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Minimum-weight seismic design of a moment-resisting frame accounting for incremental collapse

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Abstract. It was shown in the previous study (Lee and Bertero 1993) that incremental collapse can lead to the exhaustion of the plastic rotation capacity at critical regions in a structure when subjected to the number of load cycles and load intensities as expected during maximum credible earthquakes and that this type of collapse can be predicted using the shakedown analysis technique. In this study, a minimum-weight design methodology, which takes into account not only the prevention of this incremental collapse but also the requirements of the serviceability limit states, is proposed by using the shakedown analysis technique and a nonlinear programming algorithm (gradient projection method).

Key words: shakedown analysis; incremental collapse; earthquake load; plastic hinge; nonlinear programming.

1. Introduction

Many researchers (Bertero *et al.* 1976, Popov and Bertero 1973, and Zohrei 1982) in earthquake engineering have studied the problems arising in the inelastic response of structures to variable repeated loadings such as those generated by earthquake ground excitations. In particular, Bertero (1976) mentioned the possibility of incremental collapse or crawling failure caused by impact-type earthquake loads induced by near-field fault ruptures. Recently, the earthquakes in Northridge and Kobe implemented serious damages to the moment connections (Bertero *et al.* 1994, NSF & EERC 1995). However, the causes of this failure are now generally known to be the lack of toughness at the connections to accommodate the violent load reversals, particularly the lack of plastic rotation capacities (Engelhardt and Husain 1993). The failures by incremental collapse and low cycle fatigue have been thought of as some of the most probable types of structural failure under the maximum credible earthquake ground shakings (Bertero *et al.* 1976). Nevertheless, it is surprising to note that very little effort, except a few trials by Guralnick *et al*. (1984, 1991, and 1998), has been devoted to solving these problems through the use of the shakedown analysis technique itself (Hodge 1959, Horne 1979, Massonnet and Save 1965, Neal 1977).

Guralnick *et al.* (1984, 1991, and 1998) have shown, by using linear programming and energy approach for shakedown analysis, that much of the hysteretic energy dissipation is concentrated in relatively few dominant plastic hinges, and that strengthening structural components at those locations can significantly alter the incremental collapse behavior of the entire structure. However,

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the assumed load history was cyclic, but not of load-reversed type.

In the previous study (Lee and Bertero 1993), a simple one-bay three-story example structure was taken and the behavior of this structure was investigated under a pattern of fixed vertical and alternating lateral loads by first using static event-to-event nonlinear analysis with respect to shakedown phenomena, since there was little information on the plastic rotations under the state of shakedown, alternating plasticity, or incremental collapse. Next, time history nonlinear analyses with earthquake ground accelerograms of different characteristics were conducted to confirm the shakedown phenomena found in static analysis. Throughout this investigation it was found that shakedown analysis not only makes it possible to predict load factors of incremental collapse and alternating plasticity under variable repeated loading like earthquake excitations, but because it involves both elastic analysis and plastic analysis, it can also be used to predict and control the behaviors in both elastic and inelastic ranges. Therefore, it appears that the shakedown analysis technique provides a promising potential towards achieving the best or minimum-weight design among all the designs which satisfy all the requirements of multiple limit states, such as the ultimate limit state and the serviceability limit state if a nonlinear programming algorithm can be applied to solve the optimization problem. Thus, the objective of this study is to show the possibility of utilizing the shakedown analysis technique along with a nonlinear programming algorithm to obtain this minimum-weight design.

2. Shakedown problem under earthquake loading

2.1 Example structure and shakedown analysis

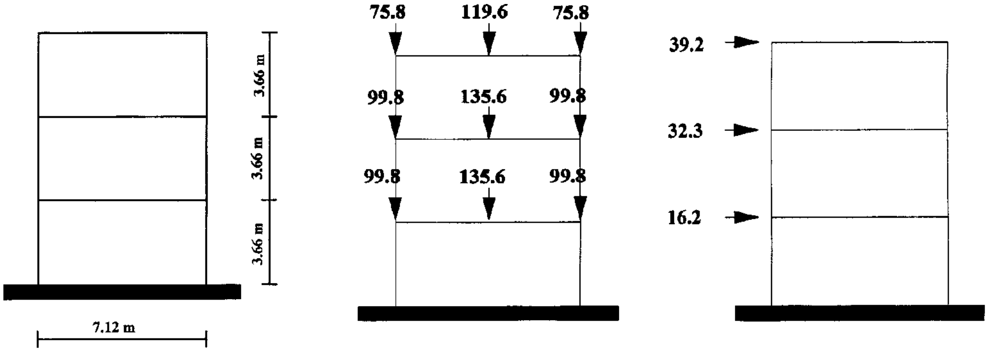
In order to present the shakedown problems (i.e., incremental collapse and alternating plasticity) and to demonstrate the reliability and advantages of the proposed design method more clearly, the same structure and design codes (*Uniform* 1982 and *Specification* 1980) as taken in the previous study (Lee and Bertero 1993) are used in this paper, though the current seismic design codes such as UBC 1997 or IBC 2000 have evolved through several revisions so far. However, the use of the new codes does not require any revision of the proposed design methodology. The geometry of an example structure and assumed gravity loads are shown in Fig. 1(a) and Table 1, respectively. By using an assumed gravity load and 1982 UBC earthquake regulations (*Uniform* 1982), the lateral load distribution equivalent to the earthquake load at service level is shown in Fig. 1(c), while the gravity loads are assumed to be concentrated at the ends and midspan of girders as shown in Fig. 1(b).

For simplicity of design, the structure is assumed to have two design variables, girder plastic moment, M_P^G, and column plastic moment, M_P^C, under the load conditions given by Part Two in

Table 1 Load assumptions for one-bay three-story structure (unit: kPa)

	Floor	Roof	Ext. wall
Dead load	4.309	3.591	1.436*
Live load	1.772	1.772	

*for vertical surface



(a) Geometry (bent spacing: 6.10 m) (b) Gravity load (c) Earthquake load

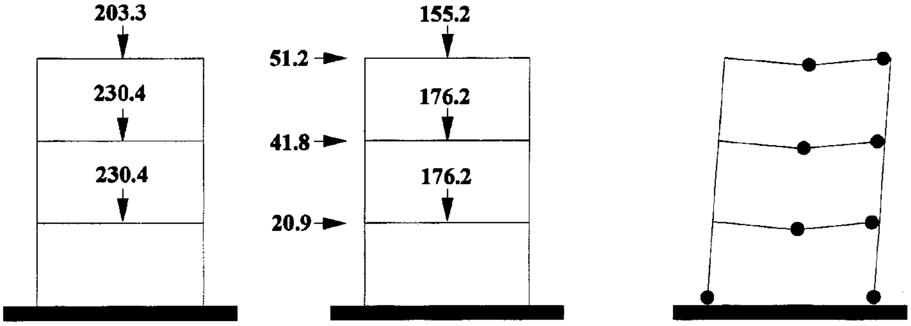
Fig. 1 Geometry of example structure and load conditions (unit: kN)

AISC Specifications (*Specification* 1980), as shown in Fig. 2(a) and (b). The minimum-weight procedure based on plastic design has resulted in the selection of compact sections, W16 × 31 (M_p = 219.7 kN-m) for girder and W14 × 26 (M_p = 164.1 kN-m) for column. The critical failure mechanism for this design is shown in Fig. 2(c).

The failure mechanisms and corresponding load factors in Fig. 4 were obtained through shakedown analysis using the load condition of 1.0(D+L) ± λ(EQ) with the load pattern and the assumed moment-curvature relation as shown in Fig. 3. In this case, the shakedown load factor, $\lambda_{s.d.}$, is the smaller of the incremental collapse load factor, $\lambda_{inc.c.}$ = 1.654, and the alternating plasticity load factor, $\lambda_{a.p.}$ = 1.550, i.e., $\lambda_{s.d.}$ = 1.550.

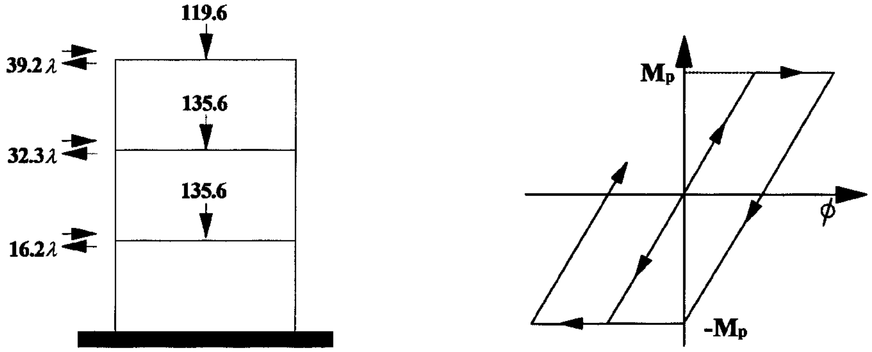
2.2 Shakedown phenomena from static nonlinear analysis

Variable repeated loading or generalized loading in shakedown theorem comprises any load history as long as the assumed load pattern is retained. However, for the convenience of illustration, the history of variable repeated loading is simplified in such a way that the lateral earthquake load is applied back and forth repeatedly with the same intensity while the gravity load remains constant. The program INSA (Powell 1985) was used to conduct nonlinear event-to-event analyses.



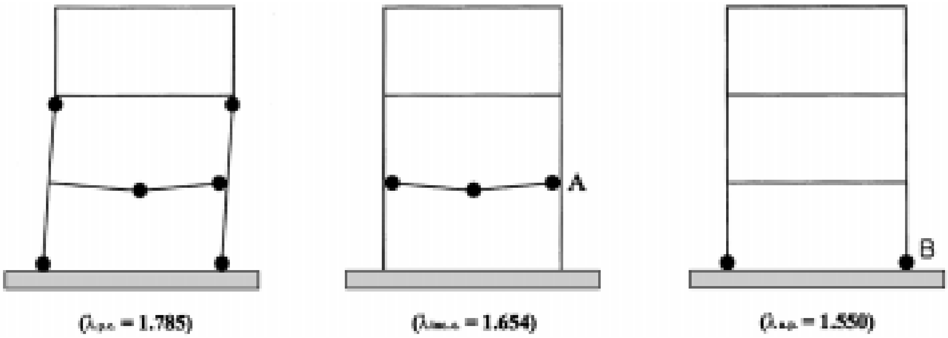
(a) 1.7 (D + L) (b) 1.3 (D + L + EQ) (c) Mechanism for 1.324 (D + L + EQ)

Fig. 2 Load cases and controlling mechanism for minimum-weight plastic design (unit: kN)



(a) Load pattern (unit: kN) (b) Moment-curvature relationship

Fig. 3 Load pattern and assumed moment-curvature relationship for shakedown analysis



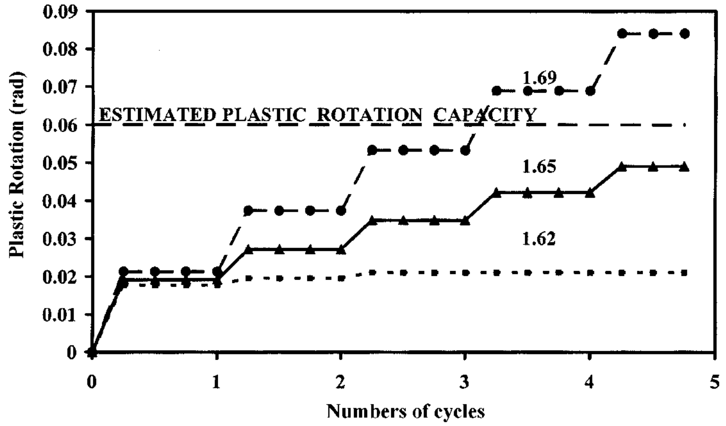
(a) Plastic collapse (b) Incremental collapse (c) Alternating plasticity

Fig. 4 Failure mechanisms and load factors obtained through shakedown analysis

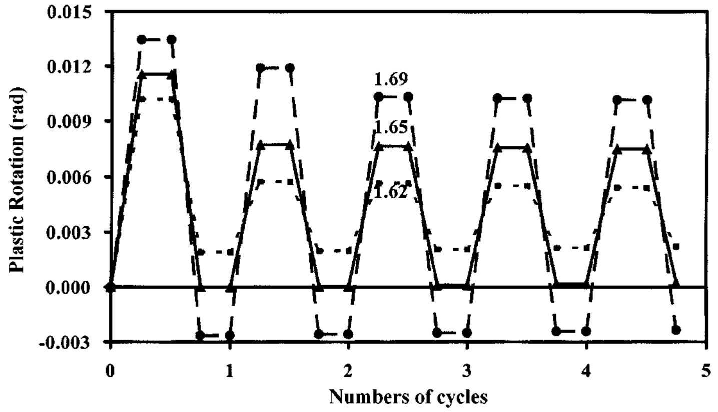
Three load factors ($\lambda_1 = 1.69$, $\lambda_2 = \lambda_{inc.c.} = 1.654$, and $\lambda_3 = 1.62$) are used to perform these nonlinear analyses. The histories of plastic rotation at the right end of the second-floor girder and at the right base support of the structure are shown in Fig. 5(a) and (b), respectively. It should be noted that the load factor $\lambda = 1.69$ is only 2% larger than the load factor of incremental collapse, $\lambda_{inc.c.} = 1.654$, whereas load factor $\lambda = 1.62$ is 2% smaller. The accumulated plastic rotation per cycle at the plastic hinge formed at the right end of the second-floor girder under the load factor $\lambda = 1.69$ is so large that only four load cycles can lead to the exhaustion of the estimated plastic rotation capacity, 0.060 radian (Lee 1989); whereas, the plastic behavior corresponding to $\lambda = 1.62$ reveals no increase of the plastic rotation after reaching a certain value. However, in Fig. 5(b), all three load factors cause alternating plasticity at the right base of the structure because they are all larger than the load factor of alternating plasticity, $\lambda_{a.p.} = 1.55$, the corresponding mechanism of which includes this plastic hinge.

2.3 Shakedown phenomena from time history nonlinear analyses

In static nonlinear analyses, the lateral load distribution has been assumed to have a constant



(a) Plastic rotations at plastic hinge A in Fig. 4(b)



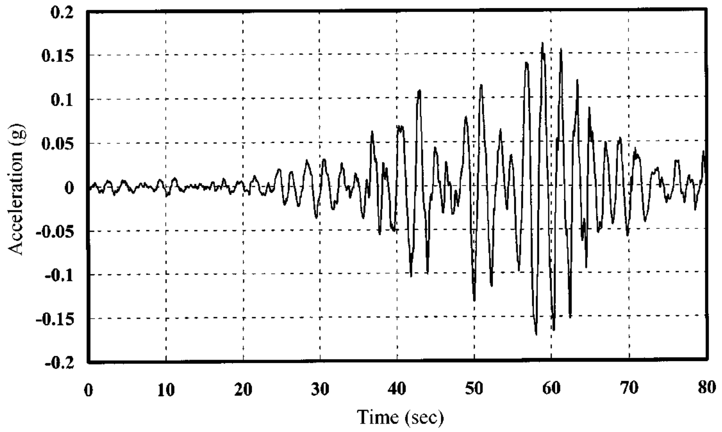
(b) Plastic rotations at plastic hinge B in Fig. 4(c)

Fig. 5 Incremental collapse and alternating plasticity

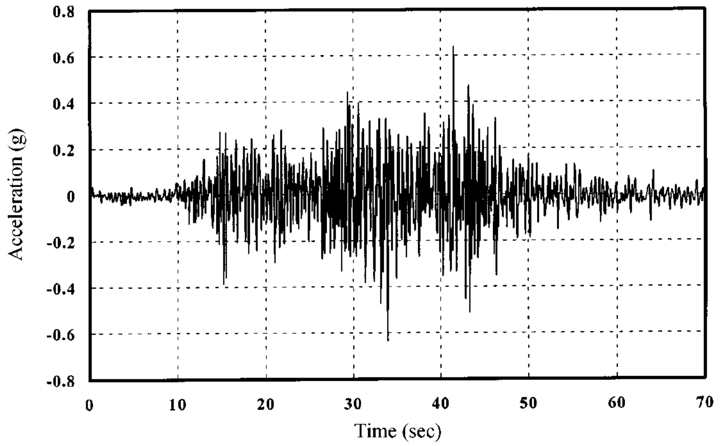
shape that goes from linear elastic behavior to the plastic collapse. This clearly does not represent real earthquake excitation. Therefore, to show that the phenomena of incremental collapse and alternating plasticity occur at the locations predicted by the previous shakedown analysis the example structure was analytically tested by using the program DRAIN-2D (Kanaan and Powell 1973) with two recorded earthquake ground accelerograms; SCT EW component, Mexico City in 1985, and N10E component, LLolleo Chile in 1985. These two accelerograms have quite different characteristics (See Fig. 6). The damping ratio is assumed to be 2% and the hysteretic behavior of the member (critical regions) linear-elastic plastic with a deformation hardening ratio of 1%.

2.3.1 1985 Mexico City earthquake (SCT, EW)

The time histories of the plastic hinge rotation at the right end of the girder at each floor and at the support of the right column are shown in Fig. 7. From this figure, it can be clearly observed that both B and C have undergone the phenomena of incremental collapse while D has never experienced inelastic deformation. The maximum plastic rotation at the right end of the girder at B is shown to be over 0.05 radian, which has almost reached the estimated capacity of 0.06 radian. It



(a) 1985 Mexico City earthquake (SCT, EW component, PGA = 0.172 g)



(b) 1985 Chile LLolleo earthquake (N10E component, PGA = 0.668 g)

Fig. 6 Earthquake ground accelerograms used for time history analyses

is also interesting to note in Fig. 7 that the manner of plastic rotation at the support of the structure (plastic hinge A) is always alternating. Therefore, if the lateral force distribution and the load pattern can be assumed to be similar to the actual one, then the shakedown analysis appears to give a satisfactory prediction on the critical locations of the structure and their probable failure modes regarding inelastic behaviors.

2.3.2 1985 Chile LLolleo earthquake (N10E)

In Fig. 8, the accumulation of plastic hinge rotation by the phenomenon of incremental collapse occurred at B and C while there occurred no plastic rotation at D. The time history of plastic rotation at the right support of the structure (Curve A) in Fig. 8 clearly represents the phenomenon of alternating plasticity. Basically, the same observations as in the case of the Mexico City record can be made for this earthquake ground motion.

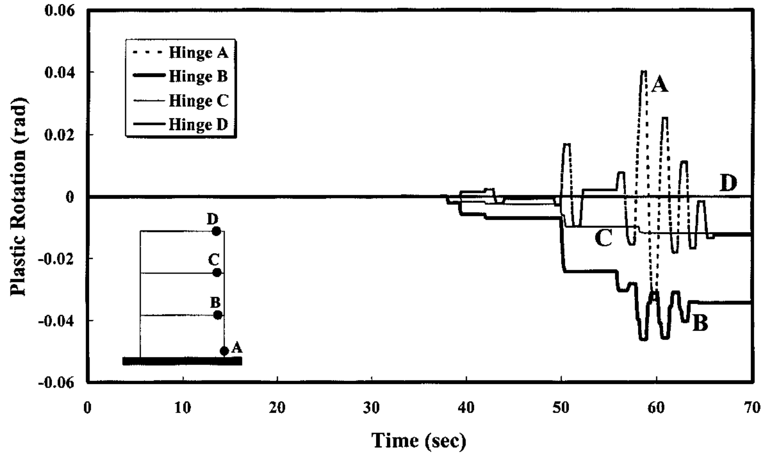


Fig. 7 Time histories of plastic rotation at plastic hinges for Mexico City earthquake (PGA = 0.172 g)

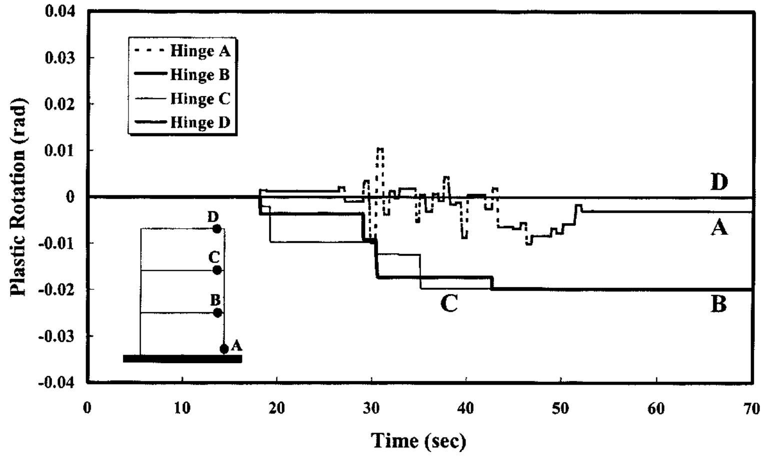


Fig. 8 Time histories of plastic rotation at plastic hinges for Chile earthquake (PGA = 0.54 g)

3. Significance of shakedown analysis technique for seismic design

Shakedown analysis based on the shakedown theorem provides the shakedown load under which a structure eventually behaves completely elastically after undergoing unknown finite plastic rotations. But engineers, who are more concerned with the energy dissipation capacity of structure through inelastic deformations under a severe earthquake, are generally not so interested in the eventual elastic behavior, and thus in the shakedown load itself. Furthermore, as stated before, alternating plasticity is one of the main sources of energy dissipation without causing the rapid exhaustion of plastic rotation capacity at critical regions and therefore, it should be allowed for the minimization of structural weight because as more energy is dissipated, less strength will be required and thus, less material will be used. Also, as pointed out before, incremental collapse should be prevented because it causes explosiveness in the accumulation of plastic rotation which leads to rapid exhaustion of available plastic rotation capacity and excessive permanent deflection.

Therefore, it is desirable to design a structure to have as many plastic hinges of alternating plasticity as possible. From this point of view, the most efficient collapse mode for an alternating load pattern, such as earthquake loading, is the sidesway mechanism of the whole structure. However, in order to ensure this mechanism, it is necessary to use the shakedown analysis technique to predict the incremental collapse load factor. Then, this incremental collapse can be prevented by introducing the constraint of $\lambda_{inc.c.}$ (Incremental collapse) $\geq \lambda_{p.c.}$ (Sidesway mechanism of the whole structure) in the design procedure (Lee and Bertero 1993).

Here, there remains one important problem to overcome. The design against the incremental collapse load needs nonlinear programming, while the analysis requires only linear programming because the elastic moments used in shakedown analysis are proportional to the displacements and moments of inertia (the design variables in case of moment-resisting steel frame) which have nonlinear relations with respect to each other in the elastic equilibrium equations. In the following, the procedure to reach the minimum-weight design using the nonlinear programming will be presented.

4. Minimum-weight design accounting for incremental collapse and serviceability limit states

The problem in applying shakedown analysis to the earthquake-resistant design is the use of nonlinear programming in optimization. Actually, the minimum-weight design under the assumption of the linear elastic behavior of a structure can be achieved only with the nonlinear programming; whereas the minimum-weight plastic design needs linear programming. Therefore, if a structural optimization program can cover the constraints on the linear elastic behavior of the structure, in other words, if it uses the technique of nonlinear programming as an algorithm of optimization, then to extend this program to solve shakedown problems and to incorporate the plastic design concept would not be so difficult. In the following sections, the mathematical model for this nonlinear programming problem specialized for this case will be formulated. However, the algorithm used to solve this problem is the gradient projection method, which has been widely known in the field of optimization (Haug and Arora 1979, Arora 1980, and Bradley *et al*. 1977). Only the formulation of the mathematical model and the resulting design will be presented in this paper.

4.1 Formulation of mathematical model

4.1.1 Design variable and state variable

The behavior of most engineering systems is governed by some law of physics. This behavior is described analytically by a set of variables called *state variables*. For structural systems, state variables may include displacements at certain points, eigenvectors, eigenvalues, etc. In this case, let $\boldsymbol{z} \in R^n$ be a state variable vector representing displacements at key points of structure, and let $\boldsymbol{y} \in R^n$ and ζ represent an eigenvector and the corresponding eigenvalue, respectively.

There is a second set of variables called *design variables* that describes the system. These variables are chosen by the designer and serve to assure specifications for fabrication. The equations that determine the state of structural systems generally depend on the design variables, so these two sets of variables are related. Member thickness, cross-sectional area, moment of inertia, flange, web thickness, and so on can be design variables. Here, let $\boldsymbol{b} \in R^k$ represent a vector of design variables which mean the moment of inertia's in the case of moment-resisting steel structures.

It is necessary to relate the other member properties such as section area, elastic section modulus, and plastic section modulus with the corresponding moment of inertia to control the elastic stresses and the plastic mechanisms. The relations are represented as equations determined by regression of data on commercial wide flange steel sections. For example, the equations for economy beam sections are given in Eq. (1) (Haug and Arora 1979).

$$\left\{\begin{array}{l}\text{Section area}\\ \text{Elastic section modulus}\\ \text{Plastic section modulus}\end{array}\right\} = \begin{Bmatrix} A_i \\ S_i \\ Z_i \end{Bmatrix} = \begin{Bmatrix} 0.58(b_i)^{0.50} \\ 0.58(b_i)^{0.75} \\ 1.15S_i \end{Bmatrix} \tag{1}$$

where b_i means the i-th design variable.

All the dimensions in Eqs. (1) and (2) are given in the units of in.^{2}, in.^{3}, and in.^{4} for A_i, S_i (Z_i), and b_i, respectively (1 in. = 2.54 cm). However, these equations should be changed if different types of steel sections are considered.

4.1.2 Objective function

The objective of structural optimization is assumed to be the minimization of the total weight of structural members. Then, the objective function ψ_0 is represented in Eq. (2) using Eq. (1).

$$\psi_0(\boldsymbol{b}) = \sum_i^k \rho_i L_i A_i = \rho \sum_i^k L_i 0.58(b_i)^{0.50} \tag{2}$$

where ρ_i and L_i are the specific weight (pound per in.^{3}) and the length of members (in.), respectively, corresponding to the i-th design variable.

Here, the specific weight usually has the same value of ρ for all the design variables. In other words, the minimization of weight refers to structural volume.

4.1.3 Constraint type I

The most practical way of analyzing the linear elastic behavior of a large and complex structural system is by using the finite element approach. The governing equilibrium equations for a finite element model of a structure subjected to quasi-static load are:

$$h\ (\boldsymbol{b}, \boldsymbol{z}) = \boldsymbol{K}(\boldsymbol{b})\boldsymbol{z} - \boldsymbol{f} = \boldsymbol{0} \tag{3}$$

where

$\boldsymbol{K}(\boldsymbol{b})$ = an $n \times n$ symmetric nonsingular structural stiffness matrix,

$\boldsymbol{f}$ = an n- vector representing nodal loads for the finite element model.

Using the displacements found from Eq. (3), elastic stresses, σ, at various points of the structure are readily computed. The general mathematical expressions of design constraint on the elastic stresses and deflections are shown in Eqs. (4) and (5), respectively.

$$\sigma_i\ (\boldsymbol{b}, \boldsymbol{z}) \le \sigma_{allow,\, i}\ (\boldsymbol{b}) \tag{4}$$

$$\sum_{j=1}^{n} a_j z_j \le \delta_{allow} \tag{5}$$

where

i: the i-th design variable,

j: the j-th degree of freedom,

a_j: constant for the displacement of j-th degree of freedom (−1 or 0 or 1),
d_{allow}: allowable limit on deflection or interstory drift.

4.1.4 Constraint type II

Since the incremental collapse load factor, $\lambda_{inc.\,c.}$, is dependent on elastic moments at plastic hinges, this factor is the function of both the design variable vector, $\boldsymbol{b}$, and the state variable vector, $\boldsymbol{z}$. However, $\lambda_{p.c.}$ corresponding to the sidesway mechanism of the whole structure is the function of the design variable vector, $\boldsymbol{b}$, only. Therefore, to make the incremental collapse load factor, $\lambda_{inc.c.}$, larger than that of the sidesway mechanism of the whole structure, $\lambda_{p.c.}$, the constraint will be as follows:

$$\lambda_{inc.c.}\ (\boldsymbol{b}, \boldsymbol{z}) \geq \alpha\ \lambda_{\mathrm{p.c.}}\ (\boldsymbol{b}) \tag{6}$$

where α ($\geq$1) is the uncertainty factor accounting for the variation in the lateral force distribution during earthquake ground shakings and the discreteness in the properties of commercial sections.

The soft-story collapse load factor, $\lambda_{s.s.}$, and the load factor of the sidesway mechanism of the whole structure, $\lambda_{p.c.}$, are functions of design vector, $\boldsymbol{b}$, only. Therefore, to prevent the soft-story collapse mechanism use:

$$\lambda_{\mathrm{s.s.}}\ (\boldsymbol{b}) \geq \alpha\ \lambda_{\mathrm{p.c.}}\ (\boldsymbol{b}) \tag{7}$$

where α is an uncertainty factor which takes into account the real lateral force distribution and the discrete properties of commercial steel sections.

4.2 Example constraint equations for the one-bay three-story structure

Now, the one-bay three-story example structure will be used again to demonstrate how the constraint type II can be implemented:

(1) $\lambda_{inc.c.}$ of the girder at the second floor, whose structural geometry is shown in Fig. 9(a), can be calculated by the shakedown analysis. To obtain $\lambda_{inc.c}$, we calculate the load factor λ satisfying the following equation:

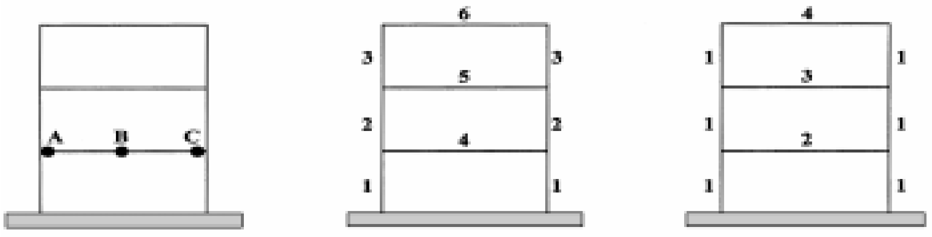
$$\sum_j \begin{Bmatrix} \lambda M_j^{\max} \\ \lambda M_j^{\max} \end{Bmatrix} \theta_j = \sum_j M_{pj} |\theta_j|$$

$M_j^{\max}$ and $M_j^{\min}$ refer to the elastic moments at the location of the plastic hinge j when the structure is subjected to an earthquake load, and where $\lambda M_j^{\max}$ is taken when θ_j is positive and $\lambda M_j^{\min}$ when θ_j is negative. However, the terms $\lambda M_j^{\max}$ and $\lambda M_j^{\min}$ in the left parenthesis should be modified for the load pattern of the fixed gravity load and the variable repeated lateral earthquake load as follows:

$$\sum_j \begin{Bmatrix} M_{j,G}^{\max} + \lambda_{inc.c.} M_{j,EQ}^{\max} \\ M_{j,G}^{\min} + \lambda_{inc.c.} M_{j,EQ}^{\min} \end{Bmatrix} \theta_j = \sum_j M_{pj} |\theta_j| \tag{8}$$

where subscripts G and EQ denote the gravity and earthquake load, respectively. In the case of the second-floor girder as shown in Fig. 9(a), Eq. (8) can be converted to Eq. (9).

$$(M_{A,G} + \lambda_{inc.c.} M_{A,EQ}^{\min})(-\theta) + (M_{B,G} + \lambda_{inc.c.} M_{B,EQ}^{\max})(2\theta) + (M_{C,G} + \lambda_{inc.c.} M_{C,EQ}^{\min})(-\theta) = 4M_p\theta$$



(a) Definition of plastic hinges (b) Definition of design variables in design case I to III (c) Definition of design variables in design case IV to V

Fig. 9 Definitions in an example of constraint on incremental collapse

or

$$\lambda_{inc.c.}(-M_{A,EQ}^{\min}+2M_{B,EQ}^{\max}-M_{C,EQ}^{\min})+(-M_{A,G}+2M_{B,G}-M_{C,G})=4M_p \tag{9}$$

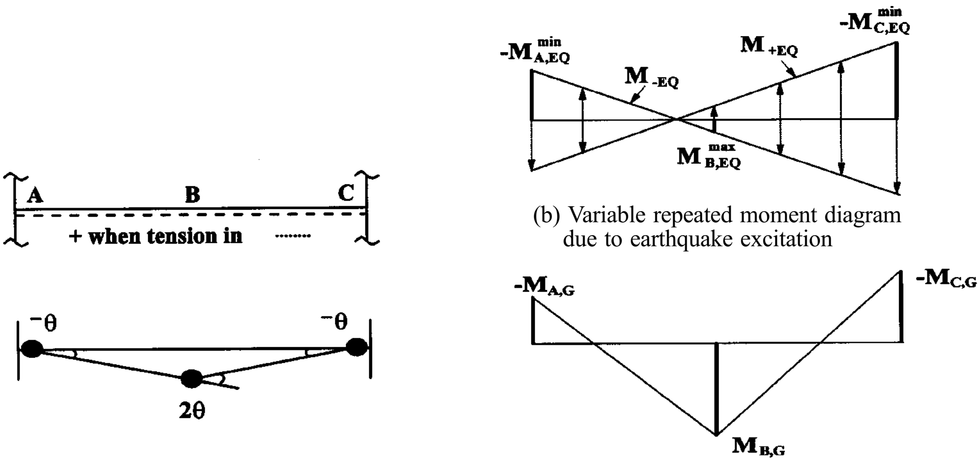
where the definitions of all the moments ($M_{A,G}$, $M_{B,G}$, $M_{C,G}$, $M_{A,EQ}^{\min}$, $M_{B,EQ}^{\max}$ and $M_{C,EQ}^{\min}$) and the sign convention in this particular case are given in Fig. 10.

(2) Load factor $\lambda_{p.c.}$ corresponding to the sidesway mechanism of the whole structure as shown in Fig. 11(a) is given as follows:

$$\lambda_{p.c.}=\frac{2M_{p,1}+2M_{p,3}+2M_{p,4}+2M_{p,5}}{16.2(3.66)+32.3(7.32)+39.2(10.98)} \tag{10}$$

with the constraint $M_{P,6}\geq M_{P,3}$. Here the subscripts $1, 2,\ldots, 6$ define the linked design variables in the example structure in Fig. 9(b) and the denominator equals the external work done by the earthquake load.

(3) Substitute $\lambda_{p.c.}$ in Eq. (10) to $\lambda_{inc.c.}$ in Eq. (9) and put the α factor in front of $\lambda_{p.c}$ and change = in Eq. (9) to ≤, then the Eq. (11), equivalent to Eq. (6), will be obtained as follows:



(a) Sign convention for moment and plastic rotation

(b) Variable repeated moment diagram due to earthquake excitation

(c) Constant moment diagram for gravity load

Fig. 10 Definition of moments and plastic rotations in Eq. (9)

$$\alpha\lambda_{p.c.}(-M_{A,EQ}^{\min}+2M_{B,EQ}^{\max}-M_{C,EQ}^{\min})+(-M_{A,G}+2M_{B,G}-M_{C,G})\leq 4M_p \tag{11}$$

It should be noted that this constraint equation has implicit nonlinearity with respect to the design variables.

(4) $\lambda_{s.s.}$ can be obtained for each soft story mechanism. For example, $\lambda_{s.s.}$ corresponding to the soft story mechanism as shown in Fig. 11(b) can be calculated as follows:

$$\lambda_{s.s.}=\frac{2M_{p,1}+2M_{p,2}+2M_{p,4}}{16.2(3.66)+32.3(7.32)+39.2(7.32)} \tag{12}$$

Therefore, it can be observed that Eq. (7) is just a linear contribution of design variables when Eq. (10) and (12) are substituted to Eq. (7).

4.3 Minimum-weight design using developed optimization program

The mathematical model established in the above can be solved by using a nonlinear programming technique which consists of the design sensitivity analysis and the gradient projection method (Arora 1980). The program OPTIMUM was developed by the writer and the details of the algorithm are explained in the reference (Lee 1989). The same geometry and load conditions as in the previous example structure are used and the imposed constraints are as follows:

Constraint type I: to control the deflections and stresses under service level of load.

(1) Moment: $f_s \leq 0.66F_y$ (compact section)

(2) Deflection: Interstory drift ratio ≤ 0.0025

Constraint type II: to ensure the sidesway mechanism of the whole structure.

(3) Soft story failure load factor $\lambda_{s.s.} \geq 1.1\ \lambda_{p.c.}$

(4) Incremental collapse load factor $\lambda_{inc.c.} \geq 1.1\ \lambda_{p.c.}$

where $\lambda_{p.c.}$ is the instantaneous collapse load factor of the sidesway mechanism of the whole structure.

It should be noted that constraints (3) and (4) have the uncertainty factor, 1.1, which takes into account the uncertainty of the lateral force distribution and the discreteness in the properties of the commercial steel sections, so to ensure the prevention of the soft story failure and the incremental collapse.

As expected, when the number of design variables is reduced from six to four [see Fig. 9(b) and (c)], the volume of structure increases as shown in Table 2. Also, as the number of constraints increases, the volume also increases. In this particular case, the deflection constraints (2) only (case II in Table 2) are more restrictive than the moment constraints (1) only (case I in Table 2). However, the constraint for the prevention of the incremental collapse turns out to be inactive in the case V design as indicated in Table 3(b).

The shadow price (Bradeley *et al.* 1977) as shown in Table 3(b) implies the amount of decrease in the structural volume which could be expected if the normalized constraints were released by one unit, though these values make sense only within the small subspace in the design space where the linearization of nonlinear functions could be acceptable. The shadow price of the fourth constraint, that is, the constraint which ensures that 1.1 times the load factor of the sidesway mechanism of the whole structure shown in Fig. 11(a) should be smaller than that of the failure mode shown in Fig. 11(b), is three times as much as the second largest shadow price. This means that the fourth constraint is the most restrictive design consideration. From Table 3(b), designers can identify which design constraints participate in the final optimum design and which design constraint is the most controlling

Table 2 Volumes of minimum-weight designs with different constraints and numbers of design variables

Case	No. of design variables	Imposed constraints	Volume of structure (m^3)
0	2	Strength only (plastic design)	0.2870
I	6*	(1)	0.3504
II	6*	(2)	0.3917
III	6*	(1) + (2)	0.3979
IV	4⊥	(2)	0.4071
V	4⊥	(1) + (2) + (3) + (4)	0.4518

*See Figure 9(b) for the definition of design variables.
⊥ See Figure 9(c) for the definition of design variables.

Table 3 Minimum-weight design for case *V* in Table 2

(a) Section properties of minimum-weight design

Design Variable*	Moment of inertia (mm^4) obtained by OPTIMUM	Actually Selected Section	Moment of inertia (mm^4) of Selected Section
1	371.2×10^6	W18 × 55	370.4×10^6
2	343.7×10^6	W18 × 50	333.0×10^6
3	289.8×10^6	W18 × 46	296.4×10^6
4	171.8×10^6	W18 × 35	212.3×10^6

*See Fig. 9(c) for the definition of design variables.

(b) Active constraints at final stage of optimization

Content of constraint	Shadow price (m^3)
Moment of design variable 3 for D + L + EQ ≤ $0.66M_y$	0.1158
Moment of design variable 4 for D + L + EQ ≤ $0.66M_y$	0.0756
I.D.I. of second story ≤ 0.0025	0.0660
$1.1\ \lambda_{p.c.} \le \lambda_{s.s}$*	0.3655

*See Figure 11 for definition of $\lambda_{p.c.}$ and $\lambda_{s.s.}$.

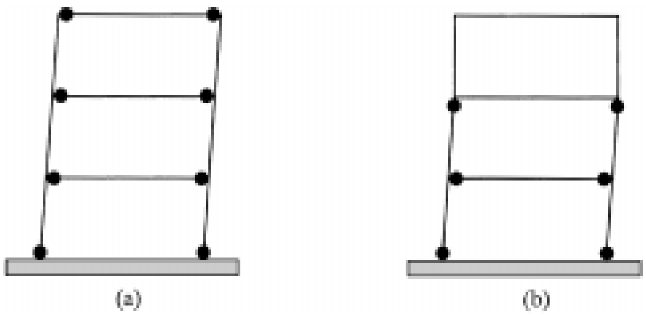
one. In other words, it is possible to analyze the design from the economic point of view.

4.4 Reliability analysis by realistic earthquake ground motions

One of the most important tasks of this study is to show that the structure designed by the proposed concepts and the algorithm behaves as desired under the realistic conditions of earthquake excitation. Thus, the same types of earthquakes that were used to show the phenomena of incremental collapse have been adopted to test the behaviors of the design case *V* as shown in Table 3(a).

4.4.1 1985 Mexico City earthquake (SCT, EW)

(See Fig. 12) The structure resists 0.50 g of peak ground acceleration (PGA). Considering that the previous example structure resisted, at most, 0.172 g of PGA, the case *V* design is about three times more efficient than the original design from the standpoint of strength with the increase of structural



(a) $\lambda_{\text{p.c.}} = 4.072$ (b) $\lambda_{\text{s.s.}} = 4.479$

Fig. 11 Collapse modes and load factors in the fourth constraint in Table 3(b):

volume being only 57%. This is because the plastic hinges are distributed over the whole structure and; therefore, the whole capacity of the structure can be mobilized to resist the critical earthquake ground acceleration. On the contrary, it should be noted that in the case of the previous design, only a few members develop the maximum resistant capacity through inelastic deformations, while the other members remain intact or behave elastically as shown in Fig. 7.

Another observation is that the structure dissipates the earthquake input energy by alternating plasticity at every plastic hinge. Thus, there is neither excessive accumulation of plastic rotation nor deflection instability. This earthquake is characterized by a few cycles of high acceleration for the duration of 10 seconds (55 to 65 seconds). Hence, the significant inelastic behavior was conspicuous only in this critical range of time. It is obvious from Fig. 12 that the structure forms the sidesway mechanism of the whole structure, around, at $t = 58$ seconds. The maximum drift index of the roof with respect to the height of the structure is 0.032 which is twice as large as the 0.015 limit given by ATC 3-06 (ATC 1978).

4.4.2 1985 Chile LLolleo earthquake (N10E)

(See Fig. 13) The major difference between the Chile LLolleo earthquake and the Mexico City earthquake is that the predominant period of the former is much shorter than that of the latter; and therefore, the input energy per load cycle of the Chile LLolleo earthquake is smaller than that of Mexico City earthquake, though the number of cycles of significant acceleration in the Chile LLolleo earthquake is much larger than that of the Mexico City earthquake.

Hence, the inelastic structural response reflects these characteristics of the Chile LLolleo earthquake very clearly. The number of reversals in plastic rotations (Fig. 13) is much larger than that of the Mexico City earthquake (Fig. 12). However, the amount of plastic rotation per cycle is smaller than that of the Mexico City earthquake, even though the applied PGA of the Chile LLolleo earthquake (1.0 g) is much higher than that of the Mexico City earthquake (0.50 g).

Similarly, the structure dissipates the input earthquake energy through alternating plasticity at all plastic hinges; also, the phenomenon of incremental collapse which occurred with the applied PGA of 0.54 g in the original design does not occur with the applied PGA of 1.0 g in the design case V at all. The maximum drift index of the roof with respect to the height of the structure is 0.019, and thus the limit value, 0.015, allowed by ATC 3-06 is exceeded.

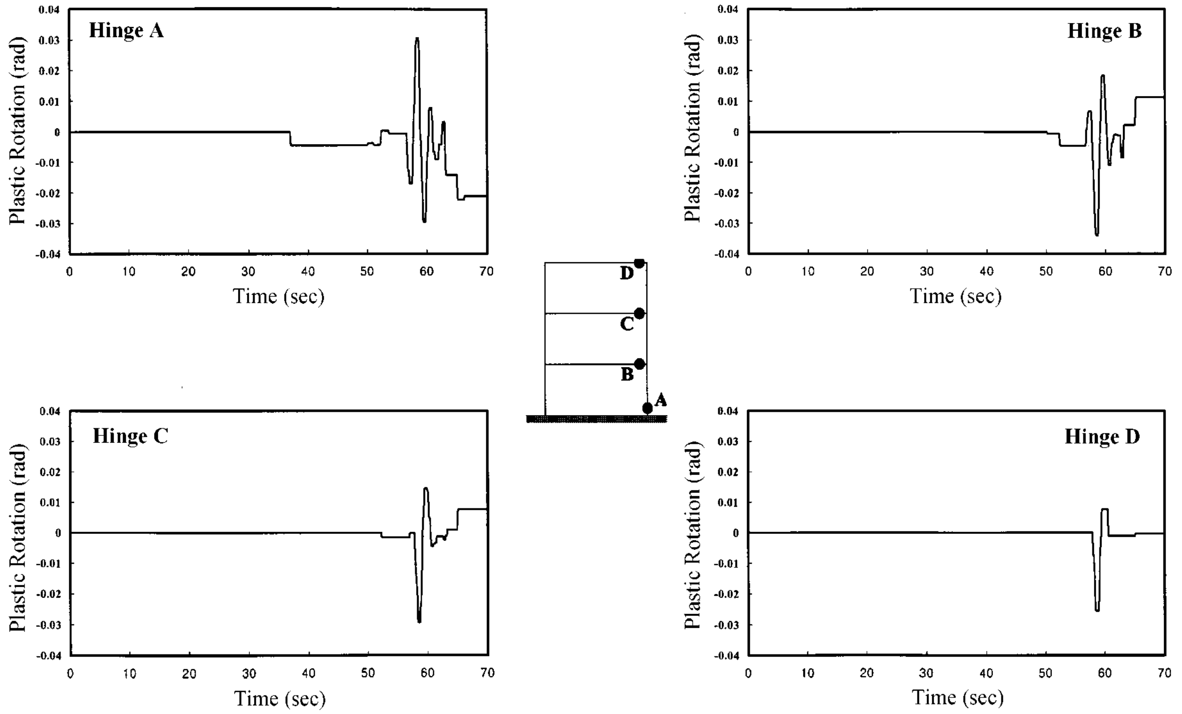


Fig. 12 Time histories of plastic rotations at plastic hinges in the design case V for Mexico City earthquake (PGA=0.5 g)

5. Conclusions

From the view point of earthquake engineering, the most efficient collapse mode is the sidesway mechanism of the whole structure. However, this mechanism cannot be guaranteed since it is impossible to prevent the excessive accumulation of plastic rotations at some plastic hinges, caused by incremental collapse, when the design procedures, which do not usually consider these shakedown problems directly, are adopted. In this paper, it has been demonstrated through a simple example that the excessive accumulation of plastic rotation at certain plastic hinges due to the incremental collapse can be prevented by utilizing the techniques of shakedown analysis and nonlinear programming and; therefore, the sidesway mechanism of the whole structure can be ensured.

Since the same technique of nonlinear programming used for the prevention of the incremental collapse can be applied to the constraints of serviceability limit states such as the limits on the allowable stresses and the allowable deflections, which are generally imposed in the ordinary design procedures, it is feasible in the proposed design approach to deal with the requirements of the serviceability limit states and the ultimate (collapse) limit state simultaneously.

6. Further research needs

Considering that the applied PGA's in earthquake ground motions are unrealistically high in the above demonstration, the control of the maximum drift of the structure subjected to rare extreme

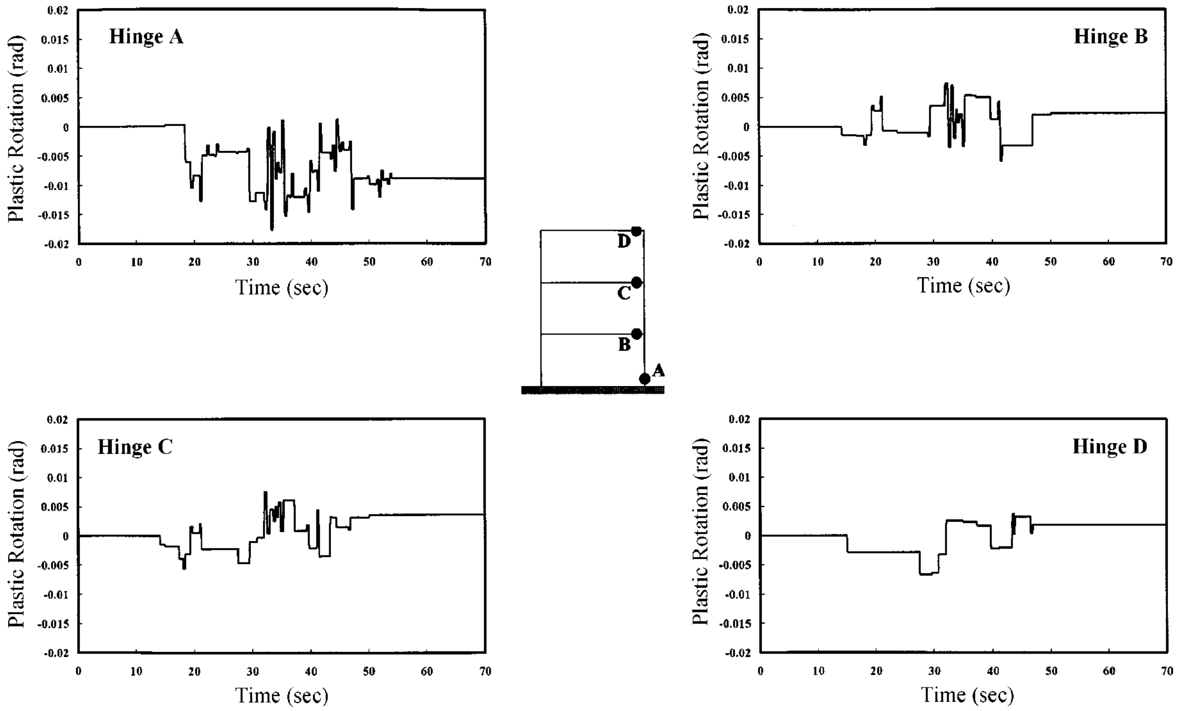


Fig. 13 Time histories of plastic rotations at plastic hinges in the design case *V* for Chile Lolleo earthquake (PGA = 1.0 g)

earthquake ground motions should be conducted in association, not only with the strength, stiffness and energy dissipation capacity of the structure, but also with the characteristics of expected earthquake ground shakings themselves. Therefore, this topic needs additional study as follows:

(1) Determination of design earthquake: The critical response of the structure depends on the characteristics of applied earthquake ground shakings. In particular, the maximum drift and the required strength of the structure are highly dependent on the intensity, frequency content, and duration of strong motion of the applied earthquake ground shakings. On the other hand, failure by low cycle fatigue, if it happens at all, is mainly caused by the intensity and long duration of strong ground excitations which are random in character and can be different even at the same site. The determination of these characteristics for the design of the structure requires a statistical analysis of recorded earthquake ground motions considering geological and tectonic conditions.

(2) Determination of critical lateral load distribution for shakedown analysis: In this design approach, the lateral load distribution representing the earthquake load is assumed to be constant from elastic behavior to the ultimate plastic collapse of a structure. This assumption clearly does not describe the real excitation, particularly for the high-rise building structures. Although the critical shapes of lateral load distributions can vary for each component of a structure, simplified force distributions, which should be as realistic as possible, are needed.

(3) Alternating plasticity as a limit state: In this design approach, alternating plasticity has been assumed as a desirable structural behavior as far as energy dissipation and deflection stability are concerned. However, it is necessary to clarify the allowable limits on the number of yield reversals and yield deformations concerning lateral and local buckling, and the possible low cycle fatigue at

the critical regions, usually located at the beam-column and base connections. Thus, if these limits were known, then it may be possible to limit the level of damage which an earthquake can cause to a structure with respect to hysteretic energy dissipation.

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Definition of terminology

Terminology frequently used in this paper is defined in this appendix for clarity.

Shakedown phenomenon (Massonnet and Save 1965): Individual loads or groups of loads may vary independently between limiting intensities that may involve the reversal of their sense of action. The limiting load intensity for repeated and reversed loading of these kinds may well be smaller than the limit load for simple proportional loading to plastic collapse. The structure would then fail in one of the following ways:

(1) Plastic rotations may accumulate during successive cycles of loading and may accumulate to cause failure by progressive plastic rotation, which is termed *incremental collapse*.

(2) No large overall deformations will develop because a plastic rotation in one sign during the first half of a cycle is more or less canceled by a plastic rotation in the opposite sign during the second half. However, These repeated plastic rotations of alternating sign are likely to cause a fatigue type of failure, sometimes called *low cycle fatigue*. However, in this paper the term *failure by alternating plasticity* is used.

By choosing a sufficiently small load factor, we may rule out failure by incremental collapse or alternating plasticity, but this need not mean that no plastic rotations will occur when the loads vary within the ranges corresponding to the chosen value of this factor. The amounts of plastic rotation that occur in consecutive cycles of loading may decrease as the terms of a convergent infinite series, or plastic rotation may stop altogether after the first cycle or the first few cycles of loading. In either case, the structure is said to *shake down*. The determination of the shakedown factor, that is, the greatest value of the load factor for which the structure will shakedown involves *shakedown analysis* using fundamental theorems (Hodge 1959, Horne 1979, Massonnet and Save 1965, Neal 1977).

Design limit state (Macgregor 1997): When a structure or structural element becomes unfit for its intended use, it is said to have reached a *limit state*. The limit states for general structures can be categorized into two basic groups:

(1) *Ultimate limit states*: These involve a structural collapse of part or all of the structure. The major ultimate states include the formation of a plastic mechanism and the rupture of critical parts of the structure leading to partial or complete collapse and so on.

(2) *Serviceability limit states*: These involve disruption of the functional use of the structure but not collapse per se. One of the major serviceability limit states is the excessive deflections for

normal service.

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