Structural Engineering and Mechanics, Vol. 13, No. 1 (2002) 1-16
DOI: http://dx.doi.org/10.12989/sem.2002.13.1.001

Connections of sleeve joint purlin system

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Abstract. This paper presents the findings of an investigation carried out to determine the most appropriate connections, in terms of rotational stiffness, to use for the optimum design of cold-formed Zed section sleeve joint purlin system. Experiments and parametric studies were conducted to investigate the effects of geometric variables on the behavior of the sleeve-purlin and cleat-purlin connections of the sleeve joint purlin system. The variables considered were purlin size and thickness, sleeve size, thickness, length and bolt position. The test results were used to verify the empirical expressions, developed herein, employed to determine the rotational stiffness of connections. With the predicted connection stiffness, the most suitable sleeve-purlin and cleat-purlin connections can be selected so as to produce an optimum condition for the sleeve joint purlin system.

Key words: connection; cleat-purlin; cold-formed; joint; optimum design; purlin; rotational stiffness; sleeve; sleeve-purlin; Zed section.

1. Introduction

A typical roof construction in a low-rise building usually consists of components such as portal frame and roof structure. Fig. 1 shows a one-span assembly, commonly known as a sleeve joint purlin system, of a typical roof structure. The sleeve joint purlin system usually consists of two or more thin-walled cold formed purlins overlapping each other or connected with cleats and sleeves by bolts, e.g., cleat-purlin connection and sleeve-purlin connection, as shown in Fig. 1.

Better structural performance resulting from a more evenly distributed moment over the span can be achieved by utilizing cleat-purlin and sleeve-purlin connections of appropriate rotational stiffness. Although major codes of practice such as AISI (1986) and BS 5950 Part 5 (1987) provide information and guidance on the use of above connections, little details are given. In an attempt to provide more specific details, Bryan (1993) developed a set of design equations to characterize the effects of simple lap-bolted connections for cold-formed flat members only.

In this present investigation, experiments and parametric studies were carried out to determine the effects of geometric variables on the sleeve joint purlin systems. The variables considered were purlin size (depth of purlin) and thickness, sleeve size, thickness, length and bolt position. The test results were used to verify the empirical expressions, developed herein, employed to determine the rotational stiffness of the standard size Zed section cold-formed cleat-purlin and sleeve-purlin connections. With the predicted connection rotational stiffness, the most suitable connection can be

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selected so as to produce an optimum condition for the sleeve joint purlin system. It should be noted that all the purlins, sleeves and cleats used in this investigation are of standard sizes used in the industries.

2. Experimental investigation

The objective of the experimental investigation is to determine the rotational stiffness of cleat-purlin and sleeve-purlin connections and to study the effects of parameters, such as thickness and size of purlin, thickness, size and length of sleeve and position of the bolts, on the rotational stiffness of the connections.

Table 1 Sleeve-purlin connection test configuration

Test No.	Purlin	Purlin	Sleeve	Remarks
SPT1	Z15016/1	Z15016/2	S15020/1	Purlin thickness comparison:
SPT2	Z15020/1	Z15020/2	S15020/2	Purlin thickness of 1.6 mm,
SPT3	Z15025/1	Z15025/2	S15020/3	2.0 mm and 2.5 mm.
SPS1*	Z15020/1	Z15020/2	S15020/2	Purlin size comparison:
SPS2	Z20020/1	Z20020/2	S20020/1	Purlin size of 150 mm,
SPS3	Z25020/1	Z25020/2	S25020/1	200 mm and 250 mm.
SST1	Z15020/3	Z15020/4	S15016/1	Sleeve thickness comparison:
SST2*	Z15020/1	Z15020/2	S15020/2	Sleeve thickness of 1.6 mm,
SST3	Z15020/5	Z15020/6	S15025/1	2.0 mm and 2.5 mm.
SSL1	Z15020/7	Z15020/8	S15020/4	Sleeve length comparison:
SSL2*	Z15020/1	Z15020/2	S15020/2	Sleeve length of 900 mm, 600 mm
SSL3	Z15020/9	Z15020/10	S15020/5	and 300 mm.
SBP1*	Z15020/1	Z15020/2	S15020/2	Bolt position comparison: Bolt near
SBP2	Z15020/11	Z15020/12	S15020/6	upper flange (normal position), lower
SBP3	Z15020/13	Z15020/14	S15020/10	flange and at middle of web.

*Same test

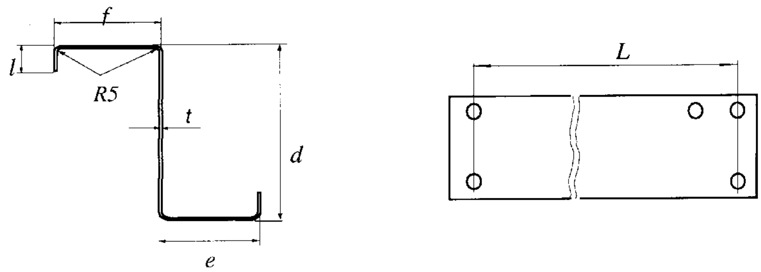
Table 2 Cleat-purlin connection test configuration

Test No.	Purlin	Remarks
CPT1	Z15016/3	Purlin thickness comparison:
CPT2*	Z15020/15	Purlin thickness of 1.6 mm,
CPT3	Z15025/3	2.0 mm and 2.5 mm.
CPS1	Z10020/1	Purlin size comparison:
CPS2*	Z15020/15	Purlin size of 100 mm, 150 mm,
CPS3	Z20020/3	200 mm and 250 mm.
CPS4	Z25020/3	

*Same test

Test series SPS was carried out to investigate the influence of purlin size on the rotational stiffness and behavior of the sleeve-purlin connection. The details of the sleeve-purlin connection test are shown in Table 1. In the test number, the first alphabet S or C indicates the type of connection, i.e., sleeve-purlin connection or cleat-purlin connection respectively. The second

Table 3 Dimensions of purlins

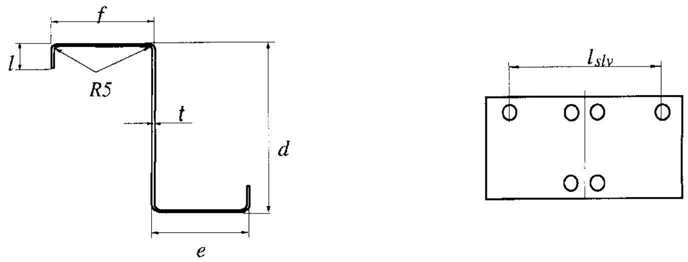


Specimen Number	*d* (mm)	*e* (mm)	*f* (mm)	*l* (mm)	*L* (mm)	*t* (mm)
Z10020/1	103.5	45.48	48.51	20.03	928.4	2.042
Z15016/1	152.3	61.72	67.51	18.29	930.5	1.655
Z15016/2	153.0	58.47	68.95	21.31	931.7	1.650
Z15016/3	151.8	57.93	68.42	22.58	933.4	1.652
Z15020/1	152.6	59.23	67.54	20.37	930.8	2.053
Z15020/2	152.1	60.22	65.33	19.01	932.5	2.041
Z15020/3	152.3	60.07	67.82	19.33	934.3	2.050
Z15020/4	152.5	58.92	66.42	19.57	928.7	2.048
Z15020/5	153.1	59.12	67.87	18.59	929.3	2.048
Z15020/6	153.6	60.35	68.10	21.06	931.5	2.049
Z15020/7	151.3	58.13	67.09	20.37	933.1	2.050
Z15020/8	152.8	61.76	66.55	18.88	931.7	2.054
Z15020/9	152.5	60.32	66.18	18.26	930.4	2.052
Z15020/10	151.3	59.61	67.07	19.41	930.6	2.048
Z15020/11	153.1	59.95	66.92	19.60	929.8	2.052
Z15020/12	153.9	61.33	65.97	21.57	933.2	2.050
Z15020/13	152.7	60.60	66.33	20.31	932.4	2.048
Z15020/14	154.1	58.47	68.02	20.43	930.5	2.049
Z15020/15	153.2	59.92	66.17	19.99	934.1	2.047
Z15025/1	153.9	61.72	67.15	15.21	932.7	2.537
Z15025/2	155.7	60.65	66.32	16.45	930.4	2.538
Z15025/3	155.2	61.54	65.99	15.68	931.1	2.535
Z20020/1	202.2	76.60	78.96	19.39	928.4	2.015
Z20020/2	203.1	75.38	79.65	20.52	931.2	2.014
Z20020/3	204.1	75.46	79.13	19.73	933.4	2.010
Z25020/1	250.6	72.63	81.43	23.59	931.4	2.024
Z25020/2	253.4	72.63	81.03	22.42	931.4	2.022
Z25020/3	251.9	72.63	80.56	23.67	931.4	2.029

alphabet P, S or B indicates the type of member, i.e., purlin, sleeve or bolt, respectively while the third alphabet denotes the variable under investigation. Size is denoted by S, thickness by T, length by L and bolt position by P. The numeral following the alphabets indicates the order of each test with respect to the variable to be investigated. Purlins and sleeves are also assigned a number to indicate the particular purlin or sleeve used. The test consists of five groups, i.e., comparison of purlin thickness, purlin size, sleeve thickness, sleeve length and bolt position.

Test series CPS was designed to investigate the effects of purlin size on the rotational stiffness of cleat-purlin connection. The details of the cleat-purlin connection tests are presented in Table 2. Two groups of cleat-purlin connection tests were designed to study the effects of purlin size and

Table 4 Dimensions of sleeves



Specimen Number	d (mm)	e (mm)	f (mm)	l (mm)	l_{slv} (mm)	t (mm)
S15016/1	152.6	58.02	67.73	18.27	931.4	1.650
S15020/1	152.4	60.38	66.43	20.81	932.6	2.053
S15020/2	153.9	60.45	65.78	21.32	930.8	2.047
S15020/3	154.2	59.72	66.11	19.05	930.2	2.057
S15020/4	153.2	58.41	67.32	19.34	932.5	2.052
S15020/5	151.9	60.78	66.08	20.71	931.7	2.051
S15020/6	152.5	59.22	65.46	18.97	930.1	2.053
S15020/7	153.1	59.68	66.96	19.32	929.8	2.048
S15025/1	155.5	60.34	67.10	17.24	931.3	2.533
S20020/1	202.6	73.74	79.68	20.41	933.1	2.024
S25020/1	252.7	73.12	80.96	23.85	931.6	2.000

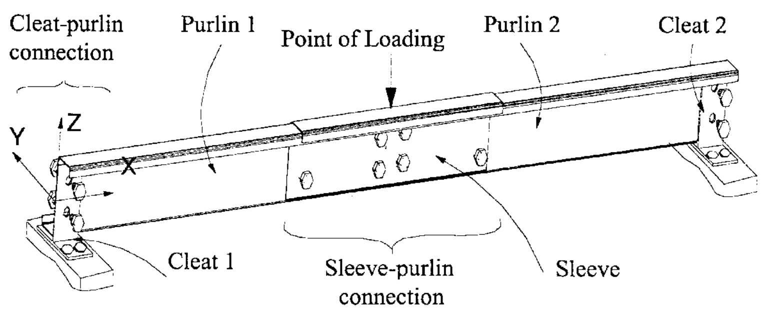


Fig. 1 Sleeve-purlin test setup

thickness on the rotational stiffness of the cleat-purlin connections. The length and cross-sectional dimensions of the purlins and sleeves are tabulated in Tables 3 and 4 respectively.

The basic setup for the sleeve-purlin connection test is shown in Fig. 1. For each purlin, one end was bolted to the cleat while the other was bolted to the sleeve. To simulate the simple support condition at the two ends, the bolts at the cleat-purlin connections were "hand" tightened. All other bolts were fully tightened with a torque of 800 Nm as recommended by the supplier of the purlins and sleeves. A loading attachment at the mid-span allows a vertical concentrated load to be applied by means of a servo-controlled hydraulic system. The loading rate was set to 10 mm/min. Another major function of the loading attachment was to prevent lateral buckling and twisting of the sleeve and purlins during testing. The applied load and corresponding vertical deflection was recorded. Loading was increased till ultimate failure occurred.

In the cleat-purlin connection test, only one purlin and cleat were used. The assembled specimens consisted of a purlin bolted to a cleat in a cantilever type arrangement. The single bolt at the free end of the purlin was "hand" tightened to allow rotation and transfer of the vertically applied concentrated load from the loading attachment to the purlin without causing any restraint moment. The test procedure was similar to the sleeve-purlin connection test.

3. Experimental results and discussion

3.1 Cleat-purlin connection

Fig. 2(a) shows the load-deflection plot for purlin size of 100 mm, 150 mm, 200 mm and 250 mm. It can be seen that the slip load increased from 0.7 kN to 2 kN when larger purlin size was used. The slip varied greatly from about 15 mm for CPS4 to about 100 mm for CPS1. The large initial deflection in the latter was due to large initial rotation of the purlin relative to the cleat at the connection. This effect resulted in contact between the bolt heads at the base plate (for bolting of cleat) and the lower flange of the purlin before the bolts at the cleat-purlin connection slipped into bearing and consequently increasing the rigidity of the connection. At a constant load of 2.5 kN, the deflection in CPS1 was three times that of CPS4. Therefore, purlin size has a significant effect on

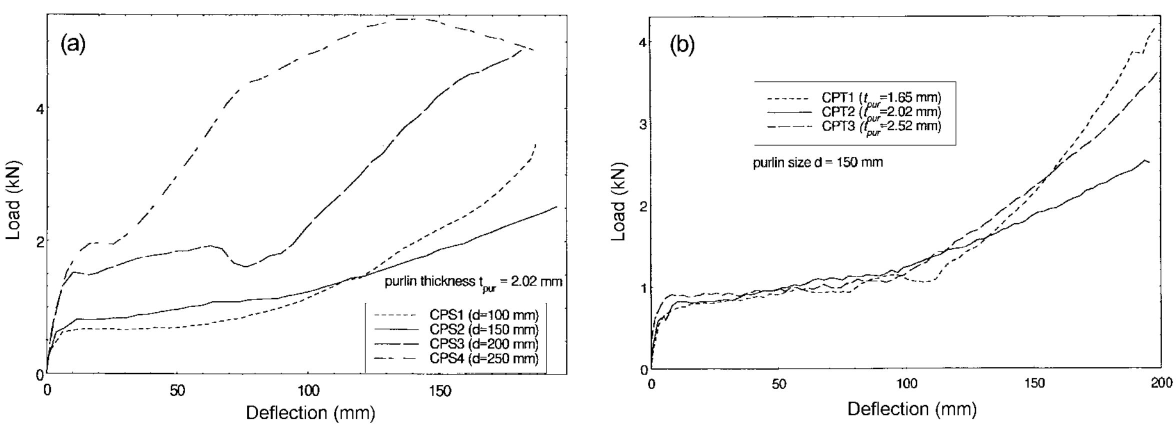


Fig. 2 (a) Effect of purlin size on cleat-purlin connection, (b) effect of purlin thickness on cleat-purlin connection the behavior of cleat-purlin connection.

In test series CPT, three connections of similar purlin size but different thickness of 1.6 mm, 2.0 mm and 2.5 mm, were tested to determine the influence of purlin thickness on the rotational stiffness of the cleat-purlin connection. The load-deflection curves are shown in Fig. 2(b). The slip load and initial deflection were almost identical for all the three tests. In tests CPT1 and CPT3, the bolt heads at the base plate were observed to be in contact with the lower flange of the purlin after the latter had slipped into bearing. The end result was an increase in the rotational stiffness of the connection. At a constant load of 2 kN, the difference in the deflection of the three tests were comparatively small. Thus, the influence of purlin thickness on the rotational stiffness of the cleat-purlin connection is very much less significant compared to purlin size.

Two failure modes were observed in the cleat-purlin connection tests and these were cleat failure and bolt hole failure. Cleat failure was observed to have taken place in tests with bigger purlin sizes (200 mm or 250 mm). This was most probably due to the serious lateral bending of cleats. Bolt hole failure occurred when purlin size of 100 mm or 150 mm was used, i.e., in tests CPS1, CPS2, CPT1 and CPT3. In these cases, the purlin remained undamaged as a whole except for failure at the bolt holes due to high shearing stresses.

3.2 Sleeve-purlin connection

The load-deflection results of the purlin size comparison test are shown in Fig. 3a. The curves reveal a constant trend in which the connection rotational stiffness increases while the deflection at the mid-span decreases with increase in purlin web size. Due to weakness of the cleats in tests SPS2 and SPS3, unexpected failure of the cleats occurred in the course of the tests. To fully utilize the loading capacity of purlin with large web size, such as in tests SPS2 and SPS3, stronger and stiffer cleats have to be used.

In test series SSL, the sleeve length was varied from 300 mm in SSL1 to 900 mm in SSL3. Fig. 3b shows the load-deflection curves for the three tests. It can be seen that a very significant increase in the system stiffness when longer sleeve was used, especially from 300 mm to 600 mm. The results tend to suggest that the rotational stiffness of the sleeve-purlin connection would increase by about 150% if the sleeve length is increased by 100%.

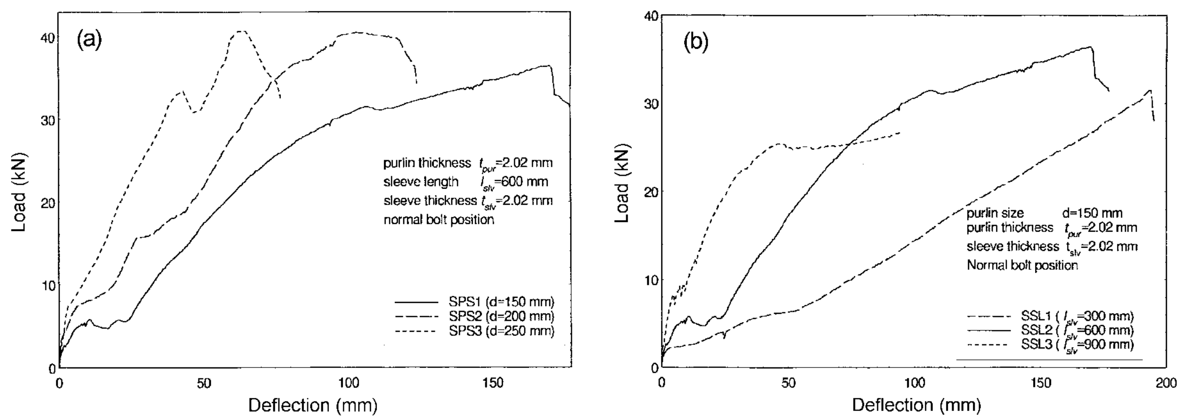


Fig. 3 (a) Effect of purlin size on sleeve-purlin connection, (b) effect of sleeve length on sleeve-purlin connection

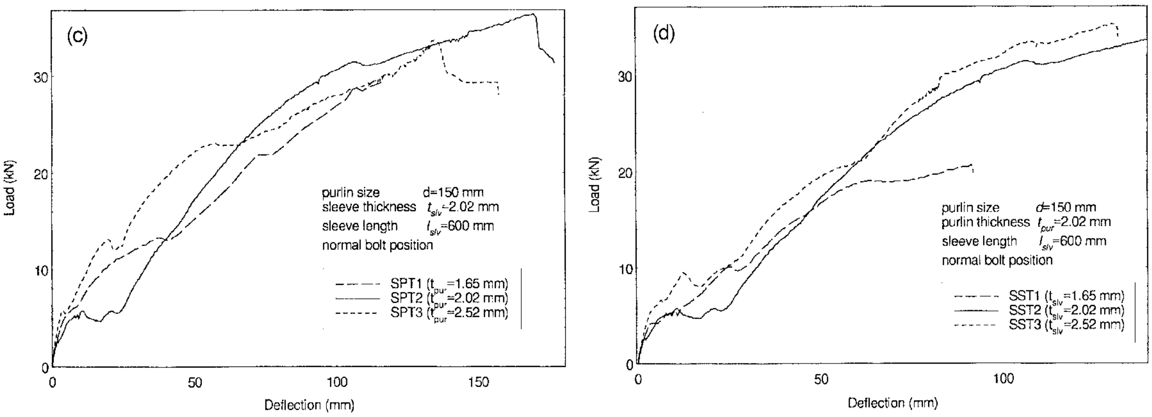


Fig. 3 (c) Effect of purlin thickness on sleeve-purlin connection, (d) effect of sleeve thickness on sleeve-purlin connection

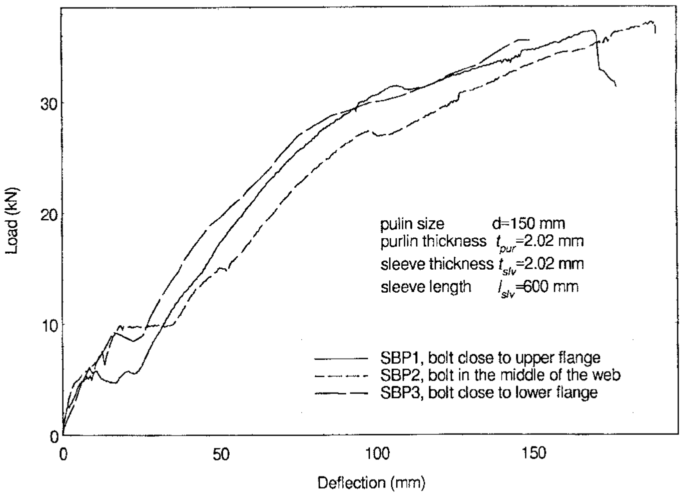


Fig. 3e Effect of bolt position on sleeve-purlin connection

An investigation on the effects of purlin thickness was performed in test series SPT. The various thickness considered were 1.6 mm, 2.0 mm and 2.5 mm. The results are presented in Fig. 3c. It can be observed that the gradient of the curve did not increase linearly with respect to purlin thickness. Similar conclusion can be drawn from the test series SST (Fig. 3d), which was designed to study the influence of sleeve thickness on the rotational stiffness of the connection. It was found that depending on the ratio of purlin thickness to sleeve thickness, different mode of failure would occur and consequently affect the rotational stiffness of the connection. Better performance can be achieved in the sleeve-purlin connection with a combination of thicker sleeve with relatively thinner purlin.

In order to investigate the influence of the bolt position on the rotational stiffness and behavior of the sleeve-purlin connections, three bolt positions were arranged in test series SBP. The load-deflection curves are shown in Fig. 3e. It can be seen that the influence of the bolt position on the rotational stiffness of the sleeve-purlin connection is marginal and limited since variation between the curves is small.

4. Connection rotational stiffness empirical expression

From the results of some 230 tensile tests on bolted lap joints, Bryan (1993) established a set of empirical expressions to characterize two-bolt and three-bolt connections under moment. The expressions of the rotational stiffness of the respective connections are as follows.

$$K_{TWO} = \frac{d_{bolt}^2}{4c} \tag{1}$$

$$K_{THREE} = \frac{(r_1^2 + r_2^2 + r_3^2)}{c} \tag{2}$$

where d_{bolt} is the distance between the two bolt holes on the cleat and r_i is the distance between bolt i to the elastic center of rotation (see Fig. 4). For a single bolted lap joint in tension with sheet thickness t_1 and t_2 the joint flexibility c is given as

$$c = 5n\left(\frac{10}{t_1} + \frac{10}{t_2} - 2\right) \times 10^{-6}\ \text{mm/N} \tag{3}$$

The factor n is dependent on the position of the shear plane and the number of bolts. The value of n suggested by Bryan (1993) is listed in Table 5. The above equations will be used in the present study with some modifications as the test conditions are different.

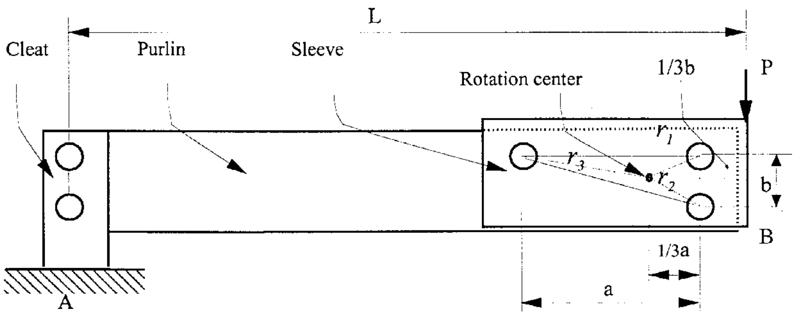


Fig. 4 Details of sleeve-purlin connection (symmetrical half)

Table 5 Values of n for bolted connection under moment

	Simple bolted joints	Joints which nest or interlock	Joints which nest and interlock
Shear plane on plain shank	1.8	1.4	1.2
Shear plane on thread	3.0	2.4	2.0

4.1 Rotational stiffness of cleat-purlin connection

The rotational stiffness of the cleat-purlin connection is contributed by two components: firstly, by the stiffness of the cleat which is affected by its dimensions and the material properties, and secondly, by the stiffness of the two-bolt connection which can be determined from Eq. (1). Hence, the design expression for the cleat-purlin connection will include the parameters of the two-bolt connection, i.e., the distance between the two bolts and the thickness of the purlin and cleat, as well as the parameters of the cleat, for example, the effective height, material and geometric properties, etc.

From the cleat-purlin connection tests, the moment-rotation relationship for a two-bolt connection given by Eqs. (1) and (3) is still valid but some modification are required. One major difference between the present study and Bryan's work is that the diameter of the bolts used in the latter was 12 mm instead of 16 mm. This difference attributes to the modification of the coefficient from 5 to 4 and the value of *n* from 3.0 to 3.5 in Eq. (3). Hence

$$K_{TWO}' = K_{TWO} = \frac{d_{bolt}^2}{4c} \tag{4}$$

and

$$c = 14\left(\frac{10}{t_{pur}} + \frac{10}{t_{clt}} - 2\right) \times 10^{-6} \ \text{mm/N} \tag{5}$$

where t_{pur} and t_{clt} are the thickness of the purlin and cleat respectively. Fig. 5 shows a cleat subjected to moment *M*. The effective height h_e of the cleat is the distance from the base of the cleat to mid-point *O* between the two bolt holes. φ is the angle of rotation of the center line at point *O*. From simple beam theory, the angle of rotation of the center line of the cleat can be found from

$$\varphi = \frac{h_e M}{E_c I_c} \tag{6}$$

The rotational stiffness of the cleat is given by

$$K_C = \frac{M}{\varphi} = \frac{E_c I_c}{h_e} \tag{7}$$

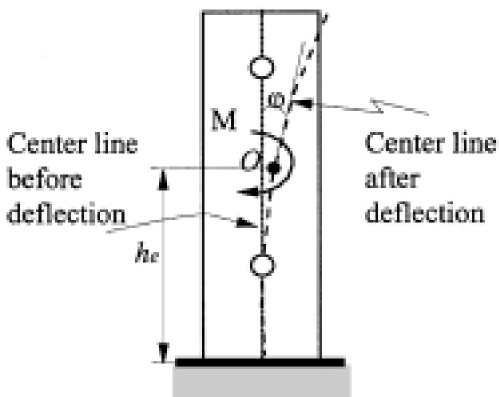


Fig. 5 Effect of stiffness of cleat

Similarly, the out-of-plane rotational stiffness K_C' of the cleat is

$$K_C' = \frac{E_c I_c'}{h_e} \tag{8}$$

The modulus of elasticity of the cleat is E_c. The in-plane and out-of-plane second moment of area of the cleat are I_C and I_C' respectively. Using superposition, the rotational stiffness of the cleat-purlin connection K_{CP} is

$$\frac{1}{K_{CP}} = \frac{1}{K_{TWO}'} + \frac{1}{K_C} + \beta\frac{1}{K_C'} \tag{9}$$

where β is an empirical factor which accounts for the lateral instability of the cleat and is defined as

$$\beta = 0 \text{ for } d = 0 \text{ and } 100 \text{ mm}$$
$$\beta = 10^{-\frac{150}{d}} \text{ for } d = 200 \text{ mm and } 250 \text{ mm}$$

4.2 Rotational stiffness of sleeve-purlin connection

Fig. 4 shows the symmetrical half of the sleeve-purlin connection under investigation. The rotational stiffness of the sleeve-purlin connection K_{SP} can be obtained by modification of Eq. (2), i.e.,

$$K_{SP} = \frac{1}{2} \times \frac{(r_1^2 + r_2^2 + r_3^2)}{c} = \frac{(a^2 + b^2)}{3c} \tag{10}$$

where a and b are the longitudinal and vertical distance between the bolt holes respectively as shown in Fig. 4. The coefficient of Eq. (3) is modified from 5 to 4 to account for the smaller bolt used. The value of n is modified from 2.4 to 2.5 due to the effect of interlock. Since the rotational stiffness of the connection is more sensitive to purlin thickness than sleeve thickness t_{slv}, Eq. (3) becomes

$$c = 10\left(\frac{10}{t_{pur}} + \frac{12}{t_{slv}} - 2\right) \times 10^{-6} \text{ mm/N} \tag{11}$$

5. Connection rotational stiffness from experiment

5.1 Cleat-purlin connection

The cleat-purlin connection experimental test can be represented schematically by a cantilever with a rotational spring as shown in Fig. 6. By assuming elastic connection behavior and ignoring the lateral deflection, shearing effect and cross-section distortion, the rotational stiffness of the connection can easily be shown to be

$$K_{CP} = \frac{L^2}{\frac{1}{\lambda} - \frac{1}{\lambda_0}} \tag{12}$$

where $\lambda = P/\Delta_B$ and $\lambda_0 = 3EI/L^3$. The purlin length is L and P is the applied point load at the free end. The modulus of elasticity and in-plane second moment of area of the purlin are E and I

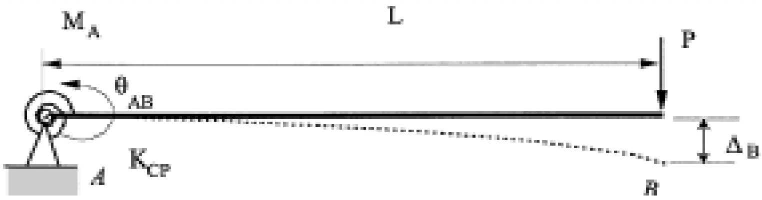


Fig. 6 Schematic of cleat-purlin connection

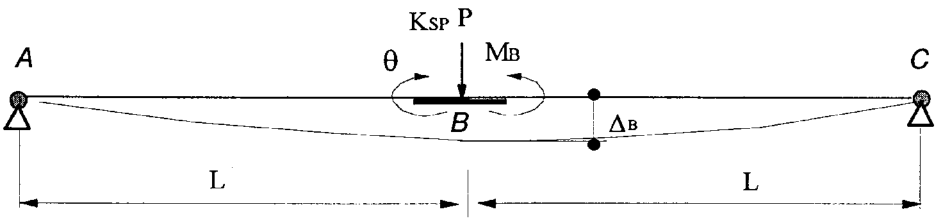


Fig. 7 Schematic of sleeve-purlin connection

respectively. Δ_B is the experimental vertical deflection at the free end. Knowing displacement Δ_B and corresponding load P allows the rotational stiffness of the cleat-purlin connection to be computed from Eq. (12).

5.2 Sleeve-purlin connection

The sleeve purlin connection under investigation can be considered as a continuous beam simply supported at both ends and subjected to a point load P at the mid-span as shown in Fig. 7. Due to the sleeve, the second moment of area of the cross section at the sleeve-purlin connection is assumed to be twice that of the purlin if the sleeve and purlin are of similar thickness (usually the case). From elastic theory, the mid-span vertical deflection can be shown to be

$$U_o = P[L^3 + (L - l_{slv})^3]/(12EI) \tag{13}$$

By assuming elastic connection behavior and ignoring the lateral deflection, shearing effect and cross-section distortion, the rotational stiffness of the connection can be obtained by considering U_o and Δ_B, i.e.,

$$K_{SP} = \frac{L^2}{2(1/\lambda - 1/\lambda_1)} \tag{14}$$

where $\lambda = P/\Delta_B$ and $\lambda_1 = P/U_0$. The experimental vertical deflection at the point load is denoted by Δ_B.

6. Comparison of results from empirical expression and experiment

6.1 Cleat-purlin connection

The comparisons between the predicted rotational stiffness of the cleat-purlin connection determined from Eq. (9) and that obtained from the experiment using Eq. (12) are shown in Figs. 8a and 8b. It can be seen that the agreement is rather good. The maximum error is about 16%. The results also reaffirm that purlin size has a more significant effect on the connection rotational stiffness compared to purlin thickness.

6.2 Sleeve-purlin connection

The comparisons between the predicted rotational stiffness of the sleeve-purlin connection determined from Eq. (10) and its the corresponding experimental value using Eq. (14) are shown in Fig. 9a to 9d. Except for SSL3 (sleeve length of 900 mm) which has an error of about 33%, the maximum error is about 17%. Generally, the rotational stiffness of the sleeve-purlin connections predicted by the empirical expression is acceptable and in good agreement with the corresponding experimental values.

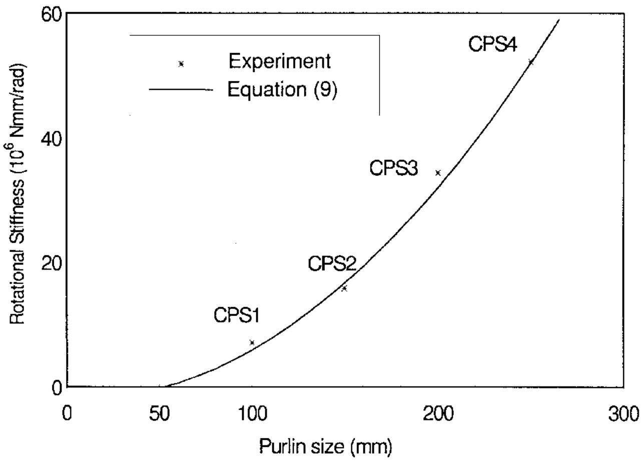


Fig. 8a Cleat-purlin connection rotational stiffness comparison: Effect of purlin size

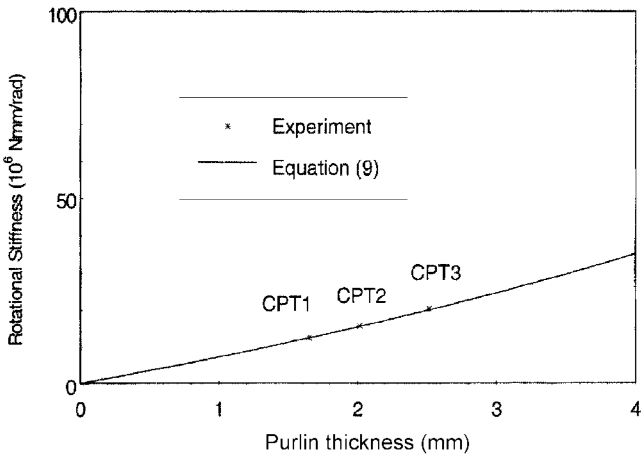


Fig. 8b Cleat-purlin connection rotational stiffness comparison: Effect of purlin thickness

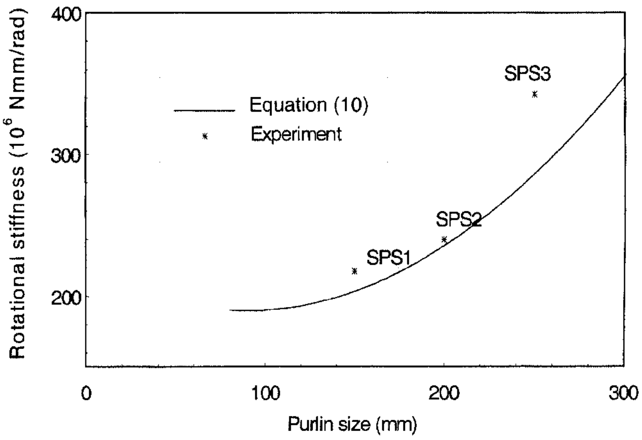


Fig. 9a Sleeve-purlin connection rotational stiffness comparison: Effect of purlin size

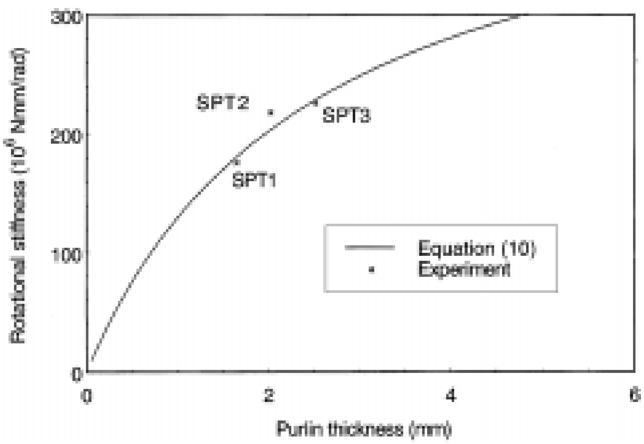


Fig. 9b Sleeve-purlin connection rotational stiffness comparison: Effect of purlin thickness

7. Sample design of purlin system

Using the empirical expressions of the cleat-purlin and sleeve-purlin connections, a multi-span sleeve joint purlin system can be designed and optimized by adjusting the parameters of the connections. A commonly used three-span Zed-section purlin system with cleat-purlin and sleeve-purlin connections is shown in Fig. 10. The following data of the system was obtained from a supplier dealing with purlin systems.

Zed section: Span L = 6 m; 150 mm depth (size) $\times$ 2.02 mm thickness
Second moment of area: $I = 2.111 \times 10^6$ mm^4
Modulus of elasticity: E = 205 kN/mm^2
Sleeve: Three-bolt type as shown in Fig. 1
Cleat: 7.8 mm in thickness as used in experimental test
Loading: Uniform distributed load: w = 1.5 kN/m

The load carrying capacity of the system can be maximized by adjusting the rotational stiffness of

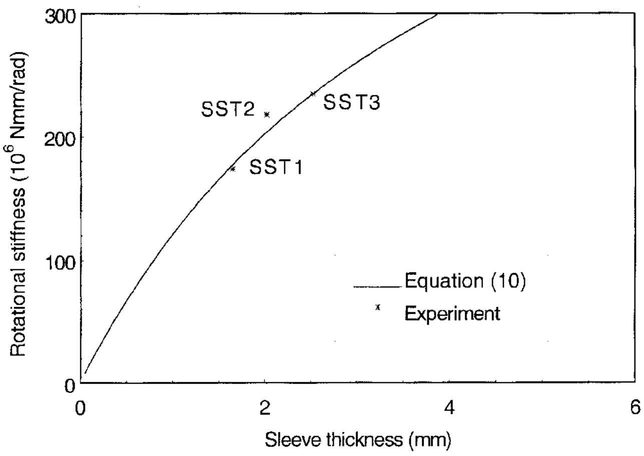


Fig. 9c Sleeve-purlin connection rotational stiffness comparison: Effect of sleeve thickness

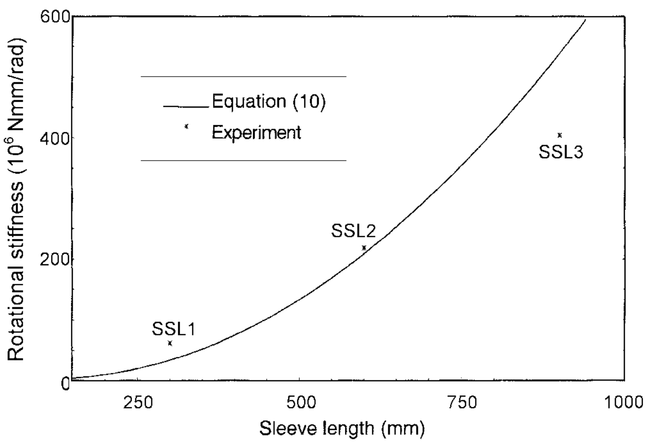


Fig. 9d Sleeve-purlin connection rotational stiffness comparison: Effect of sleeve length

the connections, thus, optimizing the moment distribution along the beam. Assuming elastic behavior, the relationship of the restraint moment (M_i) and connection rotational stiffness (K_i) is given by

$$\left|\begin{array}{cccc} 6EI/LK_A+2 & 1 & 0 & 0 \\ 1 & 6EI/LK_B+4 & 1 & 0 \\ 0 & 1 & 6EI/LK_C+4 & 1 \\ 0 & 0 & 1 & 6EI/LK_D+2 \end{array}\right| \begin{Bmatrix} M_A \\ M_B \\ M_C \\ M_D \end{Bmatrix} = \frac{wL^2}{4} \begin{Bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{Bmatrix} \tag{15}$$

As in actual practice, similar cleat-purlin connections are used at A and D, and similar sleeve-purlin connections at B and C. The bending moment distribution of the system is shown in Fig. 11. For optimum condition, the bending moment at each support and mid-point of each span should be equal. Hence, simplifying Eq. (15) gives

$$\left(\frac{6EI}{LK_A} + \frac{5}{3}\right) M_A = \frac{wL^2}{6} \tag{16a}$$

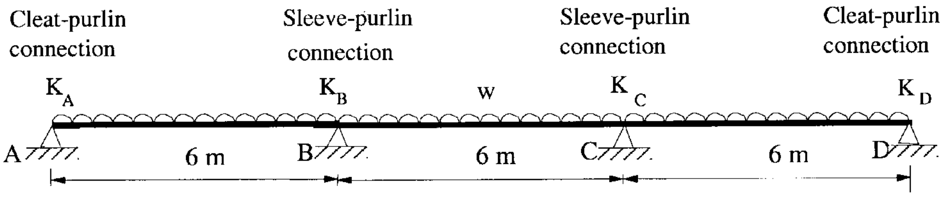


Fig. 10 Three-span sleeve joint system

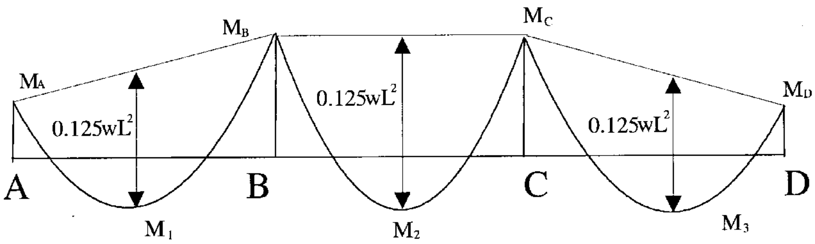


Fig. 11 Bending moment diagram

$$\left(\frac{6EI}{LK_B} + 2\right)M_B = \frac{wL^2}{4} \tag{16b}$$

For a purlin size of 150 mm and using standard cleats, the rotational stiffness of the connection at A as determined from Eq. (9) is $K_A = K_{CP} = 10.729$ Nmm/rad. The bending moment at A and B are $M_A = 214.1 \times 10^6$ Nmm and $M_B = 442.6 \times 10^6$ Nmm respectively. From Eq. (16b), the rotational stiffness of the sleeve-purlin connection is $K_B = 412.4 \times 10^6$ Nmm/rad. In order to provide this rotational stiffness, the sleeve length can be adjusted. Substituting K_B into Eq. (10) gives $a = 312.87$ mm (see Fig. 4). Together with the standard dimensions of a 150 mm sleeve, the required sleeve length is 775 mm.

Although the same rotational stiffness of the sleeve-purlin connection can be achieved by changing other parameters, for example, purlin size or thickness, the most convenient and practical solution is to vary the sleeve length.

8. Conclusions

From the investigation conducted, the following conclusions can be drawn. It should be noted that the conclusions are applicable only to the range of Zed section purlins and connections investigated.

1. Of the two parameter investigated, the size of the purlin has a more pronounced effect on the rotational stiffness of cleat-purlin connection compared to purlin thickness.
2. The cleats play an important role in the structure’s overall performance. In order to prevent the premature failure at the cleats, stronger cleats are recommended for the connections when purlin size larger than 200 mm is used.
3. The rotational stiffness of the sleeve-purlin connection is affected to large extend by the sleeve length.
4. A set of empirical expressions capable of predicting the rotational stiffness of cleat-purlin and

sleeve-purlin connections has been developed. The agreement between the predictions and experimental results is rather good. With the empirical expressions, the most suitable connections can be selected to provide optimize condition for sleeve joint purlin system.

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