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Fuzzy logic based estimation of effective lengths of columns in partially braced multi-storey frames

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Abstract. Columns in multi-storey frames are presently categorised as either braced or unbraced, usually by means of the stability index criterion, for estimating their effective length ratios by design aids such as 'alignment charts'. This procedure, however, ignores the transition in buckling behaviour between the braced condition and the unbraced one. Hence, this results in either an overestimation or an underestimation of effective length estimates of columns in frames that are in fact 'partially braced'. It is shown in this paper that the transitional behaviour is gradual, and can be approximately modelled by means of a 'fuzzy logic' based technique. The proposed technique is simple and intuitively agreeable. It fills the existing gap between the braced and unbraced conditions in present codal provisions.

Key words: effective length; *K*-factor; column; multi-storey frame; partial bracing; stability index; fuzzy logic; membership function; degree of bracing.

1. Introduction

The concept of 'effective length' is fundamental in the design of columns, and is relied upon heavily by codes of practice (Wood 1974). The effective length of a column in a frame may be defined as the length of a fictitious pin-ended braced column whose Euler buckling load is equal to the critical load acting on the given column when the frame buckles. The ratio of the effective length, l_e, to the actual length, h, of the column is termed the 'effective length factor' (or ratio), k_e, and is also known as the '*K*-factor'.

The value of k_e varies theoretically between 0.5 and 1.0 for columns in frames 'braced' against sidesway, and between 1.0 and ∞ for columns in 'unbraced' frames. The exact value of k_e depends on the degrees of rotational restraint at the two ends of the column, as well as on the degree of translational restraint against sidesway. These elastic restraints may be modelled by means of two rotational springs (at the top and bottom ends), and a single translational spring at the top end, in the structural model of an isolated column (Fig. 1). However, in the simplified methods of k_e estimation recommended by codes (such as ACI 318 1992), it is conveniently assumed that sidesway is either totally inhibited or totally uninhibited. This is tantamount to assuming that the translational spring stiffness, S, in the idealised model shown in Fig. 1 is either infinity or zero. The condition of 'partially inhibited sway' has not received much research attention, except recently (Aristizabal-Ochoa 1995), and has been largely ignored in practice, for want of suitable design aids recommended by codes.

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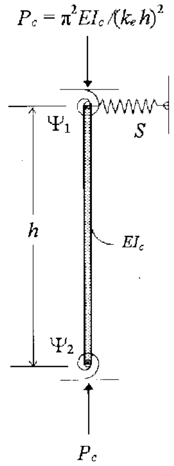

Fig. 1 Idealized model of a column for the purpose of simplified elastic stability analysis

Codes do recognise, however, that “in an actual structure, there is rarely a completely braced or a completely unbraced frame” (ACI 318 1992). The modelling of an actual frame (which is *partially braced*) one way or the other can lead to either an overestimation of k_e (if the ‘unbraced’ condition is assumed) or an underestimation (if the ‘braced’ condition is assumed). The error becomes particularly significant when the ‘degree of bracing’ is such that the actual value of k_e is midway between the two idealised extremes, $k_{e,b}$ (fully braced) and $k_{e,ub}$ (fully unbraced). This error has serious significance in the codal estimates of second-order moments in slender columns of multi-storey frames, involving moment magnifier factors, δ_b and δ_s (for nonsway and sway effects) which depend on proper estimates of k_e values.

The most widely used design aids for estimating k_e values of columns in multi-storey frames are the nomograms or ‘alignment charts’, developed originally by Julian and Lawrence in 1959 (Johnston 1976). These charts, recommended by many international codes dealing with the design of concrete and steel structures, provide an easy graphical means of estimating k_e for any column in a multi-storey frame. (The charts developed by Wood (1974) are favoured by some codes, such as the British and Indian codes, as an alternative to the ‘alignment charts’). Given the measures of the rotational restraints, expressed in terms of the factor, $\Psi \equiv \Sigma(I_c/h)/\Sigma(I_b/L)$, at the top and bottom ends (Ψ_1 and Ψ_2) of the column (Fig. 1), the value of k_e can be easily located. The extreme values, Ψ=0 and Ψ=∞, represent rotationally fixed and hinged end conditions respectively. There are two charts: one for columns in ‘braced frames’, and the other for columns in ‘unbraced frames’, neither of which is appropriate for partially braced frames. The two values of k_e, viz. $k_{e,b}$ and $k_{e,ub}$, can be significantly different, as shown in Table 1 (for six typical combinations of Ψ_1 and Ψ_2). For instance, the ratio, $k_{e,ub}/k_{e,b}$ is 2.00 for the ‘fixed-fixed’ case (Ψ_1=Ψ_2=0); it is 2.86 for the ‘hinged-fixed’ case (Ψ_1=∞, Ψ_2=0), and takes on very high values when there is high rotational flexibility at both ends of the column. For a column in a ‘partially braced’ frame, the true value of k_e is obviously in between $k_{e,b}$ and $k_{e,ub}$.

The ACI Code Commentary (ACI 318 1992) suggests that a storey in a multi-storey frame can be considered to be braced if the second-order (P–Δ) moments do not exceed 5 percent of the first-order moments. Two guidelines are offered: one in terms of the ‘stability index’, Q (introduced originally by MacGregor and Hage 1970), and another in terms of the ratio of the translational storey stiffness of the bracing elements to the total column stiffness in a storey. Columns in a given storey may be considered to be 'adequately braced', if the former (stability index) does not exceed 0.04[†], or alternatively, if the latter (stiffness ratio) is at least 6.0. These guidelines are useful and frequently adopted in practice, but raise questions about their validity when the stability index is in the close neighbourhood of 0.04 (or the stiffness ratio in the neighbourhood of 6.0). It does not seem reasonable to postulate that a stability index of 0.04 (or a stiffness ratio of 6.0) is indicative of a braced frame, whereas a stability index of 0.041 (or a stiffness ratio of 5.9) signifies an unbraced frame. Yet, the recommendations must allow for such narrow interpretations that run contrary to common-sense judgement. This kind of problem, commonly encountered in engineering and other disciplines, is not really a problem of identifying the most accurate or appropriate cut-off value separating two significantly different categories. Rather, it is a fundamental problem of the inadequacy of conventional binary logic to accommodate the grey region in which the two categories tend to overlap. This is a 'fuzzy' region, which is ideally suited for the application of 'fuzzy logic' concepts (Zadeh 1965). In this instance, the fuzziness lies in the idea of what constitutes 'adequate bracing', and in the notion of a limiting 5 percent P–Δ moment effect (which forms the basis for the stability index criterion).

This paper attempts to resolve that fuzziness, and shows how the ACI Code Commentary (ACI 318 1992) recommendation based on the 'stability index' criterion can be significantly improved to account for 'partial bracing' in multi-storey frames. The use of 'fuzzy logic' concepts enables a realistic estimation of effective length factors of such columns.

The text of this paper is organised as follows: the concept of 'partial bracing' and the behaviour of partially braced frames are first explained; the basis for the 'stability index' criterion is discussed; a 'fuzzy measure' is defined for partial bracing in a storey, in terms of its stability index; a simple technique is proposed to apply the fuzzy measure to estimate k_e values; finally, some typical examples of partially braced columns are illustrated to demonstrate the proposed method, and to compare with the existing codal practice.

2. Significance of the topic

This paper attempts to resolve the problem of determining effective lengths of columns in 'partially braced' multi-storey frames. At present, the widely used 'alignment charts' (or their equivalent formulas) for estimating k_e do not have any provision for accommodating the concept of 'partial bracing', and no modification to the 'alignment chart' procedure has yet been proposed for this purpose. As a result, the designer is confronted with a choice of two disparate values of k_e, that will result in either a conservative design or an unconservative one. This has significant implications with regard to economy and safety in design. This paper highlights the importance of providing for a smooth transition between the 'braced frame' and 'unbraced frame' conditions. 'Fuzzy logic' concepts are used to arrive at a simple procedure for estimating k_e values. The proposal is an improvisation of the ACI Code Commentary guidelines based on the concept of 'stability index' of a storey. The idea proposed is both simple and intuitively agreeable, and holds out promise for extension to other problems of similar nature in civil engineering.

[†]Modified as 0.05 in the recent 1995 revision of ACI 318.

3. Behaviour of partially braced frames

The concept of 'partial bracing' is relatively new in the sense that it has not received much attention, except in the recent past (Aristizabal-Ochoa 1994, 1995), in the existing literature on stability analysis. This concept, which deals with the translational stiffness of a frame with some bracing, must not be confused with the concept of 'semi-rigid' beam-column connections studied extensively in metal structures (Johnston 1976).

The buckling behaviour of 'partially braced' frames is explained by a simple illustrative example of a single-bay symmetrical portal frame. The bracing in the frame may be in the form of an infilled masonry wall, which can be modelled by means of a diagonal axial element (Smith and Carter 1969) whose horizontal translational stiffness, S, is known (Fig. 2a). When the loading is symmetrical ($P_1 = P_2 = P$), the two columns behave identically, with the frame buckling either in the antisymmetric or the symmetric mode as shown in Fig. 2b; hence, each column may be considered in isolation using the model shown in Fig. 1.

The typical variation of the effective length factor k_e with S is as shown in Fig. 2c. When S is equal to zero (which is tantamount to having no bracing), the sidesway becomes uninhibited, and the frame buckles in the fully unbraced antisymmetric mode, with a critical load, $P_c = \pi^2 EI_c/(k_e h)^2$

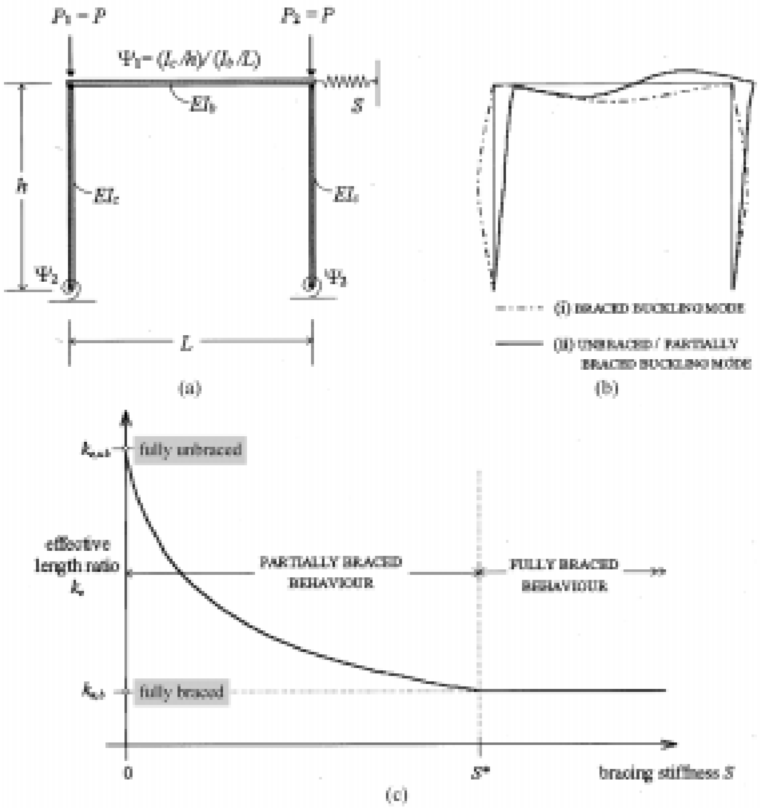

Fig. 2 (a) model of a partially braced portal frame for buckling analysis, (b) typical buckling modes under (i) braced and (ii) unbraced or partially braced conditions, (c) typical variation of effective length ratio with translational bracing stiffness and k_e=$k_{e,ub}$. The value of $k_{e,ub}$ may be conveniently obtained from the conventional alignment chart. With increase in stiffness, S, the sway becomes inhibited, and the buckling load increases. The corresponding fall in k_e can be found to be steep initially (at relatively low values of S) and gradual at higher values of S. When the translational stiffness, S, equals a certain value, S^*, the value of k_e becomes equal to the effective length factor, $k_{e,b}$, corresponding to the symmetric braced mode of buckling. This value of $k_{e,b}$ may be obtained from the alignment chart or from Newmark's formula (Newmark 1949). For values of $S > S^*$, the frame buckles in the symmetric braced mode (i.e., with $k_e = k_{e,b}$), as the buckling load corresponding to the sway mode turns out to be higher than that corresponding to the nonsway mode. Thus, for values of $0 < S < S^*$, the frame behaves as a 'partially braced frame' with effective length factors in the range $k_{e,b} < k_e < k_{e,ub}$. The minimum bracing stiffness required to correspond to 'adequately braced' behaviour is given by S^*.

The results of elastic stability analyses for the six column end factor combinations (Ψ_1, Ψ_2) considered in Table 1 are depicted graphically in Fig. 3. The spring stiffness, S, is nondimensionalised, for convenience, by dividing by (EI_c / h^3), as indicated by the parameter, s, in the x-axis in Fig. 3. In this problem, the values of the minimum bracing stiffness $s^* = S^*/(EI_c /h^3)$ required to ensure the

Table 1. Comparison of effective length ratios of columns in braced and unbraced frames

Case	Column end factors (ψ_1, ψ_2)	Braced frame $k_{e,b}$	Unbraced frame $k_{e,ub}$	$\frac{k_{e,ub}}{k_{e,b}}$
1. hinged-hinged	(∞, ∞)	1.00	∞	∞
2. not fully hinged (both ends)	(10, 10)	0.97	3.00	3.09
3. hinged-fixed	(∞, 0)	0.70	2.00	2.86
4. partially hinged (both ends)	(2, 2)	0.86	1.60	1.86
5. partially fixed (both ends)	(1, 1)	0.78	1.32	1.69
6. fixed-fixed	(0, 0)	0.50	1.00	2.00

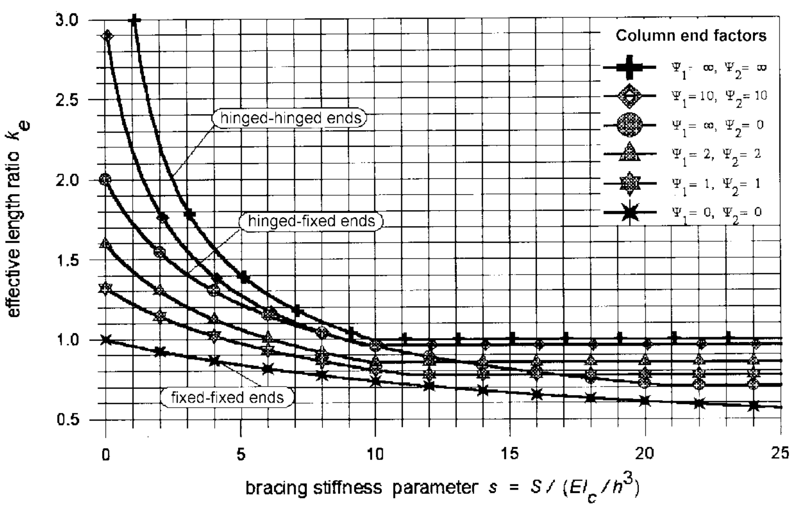

Fig. 3 Variation of k_e with bracing stiffness parameter, s, for various combinations of Ψ_1 and Ψ_2

adequately braced behaviour is found to vary between 9.6 and 35.5, with the higher values corresponding to conditions of rotational fixity (Ψ=0). In other words, when the connecting beams are very stiff compared to the columns, the bracing also must be correspondingly very stiff in order to induce a 'braced' buckling mode. However, this dependence of S^* on Ψ is ignored in the ACI Code Commentary guideline that suggests that a storey in a multi-storey frame can be treated as adequately braced if the total horizontal stiffness of the bracing elements is at least six times the total lateral stiffness of the columns.

From the curves shown in Fig. 3, it is interesting to note that even a nominal amount of bracing stiffness can substantially reduce the effective length ratio k_e (compared to $k_{e,ub}$). The value of k_e drops to midway between $k_{e,ub}$ and $k_{e,b}$ when the bracing stiffness S is 20 to 30 percent of S^*. This is an important point to note, as in practice very often, the beneficial influence of infilled masonry walls is usually neglected, resulting in conservative estimates of k_e (= $k_{e,ub}$).

When the stiffness properties of the two columns are different, or when the axial loads are unequal ($P_1 \neq P_2$ in Fig. 2a), their effective lengths will be different. Under such circumstances, each of the two columns is, in effect, partially braced by the other, and this coupling effect must be properly accounted for, in order to avoid errors (Aristizabal-Ochoa 1994, Cheong-Siat-Moy 1986) in the simplified estimates of k_e.

The above behaviour of a single-bay single-storey portal frame may likewise be extended to a multi-bay multi-storey frame, although the analysis becomes more complicated. It is possible to simplify the problem and view each storey in isolation; a simplified method of determining k_e values for columns in a storey with inhibited sway has been proposed by Aristizabal-Ochoa 1994. However, the accuracy in the solution depends, among other factors, on the accuracy in the estimate of the translational inter-storey stiffness, S, provided by the various bracing elements.

The concept of 'partial bracing' is not generally adopted in practice for want of design aids and codal guidelines for determining k_e values. The common practice is to assume that the frame is either braced or unbraced; in the case of multi-storey frames, the 'stability index' criterion recommended by the ACI Code Commentary is used as a guideline. It is instructive to study the basis of this guideline, which is done in the next section.

4. The stability index criterion

It has been shown by various researchers (Stevens 1967, Rosenbleuth 1967, MacGregor and Hage 1970) that there is a strong relationship between the total critical load ΣP_c in all the columns of a storey in a multi-storey frame and the inter-storey translational stiffness, K:

$$(\Sigma P_c)=\frac{Kh}{\gamma} \tag{1}$$

Here, h denotes the storey height, and the factor γ is a parameter that accounts for the elastic restraints at the column ends. γ is found to vary between 1.0 (for $\Psi \to 0$) and 1.22 (for $\Psi \to \infty$). It may be noted here that the stiffness K includes not only the contribution S from the bracing elements, but also the inherent translational stiffness due to the stiffnesses of the various columns in the frame. It can be obtained from a first-order elastic analysis of the inter-storey drift, Δ, due to a lateral load, H, acting on the storey under consideration:

$$K = \frac{H}{\Delta} \tag{2}$$

The value of ΣP_c (Eq. 1) finds application in the 'moment magnifier' method (MacGregor and Hage 1977), according to which, the second-order moments, M_2, in a 'sway frame' are approximately obtainable from the first-order moments, M_1, in terms of a 'moment magnifier', δ, as follows:

$$M_2 = \delta \cdot M_1 \tag{3}$$

$$\delta = \frac{1}{1 - \dfrac{\Sigma P}{\Sigma P_c}} \tag{4}$$

The term ΣP denotes the sum of the actual column loads acting on the storey. The ACI Code Commentary (ACI 318 1992) uses the above concept to define the 'sway magnifier factor', δ_s, which is to be applied under factored (ultimate) load conditions. Combining Eqs 1, 3 and 4,

$$\delta = \frac{1}{1 - \dfrac{(\Sigma P)\Delta}{Hh}\gamma} = \frac{1}{1 - Q\gamma} \tag{5}$$

where the term 'stability index' is used to describe the parameter Q:

$$Q \equiv \frac{(\Sigma P)\Delta}{Hh} \tag{6}$$

Assuming that a moment magnification of less than 5 percent is negligible for the purpose of categorising a frame as 'adequately braced' (MacGregor and Hage 1970), the limiting value of Q (corresponding to $\delta = 1.05$ in Eq. 5) is found to lie between 0.039 and 0.048 for values of γ between 1.0 and 1.22. The ACI Code Commentary (ACI 318 1992) recommends a value of 0.04.

The stability index criterion may be summarised as follows:

$$Q \equiv \frac{(\Sigma P_u)\Delta_u}{H_u h} \begin{array}{l} \leq 0.04 \Rightarrow \text{ 'braced' frame} \\ > 0.04 \Rightarrow \text{ 'unbraced' frame} \end{array} \tag{7}$$

where the subscript u is used by the code to emphasise that the first-order analysis is to be done under ultimate (factored) loads.

In practice, it is possible (Pillai and Menon 1998) to have values of Q in the close neighbourhood of the critical value 0.04. A strict interpretation of Eq. 7 suggests that if the value of Q equals 0.04, the frame can be taken to be braced, but if the value is 0.041, the frame should be taken as 'unbraced'. The resulting disparities in the estimates of k_e can be quite significant, as discussed earlier (refer Table 1).

Thus, the prevailing stability index criterion is not very satisfactory in such situations. It has two fundamental shortcomings:

1. It does not provide for a gradual transition from the braced condition to the unbraced one, and thus is not able to account for the practical condition of 'partial bracing'.

2. It is based on the rather subjective judgement that a frame can be considered to be adequately braced if the moment magnification does not exceed 5 percent; i.e., if the stability index, Q, does not exceed a limit of 0.04.

The above shortcomings may be overcome by incorporating the concept of ‘fuzzy logic’, which is ideally suited to deal with such situations involving fuzziness and subjective judgement. This is described in the sections to follow.

5. A Fuzzy measure of partial bracing

‘Fuzzy logic’ was first proposed by Lofti Zadeh (1965), and although it took a couple of decades to gain acceptance, it is now recognised as a path-breaking mathematical discipline (‘fuzzy sets theory’), with a very wide range of applications. Conventional (Aristotelian) ‘binary logic’ holds that an assertion, such as the proposition, “X is a member of the set A”, must be either true or false. As opposed to this, ‘fuzzy logic’ holds that this need not be so, and that the assertion may be only partly true (i.e., partly false). A ‘degree of truth’ (Klir and Yuan 1995) may be associated with every such assertion, and may be quantified as the degree to which X is actually a member of the set A. The latter is called a ‘fuzzy set’, because its boundaries are generally not precise. The capability of fuzzy sets theory to express a gradual transition from full membership to nonmembership renders it a powerful tool for modelling uncertainties (Klir and Yuan 1995). It also provides for a meaningful representation of vague concepts expressed in natural language (in the present context, terms like ‘*adequate* bracing’, and ‘*negligible* moment magnification’).

Fuzzy logic has many concepts and applications, and some of these are beginning to find a place in civil and structural engineering (Albert and Schnellenbach-Held 1997). In the present context, we need to employ only the most fundamental concept of fuzzy set theory, viz. the concept of *membership function*, and this may be explained with reference to the idea of ‘partial bracing’.

According to the prevailing binary logic, any frame X is to be considered as either ‘braced’ or ‘unbraced’, for the purpose of estimating k_e by ‘alignment charts’. Let A denote the set of ‘braced’ frames, and its complementary set $\bar{A}$ the set of ‘unbraced frames’. According to the ‘stability index’ criterion, any storey X in a multi-storey building can be categorised as a member of the set A, if its stability index $Q_X \leq 0.04$. Should Q_X exceed 0.04, the frame X is categorised as a member of the complementary set $\bar{A}$.

Let us now view A (the set of braced frames) as a ‘fuzzy set’ (or, more precisely, ‘fuzzy subset’), which means that it now accommodates ‘partially braced’ frames‡. We may define a ‘membership function’, q_X, (whose value ranges from zero to unity) to quantify the degree of membership of any frame X. The larger the value of q_X, the greater is the degree of membership. A membership of zero indicates that the frame is in fact fully unbraced, whereas a membership of 1.0 indicates that the frame is adequately braced. A membership of say, $q_X = 0.45$, indicates a state of partial bracing, and the number 0.45 conveys the relative degree of bracing in the frame X. It is interesting to note that the concept of the membership function covers all possibilities, including the conventional binary logic outcomes. The underlying concept is therefore quite generalised, and at the same time simple and intuitively agreeable.

What now remains to be formulated is a mathematical description of the membership function, q_X, in terms of the stability index, Q_X. The function must provide for a gradual transition from the ‘braced frame’ condition to the unbraced frame condition. As pointed out earlier, the prevailing cut-off value of $Q_X = 0.04$ corresponds approximately to a moment magnification of 5 percent due to P–

‡The complementary set, $\bar{A}$, also becomes a fuzzy set.

Δ effects. However, the choice of δ=1.05 is clearly a matter of subjective judgement, and it is certainly unreasonable to claim that if the moment magnification factor, δ, exceeds 1.05 ever so slightly, the frame is to be treated as unbraced, while it is may be treated as braced if δ is exactly 1.05 or less. Strictly, the horizontal deflection, Δ, must be equal to zero, and hence δ=1.00 (Eq. 5), for a frame to be treated as 'braced'. However, from the point of view of engineering design, it is recognised by codes that the condition, $\Delta = 0$, is almost impossible to achieve in practice, and hence, this criterion is relaxed. Moreover, from an understanding of the buckling behaviour of partially braced frames (Fig. 2), we recognise that if a frame is adequately braced (bracing stiffness $S > S^*$), it will buckle in the nonsway (braced) mode, even though its translational stiffness is finite ($S < \infty$). If the bracing were to fall short of the minimum requirement for 'adequately braced' frame action, the frame would buckle in an 'inhibited sway' mode, which is qualitatively different from the 'unbraced' frame behaviour.

In view of the above, it is proposed in this paper that, instead of adopting a sharp cut-off value of δ=1.05, it is more reasonable to consider a range, such as $1.025 < \delta < 1.075$, where the frame is conceived to be 'partially braced'. The selection of these limits, corresponding to moment magnifications of 2.5 percent and 7.5 percent, are no doubt subjective and a matter of engineering judgement, but they lead to results that are definitely more meaningful than are possible at present. In terms of the stability index, this range corresponds approximately to $0.02 < Q_X < 0.06$ (Eq. 5). The frame is 'adequately braced' if its stability index, Q_X, is less than 0.02; as 'unbraced' if Q_X exceeds 0.06; and as 'partially braced' if Q_X, lies in the intermediate domain [0.02, 0.06]. The degree of bracing of the frame can be expressed in terms of its membership function, q_X, which is assigned values of 1.0 and 0.0 at the two extreme ends ($Q_X = 0.02$ and 0.06 respectively), and an average value of 0.5 at the middle (i.e., $Q_X = 0.04$). Although, a wide variety of curves can be proposed to model the variation of q_X, perhaps the simplest is a parabolic variation commonly used in fuzzy logic applications (Klir and Yuan 1995, Nambudiripad *et al*. 1998), having a zero gradient at the points corresponding to $Q_X = 0.02$ and $Q_X = 0.06$. Other (higher-order) variations are not likely to significantly affect the results. The proposed formulation may be mathematically expressed as follows:

$$q_X = \begin{cases} 1.0 & : Q_X \le 0.02 \\ 1.0 - 0.5(Q_X/0.02 - 1.0)^2 & : 0.02 < Q_X \le 0.04 \\ 0.5(3.0 - Q_X/0.02)^2 & : 0.04 < Q_X \le 0.06 \\ 0.0 & : Q_X \ge 0.06 \end{cases} \tag{8}$$

The proposed gradual variation of q_X, with Q_X is illustrated in Fig. 4. Also shown in Fig. 4 is the existing stability index criterion (Eq. 7), which depicts an abrupt transition from the braced behaviour to the unbraced behaviour (at $Q_X = 0.04$). The contrast between the existing criterion and the proposed criterion is quite marked. The latter is not only intuitively more agreeable, but also conforms approximately to the actual behaviour of partially braced frames. However, it is important to note that the proposed description of the membership function is in no way 'exact', as it is based on a subjective engineering judgement. The use of the membership function is demonstrated in the next section.

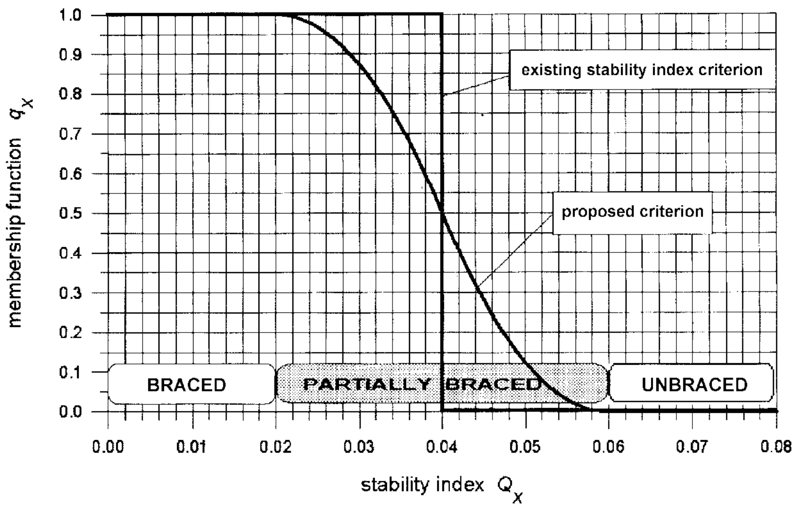

Fig. 4 Variation of membership function, q_X, with stability index, Q_X

6. A simple procedure to estimate effective length ratios

The effective length ratio, k_e, of a partially braced frame lies in between the values, $k_{e,b}$ (for braced frames) and $k_{e,ub}$ (for unbraced frames), and its magnitude obviously depends on the extent to which it is braced (or, in a complementary sense, unbraced). For a column in a multi-storey frame, the membership function q_X (as given by Eq. 8) gives a measure of the degree of membership in the set A of 'braced frames'. According to fuzzy sets theory, this also implies that the degree of membership of the frame in the complementary set $\bar{A}$ of 'unbraced frames' is given by $\bar{q}_X = q - q_X$. Applying this interpretation of membership function, the desired value of k_e may be simply obtained as a weighted average of $k_{e,b}$ and $k_{e,ub}$, with the two membership functions, q_X and $\bar{q}_X$, serving as the weighting functions:

$$\begin{aligned} k_e &= q_X \cdot k_{e,b} + \bar{q}_X \cdot k_{e,ub} \\ &\Rightarrow k_e = k_{e,ub} - q_X\,(k_{e,ub} - k_{e,b}) \end{aligned} \tag{9}$$

Using Eqs. 8 and 9, the effective length ratio of a column in a partially braced multi-storey frame can be easily computed. The values of $k_{e,b}$ and $k_{e,ub}$, are obtainable from the alignment charts or other design aids recommended by various codes.

The proposed fuzzy logic based procedure may be demonstrated by applying it to the columns with the typical end restraint conditions considered earlier (Table 1). The results are graphically depicted in Fig. 5. The various curves demonstrate the gradual variation of the effective length ratio, k_e, with reduction in the degree of bracing, measured in terms of the storey stability index, Q_X. For any given value of Q_X in the partially braced domain [0.02, 0.06], the value of k_e is obtained as a value intermediate to $k_{e,b}$ and $k_{e,ub}$. The transition from the 'braced' to the 'partially braced' regions is gradual, and the transition from the 'partially braced' to the 'unbraced' regions is also gradual.

Considering a typical case, such as the 'hinged-fixed' case, in Fig. 5, the proposed procedure provides for a gradual transition between $k_{e,b} = 0.70$ and $k_{e,ub} = 2.00$; this contrasts sharply with the existing stability index criterion (also indicated in Fig. 5) which suggests that k_e is either equal to

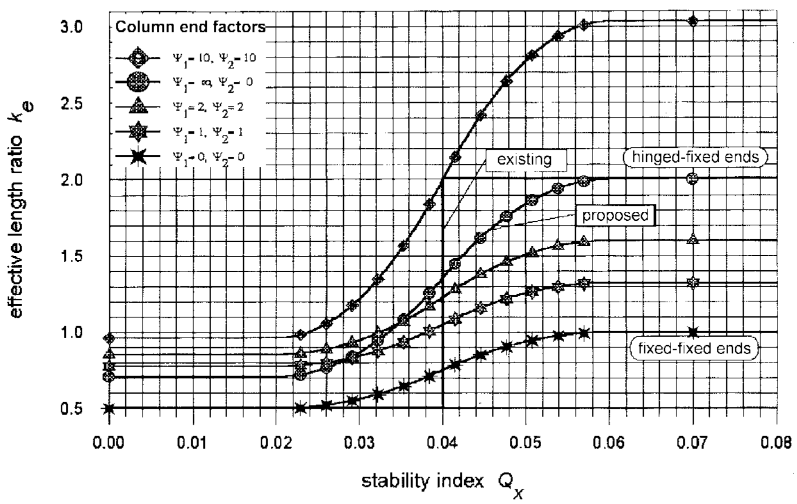

Fig. 5 Variation of k_e with stability index, Q_X, for various combinations of Ψ_1 and Ψ_2

0.70 or 2.00, depending on which side of $Q = 0.04$ the stability index, Q_X lies. Comparing the new proposal with the existing method, it is seen that when the degree of bracing in the frame is moderately high such that $0.02 < Q_X < 0.04$ (i.e., $1.025 < \delta < 1.05$), the proposed method attempts to rectify the unconservatism in the existing method which prescribes a low value, $k_e = k_{e,b} = 0.70$. On the other hand, when the degree of bracing in the frame is such that $0.04 < Q_X < 0.06$ (i.e., $1.05 < \delta < 1.075$), the proposed method attempts to rectify the overconservatism in the existing method which prescribes a high value, $k_e = k_{e,ub} = 2.00$. This is justifiable, considering that even a small amount of bracing can significantly reduce the value of k_e (Fig. 2c).

7. Conclusions

The widely used ‘alignment chart’ (or equivalent) method of estimating the effective length ratios, k_e, of columns in a multi-storey frame is based on the idealisation that the frame is either ‘braced’ or ‘unbraced’, the two conditions leading to widely disparate values (Table 1). When the frame is in fact ‘partially braced’ (with inhibited sidesway), such an idealisation will result in either an underestimate or an overestimate of the true value of k_e; this has significant implications with regard to economy and safety in design.

A study of the buckling behaviour of a partially braced frame (Fig. 2) indicates that there is in fact a gradual transition between the unbraced condition and the adequately braced condition. It is seen that even a nominal amount of lateral bracing stiffness, S, can significantly reduce the value of k_e (with respect to the unbraced condition).

The stability index criterion suggested by the ACI Code Commentary, as a means of deciding whether a storey in a multi-storey frame can be considered to be ‘braced’ or ‘unbraced’, can be significantly improved to include the condition of ‘partial bracing’, by incorporating ‘fuzzy logic’ concepts. The present criterion is based on the rather subjective judgement that a frame can be considered to be ‘braced’ if the moment magnification due to P–Δ effects is less than 5 percent; otherwise it is to be considered ‘unbraced’. It is proposed that, instead of fixing a sharp cut-off value of a 5 percent moment magnification, it makes better sense to broaden the cut-off range to make it representative of the fuzzy region in which the braced and unbraced conditions overlap. A fuzzy measure of the 'degree of bracing', q_X, in a storey, X, is proposed in terms of its stability index, Q_X (Eq. 8), and using this, the effective length ratios, k_e, of columns can be estimated as a weighted average of the values corresponding to the braced and unbraced conditions.

The proposal is simple and intuitively agreeable, and is essentially an improvisation of the existing stability index criterion specified in the ACI Code Commentary. It attempts to rectify the present disparities associated with simplified estimates of effective length ratios of columns in multi-storey frames that are partially braced.

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Notation

A : set of braced frames
$\bar{A}$: set of unbraced frames

E	: modulus of elasticity (secant modulus of concrete)
h	: height of a column or a storey
H	: lateral horizontal load acting on a storey
I_b	: second moment of area of a beam section
I_c	: second moment of area of a column section
k_e	: effective length ratio (or factor) of a column
$k_{e,b}$	: value of k_e for a column in a braced frame
$k_{e,ub}$	: value of k_e for a column in an unbraced frame
K	: inter-storey translational stiffness against sidesway
l_e	: effective length of a column
L	: span of a beam or a bay
M_1	: first-order moment
M_2	: second-order moment
P	: axial load on a column
P_c	: critical buckling load on a column
q_X	: membership function, a fuzzy measure of the degree of bracing in a frame X
Q_X	: stability index of a storey X
X	: frame or storey in a multi-storey frame
s	: nondimensional bracing stiffness parameter = $S/(EI_c/h^3)$
S	: translational bracing stiffness against sidesway
S^*	: value of S corresponding to the condition of 'adequate bracing'
δ	: moment magnification factor
Δ	: inter-storey lateral drift due to the action of H
γ	: a nondimensional factor (accounting for variable column end restraint)
$\psi_{1,2}$	: column end factors at ends 1 and 2 (for use in 'alignment charts')