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Slenderness ratio of telescopic cylinder-columns

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Abstract. The present paper deals with the effective slenderness ratio of telescopic cylinders as a long column having different cross sections. Firstly, the slenderness ratio defined in the current standard for the telescopic cylinders is discussed to point out some difficulties which arise when the ratio is applied to the column having different cross sections. Secondly, a new effective slenderness ratio is proposed for columns having different cross sections by introducing a partial effective slenderness ratio. Finally, the proposed slenderness ratio is applied, for extending and development of discussion, to a two-staged column having piece-wise constant cross sections and a cylindrical column having linearly varying diameters.

Key words: column; telescopic cylinder; buckling; slenderness ratio; partial effective slenderness ratio; cylinder-column; two-staged column.

1. Introduction

Buckling of compressed structural members has assumed ever increasing importance in structural design (Timoshenko and Gere 1961, Chen and Lui 1987, Fukumoto 1997, Singer *et al.* 1998). The intended aim of the present paper is to discuss the slenderness ratio of telescopic cylinder-columns under the recent requirement for more light-weight design and longer spans.

Oil-hydraulic telescopic cylinders consist of multistage plungers having different cross sections. Due to the compactness of the cylinders, they are widely used as an actuator in cranes, dump trucks, oil-hydraulic elevators and so on. The light-weight design has been requested for the telescopic cylinders in recent applications. It is thus necessary to exactly estimate the buckling load and the effective slenderness ratio of the telescopic cylinders. The authors have compiled the computer programs for the Euler buckling load of the telescopic cylinders (Timoshenko and Gere 1961, Chen and Lui 1987, Ohtomo and Sugiyama 1997). As for the single stage oil-hydraulic cylinders consisting of a single stage plunger having constant cross section, an equation is given in “Guidance of technical standard for elevators” (Japan Elevator Association 1994) in order to calculate the effective slenderness ratio and the calculated value is defined to be under 250. However, “Guidance of technical standard for elevators” does not refer to the equation of the effective slenderness ratio of the telescopic cylinders having multistage plungers with different cross sections. The effective slenderness ratio of the telescopic cylinders has not been discussed by engineers, e.g., even in the paper by Miyasako *et al.* (1991) which was presented recently. Under these circumstances, the next second section describes that the current standard (European Standard

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1998, Miyasako *et al.* 1991) is applicable to the column having different cross sections under some limited conditions and there are some difficulties in applying it to the column. The third section proposes a new concept of effective slenderness ratio, which is generally applicable to a column under any conditions, by introducing a partial effective slenderness ratio. The fourth section demonstrates applications of the new effective slenderness ratio to a two-staged column and a cylindrical column having linearly varying diameter.

2. Current effective slenderness ratio of the column with different cross sections

2.1 Discussion on current standard (European Standard 1998 and Miyasako et al. 1991)

Let us consider a two-staged column carrying the axial compressive load P as shown in Fig. 1. It is now assumed that the buckling load P_{cr} has been already calculated (Timoshenko and Gere 1961, Chen and Lui 1987, Ohtomo and Sugiyama 1997). It is assumed that the buckling load P_{cr} of two-staged columns with total length l can be expressed in a similar formula to uniform/single-staged columns as follows;

$$P_{cr}=\alpha_i^2\frac{EI_i}{l^2}=\varphi_i\frac{\pi^2 EI_i}{l^2}, \tag{1}$$

where α_i^2 is the buckling load factor, while φ_i is the buckling coefficient (Fukumoto 1997).

The subscript i corresponds to span i (i=1, 2). It is noted that the buckling coefficient φ_i is the buckling load factor normalized by π^2, i.e., the buckling load factor of simply supported single-staged columns. In case of two-staged columns, there are two bending stiffness, EI_1 for the span 1 and EI_2 for the span 2. Therefore there can be considered two buckling coefficients, φ_1 for EI_1 and

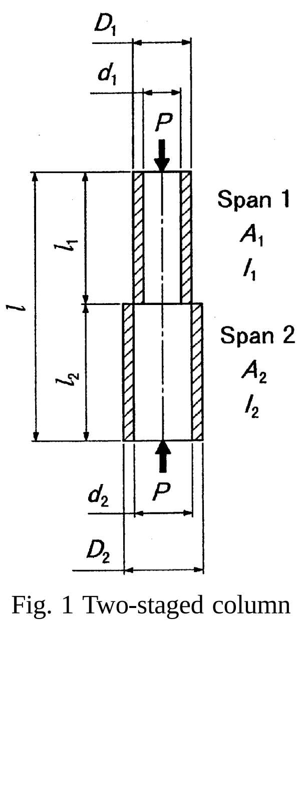

Fig. 1 Two-staged column

φ_2 for EI_2. In the current standard, EI_2 is considered more important than EI_1.

Introducing a buckling coefficient φ_2 in reference to span 2, the buckling load P_{cr} can be written in the form

$$P_{cr}=\varphi_2\frac{\pi^2 EI_2}{l^2}. \tag{2}$$

Eq. (2) represents Euler buckling load for a column which has the total length l, a uniform cross section of moment of inertia $\varphi_2 \cdot I_2$, modulus of elasticity E, and both ends simply supported. The equivalent model is shown in Fig. 2, where I_2' is the equivalent moment of inertia, A_2' is the equivalent cross sectional area, d_2' and D_2' are the equivalent inner and outer diameter, respectively.

Under the assumption $d_2/D_2=d_2'/D_2'$, the following equations can be established;

$$D_2'=\sqrt[4]{\varphi_2}\cdot D_2, \tag{3}$$

$$d_2'=\sqrt[4]{\varphi_2}\cdot d_2, \tag{4}$$

$$A_2'=\sqrt{\varphi_2}\cdot A_2. \tag{5}$$

Then, the equivalent slenderness ratio λ_2' of the equivalent column is given by

$$\lambda_2'=l\cdot\sqrt{\frac{A_2'}{I_2'}}=\frac{l}{\sqrt[4]{\varphi_2}}\cdot\sqrt{\frac{A_2}{I_2}}=\frac{l}{\sqrt[4]{\varphi_2}}\cdot\frac{4l}{\sqrt{D_2^2+d_2^2}}. \tag{6}$$

This expression is the formula to determine a slenderness ratio of a column with different cross sections, according to the current European Standard (1998) on the telescopic cylinders.

2.2 Some difficulties in the current standard

2.2.1 Buckling stress

The buckling stress σ_2' of the equivalent model as shown in Fig. 2 is

$$\sigma_2'=\frac{P_{cr}}{A_2'}=\frac{P_{cr}}{\sqrt{\varphi_2}\cdot A_2}=\frac{\sigma_2}{\sqrt{\varphi_2}}, \tag{7}$$

where σ_2 is the buckling stress in the span 2 on the two-staged column shown in Fig. 1. It is seen from Eq. (7) that σ_2' is not equal to actual buckling stress.

2.2.2 Equivalent slenderness ratio and buckling stress in reference to span 1

The equivalent slenderness ratio λ_1' in reference to span 1 can be easily introduced and represented, just like Eq. (6), in the form

$$\lambda_1'=\frac{1}{\sqrt[4]{\varphi_1}}\cdot\frac{4l}{\sqrt{D_1^2+d_1^2}}, \tag{8}$$

where φ_1 is the buckling coefficient satisfying the following equation;

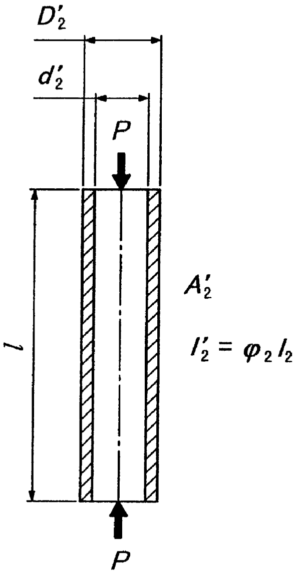

Fig. 2 Equivalent model

$$P_{cr}=\varphi_1 \cdot \frac{\pi^2 EI_1}{l^2}. \tag{9}$$

However, the current standard does not refer to the equivalent slenderness ratio in reference to span 1 as given by Eq. (8). And also, buckling stress σ_1' in reference to span 1 can be given by

$$\sigma_1'=\frac{P_{cr}}{\sqrt{\varphi_1} \cdot A_1}=\frac{\sigma_1}{\sqrt{\varphi_1}}. \tag{10}$$

where σ_1 is the actual buckling stress in span 1 shown in Fig. 1.

The buckling stress σ_1 in the smallest cross sectional area should be taken into account in the design of the telescopic cylinders. However, the current standard contains no mention of the buckling stress in the smallest cross sectional area.

2.2.3 Support condition and span length

The equivalent slenderness ratio presented in the current standard as given by Eq. (6) is defined under the following conditions;

Support end condition: Both ends simply supported

Span length : $l_1=l_2$

And also, as for a three-staged column as shown in Fig. 3, Eq. (6) can be applied under the conditions;

Support end condition : Both ends simply supported

Span length: $l_1=l_2=l_3$

Moment of inertia: $I_3=I_2$

Accordingly, Eq. (6) can not be applied to general support end conditions except simply supported ends, and to long columns having different span length such as $l_1 \neq l_2 \neq l_3$ and to multistage

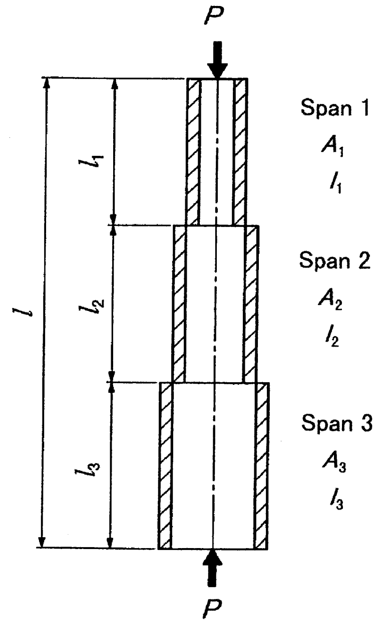

Fig. 3 Three-staged column

columns exceeding four-staged columns inclusive.

3. New concept of effective slenderness ratio

3.1 Partial effective slenderness ratio

Effective slenderness ratio λ_l of a column having uniform cross section is given by

$$\lambda_l = \frac{1}{\sqrt{\varphi}} \cdot \frac{l}{\kappa} = \frac{l}{\sqrt{\varphi}} \cdot \sqrt{\frac{A}{I}}, \tag{11}$$

where A : Cross sectional area,
I : Moment of inertia,
l : Length of column,
κ : Radius of gyration of area,
φ : Buckling coefficient.

A column with varying cross section carrying the axial compressive load P is shown in Fig. 4. It is now assumed that the buckling load P_{cr} has been already known. Dividing this column into a finite number of column elements with short length, each cross section of the column element is now assumed to be constant. Thus, we can consider that the column having varying cross section is composed of column elements with constant cross section. Each column element is assumed to be rigidly connected with others.

Introducing a buckling coefficient φ_k in reference to column element of length l_k having constant

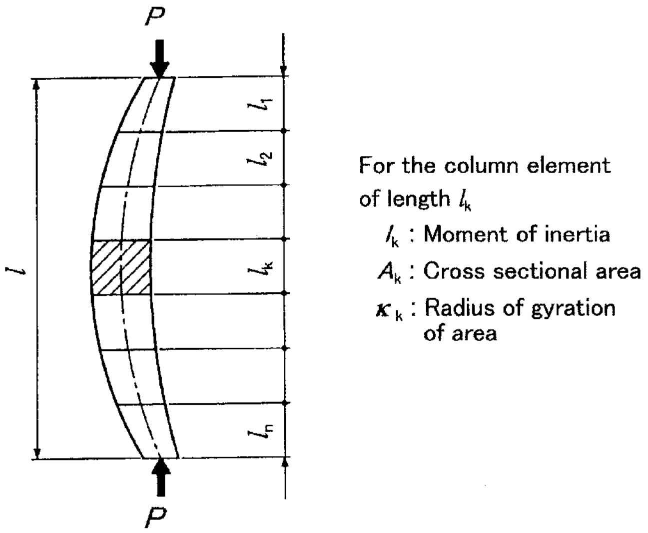

Fig. 4 Column with varying cross section

cross section, then we have

$$P_{cr}=\varphi_k \cdot \frac{\pi^2 E I_k}{l^2}, \tag{12}$$

where φ_k represents the buckling coefficient for a column of length l with the same constant cross section as column element of length l_k.

The effective slenderness ratio of a column element of length l_k with constant cross section is given by

$$\lambda_k=\frac{1}{\sqrt{\varphi_k}} \cdot \frac{l_k}{\kappa_k}=\frac{l_k}{\sqrt{\varphi_k}} \cdot \sqrt{\frac{A_k}{I_k}}, \tag{13}$$

where λ_k is a new ratio, now referred to as partial effective slenderness ratio.

Then, the overall effective slenderness ratio λ_l of the whole column with varying cross section can be given by

$$\lambda_l=\sum_{k=1}^{n} \lambda_k=\sum_{k=1}^{n} \frac{l_k}{\sqrt{\varphi_k}} \cdot \sqrt{\frac{A_k}{I_k}}. \tag{14}$$

The buckling stress σ_k of column element of length l_k is, from Eqs. (12) and (13), given by

$$\sigma_k=\frac{P_{cr}}{A_k}=\left(\frac{l_k}{l}\right)^2 \cdot \frac{\pi^2 E}{\lambda_k^2}. \tag{15}$$

3.2 Effective slenderness ratio of two-staged column

A buckling load P_{cr} for two-staged column as shown in Fig. 1 can be calculated by using a computer programs compiled by the well known method for buckling loads of continuous beam-columns (Timoshenko and Gere 1961, Chen and Lui 1987, Ohtomo and Sugiyama 1997). Introducing the buckling coefficients φ_1 and φ_2 in reference to span 1 and span 2, respectively, the buckling load P_{cr} for the column is given by

$$P_{cr}=\varphi_1 \cdot \frac{\pi^2 EI_1}{l^2}=\varphi_2 \cdot \frac{\pi^2 EI_2}{l^2}. \tag{16}$$

Then, the partial effective slenderness ratios are

$$\lambda_1=\frac{1}{\sqrt{\varphi_1}} \cdot \frac{l_1}{\kappa_1}=\frac{l_1}{\sqrt{\varphi_1}} \cdot \sqrt{\frac{A_1}{I_1}} \qquad \text{for span 1,} \tag{17}$$

$$\lambda_2=\frac{1}{\sqrt{\varphi_2}} \cdot \frac{l_2}{\kappa_2}=\frac{l_2}{\sqrt{\varphi_2}} \cdot \sqrt{\frac{A_2}{I_2}} \qquad \text{for span 2.} \tag{18}$$

Overall effective slenderness ratio λ_l of two-staged column shown in Fig. 1 now can be given by

$$\lambda_l=\lambda_1+\lambda_2=\pi\sqrt{\frac{E}{P_{cr}}}\left(\sqrt{A_1} \cdot \frac{l_1}{l}+\sqrt{A_2} \cdot \frac{l_2}{l}\right). \tag{19}$$

Actual buckling stress σ_1 and σ_2 at span 1 and span 2, respectively, are

$$\sigma_1=\left(\frac{l_1}{l}\right)^2 \cdot \frac{\pi^2 E}{\lambda_1^2}=\frac{P_{cr}}{A_1}, \tag{20}$$

$$\sigma_2=\left(\frac{l_2}{l}\right)^2 \cdot \frac{\pi^2 E}{\lambda_2^2}=\frac{P_{cr}}{A_2}. \tag{21}$$

These equations have cleared out the difficulties in the current standard which are discussed in the previous section, i.e., buckling stress can be specified for respective span.

4. Example

4.1 Effective slenderness ratio of two-staged column

Fig. 5 shows a typical model of a two-staged column. Now let us evaluate the effective slenderness ratio for extending a discussion by making a comparison between the ordinary and newly proposed slenderness ratios. For the case of simply supported ends, the effective slenderness ratio λ for the typical model is shown in Fig. 6, while for the case of fixed ends in Fig. 7. Here, the buckling load P_{cr} is calculated with compiled programs (Timoshenko and Gere 1961, Chen and Lui 1987, Ohtomo and Sugiyama 1997).

It is apparent from Figs. 6 and 7 that an overall effective slenderness ratio λ_l can be obtained for any range of l_1/l. It is noted that the new ratio is more general than the one given by the current standard in which the ratio is defined at only one point of $l_1/l=0.5$ in case of simply supported ends.

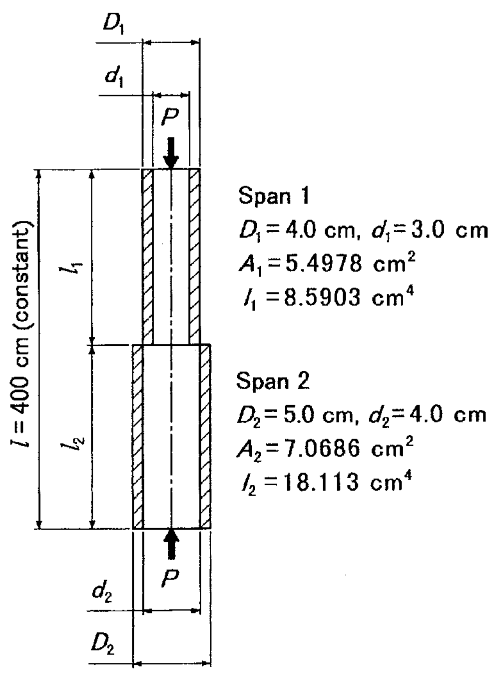

Fig. 5 Typical model of two-staged column

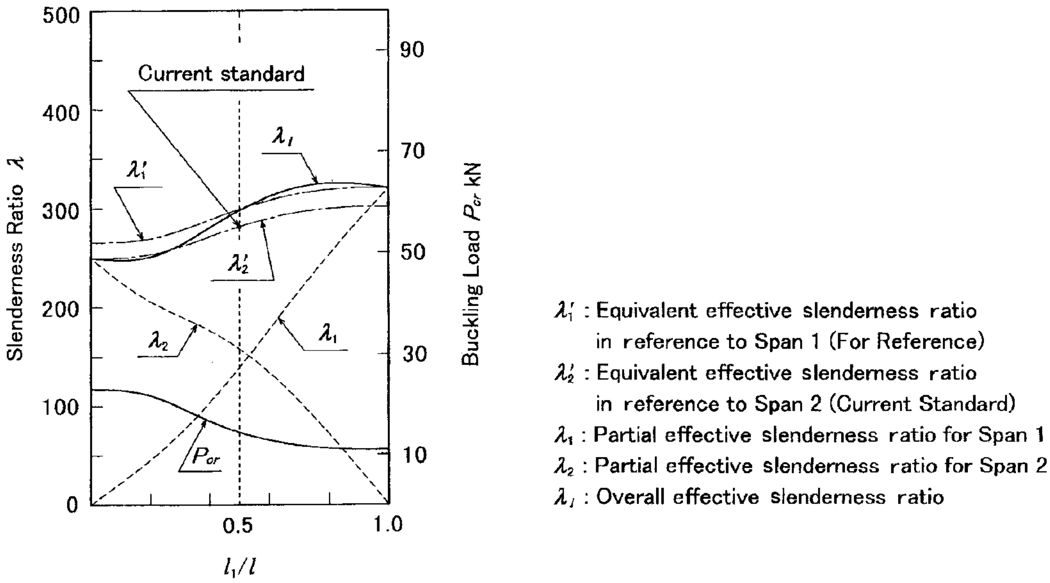

Fig. 6 Chart of slenderness ratio (Both ends simply supported)

4.2 Effective slenderness ratio of cylindrical column having linearly varying diameter

Let us consider the second example of a cylindrical column having linearly varying diameters, as

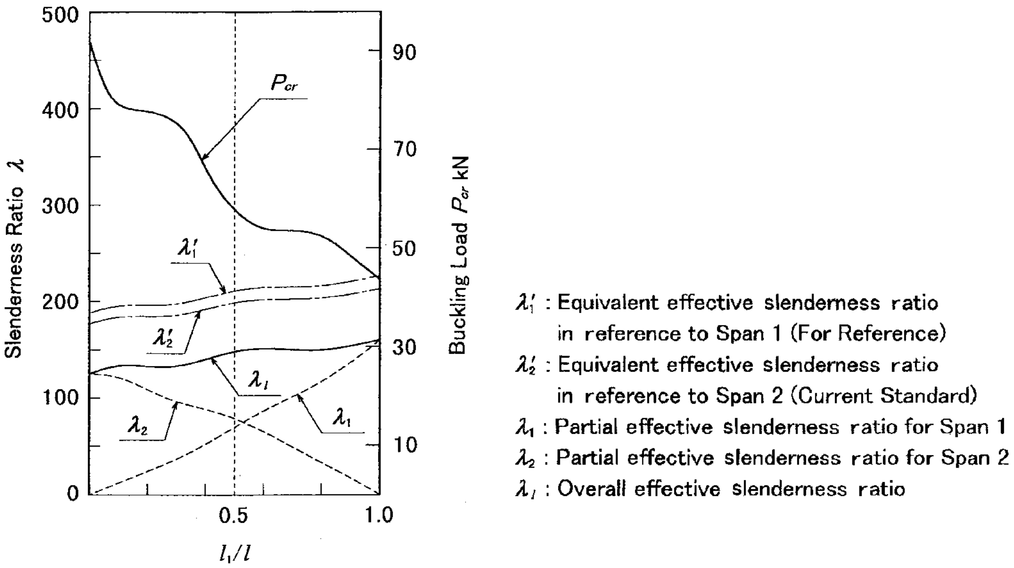

Fig. 7 Chart of slenderness ratio (Both ends fixed)

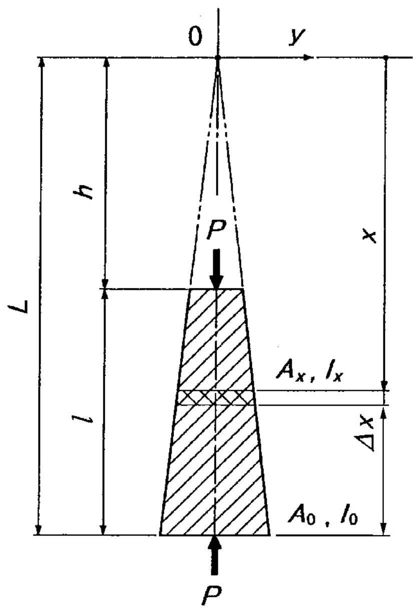

Fig. 8 Cylindrical column having linearly varying diameters

shown in Fig. 8, which has a moment inertia I_0 and a cross sectional area A_0 at the bottom. A moment of inertia I_x and a cross sectional area A_x at a distance x from the origin of the coordinate can be given by

$$I_x = I_0 \cdot \left(\frac{x}{L}\right)^4, \tag{22}$$

$$A_x = A_0 \cdot \left(\frac{x}{L}\right)^2 . \tag{23}$$

For the case of simply supported ends, the equilibrium equation of the column are given by

$$\frac{d^2y}{dx^2} = -\frac{P}{EI_x}y = -\frac{PL^4}{EI_0} \cdot \frac{1}{x^4}y \tag{24}$$

Applying the boundary conditions, we have the buckling load P_{cr} in the form

$$P_{cr} = \left(1 - \frac{l}{L}\right)^2 \frac{\pi^2 EI_0}{l^2} . \tag{25}$$

Introducing a buckling coefficient φ_x in reference to column element Δx, satisfying the following equation;

$$P_{cr} = \varphi_x \cdot \frac{\pi^2 EI_x}{l^2} , \tag{26}$$

the partial effective slenderness ratio of column element Δx is given by

$$\lambda_x = \frac{\Delta x}{\sqrt{\varphi_x}} \cdot \sqrt{\frac{A_x}{I_x}} = \frac{\pi}{l} \cdot \sqrt{\frac{E}{P_{cr}}} \cdot \sqrt{A_0} \cdot \frac{x}{L} \cdot \Delta x . \tag{27}$$

Then, overall effective slenderness ratio λ_l of whole column can be given by

$$\lambda_l = \int_h^L \lambda_x = \frac{L+h}{2L}\pi \cdot \sqrt{\frac{E}{P_{cr}}} \cdot \sqrt{A_0} . \tag{28}$$

It is seen that Eq. (28) equals to the equivalent effective slenderness ratio of the equivalent model in reference to the cross section at the half length of the cylindrical column.

5. Conclusions

A new effective slenderness ratio of the columns with different cross section is proposed by introducing a concept of partial effective slenderness ratio. It is noted that the new effective slenderness ratio is generally applicable and more reasonable than the classical slenderness ratio used in the current standard. The usefulness of the proposed effective slenderness ratio has been demonstrated by applying it to a two-staged column and a cylindrical column having linearly varying diameters. It is expected that the proposed concept of the new effective slenderness ratio will make a key measure in light-weight design of the advanced telescopic cylinders.

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