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Application of softened truss model with plastic approach to reinforced concrete beams in torsion

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Abstract. The present paper discusses the behavior of the reinforced concrete beams subjected to torsion by applying the endochronic plastic model in conjunction with the softened truss model. The endochronic constitutive equations are developed to describe the behavior of concrete. The mechanical behavior of concrete is decomposed into hydrostatic part and deviatoric part. New definition of the bulk modulus and the shear modulus are defined in terms of compressive strength of concrete. Also, new deviatoric hardening function is developed. Then, the endochronic constitutive equations of concrete are applied with the softened truss model for the behavior of the reinforced concrete beams subjected to torsion. The theoretical results obtained based on the present model are compared with the experimental data. The present model has shown the ability to describe the behavior of reinforced concrete beams subjected to torsion.

Key words: endochronic model for concrete; reinforced concrete beam; softened truss model.

1. Introduction

The present paper discusses the behavior of the reinforced concrete beams subjected to torsion by applying the endochronic constitutive equations of concrete with the softened truss model. The endochronic theory was first proposed by Valanis (1971). Next, the theory was applied to describe the mechanical behavior for different materials, as in Bazant and Bhat (1976, 1977), Bazant (1978), Valanis (1980), Wu and Yip (1981), Valanis and Fan (1983), Wu and Wang (1983), Watanabe and Atluri (1985), Wu and Aboutorabi (1988), Pan, Yang and Lu (1998), Wu and Lu (1995), Wu, Lu and Pan (1995) and Lu (1998).

Bazant and Bhat (1976) modified the endochronic theory to describe the mechanical behavior of concrete. Their model is powerful but a comply. Wu and Aboutorabi (1988) derived an endochronic model for concrete based on Gibbs formulation. Their model did not discuss the strain softening behavior of concrete. The present paper develops the endochronic constitutive equations for concrete accounting the strain softening behavior in a simple form. Following the similar procedure of Lu (1998), we have the endochronic constitutive equations for concrete. The mechanical behavior of concrete are decomposed into hydrostatic part and deviatoric part. The bulk modulus and the shear modulus used in present model are redefined in terms of compressive strength of concrete. New version of deviatoric hardening function is developed in this paper to account the strain softening

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behavior of concrete. Then, the endochronic constitutive equations of concrete are applied with the softened truss model to describe the behavior of reinforced concrete beams subjected to pure torsion.

The softened truss model was developed and improved by Hsu and his co-workers (Hsu and Mo 1985, Hsu 1988, 1991, Belarbi and Hsu 1994, 1995, Hsu and Zhang 1996 and Pang and Hsu 1996). It is based on the two dimensional equilibrium, Mohr strain compatibility and biaxial constitutive equations of materials. The softened truss model has been applied to determine the ultimate strength and the deformation throughout its post-cracking loading history of reinforced concrete beams in torsion. In present paper, the endochronic constitutive equations of concrete are applied with the softened truss model to describe the behavior of reinforced concrete beams subjected to torsion. The theoretical results are compared with the experimental data in Fang (1995). Several specimens with different properties have been included. Reasonable agreement between theory and experiment has been achieved.

2. The endochronic model for concrete

The endochronic stress-strain relationship for concrete can be obtained by following the similar procedure of Lu (1998) as

$$\varepsilon_{kk}=\frac{1}{3K}\sigma_{kk}+3B_0\gamma_{kk} \tag{1}$$

and

$$e_{ij}=\frac{1}{2G}s_{ij}+C_2p_{ij} \tag{2}$$

Also, the evolution equation of internal variables of above equations can be obtained as

$$M_0\frac{d\gamma_{kk}}{dZ_H}+E_0\gamma_{kk}-B_0\sigma_{kk}=0 \tag{3}$$

and

$$N_2\frac{dp_{ij}}{dZ_D}+F_2p_{ij}-C_2s_{ij}=0 \tag{4}$$

In Eqs. (1)-(4), ε_{ij} represents the strain, σ_{ij} represents the stress, γ_{ij} denotes the internal variable related to hydrostatic deformation and p_{ij} denotes the internal variable related to deviatoric deformation; e_{ij} and s_{ij} are the deviatoric components of ε_{ij} and σ_{ij}, respectively; B_0, E_0, M_0, C_2, F_2 and N_2 are the material constants; Z_H and Z_D represent the hydrostatic and the deviatoric intrinsic time, respectively; K denotes the bulk modulus and G is the shear modulus.

In present investigation, the mechanical response of concrete are decomposed into two parts: the hydrostatic response and the deviatoric response. Eqs. (1) and (2) are the stress-strain relation for the hydrostatic response and the deviatoric response, respectively. Eqs. (3) and (4) are the evolution form of internal variables for the hydrostatic response and the deviatoric response, respectively. To consider the effect of the compressive strength of concrete, new bulk modulus and shear modulus are defined in terms of the compressive strength of concrete in this paper. To consider the strain softening behavior of concrete, the new deviatoric hardening function is developed. These are discussed in the following sections:

2.1 The hydrostatic response

The hydrostatic strain increment $d\varepsilon_{kk}$ can be expressed in terms of the increment of hydrostatic stress $d\sigma_{kk}$ with hydrostatic response internal variable $d\gamma_{kk}$ that is related to the hydrostatic response, from Eq. (1), as

$$d\varepsilon_{kk}=\frac{1}{3K}d\sigma_{kk}+3B_0 d\gamma_{kk} \tag{5}$$

The bulk modulus K is defined in terms of the compressive strength of concrete in the present model as

$$K=K_0+K_1 f_c' \tag{6}$$

where K_0 and K_1 are material constants and f_c' represents compressive strength of concrete. Note that, from Eq. (3), the increment of internal variable $d\gamma_{kk}$ depends on the hydrostatic intrinsic time Z_H, the current state of hydrostatic stress and internal variable γ_{kk} and can be expressed as

$$d\gamma_{kk}=\left(\frac{B_0}{M_0}\sigma_{kk}-\frac{E_0}{M_0}\gamma_{kk}\right)dZ_H \tag{7}$$

To account for strain-hardening, the hydrostatic intrinsic time Z_H is scaled by

$$dZ_H=\frac{d\zeta_H}{h(\theta_{kk})} \tag{8}$$

where the ζ_H represents the hydrostatic intrinsic time scale and the function $h(\theta_{kk})$ represents volumetric hardening; θ_{kk} is the plastic volumetric strain-like tensor and its increment is defined by

$$d\theta_{kk}=d\varepsilon_{kk}-k_1\frac{d\sigma_{kk}}{3K} \tag{9}$$

where k_1 denotes a material constant with the value between zero and one. When k_1=1, $d\theta_{kk}$ is the plastic volumetric strain increment. The increment of the hydrostatic intrinsic time scale $d\zeta_H$ is defined interns of volumetric strain as

$$d\zeta_H=|d\theta_{kk}| \tag{10}$$

The volumetric hardening function $h(\theta_{kk})$ is defined as

$$h(\theta_{kk})=\frac{c_h-(c_h-\theta_m)e^{-\alpha\theta_{kk}}}{\theta_m-\theta_{kk}} \tag{11}$$

where c_h and α denote material constants and θ_m represents the maximum attainable plastic volumetric strain. The form of Eq. (7) was originally used for porous material by Wu and Aboutorabi (1988).

2.2 The deviatoric response

The deviatoric strain increment de_{ij} can be expressed in terms of the increment of deviatoric stress

ds_{ij} with deviatoric response internal variable dp_{ij}, from Eq. (2), as

$$de_{ij}=\frac{1}{2G}ds_{ij}+C_2dp_{ij} \tag{12}$$

Similar to the bulk modulus K, the shear modulus G is defined in terms of the compressive strength of concrete in this paper as

$$G=G_0+G_1f_c' \tag{13}$$

where G_0 and G_1 are material constants. It notes that, from Eq. (4), the increment of internal variable dp_{ij} depends on the deviatoric intrinsic time Z_D, the current state of deviatoric stress s_{ij} and the internal variable p_{ij} and can be expressed as

$$dp_{ij}=\left(\frac{C_2}{N_2}s_{ij}-\frac{F_2}{N_2}p_{ij}\right)dZ_D \tag{14}$$

The deviatoric intrinsic time Z_D is scaled by

$$dZ_D=\frac{d\zeta_D}{f(Z_D)} \tag{15}$$

where function $f(Z_D)$ represents deviatoric hardening; the $d\zeta_D$ represents the deviatoric intrinsic time scale and defined as

$$d\zeta_D=\left|dQ_{ij}\right| \tag{16}$$

with

$$dQ_{ij}=de_{ij}-k_2\frac{dS_{ij}}{2G} \tag{17}$$

where dQ_{ij} denotes the increment of plastic deviatoric strain-like tensor and k_2 denotes a material constant with the value between zero and one.

To account for the compressive strength dependence of strain hardening and strain softening of concrete, the new definition of deviatoric hardening function $f(Z_D)$ is defined in this paper as

$$f(Z_D)=\frac{F_1(Z_D)F_2(f_c',J_{2d})}{F_3(f_c',J_{2d}',J_{2d},I_{1h})} \tag{18}$$

with

$$F_1(Z_D)=c_d-(c_d-1)e^{-\beta Z_D} \tag{19}$$

$$F_2(f_c',J_{2d})=b_1+b_2f_c'+b_3J_{2d}(f_c')b_4 \tag{20}$$

$$F_3(f_c',J_{2d}',J_{2d},I_{1h})=1+(a_1-a_2f_c')\langle J_{2d}'-J_{2d}'(\varepsilon_p)\rangle\left(d_1+d_2\frac{J_{2d}}{I_{1h}^2}\right) \tag{21}$$

where c_d, β, a_i, b_i, d_i are material constants; I_{1h}, J_{2d} and J_{2d}' represent the first invariant of the hydrostatic stress, the second invariant of the deviatoric stress and the second invariant of the deviatoric strain, respectively. $J_{2d}'(\varepsilon_p)$ denotes the value of J_{2d}' with the plastic strain equal to ε_p. The mark “< >” is the Macaulay function with $<A-B>=0$ for $A\le B$ and $<A-B>=A-B$ for $A>B$.

The function $F_1(Z_D)$ in Eq. (18) is the hardening function that are originally used for metal by Wu and Yip (1981). The function $F_2(f_c',J_{2d})$ accounts for the effect of compressive strength of concrete. The function $F_3(f_c',J_{2d}',J_{2d},I_{1h})$ is the function that governs the strain softening behavior of concrete. Before the plastic strain reaching ε_p, the F_3 is equaling one.

2.3 Comparison with experimental data

We now apply the present endochronic constitutive equations to describe the mechanical behaviors of concrete. The specimens with different compressive strength are discussed. Fig. 1 shows the theoretical results fit experimental data of concrete under uniaxial loading condition. The material constants are obtained from curve fitting the theoretical results with experimental data of concrete under uniaxial loading. Note that the material constants for hydrostatic part are K_0=5.729 MPa, K_1= 0.1948, c_h=0.000316, θ_m=0.005, α=1.36, $B_0=6.234\times10^{-5}$, $E_0=6.872\times10^5$, $M_0=6.46\times10^3$, k_1=0.95 and the material constants for deviatoric part are G_0=4.64 MPa, G_1=0.1876, C_2=0.539, β=152.6, $N_2=3.458\times10^6$, $F_2=1.3264\times10^8$, c_d=3.7234, a_1=224500, a_2=2430, b_1=0.3, b_2=0.037, b_3=0.0002, b_4= 0.5544, d_1=0.135, d_2=1.84, k_2=0.882 and the ε_p in $J_{2d}'(\varepsilon_p)$ equals 0.0014. The same material constants will be used through this paper. Fig. 1 includes the specimens with concrete compressive strengths of 21 MPa, 31 MPa and 41 MPa. The experimental data are from Hognestad, Hanson and McHenry (1955). Fig. 2 shows the theoretical results and experimental data of concrete under uniaxial loading/unloading condition. It includes the specimens with concrete compressive strengths of 25.86 MPa and 40 MPa. The experimental data of concrete compressive strength equal to 25.86 MPa are from Sinha, Gerstle and Tulin (1964) and that of compressive strength equal to 40 MPa are from Spooner and Dougill (1975). Fig. 3 shows the theoretical results and experimental data of concrete under biaxial compression loading condition. The compressive strength of the concrete is 32.8 MPa and the experimental data are from Kupfer, Hilsdorf and Rusch (1969). The agreements

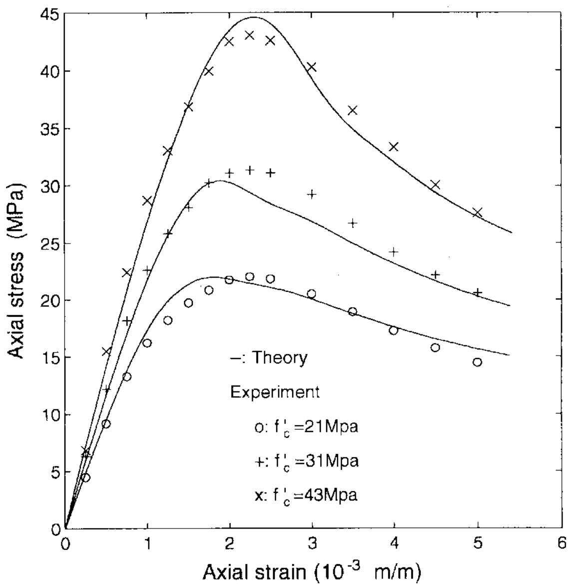

Fig. 1 Stress-strain curve for concrete under uniaxial loading

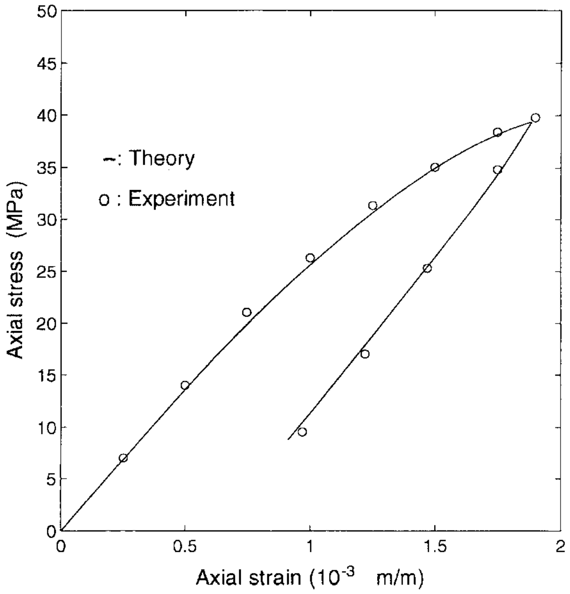

Fig. 2 Stress-strain curve for concrete under uniaxial loading/unloading

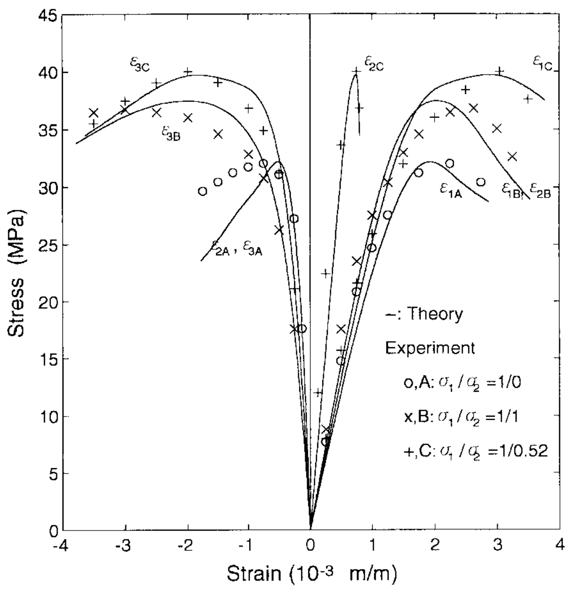

Fig. 3 Stress-strain curve for concrete under biaxial compression loading

between the theoretical and experimental results are quite good.

3. The summary of softened truss model

The softened truss model, which was developed and improved by Hsu and his co-workers, includes the stress equilibrium equations, strain compatibility equations, the constitutive equations of concrete in compressive and tension and the constitutive equations of reinforcing steel. They are summarized below.

3.1 The constitutive equations of concrete

The stress-strain curve of concrete recommended by Hsu (1996) are improved version where suggested by Belarbi and Hsu (1994, 1995)

Concrete in compression:

$$\sigma_d=\zeta\cdot f_c'\cdot\left[2\cdot\left(\frac{\varepsilon_d}{\zeta\cdot\varepsilon_0}\right)-\left(\frac{\varepsilon_d}{\zeta\cdot\varepsilon_0}\right)^2\right]\qquad \frac{\varepsilon_d}{\zeta\cdot\varepsilon_0}\le 1 \tag{22}$$

$$\sigma_d=\zeta\cdot f_c'\cdot\left[1-\left(\frac{\frac{\varepsilon_d}{\zeta\cdot\varepsilon_0}-1}{\frac{2}{\zeta}-1}\right)^2\right]\qquad \frac{\varepsilon_d}{\zeta\cdot\varepsilon_0}>1 \tag{23}$$

where ε_0 is strain at maximum compressive stress and ξ denotes softening coefficient with

$$\zeta=\frac{0.9}{\sqrt{1+400\cdot\varepsilon_r}} \tag{24}$$

Concrete in tension:

$$\sigma_r=E_c\cdot\varepsilon_r,\qquad \varepsilon_r\le 0.00008 \tag{25}$$

$$\sigma_r=f_{cr}\cdot\left(\frac{0.00008}{\varepsilon_r}\right)^{0.4},\qquad \varepsilon_r>0.00008 \tag{26}$$

where E_c is elastic modulus of concrete and f_{cr} denotes cracking stress of concrete.

3.2 The constitutive equations of steel

The simplified stress-strain relationships for longitudinal and transverse steel from Hsu (1991) are below:

$$f_l=E_s\cdot\varepsilon_l,\qquad \varepsilon_l<\varepsilon_{ly} \tag{27}$$

$$f_l=f_{ly},\qquad \varepsilon_l\ge\varepsilon_{ly} \tag{28}$$

and

$$f_t=E_s\cdot\varepsilon_t,\qquad \varepsilon_t<\varepsilon_{ty} \tag{29}$$

$$f_t=f_{ty},\qquad \varepsilon_t\ge\varepsilon_{ty} \tag{30}$$

where E_s represents the elastic modulus of steel bars; ε_{ly} and ε_{ty} are the yield strains of longitudinal and transverse steel bars, respectively; f_{ly} and f_{ty} are the yield stresses of longitudinal and transverse steel bars, respectively.

3.3 The equilibrium equations

$$\sigma_l=\sigma_d\cdot\cos^2\alpha+\sigma_r\cdot\sin^2\alpha+\rho_l f_l \tag{31}$$

$$\sigma_t=\sigma_d\cdot\sin^2\alpha+\sigma_r\cdot\cos^2\alpha+\rho_t f_t \tag{32}$$

$$\tau_{lt}=(-\sigma_d+\sigma_r)\sin\alpha\cdot\cos\alpha \tag{33}$$

$$\tau_{lt}=\frac{T}{2A_e\cdot t_d} \tag{34}$$

where σ_l and σ_t are the normal stress in the l and t directions, respectively; τ_{lt} denotes the shear stress in the l–t coordinate; σ_d and σ_r are the principal stress in the d and r directions, respectively; α denotes the angle of inclination of the d-axis with respect to l-axis; ρ_l and ρ_t are the reinforcement ratio in the l and t directions, respectively; f_l and f_t are the steel stress in the l and t directions, respectively; T represents the applied torque; A_e denotes the area within the centerline of the shear flow and t_d denotes the thickness of shear flow zone.

For pure torsion, σ_l, σ_t are nil and σ_r is neglected, i.e. $\sigma_l=0$, $\sigma_t=0$ and $\sigma_r=0$. Then, Eqs. (31), (32) and (33) can be rewritten as

$$\sigma_d\cdot\cos^2\alpha+\rho_l f_l=0 \tag{35}$$

$$\sigma_d\cdot\sin^2\alpha+\rho_t f_t=0 \tag{36}$$

$$\tau_{lt}=-\sigma_d\ \sin\alpha\cdot\cos\alpha \tag{37}$$

ρ_l, ρ_t, A_e and p_e can be expressed by t_d as

$$\rho_l=\frac{A_l}{p_e\cdot t_d} \tag{38}$$

$$\rho_t=\frac{A_t}{s\cdot t_d} \tag{39}$$

$$A_e=A_c-\frac{1}{2}p_c\cdot t_d+t_d^2 \tag{40}$$

$$p_e=p_c-4t_d \tag{41}$$

where p_e represents the perimeter of the centerline of the shear flow; p_c denotes the outer perimeter of concrete cross section and A_c denotes the area bounded by the outer perimeter of concrete cross section; A_l and A_t are cross-sectional area of steel for total longitudinal bars and one transverse hoop bar, respectively.

3.4 The compatibility equations

$$\varepsilon_l=\varepsilon_d\cdot\cos^2\alpha+\varepsilon_r\cdot\sin^2\alpha \tag{42}$$

$$\varepsilon_t=\varepsilon_d\cdot\sin^2\alpha+\varepsilon_r\cdot\cos^2\alpha \tag{43}$$

$$\gamma_{lt}=2(-\varepsilon_d+\varepsilon_r)\sin\alpha\cdot\cos\alpha \tag{44}$$

$$\Psi=\theta\cdot\sin^2\alpha \tag{45}$$

$$t_d=\frac{\varepsilon_{ds}}{\Psi} \tag{46}$$

$$\varepsilon_{ds}=2\cdot\varepsilon_d \tag{47}$$

$$\theta=\frac{p_e}{2A_e}\gamma_{lt} \tag{48}$$

where ε_l, ε_t and γ_{lt} are the average strains in the l and t directions and the average shear strains in the l–t coordinate, respectively; ε_d and ε_r are the average principal strains in the d and r directions, respectively; θ represents the angle of twist per unit length; Ψ denotes the curvature of the diagonal concrete struts and ε_{ds} denotes the maximum compressive strain.

4. Application to reinforced concrete beams

In this section, we apply the endochronic constitutive equations of concrete with the softened truss model for the behavior of reinforced concrete beams subjected to torsion. The endochronic constitutive equations of concrete developed by present paper are used instead of the elastic stress-strain relationship used by Hsu and his co-workers. The theoretical results are compared with the experimental data. Several specimens with different properties have been investigated. The experimental data are from Fang (1995). The flow chart of numerical calculation procedure is shown in Fig. 4.

4.1 The constitutive equations of concrete

In stead of Eqs. (22)-(26), the constitutive equation of concrete are used in present model and expressed as

$$\sigma_d=K_c f(\varepsilon_d) \tag{49}$$

where $f(\varepsilon_d)$ represents endochronic constitutive equations of concrete, i.e. Eqs. (5)-(21), that are derived previously. K_c is the coefficient for reinforced concrete beam and relates to the longitudinal and the transverse steel ratio of reinforced concrete beam and compression strength of concrete. In present paper, the K_c is defined as

$$K_c=\left(180\frac{\rho_l+\rho_t}{f_c'-12}+0.1\right) \tag{50}$$

with

$$K_c\leq 1 \tag{51}$$

where ρ_l and ρ_t are the longitudinal and the transverse steel ratio, respectively.

The other hand, the σ_d can be obtained from Eqs. (35) and (36) as

$$\sigma_d=\frac{-\rho_l\cdot f_l}{\cos^2\alpha} \tag{52}$$

and

$$\sigma_d=\frac{-\rho_t\cdot f_t}{\sin^2\alpha} \tag{53}$$

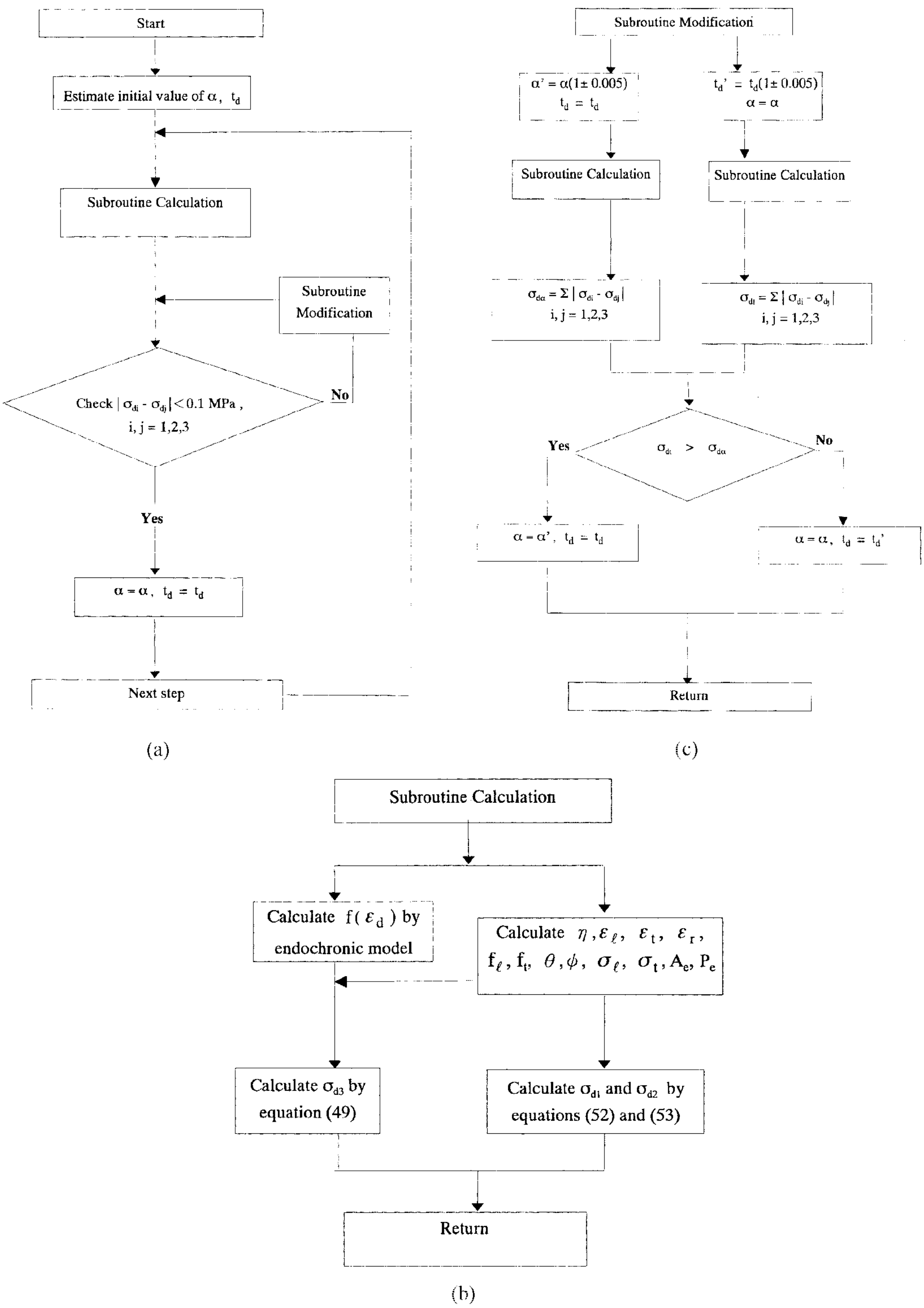

Fig. 4 (a) Flow chart of the numerical calculation procedure, (b) Flow chart of the subroutine calculation, (c) Flow chart of the subroutine modification

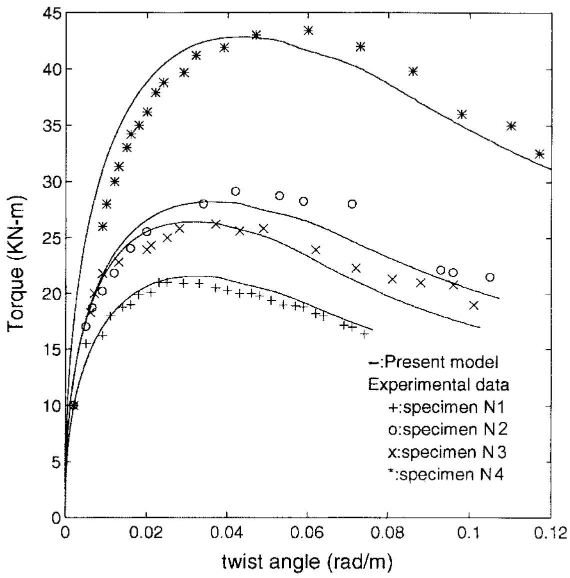

Fig. 5 Torque-twist curve for normal compression strength concrete specimens

Table 1 The properties of specimens (Fang 1995)

Specimen No.	f_c' (MPa)	x,($x1$) (mm)	y,($y1$) (mm)	Reinforcement Longitudinal bars	Reinforcement Stirrups	f_{ly} (MPa)	f_{ty} (MPa)
N1	29.3	300	180	4#5	#3@150mm	383	494
N2	27.6	300	180	4#5	#3@120mm	383	484
N3	27.6	240	240	4#5	#3@120mm	383	484
N4	27.6	240	240	4#6	#4@ 85mm	508	397
H1	81.1	300	180	4#6	#3@ 60mm	508	484
H2	81.1	240	240	4#6	#3@ 70mm	508	484
H3	81.1	240	240	4#7	#4@ 60mm	475	397
H4	75.9	240	240	4#8	#3@ 60mm	522	386

The σ_d from Eq. (49) should be the same as those from Eqs. (52) and (53). The accepted tolerances in calculation are 0.1 MPa.

Also, the angle of twist is modified in present model as

$$\theta'=\theta\left(0.006f_c'+76\frac{\sqrt{\rho_l+\rho_t}}{f_c'}\right)\left(\frac{\varepsilon_d}{\varepsilon_m}\right)^{1.4} \quad \text{for } \frac{\varepsilon_d}{\varepsilon_m}<1 \tag{54}$$

and

$$\theta'=\theta\left(0.006f_c'+76\frac{\sqrt{\rho_l+\rho_t}}{f_c'}\right) \quad \text{for } \frac{\varepsilon_d}{\varepsilon_m}\geq 1 \tag{55}$$

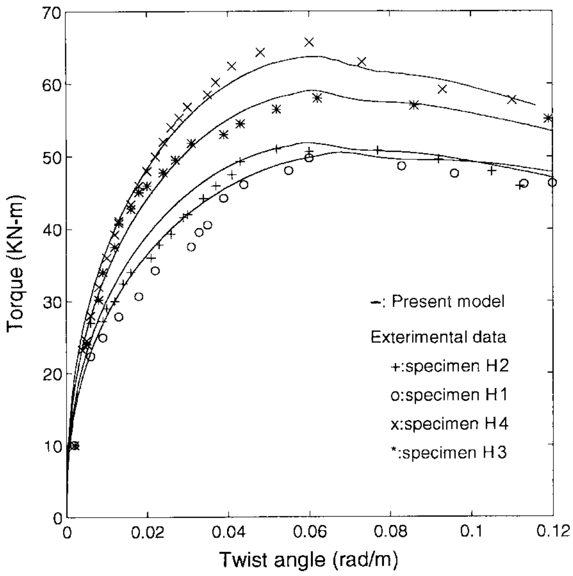

Fig. 6 Torque-twist curve for high compression strength concrete specimens

where ε_m denotes the strain in maximum stress.

4.2 Comparison with experimental data

In this application, ε_d, f_c', A_c, p_c, A_l, A_t, s, f_{ly} and f_{ty} are known, t_d and α are estimated initially. The flow chart of numerical calculation procedure is present in Fig. 4(a) and 4(b). If the differences between the σ_d's from Eqs. (49), (52) and (53) are smaller than the accepted tolerance 0.1 Mpa, the initial value of t_d and α will be used in the next step. Otherwise, t_d and α will be modified by using the modification procedure shown in Fig. 4(c). Following the procedure of numerical calculation, the torque and twist angle can be obtained. The theoretical results of present model and experimental data are shown in Figs. 5 and 6. The experimental data are from Fang (1995). Fang has investigated two major groups of specimens. One group is normal compressive strength concrete specimens and the other group is high compressive strength concrete specimens. The properties of the specimens are shown in Table 1. Fig. 5 shows the theoretical results of present model and experimental data for the normal compressive strength concrete specimens. Fig. 6 shows the results for the high compressive strength concrete specimens. It is seen that the agreement is quite satisfactory for both cases.

5. Conclusions

The present paper modifies the endochronic constitutive equations to describe the mechanical behavior of concrete. The mechanical behavior of concrete investigated includes hydrostatic response and deviatoric response. The bulk modulus and the shear modulus are redefined in terms of compressive strength of concrete. To account the strain softening behavior of concrete, new version of deviatoric hardening function is developed. Then, the endochronic constitutive equations of concrete are applied with the concepts of the softened truss model to describe the behavior of reinforced concrete beams subjected to torsion. The theoretical results of the present model are compared with the experimental data. The present model has shown the ability to describe the behavior of reinforced concrete beams subjected to torsion.

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