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Use of semi-active tuned mass dampers for vibration control of force-excited structures

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Abstract. A new class of semi-active tuned mass dampers, named as “Ground Hook Tuned Mass Damper” (GHTMD) is introduced. This TMD uses a continuously variable semi-active damper (so called ‘Ground-Hook’) in order to achieve more reduction in the vibration level. The ground-hook dampers have been used in the auto-industry as a means of reducing the vibration of primary suspension systems in vehicles. This paper investigates the application of this damper as an element of a tuned damper for the vibration reduction of force-excited single degree of freedom (SDOF) models that can be representative of many structural systems. The optimum design parameters of GHTMDs are obtained based on the minimization of the steady-state displacement response of the main mass. The optimum design parameters which are evaluated in terms of non-dimensional values of the GHTMD are obtained for different mass ratios and main mass damping ratios. Using the frequency responses of the resulting systems, performance of the GHTMD is compared to that of equivalent passive TMD, and it is found that GHTMDs are more efficient. A design methodology to obtain the tuning parameters of GHTMD using the relationships developed in this paper is presented.

Key words: tuned mass damper; tuned vibration absorber; semi-active damper; vibration control.

1. Introduction

Application of tuned mass dampers (TMD) to reduce vibrations dates back to the beginning of last century (Frahm 1911), where they were used for the reduction of rolling motion of ships. Since then there have been a large number of research efforts dedicated to the understanding and various applications of TMDs to reduce vibrations. Ormondroyd and Den Hartog (1928), and Den Hartog (1947) investigated the use of tuned mass dampers for the reduction of the amplitude of vibration of un-damped single degree of freedom systems. Tuned mass dampers have been used to control the vibration level of different structural and mechanical systems. Several variations of these systems including non-linear springs (Roberson 1952), tuned liquid dampers (Yeh *et al*. 1996), and multiple-tuned mass dampers (Yamaguchi and Harnpornchai 1993) have been applied to various structural systems. In general, the tuned mass dampers work based on adding a secondary vibrating system to a main vibrating system at resonance such that the natural frequency of the secondary system is tuned to the primary system and oscillates to counteract the motion of the primary system. Typically, the secondary system has only a fraction of the primary system mass. Therefore, by adding only a small amount of secondary mass the vibration of the primary system can be reduced

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significantly.

The most widely used tuned mass dampers have been passive TMDs. For this type of device, there is not any external forces added to the system, and the characteristics of the spring and/or damping do not change with time. There have been different studies on the use of active TMDs, based on the introduction of an external source of power to produce additional forces on the primary system to reduce its levels of vibrations (Udwadia and Tabaie 1981, Lund 1980, Chang and Soong 1980). Even though the active TMDs have better performance than passive TMDs, they have several disadvantages including: need for actuators, pumps, etc., high operational costs; and high power requirements.

In order to avoid the above disadvantages a new class of tuned mass dampers have been introduced using variable damping or stiffness. This class of tuned mass dampers (so called semi-active TMDs) have simple hardware requirements, low operational costs, and low power requirements. Different variations of semi-active TMDs have been introduced for transient vibration control such as wind induced vibrations in tall buildings (Hrovat *et al*. 1983), and for the seismic protection of civil structures (Abe 1996).

There is a large body of research on the application of semi-active dampers as primary suspension systems in vehicles. They have been used to control wheelhop (the axle vertical motion), and vehicle-body acceleration (Karnopp and Crosby 1974, Crosby and Karnopp 1973, Krasnicki 1980, Miller 1988, Ahmadian and Marjoram 1989).

A class of semi-active dampers called “skyhook” and “groundhook” (will be explained later) have resulted in significant control of vehicle vibrations (Miller 1988, Ahmadian 1997). However, these dampers can not be effective where the relative motion of the system is small. Therefore, for these situations tuned mass dampers using this type of dampers are being considered.

This paper studies the application of ground hook semi-active damper in a tuned mass damper configuration to reduce the vibration of single degree of freedom (SDOF) force excited systems. The resulting device is called ‘Ground-Hook Tuned Mass Damper (GHTMD)’. The optimum design parameters of the GHTMD will be found and its effectiveness as compared to equivalent passive TMD will be investigated.

2. Description of the GHTMD

Fig. 1 shows a single degree of freedom system consisting of mass m_1, stiffness k_1, and damping c_1 subjected to a harmonic force $F=F_0 e^{i\omega t}$. When the exciting frequency (ω) becomes close to the system natural frequency (ω_1), it will be in resonance and the main mass will be subjected to large amplitudes of motion (x_1). In order to reduce the vibration level, a secondary mass (m_2), will be attached to the main mass through a spring and dashpot with the stiffness coefficient of k_2, and a variable damper of c_2, respectively.

The “ground hook control policy” assumes that the variable damping c_2 can be switched between a low state and a high state. The two common types of this semi-active damper are on-off ground hook and continuously ground hook (Miller 1988). For the on-off ground hook dampers, the damping is switched between a minimum and a maximum level. The switching is performed based on the following conditions:

$$\dot{x}_1(\dot{x}_1 - \dot{x}_2) \geq 0 \rightarrow c_2 = c_{\max}$$

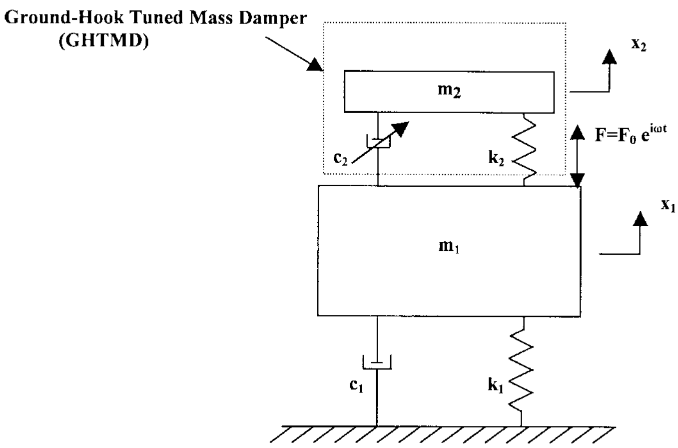

Fig. 1 Vibrating system with the GHTMD

$$\dot{x}_1(\dot{x}_1 - \dot{x}_2) < 0 \rightarrow c_2 = c_{\min} = c_{\text{off}} \tag{1}$$

where $\dot{x}_1$ and $\dot{x}_2$ are the velocities of masses m_1, and m_2, respectively.

For the continuous ground hook dampers, the damping is adjusted in the range between a minimum (off) and a maximum (on) level according to:

$$\begin{aligned} \dot{x}_1(\dot{x}_1 - \dot{x}_2) \geq 0 &\rightarrow c_2 = c_{\text{on}} = \min\{G|\dot{x}_1|,\ c_{\max}\} \\ \dot{x}_1(\dot{x}_1 - \dot{x}_2) < 0 &\rightarrow c_2 = c_{\min} = c_{\text{off}} \end{aligned} \tag{2}$$

The variable G is the gain that relates the damping level to the absolute velocity of the main mass m_1. It has been shown that the continuous semi-active damper performs better than the on-off damper (Ahmadian 1997). Therefore, this paper only studies the performance of continuous ground hook dampers as used in tuned mass dampers.

3. Response of GHTMD compared to TMD

The differential equations of motion of the system shown in Fig. 1 are as follows:

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2) = F \\ m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \end{aligned} \tag{3}$$

Since this is a nonlinear system the approximate forms of the response are:

$$\begin{aligned} x_1 = \bar{X}_1 e^{i\omega t} \\ x_2 = \bar{X}_2 e^{i\omega t} \end{aligned} \tag{4}$$

The following non-dimensional parameters as used for the passive TMDs are defined (Den Hartog 1947):

$\mu = \frac{m_2}{m_1}$ =mass ratio

$f = \frac{\omega_2}{\omega_1} = \frac{f_2}{f_1}$ =frequency ratio of the TMD or GHTMD to the main system

$\xi_1 = \frac{c_1}{2 m_1 \omega_1}$ =damping ratio of the main system

$g = \frac{\omega}{\omega_1}$ =frequency ratio of the excitation to the natural frequency of the main system

In the above $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$, where f_1, and f_2 are the natural frequencies of the main system and TMD or GHTMD individually in Hz.

Substituting (4) into equations of motion (3), and simplification of the results, the equations of motion are:

$$-g^2 \bar{X}_1 + 2ig\xi_1 \bar{X}_1 + ig\frac{c_2}{m_1\omega_1}(\bar{X}_1 - \bar{X}_2) + \bar{X}_1 + \mu f^2(\bar{X}_1 - \bar{X}_2) = \frac{F_0}{m_1\omega_1^2}$$
$$-\mu g^2 \bar{X}_2 + ig\frac{c_2}{m_1\omega_1}(\bar{X}_2 - \bar{X}_1) + \mu f^2(\bar{X}_2 - \bar{X}_1) = 0 \tag{5}$$

The displacement responses $\bar{X}_1$ and $\bar{X}_2$ in terms of non-dimensional amplification factors A_1 and A_2 are defined as:

$$\bar{X}_1 = A_1 \Delta_{st} = A_1 \frac{F_0}{k_1} = A_1 \frac{F_0}{m_1 \omega_1^2}$$
$$\bar{X}_2 = A_2 \Delta_{st} = A_2 \frac{F_0}{k_1} = A_2 \frac{F_0}{m_1 \omega_1^2} \tag{6}$$

in which Δ_{st} is the static displacement for mass m_1.

Substitution of (6) into Eq. (5) will result in:

$$-g^2 A_1 + 2ig\xi_1 A_1 + ig\frac{c_2}{m_1\omega_1}(A_1 - A_2) + A_1 + \mu f^2(A_1 - A_2) = 1$$
$$-\mu g^2 A_2 + ig\frac{c_2}{m_1\omega_1}(A_2 - A_1) + \mu f^2(A_2 - A_1) = 0 \tag{7}$$

The above equations are in terms of non-dimensional parameters except the damping term c_2. Depending on the type of damper (passive or semi-active) the above equations can be re-written as will be shown below:

3.1 Passive TMD

For the passive TMDs, c_2 is a constant term which is defined as $c_2 = 2m_2\omega_2\xi_2$, in which ξ_2 is the TMD damping ratio. Upon substitution of c_2 in Eq. (7) the following set of equations in terms of non-dimensional parameters will be obtained:

$$
\begin{aligned}
-g^2A_1+2ig\xi_1A_1+2ig\mu f\xi_2(A_1-A_2)+A_1+\mu f^2(A_1-A_2)=1 \\
-\mu g^2A_2+2ig\mu f\xi_2(A_2-A_1)+\mu f^2(A_2-A_1)=0
\end{aligned}
\tag{8}
$$

Solution of the above equations will determine the magnitudes of A_1, A_2, and the phase angle between the main mass and TMD (θ) in the following familiar forms:

$$
\begin{aligned}
A_1&=\left(\frac{c^2+d^2}{a^2+b^2}\right)^{1/2} \\
A_2&=\left(\frac{p^2+q^2}{a^2+b^2}\right)^{1/2} \\
\theta&=\tan^{-1}\frac{dp-cq}{cp+dq}
\end{aligned}
\tag{9}
$$

in which:

$$
\begin{aligned}
a&=\left(1-g^2\right)\left(f^2-g^2\right)-g^2f(\mu f+4\xi_1\xi_2) \\
b&=2g\left[\xi_1\left(f^2-g^2\right)+\xi_2 f\left(1-g^2-\mu g^2\right)\right] \\
c&=f^2-g^2 \\
d&=2gf\xi_2 \\
p&=f^2 \\
q&=2gf\xi_2
\end{aligned}
$$

3.2 Semi-active GHTMD

In the case of semi-active GHTMD, c_2 varies with time based on the rule set by Eq. (2). When the damper is off, the damping value c_2 can be defined as $c_2=2m_2\omega_2\xi_{off}$. Substituting into Eq. (7) will result in a set of equations similar to (8) with ξ_{off} replacing ξ_2:

$$
\begin{aligned}
-g^2A_1+2ig\xi_1A_1+2igf\mu\xi_{off}(A_1-A_2)+A_1+\mu f^2(A_1-A_2)=1 \\
-\mu g^2A_2+2igf\mu\xi_{off}(A_2-A_1)+\mu f^2(A_2-A_1)=0
\end{aligned}
\tag{10}
$$

According to Eq. (2) when c_2 is on, it is defined as $c_2=G|\dot{x}_1|$. In order to keep Eq. (7) in non-dimensional form, a non-dimensional parameter 'e' is defined as:

$$
e=\frac{GF_0}{m_1^2\omega_1^2}
\tag{11}
$$

Substituting the parameter 'e' and c_2 into Eq. (7) results in the following non-dimensional set of equations of motion:

$$-g^2A_1+2ig\xi_1A_1-g^2|eA_1|(A_1-A_2)+A_1+\mu f^2(A_1-A_2)=1$$
$$-\mu g^2A_2-g^2|eA_1|(A_2-A_1)+\mu f^2(A_2-A_1)=0 \tag{12}$$

Using the control policy of Eq. (2), since the damping c_2 varies with time, the GHTMD system is a nonlinear dynamic system. In general, these systems do not have any closed form solutions, however, approximate techniques have been proposed to solve the equations of motion (Miller 1988). Here, it is shown that depending on the state of the system according to Eq. (2), the equations of motion switch between Eqs. (10) and (12) in terms of non-dimensional parameters. Therefore, as will be shown later the parameters defined here can be used for the general design of GHTMDs. In order to solve the equations of motion (3), the Newmark Method (Newmark 1959), which is a direct integration technique is used. After calculation of $\bar{X}_1$ and $\bar{X}_2$, using Eq. (6) A_1 and A_2 are found for the tuned system. Other non-dimensional design parameters (e, f, ξ_{off}) are also obtained accordingly.

4. Performance of the GHTMD

In order to evaluate the efficiency of GHTMD in reducing the vibration of force-excited systems, a non-dimensional parameter (p) representing the system performance is defined as follows:

$$p=\frac{(A_1)_{\max}\text{GHTMD}}{(A_1)_{\max}\text{TMD}} \tag{13}$$

For the GHTMD to perform better than its equivalent TMD (having, the same mass ratio as the GHTMD) the value of 'p' has to be less than 1.0. The smaller the 'p', the more efficient GHTMD is as compared to its equivalent TMD. Note that in the above equation A_1 is the magnitude of the amplification factor. In addition, in the following sections of this paper A_1 (amplification factor of the main mass) and A_2 (amplification factor of the GHTMD mass) represent the magnitudes of these parameters.

5. Parametric studies

In order to study and compare the behavior of GHTMD with its equivalent TMD and establishing guidelines for the design of GHTMDs, a simple single degree of freedom system which can be representative of many structural systems is considered here. The dynamic parameters of the system are m_1=17.5 N-sec^2/mm (100 lb-sec^2/in); f_1=2 Hz; F_0=44,500N (10,000lb). It has to be noted that even though the results reported here are for a particular model since the GHTMD design properties are defined in terms of non-dimensional parameters they can be applied to any dynamic system as will be shown later in the design guide section.

In order to study the behavior of the GHTMD with different mass ratio μ, m_2 was varied to range μ from 0.01 to 0.50. In addition the main system damping ratio, ξ_1, was assumed to be 0.0, 0.01, 0.05, and 0.10 to study its effect on the system behavior. In order to perform time history analyses of the system, a time step size of 0.005 second was chosen. This selection was based on the fact

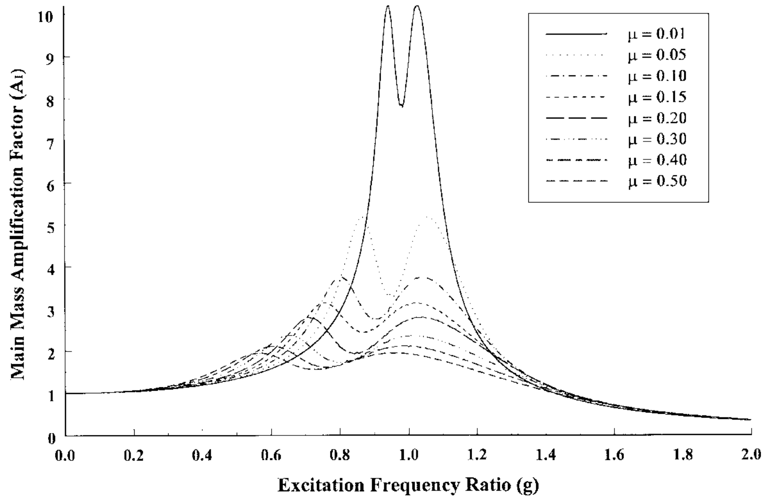

Fig. 2 Variation of the GHTMD main mass amplification factor (A_1) with excitation frequency ratio (g) for different values of μ [ξ_1=0.01]

that it takes about 0.01 second for the valves in the semi-active dampers to respond from a fully open to a fully closed position (Miller 1988). Similar to the passive TMDs (Den Hartog 1947), the optimum GHTMD design parameters f, ξ_{off}, and e were computed based on minimization of the maximum amplification factor, $(A_1)_{\max}$. In addition, in order to perform a direct comparison of the results with the passive TMD, the optimum parameters of the equivalent TMD (f, ξ_2) were calculated, where ξ_2 is the damping ratio of the TMD. Fig. 2 shows the variation of the amplification factor for system with ξ_1=0.01 with various mass ratios (μ). The maximum main mass amplification factors (A_1) reduce with an increase in the mass ratios as can be expected in TMDs. In addition, from this figure it can be concluded that the optimality condition of TMD, which is having

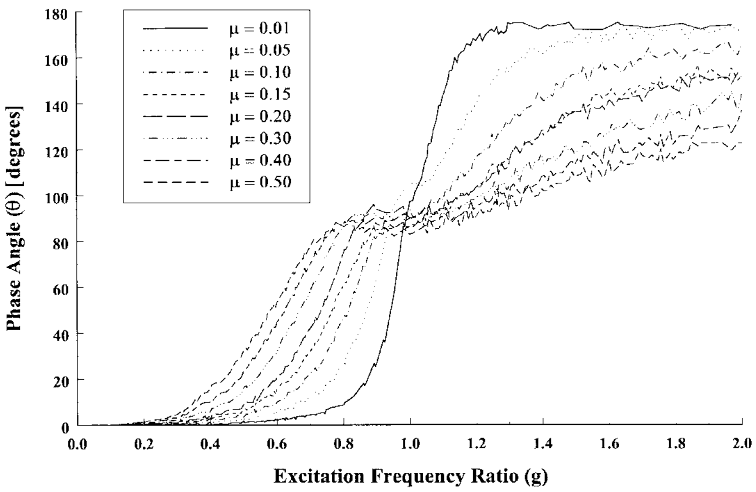

Fig. 3 Variation of the phase angle difference between the main mass (m_1), and the GHTMD mass (m_2) with excitation frequency ratio (g) for different values of μ [ξ_1=0.01]

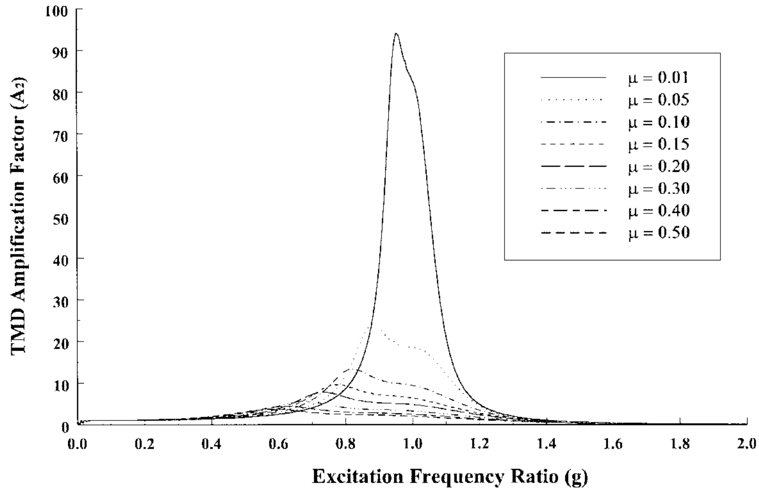

Fig. 4 Variation of the GHTMD mass amplification factor (A_2) with excitation frequency ratio (g) for different values of μ [ξ_1=0.01]

two equal peak amplitudes is also applicable to GHTMDs. Fig. 3 shows the variation of the phase angle difference between the main mass and the GHTMD mass (θ) as a function of the excitation frequency ratio (g). It has to be noted that for the system without the device the maximum amplification factor, $(A_1)_{max}$ is $1/2\xi_1$.

Fig. 4 shows the variation of the amplification factor of the GHTMD mass (A_2) with respect to the excitation frequency ratio (g). As in the case of passive TMD, the GHTMD mass has much higher amplification factor at the resonance frequency than the main mass. Figs. 5, and 6 show the

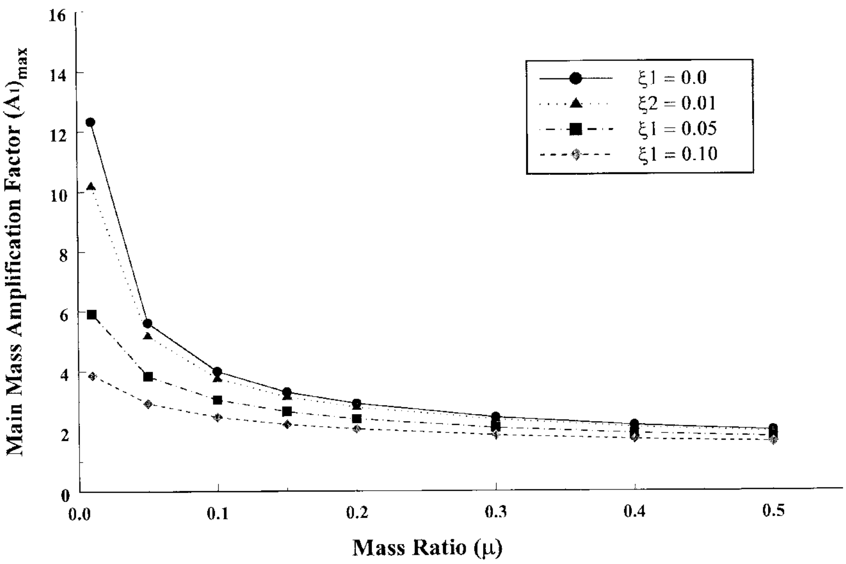

Fig. 5 Variation of the GHTMD main mass amplification factor (A_1) with mass ratio (μ) for different values of ξ_1

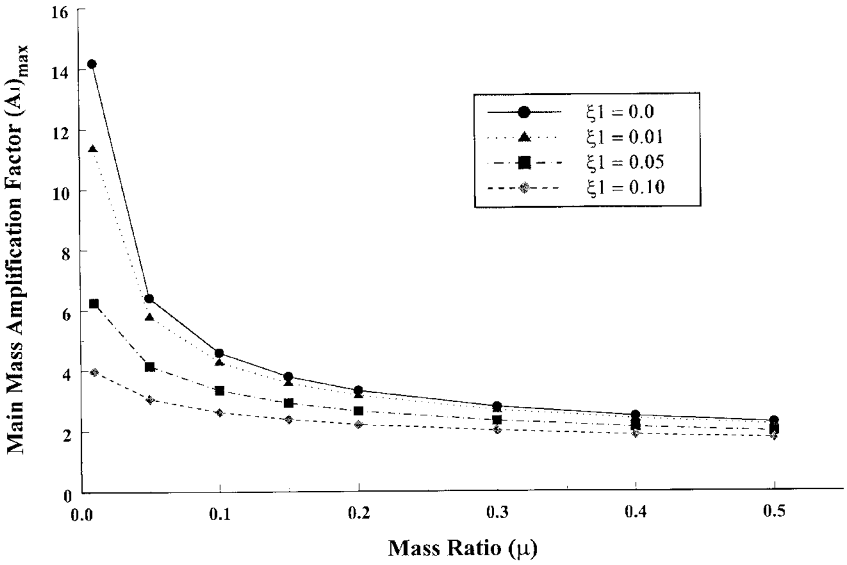

Fig. 6 Variation of the passive TMD main mass amplification factor (A_1) with mass ratio (μ) for different values of ξ_1

variation of the maximum main mass amplification factors with mass ratio (μ) for the GHTMD, and TMD respectively. Comparison of the two figures indicate similar patterns in changes of the amplification factors with respect to the variation of the primary system damping. As could be expected there is a much larger reduction in the amplification factor for the mass ratios of up to 10%. Beyond this point the increase in the secondary system mass (m_2) has less significant effect on

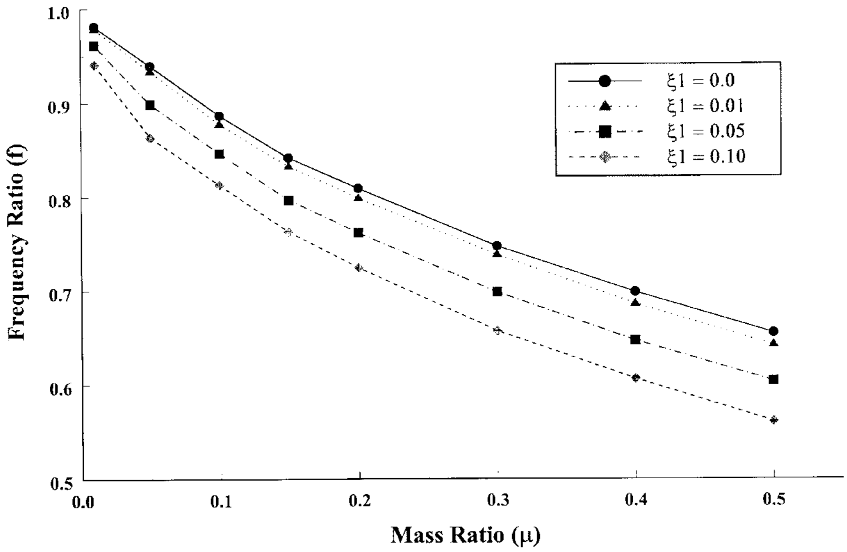

Fig. 7 Variation of the optimum frequency ratio (f) of the GHTMD with mass ratio (μ) for different values of ξ_1

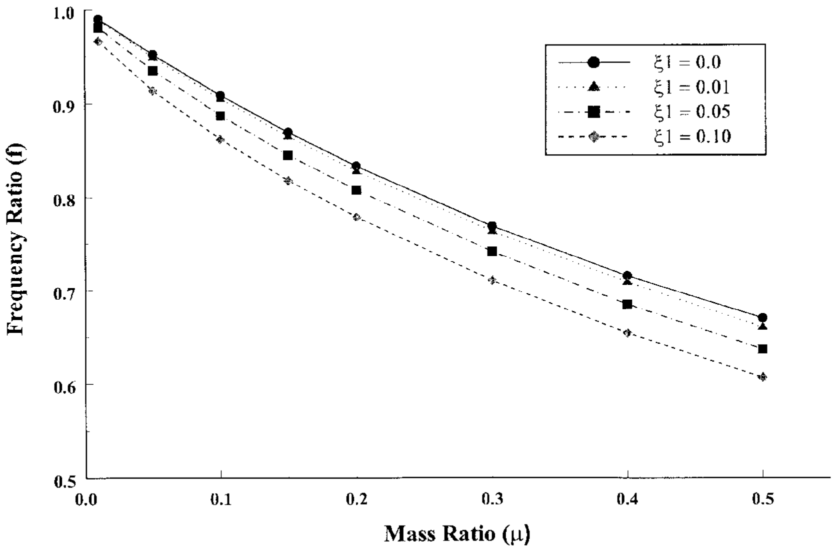

Fig. 8 Variation of the optimum frequency ratio (f) of the passive TMD with mass ratio (μ) for different values of ξ_1

the reduction of the main mass amplification factor.

Figs. 7 and 8 show the variation of the optimum frequency ratios (f) with respect to the mass ratio (μ) for the GHTMD, and TMD respectively. Comparison of these figures demonstrates the similarity of the changes in the optimum frequency ratios for the two systems. Fig. 9 shows the changes in the optimum parameter (e) of the GHTMD with mass ratio (μ). Since the gain (G) is proportional to 'e', it can be concluded that the optimum gain increases with the mass ratio (μ) and the primary system damping (ξ_1).

The variation of the optimum off-damping ratio (ξ_{off}) of the GHTMD with mass ratio (μ) is shown in Fig. 10. The off-damping as defined earlier increases with the mass ratio (μ) and decreases with the increase in the main system damping (ξ_1). The variation of performance (p) as defined in Eq. (13) with respect to the mass ratio (μ) for different values of ξ_1 is shown in Fig. 11. As is observed in this figure, the GHTMD can outperform an equivalent TMD. The GHTMD performs better with a decrease in primary system damping (ξ_1). In general, TMDs are effective only when the main system damping (ξ_1) is low. This fact shows that GHTMDs can be used as substitutes for TMDs. In the case studied here the GHTMD can outperform the equivalent TMD by about 14% where ξ_1=0, and about 12% when ξ_1=0.01.

The next two plots represent the sensitivity of the system response due to changes in the design parameters. Fig. 12 shows the variation of the main mass amplification factor $(A_1)_{max}$ with respect to changes in the frequency ratio (f) and parameter (e). Fig. 13 displays the variation of A_1 with respect to f and off-damping ratio (ξ_{off}). In both cases it can be observed that the frequency ratio is the most important design parameter, and system response varies significantly once it becomes off-tune due to changes in the frequency ratio.

One of the problems that the designer is faced when using GHTMD, is the possibility of off-tuning due to the changes in the main mass (m_1). Fig. 14 shows the variation of $(A_1)_{max}$ due to changes in m_1. As can be noticed if m_1 is more than its optimum value $(m_1)_{opt}$ the response is

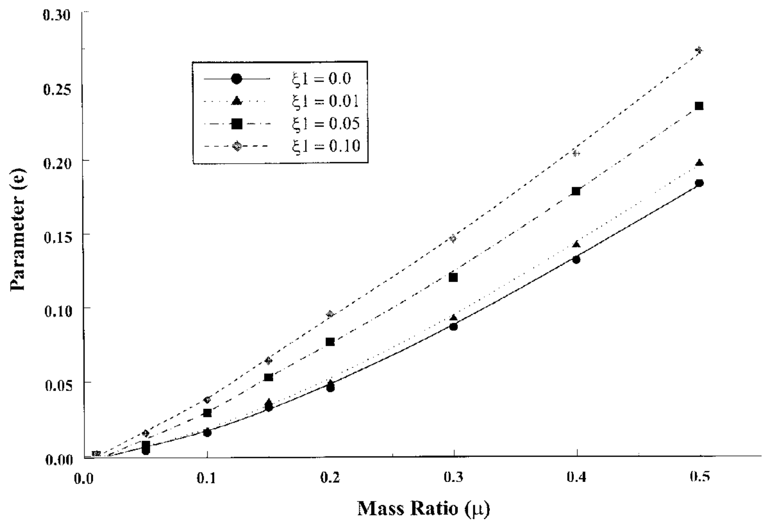

Fig. 9 Variation of the optimum parameter (e) of the GHTMD with mass ratio (μ) for different values of ξ_1

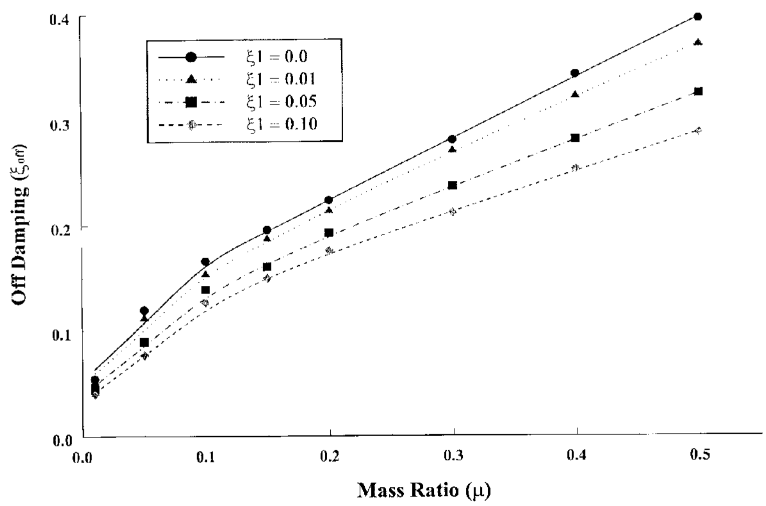

Fig. 10 Variation of the optimum off-damping ratio (ξ_{off}) of the GHTMD with mass ratio (μ) for different values of ξ_1

significantly larger than when m_1 is less than $(m_1)_{opt}$ by the same amount. Therefore, when designing a GHTMD, it is recommended to be tuned to the maximum possible main mass (m_1).

In order to compare the effects of off-tuning due to changes in frequency ratio on the performances of GHTMD and TMD, Fig. 15 is included. As can be observed the variation in the maximum amplification factor, $(A_1)_{max}$, for frequency ratios up to f_{opt} has smaller slope for GHTMD than for the equivalent TMD. However, beyond f_{opt} both lines have approximately the same slopes. Therefore, it can be concluded that a GHTMD performs better than its equivalent TMD for frequency ratios below f_{opt}.

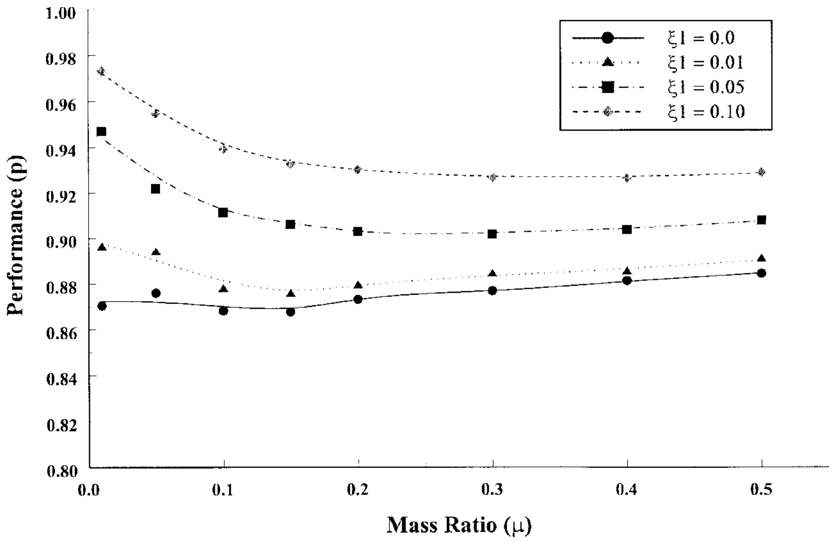

Fig. 11 Variation of the performance (p) of the GHTMD main mass with mass ratio (μ) for different values of ξ_1

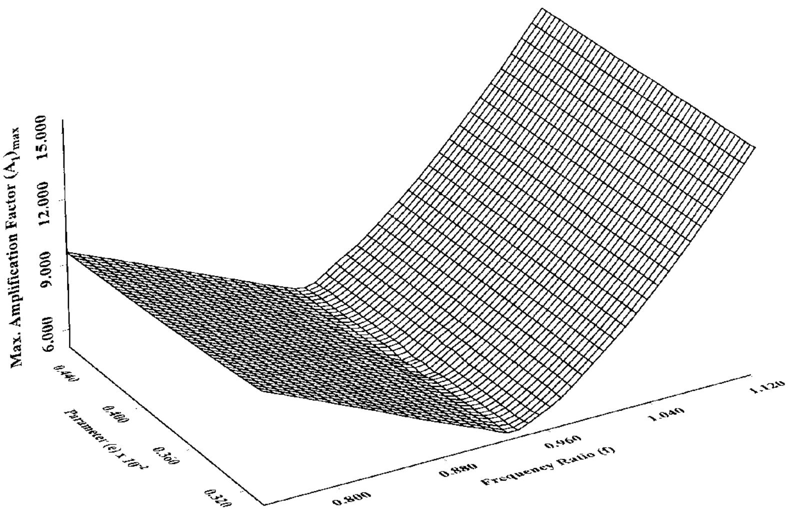

Fig. 12 Off-tuning effects on the main mass response of GHTMD with respect to changes in frequency ratio (f), and parameter (e) [μ=0.05, ξ_1=0.01]

6. Design of the GHTMD

This section details how to use the graphs in this paper to design GHTMDs. In general, the first step in the design is to select the GHTMD mass (m_2) or the mass ratio (μ) based on the desirable amplification factor. This can be accomplished using Fig. 5. Once the mass ratio is selected, using Figs. 7, 9, and 10, the non-dimensional design parameters f_{opt}, e_{opt}, and $(\xi_{off})_{opt}$ can be found. Using Eq. (11), the optimum gain (G) can be computed, and therefore, the design parameters required in

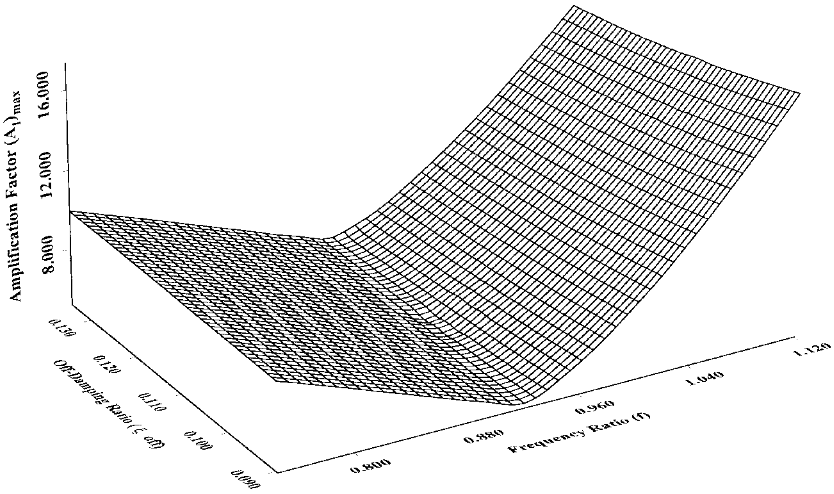

Fig. 13 Off-tuning effects of the main mass response of GHTMD with respect to changes in frequency ratio (f), and off-damping ratio (ξ_{off}) [μ=0.05, ξ_1=0.01]

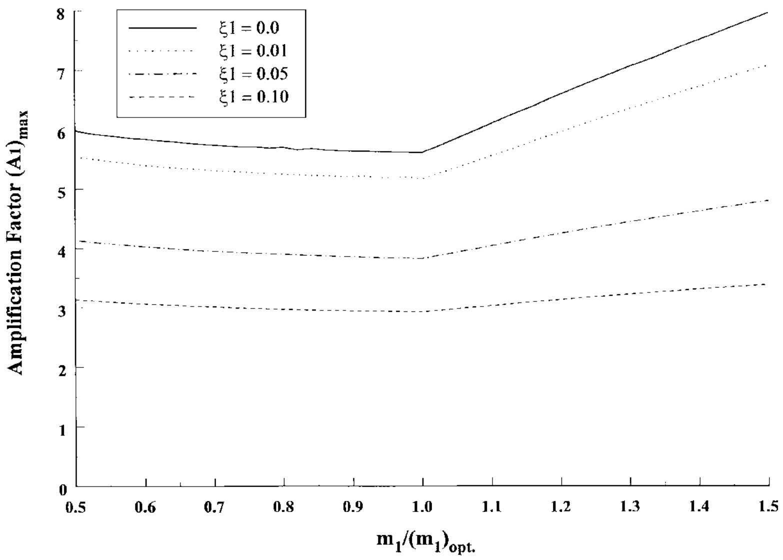

Fig. 14 Off-tuning effects on the main mass response of GHTMD with respect to changes in m_1 [μ=0.05]

Eq. (2) can be obtained.

7. Conclusions

The behavior of the Ground Hook Tuned Mass Damper (GHTMD) to reduce the vibration of structures was studied. A single degree of freedom (SDOF) system subjected to harmonic force excitation was used. Equations of motion when the GHTMD is attached to the SDOF system were

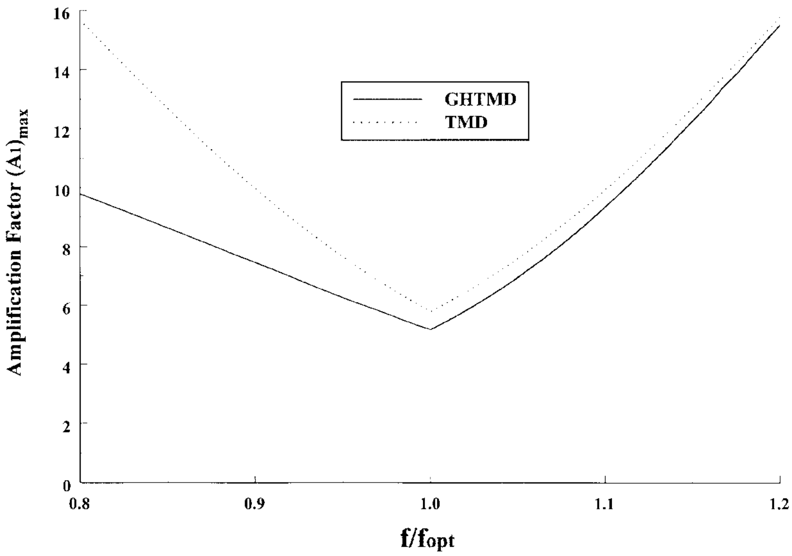

Fig. 15 Comparison of the off-tuning effects on the main mass response of GHTMD and TMD with respect to changes in frequency ratio (f) for ξ_1=0.01 [μ=0.05]

developed in terms of non-dimensional parameters. The optimum design parameters of the GHTMD were found for various mass ratios and system damping, and its performance was compared to the passive TMD with the same mass ratio. A procedure was proposed for the design of GHTMDs.

From this study the following are concluded:

(1) A GHTMD can perform better than an equivalent passive TMD in reducing the vibration. However, its effectiveness depends on the main system damping ratio. The GHTMD with the control policy used here can perform up to about 14% better than its TMD counterpart. In addition, a GHTMD is less sensitive to off-tuning due to changes in the frequency ratio (f) than an equivalent TMD.

(2) The optimum frequency ratio (f_{opt}) of the GHTMD decreases with the increase in mass ratio (μ) and main system damping ratio (ξ_1). However, the optimum value of parameter (e_{opt}) or optimum gain (G_{opt}) of GHTMD increases with the increase in the mass ratio (μ) and main system damping (ξ_1).

(3) Deviation of frequency ratio (f) from its optimum value (f_{opt}) has much more significant effect on the efficiency of GHTMD than a similar change in ξ_{off} and e. In addition, a decrease in the frequency ratio, from its optimum value (f_{opt}) has less effect on the efficiency of the GHTMD than an increase in the frequency ratio.

(4) The GHTMD should be designed considering the upper bound of ξ_{off} and e such that these design parameters will never be less than their corresponding optimum values. In addition, the GHTMD should be tuned using the maximum possible main mass (m_1).

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References

Abe, M. (1996), "Semi-active tuned mass dampers for seismic protection of civil structures", *Earthquake Eng. and Struct. Dyn.*, **25**, 743-749.

Ahmadian, M. (1997), "Semi-active control of multiple degree of freedom systems", *Proc. of DETC'97, ASME Design Engineering Technical Conference*, Sacramento, California, September.

Ahmadian, M., and Marjoram, R.H. (1989), "Effects of passive and semi-active suspensions on body and wheel-hop control", *J. of Commercial Vehicles*, **98**, 596-604.

Chang, J.C.H., and Soong, T.T. (1980), "Structural control using active tuned mass dampers", *J. of the Eng. Mech. Div.*, ASCE, **106**(EM6), Proc. Paper 15882, December, 1091-1098.

Crosby, M.J., and Karnopp, D.C. (1973), "The active damper", *The Shock and Vib. Bulletin 43*, Naval Research Laboratory, Washington, D.C.

Den Hartog, J.P. (1947), *Mechanical Vibrations*, 3rd edn, McGraw-Hill, New York.

Frahm, H. (1911), *Device for Damping of Bodies*, U.S. Patent No. 989, 958.

Hrovat, D., Barak, P., and Rabins, M. (1983), "Semi-active versus passive or active tuned mass dampers for structural control", *J. of the Eng. Mech. Div.*, ASCE, **109**(3), 691-705.

Karnopp, D.C., and Crosby, M.J. (1974), *System for Controlling the Transmission of Energy between Spaced Members*, U.S. Patent 3,807,678, April.

Krasnicki, E.J. (1980), "Comparison of analytical and experimental results for a semi-active vibration isolator", *Shock and Vib. Bulletin*, **50**.

Lund, R.A. (1980), "Active damping of large structures in winds", *Struc. Control*, H.H.E. Leipholz, ed., North-Holland Publishing Co., New York, N.Y.

Miller, L.R. (1988), "An approach to semi-active control of multiple-degree-of-freedom systems", Ph.D. Thesis, Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, North Carolina.

Newmark, N.M. (1959), "A method of computation for structural dynamics", *ASCE J. of Eng. Mech. Div.*, **85**, 67-94.

Ormondroyd, J., and Den Hartog, J.P. (1928), "The theory of the dynamic vibration absorber", *Transaction of the American Society of Mechanical Engineers*, **50**, 9-22.

Roberson, R.E. (1952), "Synthesis of nonlinear dynamic vibration absorber", *J. of Franklin Institute*, **254**, 205-220, September.

Udwadia, F.E., and Tabaie, S. (1981), "Pulse control of single degree of freedom system", *J. of Eng. Mech. Div.*, ASCE, **107**(EM6), December, 997-1009.

Yamaguchi, H., and Harnpornchai, N. (1993), "Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations", *Earthquake Eng. and Struct. Dyn.*, **22**, 51-62.

Yeh, H., Reed, D.A., Yu, J., and Gardarsson, S. (1996), "Performance of tuned liquid dampers under large amplitude excitation", *Proc. of the Second Int. Workshop on Struct. Control*, 432-443.

Notation

A_1 : amplification factor of the main mass
A_2 : amplification factor of the GHTMD mass
c_1 : damping of the main system
c_2 : damping of the GHTMD
e : non-dimensional design parameter
f : frequency ratio
F_0 : applied force magnitude

g	: excitation frequency ratio
k_1	: stiffness of main system
k_2	: stiffness of GHTMD
m_1	: main mass
m_2	: GHTMD mass
p	: performance factor
$x, \dot{x}, \ddot{x}$	: displacement, velocity, and acceleration
μ	: mass ratio
ξ_1	: main system damping ratio
ξ_{off}	: off-damping ratio
ω_1	: natural frequency of the main system
ω_2	: natural frequency of the GHTMD