

Simplified design formula of slender concrete filled steel tubular beam-columns

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Abstract. The objective of this paper is to develop a simplified method that could predict the strength of concrete filled steel tube (CFT) columns applicable to high strength material under combined axial compression and flexure. The simplified method for determining the strength of CFT columns is based on the interaction curve of the section approached by a polygonal connection of the points. These points are determined by using symmetrical properties of the CFT section. For each point, a simple equation is proposed to determine the strength of the slender columns under compression and flexure. The simple equation was adjusted with results of elasto-plastic analysis results. Validation of the simplified method is undertaken by comparison with data from the test conducted at Kyushu University. These results confirm the fact that the simplified method could accurately and reliably predict the strength of CFT columns under combined axial compression and flexure.

Key words: concrete filled steel tubular columns; simplified design formula; high strength material.

1. Introduction

The two common types of composite columns are steel reinforced concrete and concrete filled steel tubes. In a CFT column the steel tube acts as a formwork for placing the concrete and confines the filled concrete thus increasing its compressive ductility. The filled concrete delays the local buckling of the steel tube and increases the flexural stiffness of the column without increasing the size of the steel section. These advantages have been recognized and the structural system using CFT members has become popular in worldwide. It is estimated that more than 300 multi-purpose buildings have been constructed in Japan.

In the building codes of America and Japan, three different approaches were developed for the design of CFT column. The first approach modifies the equation for steel column to predict the composite column strength by transforming the concrete portion into an equivalent contribution of steel shape, such as Load and Resistance Factor Design (AISC-LRFD) Specification. In contrast, the

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second approach transforms the steel section into an equivalent amount of reinforcement and designs the composite column as an ordinary reinforced concrete member such as the Building Code Requirements for Reinforced Concrete and Commentary (ACI-318) code. The third approach uses the strength superposition concept to predict the composite column strength by adding the strength of steel and the concrete portion of the composite column, such as the Standards for Structural Calculation of Steel Reinforcement Concrete Structures (AIJ-SRC) code. It is noted that the steel column equation in the AISC-LRFD specification is significantly influenced by the residual stress in the steel shape. However, for the reinforced concrete column, the residual stress plays a much less significant role in the ACI-318 code. Thus it is felt that the direct application of the equations originally developed for the steel or reinforced concrete columns may not be appropriate for the design of CFT columns.

The introduction of high strength material has made it possible to design CFT columns more slenderly and thereby facilitate not only new architectural ideas but also economic benefits. The performance of structural elements made of high strength material has recently become major concern for design engineers. On the background, a new “Recommendations for Design and Constructions of CFT (AIJ-CFT) structures was published based on the research of CFT structure carried out after the publication of AIJ-SRC code revised in 1987. On the other hand, for those engineers who are not familiar with the Japanese building code, the use of AIJ-SRC code will become difficult task. Therefore it will be valuable that if simplified design method can be developed to evaluate the CFT column strength with high strength material.

This study intended to develop a simplified design method applicable to high strength material for CFT column.

2. Experiment

The test columns were hinged at the ends and the load was applied with the loading point at both ends (Fig. 1). The test series consisted of 18 columns. The total 18 columns constituted six types of different buckling length (L_k)-section depth (D) ratios L_k/D . Ratios L_k/D were 4, 8, 12, 18, 24 or 30. The 3 columns of each group were tested, under 3 loading cases. The first case was concentric loading. The other cases were eccentric loading with eccentricity $e=20.5$ and 61.5 mm.

A summary of the test program for the CFT columns is given in Table 1.

2.1 Specimen

All slender and stub columns were constructed using standard commercially available steel tube. It was manufactured by cold-forming and high frequency electric resistance welding. The steel tube was supplied in stocky length of 7 m. Each tube of stocky length was cut into several pieces for the preparation of the composite columns, stub steel columns and tensile coupon tests. Tensile coupon tests were tested to determine the stress - strain relation. The results of tensile coupon tests are shown in Table 2. Three vacant steel tube specimens of 375 mm long, were tested under compression to determine the carrying capacity of the steel tubes including the effect of the residual stress due to the manufacturing process. The results of vacant steel stub column tests are shown in Table 3.

The concrete mixes, designed with target compressive cylinder strength of 88 MPa were supplied in an agitator truck by Ready-mix, a supplier of concrete with significant experience in the

manufacture. The average strength of concrete at 28 days is given in Table 4. The columns were all tested after approximately 45 days. The concrete was tested at the time of the column tests and had a average cylinder compressive strength $\sigma_B = 94$ MPa.

2.2 Test setup

All of the tests were carried out in a universal testing machine with a capacity of 490 kN. The load, which was determined by measurements from an oil pressure gauge, calibrated against the testing machine prior to the column tests, was increased at a constant rate without interruption. The process of increasing the load was continued until some relatively large non-linearity was monitored or until a reasonable percentage of the predicted ultimate load was achieved. The load increment was then considerably reduced and more time was allowed to reach the equilibrium position. At the stage when the columns were unable to resist any additional load increments, the load was

Table 1 Test program

Specimen	Lk (mm)	Lk/D	e (mm)	Specimen	Lk (mm)	Lk/D	e (mm)
C4-0	500	4	0	C18-0	2250	18	0
C4-1			20.5	C18-1			20.5
C4-3			61.5	C18-3			61.5
C8-0	1000	8	0	C24-0	3000	24	0
C8-1			20.5	C24-1			20.5
C8-3			61.5	C24-3			61.5
C12-0	1500	12	0	C30-0	3750	30	0
C12-1			20.5	C30-1			20.5
C12-3			61.5	C30-3			61.5

Notes: Unfortunately, the test for C30-0 was unsuccessful in the first attempt, due to an operational error of oil installation. C30-1 was cannibalized in an attempt to replicate the test of C30-0.

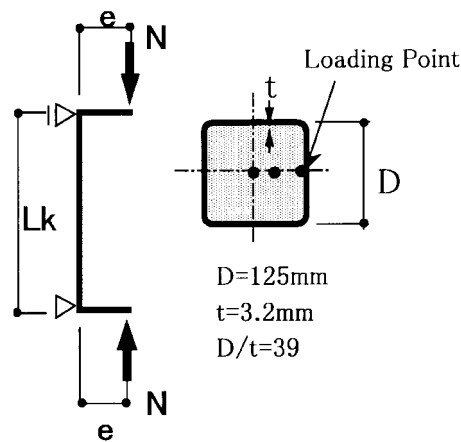


Fig. 1 Loading condition

Table 2 Tensile test results

Yield stress σ_y (MPa)	Ultimate stress σ_u (MPa)	Yield ratio σ_y/σ_u (%)	Elongation (%)
443	519	85.3	19.3

Table 3 Stub column test results

Ultimate stress σ_{cr} (MPa)	Strain at ultimate stress (%)
360	0.31

Table 4 Properties of concrete

Slump (mm)	Slump flow (mm)	Air (%)	Average strength (MPa)		
			28 days		before testing
			Standard curing	Field curing	Field curing
245	575	1.5	103	88	94

considered to be the maximum load.

3. Test results

The measured strength of each column, together with the corresponding deflection of the column at mid-height, is shown in Fig. 2. In these figures, the buckling length-section depth ratio, Lk/D , is kept constant, changing the value of eccentricity. The points marked with circles on load-lateral displacement relation curves represent the occurrence of local buckling, and the ellipses in the specimens show the locations of the local buckling recognized by mere observation.

These figures show quite clearly that, after the maximum strength of a column is attained, its capacity deteriorates much more quickly for columns of low slenderness than it does for columns of large slenderness. Furthermore it also shows that slender columns exhibit greater lateral displacements than columns of low slenderness. It is seen, therefore, that whilst low slenderness columns have a greater capacity than slender columns, they shed load much more quickly once softening occurs and are therefore less ductile.

For very slender columns ($Lk/D=24, 30$) under both concentric and eccentric load, local buckling occurs at the final stage after the ultimate load, and the dominating behavior at failure is flexural buckling mode. For short columns under concentric loads, local buckling occurs almost at the ultimate load, and the dominating behavior at failure is local buckling mode. For short columns under eccentric loads, local buckling occurs within a short period after ultimate load.

Fig. 3 shows orbits of M - N points at the mid-height, where $M=N(e+\delta)$. In these figures, solid circles represent ultimate loads, and full plastic strengths of the section are shown by dashed line. In general, full plastic strength may be calculated by assuming the rectangular stress distribution with yield stress of steel tube measured from tensile coupon test and compressive strength of concrete. However, there is a wide difference between the yield strength (443 MPa) measured from the tensile coupon test and compressive strength (360 MPa) of vacant steel stub column, because depth-

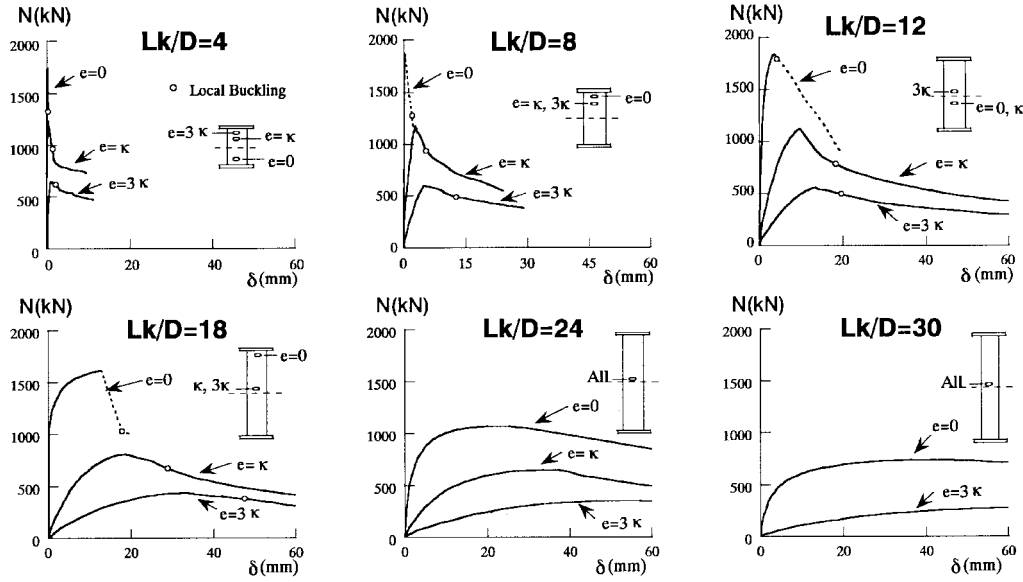


Fig. 2 Load-lateral displacement relation

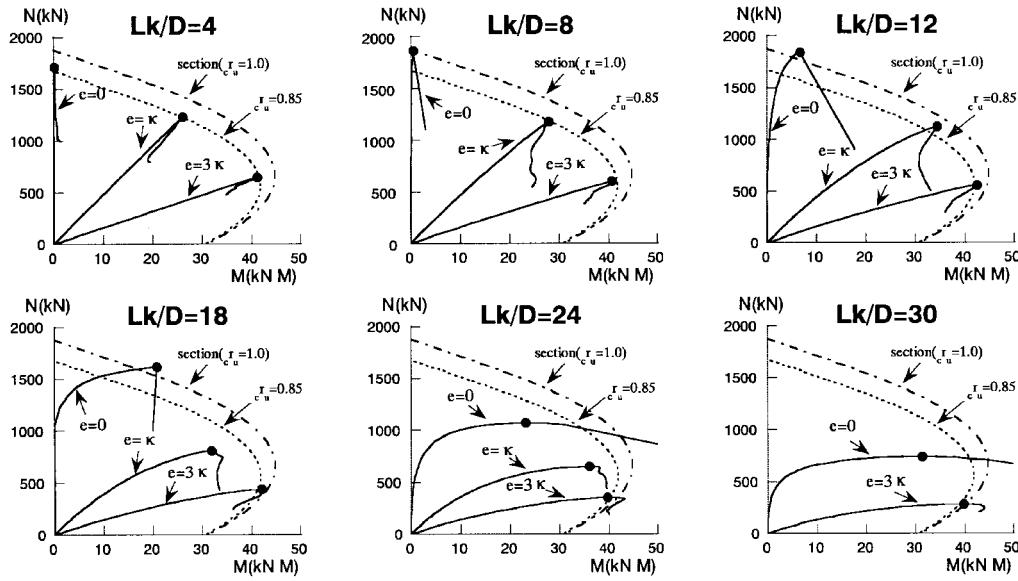


Fig. 3 Moment-axial load relation

thickness ratio is large. Therefore, compressive strength of vacant steel stub column replaced yield stress of steel tube to calculate full plastic moment of the section. For comparison, two types of full plastic strength of the section are shown. One type is calculated by compressive strength of concrete which takes the reduction factor ($r_u=0.85$) into consideration, and the other type does not.

These figures show that specimens with Lk/D ratio beyond 18 cannot attain the full plastic strength, which does not take the reduction factor into consideration, due to instability phenomenon,

as a natural result. However, specimens with no effect of instability phenomenon ($Lk/D=4, 8$) cannot also attain their full plastic strengths. Therefore, it is necessary to take the reduction factor into consideration for the concrete compressive strength to evaluate full plastic strength of a CFT section filled with high-strength concrete.

4. Computer analysis

A computer program to obtain the ultimate strength of composite columns with both ends pinned under combined bending and axial loads in single curvature was implemented by the writers. The analytical method used to develop the computer program is based on the numerical integration technique. A segmental subdivision of the column length is used to determine the complete load-moment-curvature-deflection for both short and slender column. The column cross section is divided into a number of small areas for which conditions of equilibrium and strain compatibility must be satisfied at the nodal points. The numerical technique adopted in the computer method is based on the incremental deflection approach, where an assumed deflection values is specified at a selected joint, the corresponding equilibrium loads are calculated, and the conditions of strain compatibility and equilibrium are then satisfied along the column length. The procedure successfully carried out the results when the assumed deflection values accurately matched computed deflection values with certain allowable limits. An iterative procedure to solve a system of nonlinear equation is used to obtain the solution of second-order equations generated by the finite difference methods with extremely rapid convergence. The iteration procedure achieves the maximum load.

4.1 Stress-strain relationships

The development of an appropriate analytical model to predict the behavior of CFT structural members requires a correct representation of the corresponding material characteristics. The study of an investigation set to the mechanical properties for CFT members has been reported in many researches. In AIJ-CFT, an elasto-plastic stress-strain relationship for both concrete and steel based on considerations of the previous researchers was chosen as Eq. (1) and Eq. (2). These models provided in AIJ-CFT were used in this analysis. For the stress-strain relation of the steel tube, the Ramberg-Osgood formula modeled for cold formed steel, was used.

$$\varepsilon_s = \frac{\sigma_s}{E_s} + 0.002(\sigma_s/\sigma_y)^m \quad (1)$$

where,

ε_s : steel strain corresponding to σ_s

E_s : Young's modulus of steel

m : hardening factor = 10

σ_s : steel stress at any point,

σ_y : yield stress of steel

Stress-strain relation of the concrete under compression is used as in Eq. (2).

$$\frac{\sigma_c}{\sigma_B} = 1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_B}\right)^a \quad 0 \leq \varepsilon_c \leq \varepsilon_B$$

$$\frac{\sigma_c}{\sigma_B} = 1 \quad \varepsilon_c \geq \varepsilon_B \quad (2)$$

where,

$$a = E_c \cdot \varepsilon_B / \sigma_B$$

$$\varepsilon_B = 0.93 \cdot \sigma_B^{(1/4)} \cdot 10^{-3} \quad (3)$$

σ_B : compressive strength of concrete (MPa)

σ_c : concrete stress at any point

ε_B : concrete strain corresponding to σ_B

ε_c : concrete strain corresponding to σ_c

where, elastic modulus of concrete (E_c) is as in Eq. (4).

$$E_c = (3.32 \sqrt{\sigma_B} + 6.90) \cdot 10^3 \quad (4)$$

5. Proposed method

The determination of interaction curve of a section generally requires comprehensive calculation (dotted line in Fig. 4). Therefore, an approximate method is applied to determining the strength of a CFT column section. This method is closely approximated, with the point *A* through *E* (solid line in Fig. 4).

It can be developed by assuming that the neutral axis lies exactly on the centerline, where the inner normal force and the inner moment may easily be determined using the symmetrical properties of the CFT section (Eq. 5).

$$N_{AO} = A_s \cdot \sigma_y + A_c \cdot \sigma_B$$

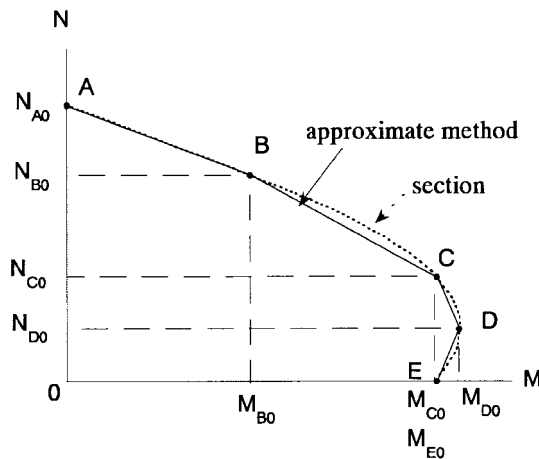


Fig. 4 Interaction curve approached by a polygonal connection of the points

$$\begin{aligned}
N_{BO} &= (N_{AO} + N_{CO})/2 \\
M_{BO} &= M_{DO} - \{2t h^2 \cdot \sigma_y + (D - 2t) \cdot h^2 \cdot \sigma_B\} \\
h &= h_e + \{(N_{AO} - N_{CO})/2D \cdot \sigma_B + 4t \cdot (2\sigma_y - \sigma_B)\} \\
N_{CO} &= A_c \cdot \sigma_B \\
M_{CO} &= M_{DO} - \{2t h_e^2 \cdot \sigma_y + (D - 2t) \cdot h_e^2 \cdot \sigma_B\} \\
h_e &= N_o(2D\sigma_B + 4t(2\sigma_y - \sigma_B)) \\
N_{DO} &= A_c \cdot \sigma_B/2 \\
M_{DO} &= M_p = Z_{ps} \cdot \sigma_y + Z_{pc} \cdot \sigma_B/2 \\
M_{EO} &= M_{CO}
\end{aligned} \tag{5}$$

where, D : Depth or Diameter of steel tube

Z_{ps} , Z_{pc} : plastic section moduli of the steel tube and concrete.

A simplified method for determining the strength of CFT columns under combined axial compression and bending moment is demonstrated using the simple function as called “column strength curve”. Usually column strength curve has been used in order to obtain the strength of column under concentric load. The column strength curve method was developed based on the postulation that an initially crooked column with the initial crookedness at the mid-span of the column equal to the deflection will fail under the combined action of an axial load and secondary bending moment ($p \cdot \delta$ effect). This manner can be used in the eccentrically loaded columns as well as in the concentrically loaded columns, because the fundamental postulation is the same that the columns will fail at the mid-span of the column. While the column strength curve to determine the lateral deflection should be changed by the eccentricity under combined action of axial compression and bending moment. It need not satisfy all eccentricity, but it can be determined directly at specific points of the interaction curve. It is summarized into four column curve formulas at specific points. For alternative buckling length, ultimate column strength can be obtained by connecting the point calculated column strength formula at alternative buckling length (Fig. 5).

The column-strength curve (Eq. 6) has been developed in terms of the slenderness ratio $\bar{\lambda}$ (Chen and Atsuta 1973). This column-strength curve is chosen for this study.

$$\frac{N_{xi}}{N_{xo}} = \frac{\alpha + \bar{\lambda}^2}{\alpha + \bar{\lambda}^2 + \bar{\lambda}^4} \tag{6}$$

$$\bar{\lambda} = \sqrt{\frac{N_o}{N_e}}$$

where

$$N_e = \frac{\pi^2 (E_s I_s + E_c I_c)}{Lk^2} \tag{7}$$

$$N_o = \sigma_y \cdot A_s + \sigma_B \cdot A_c$$

x : A, B, C, D

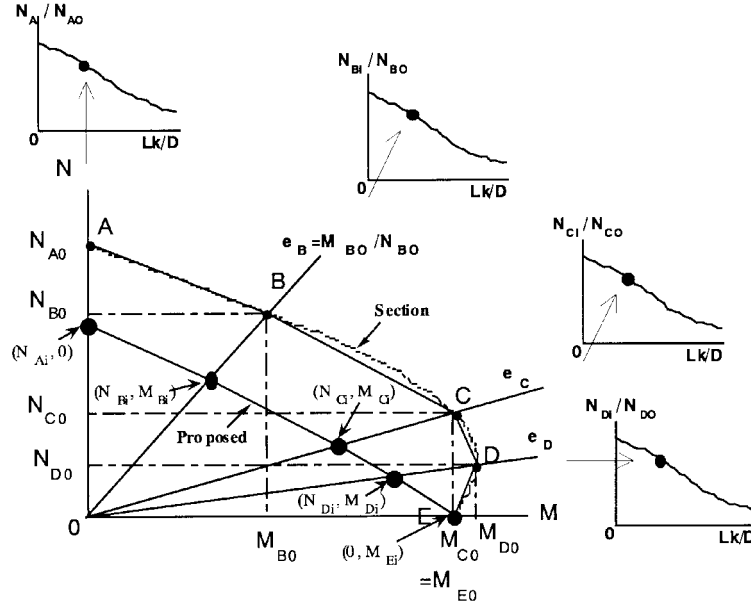


Fig. 5 Interaction curve approached by proposed method

A_s, A_c : sectional area of steel and concrete
 E_s, E_c : elastic modulus of steel and concrete
 I_s, I_c : moment of inertia of steel and concrete
 σ_y : yield stress of steel
 σ_B : compressive strength of concrete

From the formula above, the curve-fitting parameter α adjusted in conformity with the analytical result is proposed as Table 5.

The accuracy of the simplified methods is examined by comparing the analytical results on the parameters. In the comparison, outer diameter of 300 mm is constant, while the wall thickness, concrete strength and steel strength are varied. The column curves corresponding to each point in Fig. 5 are shown in Fig. 6. The results of the proposed strength of each point are the solid lines, whereas the results of the exact calculations are marked with a single dot. The exact calculations are

Table 5 The parametric values of column curves

x	α	Limit
A	0.35	$\frac{N_{Ai}}{N_{Ao}} \geq 1$
B	0	$\frac{N_{Bi}}{N_{Bo}} \geq 1$
C	0	$\frac{N_{Ci}}{N_{Co}} \geq 1$
D	0	$\frac{N_{Di}}{N_{Do}} \geq 1$ and $N_{Di} \geq \frac{N_{Ci}}{2}$

obtained from basis of elasto-plastic analysis described above. The concrete strength, yield strength of steel and rate of steel strength for squash load are shown in each figure. The calculations for circular and square CFT columns subjected to eccentric load were run for Lk/D varied from 0 to 30. For both circular and square CFT columns, the correspondence was generally good for all investigated parameters.

Comparisons between the proposed strength and the exact calculation on the interaction curve are shown in Fig. 7. In the figure, the dotted lines are the strengths of the composite section, which are computed by assuming the rectangular stress distribution with yield stress of steel tube and compressive strength of concrete. The results of the proposed method are the solid lines, whereas the results of the exact calculations which are marked with a single dot. For both circular and square CFT columns, the proposed method partially very conservative for CFT columns with the high strength concrete and the low steel strength cases. Except for this case, it was found that the proposed method gives good correlation with the exact calculation for both circular and square CFT columns.

6. Comparison of strength

All test results are compared with strengths based on the proposed method. The mean for 18 CFT columns is 1.12 and the coefficient of variation is 0.058. Fig. 8 contrasts the results of the column strength by a test and the AIJ-CFT and proposed methods on interaction curves for combinations of axial compression and end moment. The figures show the proposed strength and AIJ-CFT strength are in fairly good agreement with test results and in most cases, the experimental values are on safe side.

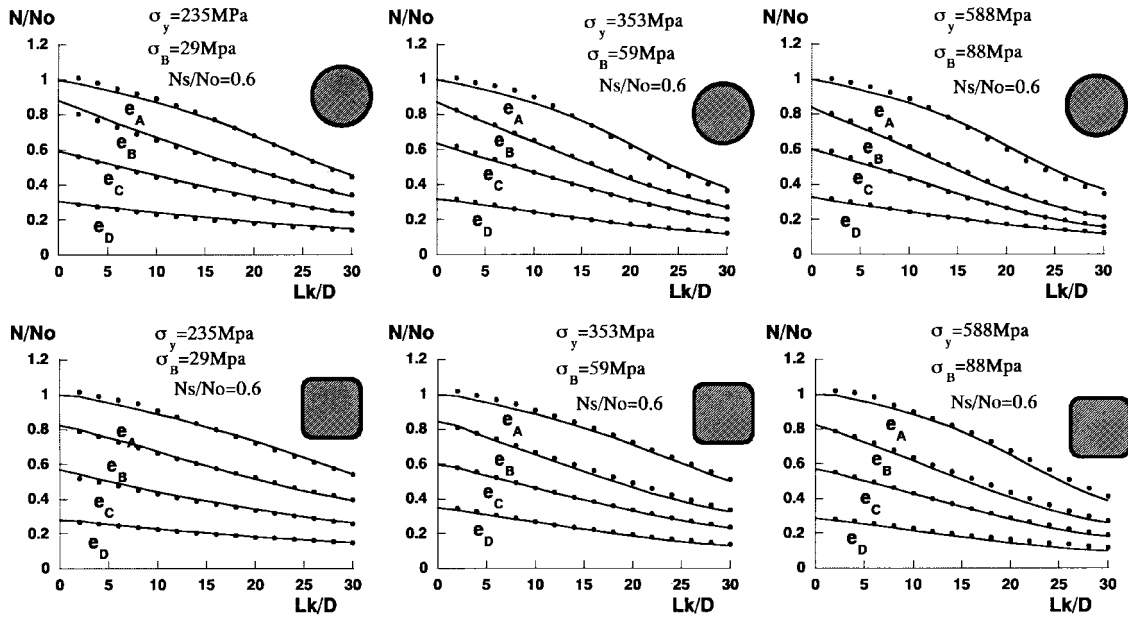


Fig. 6 Correlation with analytical results on the column curve

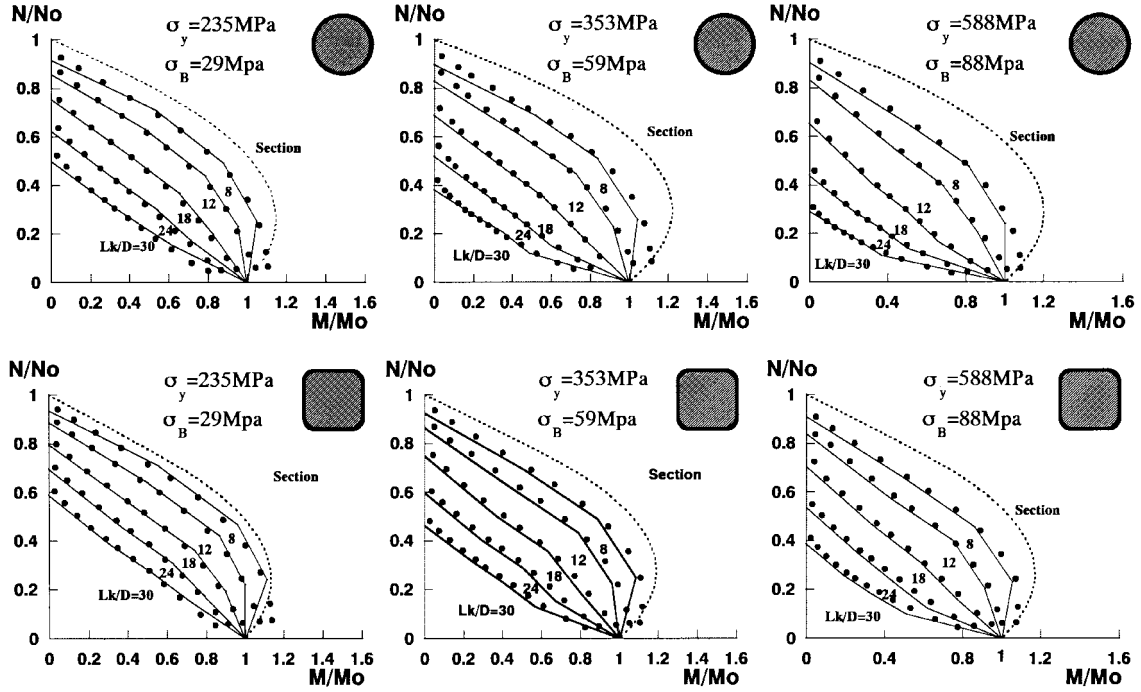


Fig. 7 Correlation with analytical results on the M-N interaction curve

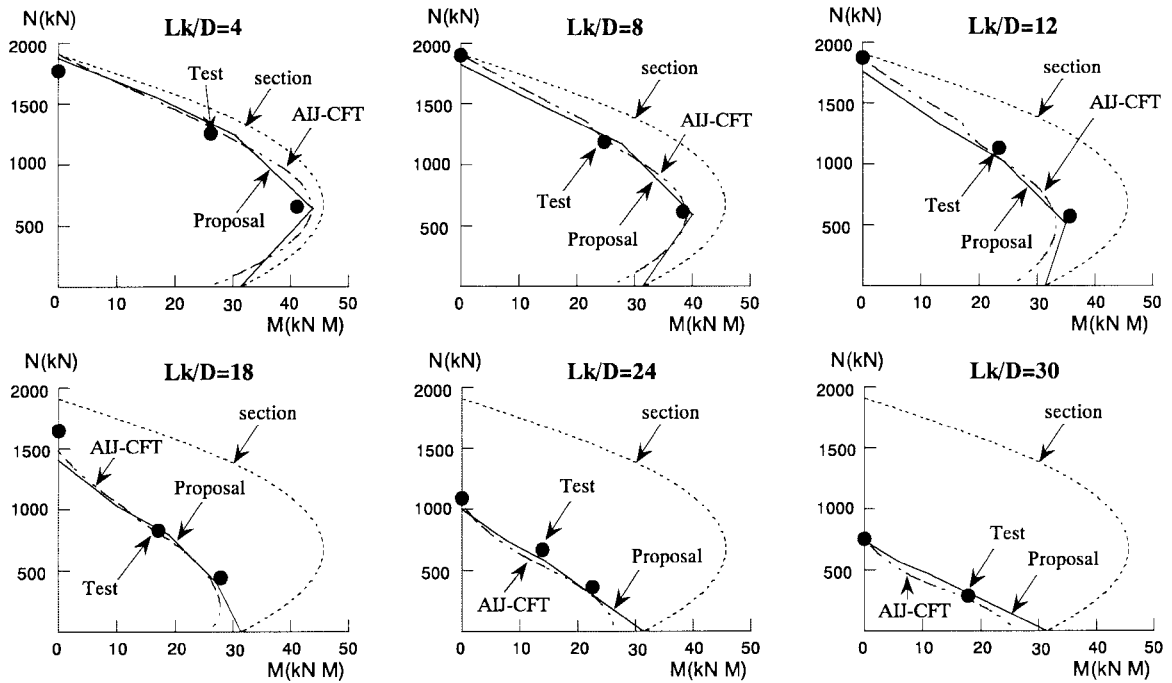


Fig. 8 Correlation with test results

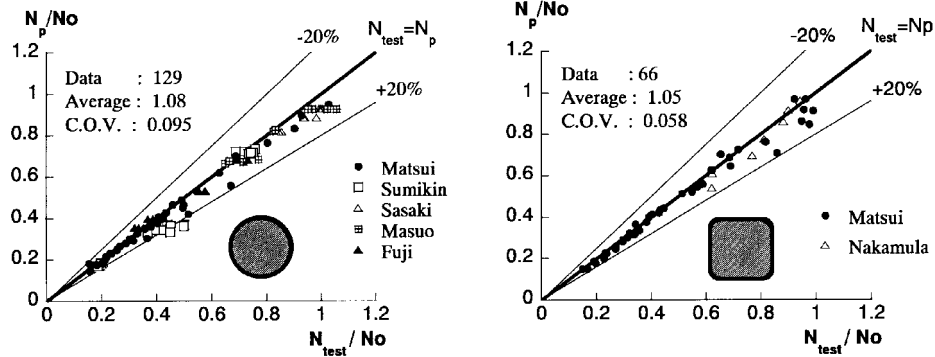


Fig. 9 Comparison of strength-reported by AIJ-CFT

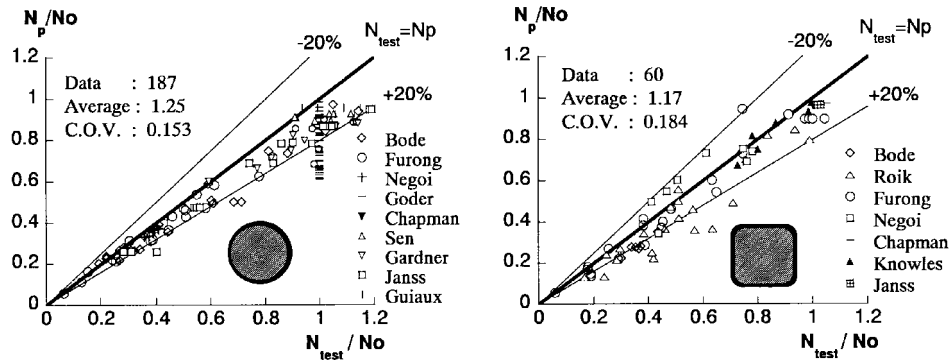


Fig. 10 Comparison of strength-reported from Europe and America

7. Summary

Fig. 9 shows the results of comparing the proposed method to experimental results reported by AIJ-CFT. In Fig. 9, the proposed strength (N_p) and tested column strength (N_{test}) was defined as nominal axial compressive strength (N_o). The mean for the circular CFT columns reported by AIJ-CFT, which included 129 test results, was 1.08, and the coefficient of variation was 0.095. The corresponding results for the square CFT columns, which were based on 66 tests, were 1.05 and 0.058, respectively.

Fig. 10 shows the results of comparing the proposed method to experimental results reported from Europe and America. The mean for the circular CFT columns reported from Europe and America, which was based on 187 test results, was 1.25, and the coefficient of variation was 0.153. The corresponding results for the square CFT columns, which included on 60 tests, were 1.17 and 0.184, respectively.

From the comparison, the strengths based on the proposed method well predict the experimental results reported by AIJ-CFT, but variations are shown in experimental results reported by Europe and America. Ideally the predictions of CFT column strength made using the proposed method would show perfect correlation with published test data. However, in practice perfect correlation is not achieved. This reason could be summarized as follows:

- the properties of constituent materials are variable.
- cross sections and test specimens are in some case very small.
- systematic errors are made in the course of scientific measurements and gross errors in experimental technique sometimes occur.

After giving due consideration to the forgoing sources errors it was concluded that the proposed method could reliably predict the strength of eccentrically loaded CFT columns bent into single curvature.

8. Conclusions

Concrete filled steel tubular members have many structural and constructional advantages, such as large strength, stiffness and ductility, high fire resistance, restraint to local buckling of the steel tube provided by the filled concrete, increase of concrete strength due to the effect of confinement provided by steel tube. In recent years the use of CFT columns has increased because of these advantages. Concurrent with this has been a trend in the construction industry towards the use of higher strength materials. Current design codes, however, exclude the use of high strength steel or high strength concrete in CFT column. Consequently, one of the primary objectives of this dissertation was to study the behavior of CFT columns with high strength material under axial compression and bending moment. In order to investigate the behavior of CFT columns with high strength material, an experimental work, which includes the theoretical analysis for test results, was undertaken. On the basis of the work carried out in the experimental study, the following conclusions were drawn:

- The eccentricity of the load and slenderness of the column have the most profound effect upon column strength.
- Improvement of strength due to the confinement effect of the concrete has not occurred for the square columns filled with high strength concrete.

The main objective of this dissertation was to develop a simplified method, which could predict the strength of CFT columns under combined axial compression and flexure. It was developed based on the interaction curve of the section approached by a polygonal connection of the points. For each point (A , B , C , D), a simple equation as called “column strength curve” was proposed to determine the strength of the slender columns under compression and flexure. The simple equation was adjusted with results of elasto-plastic analysis. The ultimate strength of concrete filled steel tube column was simply obtained using the proposed column strength curve at each point. Validation of the simplified method was undertaken by comparison with data from the test conducted at Kyushu University as well as those reported in the literature.

From the results of this paper, it was concluded that the simplified method could accurately and reliably predict the strength of CFT columns under combined axial compression and flexure.

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