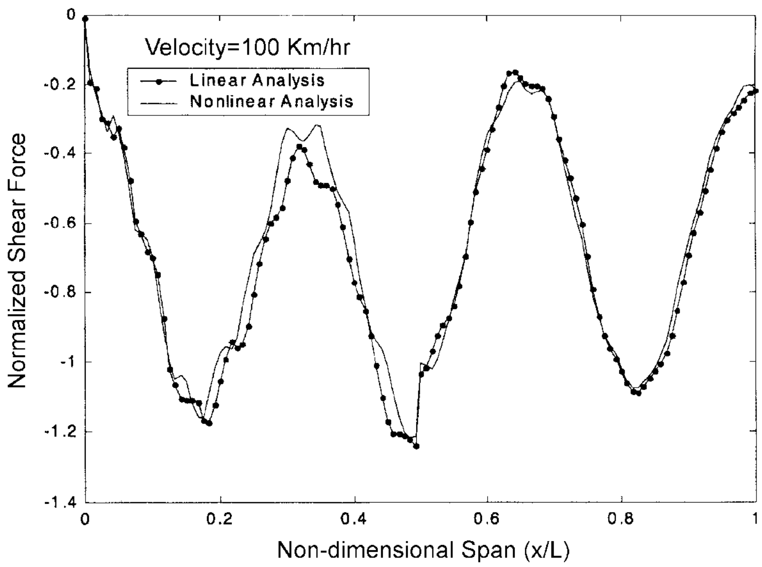


Fig. 6(b) Nonlinear effect on shear force

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + ([K] + [S])\{u\} = \{F(t)\} \tag{15}$$

where $[S]$ is a function of u and $[C]$ has been omitted in numerical examples.

Fig. 6(a) represents DLF at the midpoint of the first span. The maximum DLF created by nonlinear analysis is 2.4, and the maximum DLF created by linear analysis is 2.2. It shows that about 20% of DLF underestimated by linear analysis.

For dynamic stress, both shear force and bending moment, the geometric nonlinear effect is not

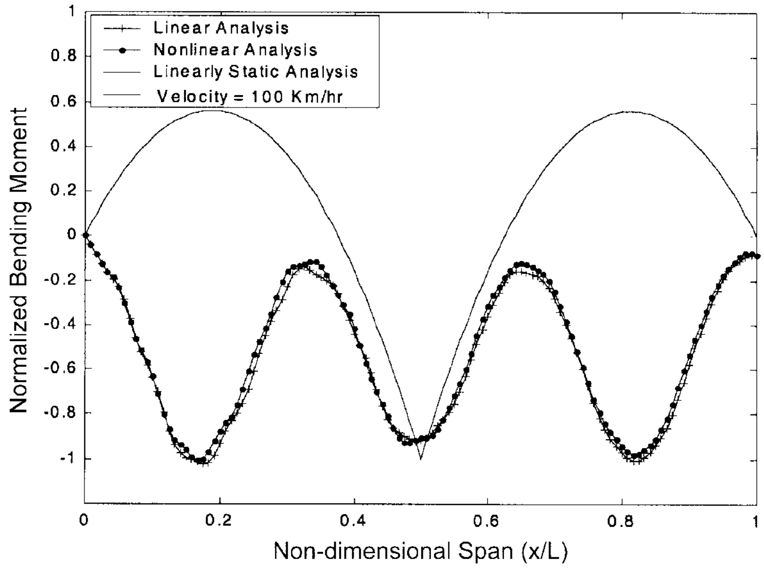


Fig. 6(c) Nonlinear effect on bending moment

significant. Figs. 6(b) and (c) show the comparison of the linear and nonlinear results at the internal support for shear force and bending moment, respectively. Both linear and nonlinear results are almost identical with a few exceptions for bending moment. In conclusion, for this type of analysis, the geometric nonlinear effect is significant for DLF but dynamic stress including shear force and bending moment.

4. Conclusions

Based on the previous works, the important conclusions can be drawn as the following.

(1) The deflection of elastic beams due to moving force is dominated by the fundamental dynamic mode, but the bending moment along the span depends on the summation of the first few modes, for example. The higher order modes have more contribution to the total response of the shear force than to the DLF or to the bending moment. For this type of structure, it is suggested that at least the first 15 modes be taken into account to accurately estimate the total shear force of the beam created by the moving loads. The first 5 modes should also be taken into account to accurately estimate the total bending moment.

(2) For linear analysis, the total response of the DLF, of the bending moments or of the shear forces can be estimated by superposition. But, the superposition is not suitable in applying to geometric nonlinear analysis. Generally speaking, the amplitude of the nonlinear numerical results is always greater than those of linear results with the exception of shear force occurring at the mid-span; in other words, nonlinear results are always underestimated if linear analysis is used.

(3) Due to the use of high strength material and computer technology, the span lengths of bridges increase; therefore, the deflection of the bridge deck subjected to moving load increases. The specification of bridge design may need to consider the dynamic response on this type of bridges.

(4) The magnitude of DLF depends on the rigidity of the structure, the span length, and the traveling

velocity of the moving load. In the further work, the mass of the traveling vehicle has to be taken into account in this type of structural systems because the mass of the vehicle may influence the global structural system, especially for the multiple traffic lanes or heavy truck load.

(5) The finite element procedures can successfully and effectively simulate this type of dynamic analysis, especially for complicated structures.

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Notation

$\dot{a}_{in}$	: velocity of the *i*th discretized mass at *n*th mode
C_j	: *j*th damping coefficient
DLF	: Dynamic Load Factor
dx	: Length between two consequent nodes
$F_i(t)$	: External load
$g(t)$	: Time function depending on the external force
k_p	: stiffness constant
$M_n(x, t)$	: Bending moment
m_i	: mass of discretized element i
N	: number of modes
$P_i(t)$	: Applied load at node i
r	: number of discretized mass
T	: Kinetic energy
t	: Traveling time
t_0	: the initial condition at time = 0

U	: Potential energy
v	: Constant vehicle velocity
$V_n(x, t)$	: Shear force
W_d	: Total energy taken by damper
W_e	: Work done by the external load $F_i(t)$
x	: Position function of the moving load defined as $x = vt$
Y	: Displacement profile function
α	: Frequency ratio
β	: Shape function
$\Phi_n(x)$	: Shape function of a simply supported beam
δ	: Displacement function of discretized system
ξ	: Damping ratio of the structure
δ_{jn}	: displacement damper j at the nth mode
$\dot{\delta}_{jn}$	: Velocity of damper j at the nth mode
δ_{pn}	: Relative displacement at the nth mode
ϕ	: Structural shape function
ω_n	: Natural frequency of the nth mode