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Structural identification of a steel frame from dynamic test-data

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Abstract. Structural identification via modal analysis in structural mechanics is gaining popularity in recent years, despite conceptual difficulties connected with its use. This paper is devoted to illustrate both the capabilities and the indeterminacy characterizing structural identification problems even in quite simple instances, as well as the cautions that should be accordingly adopted. In particular, we discuss an application of an identification technique of variational type, based on the measurement of eigenfrequencies and mode shapes, to a steel frame with friction joints under various assembling conditions. Experience has suggested, so as to restrict the indeterminacy frequently affecting identification issues, having resort to all the *a priori* acknowledged information on the system, to the symmetry and presence of structural elements with equal stiffness, to mention one example, and mindfully selecting the parameters to be identified. In addition, considering that the identification techniques have a local character and correspond to the updating of a preliminary model of the structure, it is important that the analytical model on the first attempt should be adequately accurate. Secondly, it has proved determinant to cross the results of the dynamic identification with tests of other typology, for instance, static tests, so as to fully understand the structural behavior and avoid the indeterminacy due to the nonuniqueness of the inverse problem.

Key words: structural identification; experimental modal analysis; steel frame.

1. Introduction

Structural identification via modal analysis has undoubted advantages that account for the interest it generally raises in structural mechanics, despite conceptual difficulties connected with its use. Basically, the problem of reconstructing some structural properties of a mechanical system from the measurement of certain modal quantities is an inverse problem that turns out to be highly undetermined in many practical circumstances. In fact, it is well known that existence and uniqueness results together with reconstruction techniques in the literature of inverse problems in vibration often are for special classes of mechanical systems, like beams or rods, and require knowledge of infinite data (Gladwell 1986). Real situations differ significantly from those cases. On one hand, one can measure in a simple and accurate way just the eigenparameters of the first few modes of a structure. On the other hand, analytical models of vibrating systems tend to become inadequate to describe the modes of higher order. Thus, one has only a finite amount of significant spectral data and so many solutions may exist.

Taking into account this lack of satisfactory framework of general properties, most of the identification techniques via modal analysis are based on an optimality criterion where a preliminary

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model of the system is updated so that the modal parameters of the first few vibrating modes closely match with the measured ones (Capecchi and Vestroni 1986, Hearn and Testa 1991, Capecchi and Vestroni 1993). Analyses performed on simple structures such as steel beams or frames, showed that the results of this type of identification techniques strictly depend on the accuracy of the structural analytical model that one uses for the interpretation of the experiments and on the choice of simple models to describe the dynamic behavior of the mechanical system (Davini *et al.* 1995). This remark is relevant because, in undetermined problems like those posed by diagnostics and monitoring, it may be crucial to keep the physical model as simple as possible in order to get useful information. Furthermore, basic questions arise as to whether this type of analysis can be applied to field measurements and complex structures, and how the results of the identification are affected by the accuracy of the structural modeling (Morassi and Rovere 1997). Answer to these questions would be desirable, but no general assessment of the matter seems to be available.

Bearing in mind the hitherto described aspects, the present work is concerned with the outlining and enquiry into the results related to the dynamic identification of a steel rectangular cell having bolted joints. The structural typology in question is of remarkable interest within aeronautics and civil engineering, (Bernelli-Zazzera *et al.* 1992), in that it represents the elementary modulus for a category of such structures as trusses or frames which are achieved by assembling a number of cells being nominally alike. The structural identification via dynamic analysis has called for specific contrivances due to the somehow unexpected complexity of the structure. The interpretation of a series of dynamic tests along with the employment of an identification technique, which is variational in its character and based on the sensitivity of natural frequencies, has allowed to characterize constructive details, such as the joints, that are not likely to be schematically surveyed via analytical approach, as well as assess the actual stiffness of the members. Furthermore, it has been borne out by working on the basis of a substructuring approach to identification that the identified model of the single cell in question can be aptly used for foreseeing the dynamic behavior of a truss achieved by assembling two cells which are nominally alike.

Experience has suggested, so as to restrict the indeterminacy frequently affecting identification issues, having resort to all the *a priori* acknowledged information on the system, to the symmetry and presence of structural elements with equal stiffness, to mention one example, and mindfully selecting the parameters to be identified. In addition, considering that the identification techniques have a local character and correspond to the updating of a preliminary model of the structure, it is important that the analytical model on the first attempt should be adequately accurate. Secondly, it has proved determinant to cross the results of the dynamic identification with tests of other typology, for instance, static tests, so as to fully understand the structural behavior and avoid the indeterminacy due to the nonuniqueness of the inverse problem.

2. Dynamic tests

2.1 Experimental model and description of the experiments

The experimental investigation has been performed for the steel rectangular cell represented in Fig. 1. Horizontal beams and columns consist of two steel beams of the series UNI 5787-73, while the brace is a single beam of the same series. All the members have their ends bolted to a joint

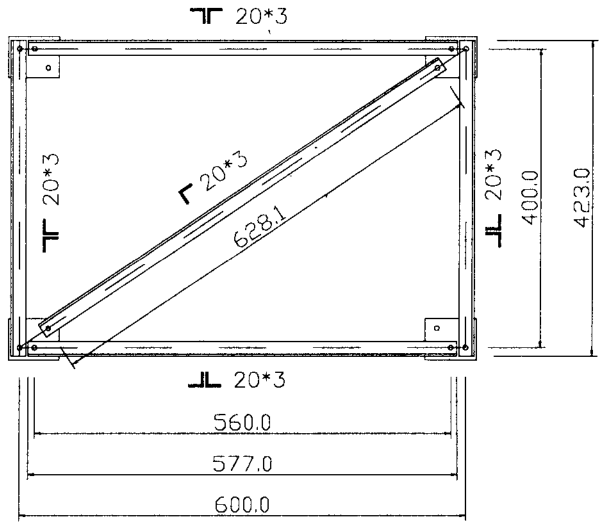


Fig. 1 Experimental model of the cell with brace (lengths in mm)

plate by means of a single bolt (with 6 mm diameter and of the series 8.8) fastened with a torque of magnitude 17 Nm. Structural joints are of friction type and the efficiency of the rotational constraint between connected members basically depends on the importance of friction forces on the contact surface. Mechanical properties of the structural elements were deduced from a preliminary series of dynamic tests performed on sample beams, see Table 1.

The analysis intended to identify an accurate analytical model for the in-plane dynamic behavior of the steel frame and in order to do so structural identification was worked out in several steps. In brief, in the first part of the experience we gave a characterization of the cell without brace: the interpretation of a series of dynamic tests allowed to include the significant effects of some constructive details in modelization, such as an increase in stiffness due to the finite size of the beam-column joints, and to assess the actual bending stiffness of the members, cf. section 4.1. Subsequently, the behavior of the plate-brace joints was deduced from the interpretation of dynamic tests performed on the above mentioned cell, to which a brace has been added, cf. section 4.3.

During the tests the specimen was placed in a vertical plane and was suspended on a contrast frame by means of two soft springs. A suitable stiffness for the springs was chosen so that its influence on the frame's free vibrations would be negligible. Therefore, all following considerations are based on free-free boundary conditions. The tests were run according to an *impulse technique* to determine some terms of the frequency response function of the frame. The grid of the measurement points included the beam-column joints, the mid-point and the quarters of all the

Table 1 Nominal mechanical properties of structural elements

Structural element	Linear mass density ρ (kg/m)	Axial stiffness EA (N)	Bending stiffness EJ (Nm²)
Horizontal beams and columns	1.903	5.1872×10^7	1.8090×10^3
Beam-column joint	Lumped mass: 0.1082 Kg Lumped rotational inertia: 1.7803×10^{-4} kgm²		

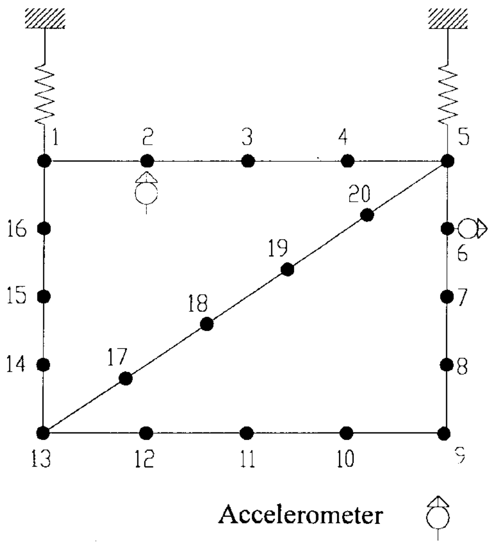


Fig. 2 Measurement points and locations of the accelerometers

members, see Fig. 2. We excited the beams transversely at each intermediate measurement point by an impulse force hammer, while horizontal and vertical excitations were introduced at beam-column joints. The structural response was acquired by two piezoelectric accelerometers, one placed in vertical direction in correspondence of node 2, the other placed horizontally at node 6, see Fig. 2. Output and input signals were weighted by a *force* and exponential window, respectively, and were processed in the frequency domain to determine the relevant frequency response terms (inertance). Considering the good degree of reproducibility of the measurements, in all cases each inertance term was evaluated as the average of ten impulsive tests. Fig. 3 shows a typical inertance term

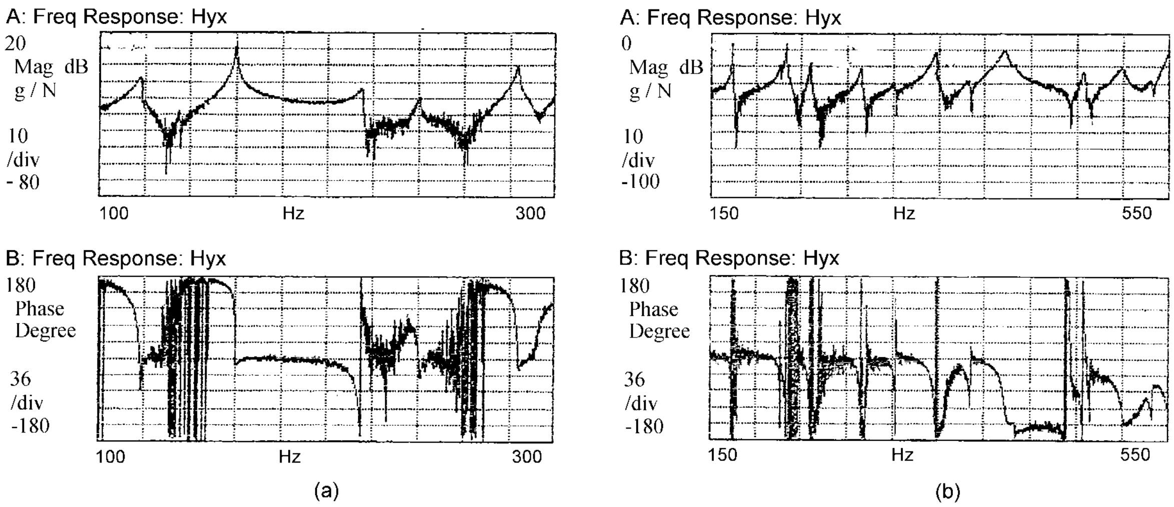


Fig. 3 Some inertance terms: (a) cell without brace (force at node 2-vertical direction; response at node 6-horizontal direction); (b) cell with brace (force at node 5-vertical direction; response at node 6- horizontal direction)

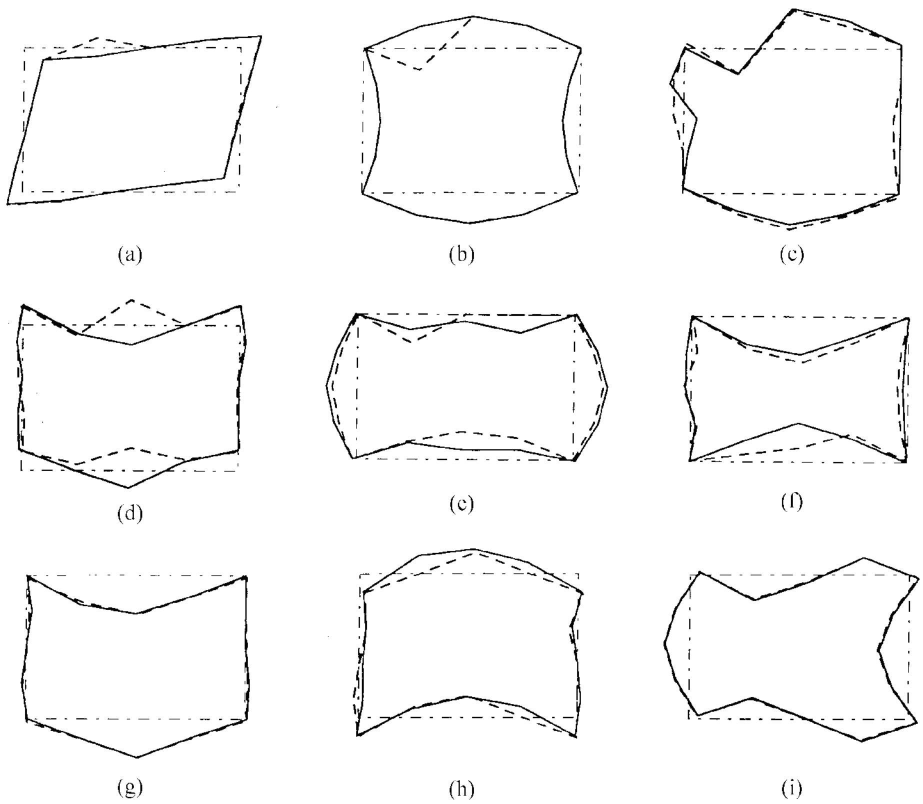


Fig. 4 Experimental normalized mode shapes *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i* of the cell without brace (Continuous line: response at node 6; dashed line: response at node 2; dot-dashed line: reference configuration)

measured during experiments.

2.2 Modal analysis results

With a view to identify vibration modes of the cell without brace, after some preliminary tests, we studied in detail the frequency range from 100 to 400 Hz. The well-separated vibration modes and the small damping enable us to adopt the single mode technique in the extraction of modal parameters from inertance measurements. The resonance frequency was made coincident with the abscissa of the inertance modulus peak; modal components were derived from peak values of inertance and during mode reconstruction only real modal deflections were considered, whose sign was given according to phase value. Modal analysis results are illustrated in Fig. 4 and Table 2 (second column). Mode shapes in Fig. 4 correspond to linear interpolations of measured modal components and have unitary norm. With the exception of a few cases, we observed a good agreement between mode shape estimates obtained considering vertical and horizontal accelerometer. Concerning frequency values, estimates have shown a good degree of reproducibility with negligible relative and absolute deviations from average values.

With the same experimental set-up described above, in the second part of the experiment we studied the cell with brace, see Mitri (1997) for more detailed description. Modal analysis results

Table 2 Experimental frequencies of the cell without and with brace

Mode shape	Frequency (Hz)	
	Cell without Brace	Cell with Brace
a	117.99	167.95
b	159.70	216.68
c	215.14	281.50
d	240.76	311.11
e	284.30	345.34
f	310.89	377.19
g	335.63	406.54
h	348.02	417.66
i	392.69	472.17

concerning this structural configuration are illustrated in Table 2 (third column) and in Fig. 5.

2.3 Interpretation of dynamic tests

We start considering in detail the interpretation of the dynamic measurements performed on the cell without brace. To select from the several experimental mode shapes those effectively corre-

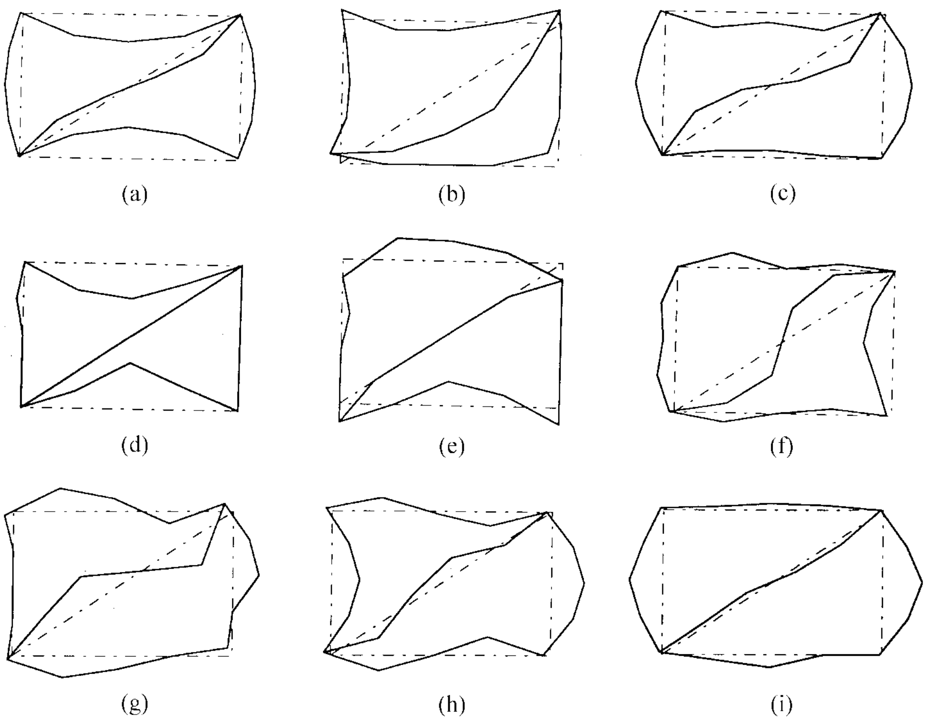


Fig. 5 Experimental normalized mode shapes *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i* of the cell with brace (Continuous line: response at node 6; dot-dashed line: reference configuration)

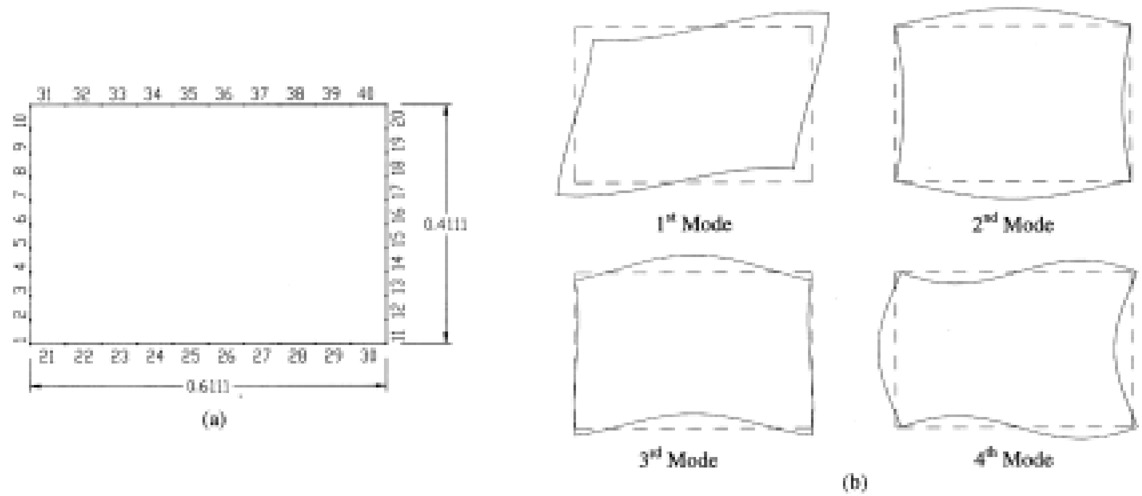


Fig. 6 Cell without brace: (a) mesh of the finite element model with element numbers and (b) first four mode shapes of the analytical model MSMP (dot-dashed line: reference configuration)

sponding to first few in-plane vibration modes of the cell, we introduced a "guess" analytical model, which shall be called by MSMP in the following, and used the Modal Assurance Criterion (*MAC*) (Ewins 1984). Model MSMP has point-like joints and structural continuity between connected members is fully restored (ideal joints). Nominal mechanical properties of Table 1 were considered and suitable inertia values concentrated in joint positions were taken into account. Fig. 6(b) shows first four vibration modes obtained considering a finite element model of the cell with ten-element-mesh for each beam, see Fig. 6(a); relative natural frequency values are reported in Table 3.

With reference to a discrete model of the cell with lumped masses located at the points of the measurement grid and considering as analytical modes $\left\{ \boldsymbol{u}_{an}^{(s)} \right\}_{s=1}^{N_{an}}$ those of the model MSMP, we calculated the matrix

$$MAC(r,s) = \frac{\left|\boldsymbol{M}\boldsymbol{u}_{sp}^{(r)} \cdot \boldsymbol{u}_{an}^{(s)}\right|^2}{\left|\boldsymbol{M}\boldsymbol{u}_{sp}^{(r)} \cdot \boldsymbol{u}_{sp}^{(r)}\right|\left|\boldsymbol{M}\boldsymbol{u}_{an}^{(s)} \cdot \boldsymbol{u}_{an}^{(s)}\right|}, \tag{1}$$

$r = 1, \ldots, N_{sp}$, $s = 1, \ldots, N_{an}$, where $\left\{ \boldsymbol{u}_{sp}^{(r)} \right\}_{r=1}^{N_{sp}}$ are the measured modes and $\boldsymbol{M}$ is the mass matrix.

Table 3 Cell without brace: comparison between first four experimental f_{exp} and analytical f_{an} frequencies (Analytical values are for preliminary model MSMP and for identified models MSM1 and MSM2 $\Delta f\,\% = 100 \times (f_{an} - f_{\text{exp}})/f_{\text{exp}}$)

Mode	f_{exp} (Hz)	MSMP		MSM1		MSM2	
		f_{an} (Hz)	Δf %	f_{an} (Hz)	Δf %	f_{an} (Hz)	Δf %
1	117.99	103.77	–12.05	114.88	–2.64	120.08	1.77
2	159.70	169.94	6.41	173.42	8.59	163.03	2.09
3	348.02	287.16	–17.49	333.58	–4.15	334.00	–4.03
4	392.69	420.73	7.14	406.19	3.44	389.62	–0.78

Table 4 MAC matrix for the cell without brace (response at node 6-horizontal direction)

Anal. Modes	Experimental modes								
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
1	**0.9750**	0.0008	0.0035	0.0039	0.0001	0.0003	0.0001	0.0039	0.0048
2	0.0000	**0.9948**	0.4113	0.0145	0.5661	0.7513	0.0036	0.0110	0.0025
3	0.0001	0.0004	0.0148	0.6445	0.0000	0.0003	0.5522	**0.8768**	0.0057
4	0.0000	0.0000	0.0284	0.0000	0.0004	0.0110	0.0007	0.0048	**0.9502**

The results shown in Table 4 allow to state that first two modes of the cell correspond to experimental modes *a* and *b*; moreover, third mode corresponds to the measured mode *h* and the fourth to the mode *i*. As a confirmation of test accuracy and of correct interpretation of the measurements, an orthogonality check between first four experimental mode shapes shows negligible deviations (about 1-2%) from zero value for the off-diagonal terms. Fig. 7 compares experimental and theoretical vibration modes: although experimental mode shapes correspond to linear interpolation of measured components, experimental and theoretical patterns are almost similar. It is good to note that measured mode shapes are approximately orthogonal at horizontal beam-column joints and this suggests that the joints are meant to fully restore the structural continuity between a horizontal beam and a column.

Table 3 (column 4) compares experimental and theoretical (MSMP model) frequency values of first four vibration modes. Deviations are important and, in particular, errors change alternatively sign with the order of the modes. The reasons for this are discussed in section 4.1, together with the

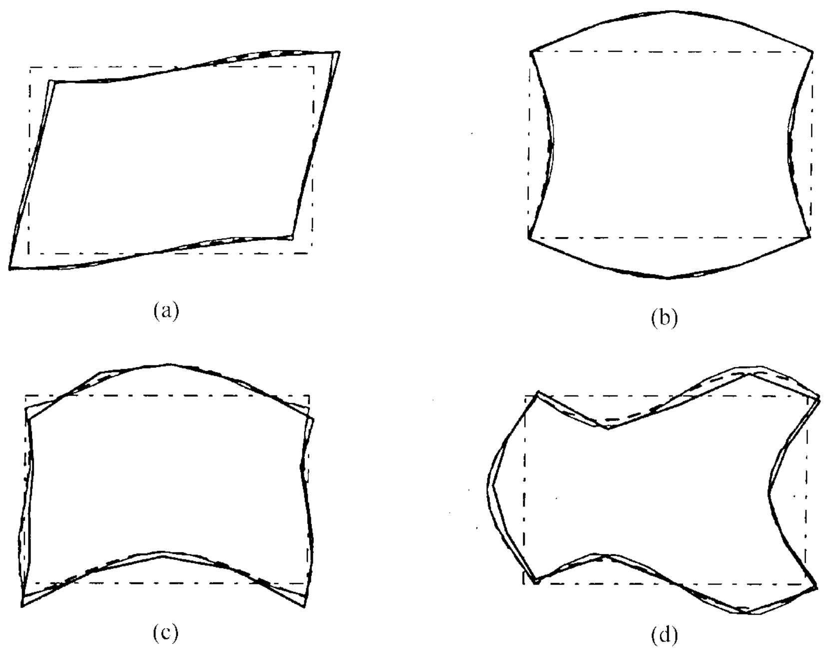


Fig. 7 First four experimental and theoretical mode shapes of the cell without brace (Continuous line: experimental values (response at node 6-horizontal direction); dashed line: preliminary analytical model MSMP; thin continuous line: optimal model MSM2; dot-dashed line: reference configuration)

determination of an accurate analytical model of the cell.

Similar considerations were developed for the interpretation of dynamic testing on the cell with brace. In that case it was found that measured vibration modes *a*, *b*, *e* and *h* of Fig. 5 correspond respectively to first four in-plane mode shapes of the cell.

3. Frequency sensitivity to stiffness changes

We start by briefly recalling some results about free in-plane vibrations of the cell and, to fix the ideas, we consider the structural configuration without a brace. Analysis carried out in Mitri (1997) shows that the lower frequency in-plane modes we are concerned with are practically insensitive to the introduction of the constraint on the axial deformations for all structural members. This is in agreement with the fact that the energy content of these modes is essentially due to the bending energy of the members of the frame. This then enables one to deal with a simpler, but at the same time accurate, class of models where axial deformations are neglected. Therefore, for the interpretation of the experiments and also in applying the identification technique of section 4.1, we adopt the Euler-Bernoulli mathematical model for beams.

Once free vibration problem for a beam-like structure S is solved, the sensitivity of the *r*-th frequency ω_r to a bending stiffness variation *h* can be explicitly determined. In fact, the partial derivative $d_{EJ}\omega_r^2$ of ω_r^2 with respect to bending stiffness *EJ* is given by the scalar product between the gradient of ω_r^2,

$$\frac{\partial \omega_r^2}{\partial EJ(x)} = \left(u^{(r)''}(x, EJ) \right)^2, \tag{2}$$

and the stiffness variation *h*, that is

$$d_{EJ}\omega_r^2(h) = \int_S h(x) \left(u^{(r)''}(x, EJ) \right)^2 dx, \tag{3}$$

where $u^{(r)}$ is the *r*-th normalized mode shape of the structure S. Identities (2)-(3) are standard, see Pöschel and Trubowitz (1987, pp. 31-33) for the axial vibration case. However, for reasons of completeness we provide some details on their derivation. To fix ideas and without affecting the character of generality, let us consider a simply supported beam. The procedure can be easily extended to any beam-like system. Free-bending vibrations of the beam are governed by the following eigenvalue problem:

$$\left(EJu^{(r)''} \right)'' - \omega_r^2 \rho u^{(r)} = 0 \qquad x \in (0, L) \tag{4a}$$

$$u^{(r)} = u^{(r)''} = 0 \qquad \text{at } x = 0 \text{ and } x = L, \tag{4b}$$

where $\left(\omega_r^2,\, u^{(r)} \equiv u^{(r)}(x) \right)$ is the *r*th eigenpair, $r = 1, 2, \ldots$, and $\rho \equiv \rho(x)$ is the (continuous) linear mass density of the beam. Assume that the *r*th eigenmode $u^{(r)}$ is normalized so that $\int_0^L \rho \left(u^{(r)} \right)^2 dx = 1$. Differentiating both sides of the differential Eq. (4a) in the direction *h* we obtain

$$\left(EJd_{EJ}u^{(r)''}(h) + hu^{(r)''} \right)'' - d_{EJ}\omega_r^2(h)\rho u^{(r)} - \omega_r^2 \rho d_{EJ}u^{(r)}(h) = 0, \tag{5}$$

where the linear operator $d_{EJ}\Phi$ is the partial derivative of Φ with respect to the coefficient *EJ*. If *EJ* is twice continuously differentiable, then $u^{(r)}$ has continuous derivatives up to order four, and we may interchange differentiation with respect to x and EJ to obtain

$$\left(EJ\left((d_{EJ}u^{(r)}(h))''+hu^{(r)''}\right)\right)''-d_{EJ}\omega_r^2(h)\rho u^{(r)}-\omega_r^2\rho d_{EJ}u^{(r)}(h)=0 \tag{6}$$

Multiplying expression (6) by $u^{(r)}$ and integrating by parts on using boundary conditions (4b), we find

$$\int_0^L \left(d_{EJ}u^{(r)}(h)\right)\left(\left(EJu^{(r)''}\right)-\omega_r^2\rho u^{(r)}\right)dx+\int_0^L h\left(u^{(r)''}\right)^2dx-d_{EJ}\omega_r^2(h)=0 \tag{7}$$

Since $u^{(r)}$ satisfies (4a), the first integral vanishes and we obtain identities (2) and (3). It can be shown that (2)-(3) hold in general also for less regular coefficients. Identity (3) is useful for estimating the effect of small stiffness variations on frequencies. For example, if the stiffness variation $h\equiv\Delta(EJ)$ is constant in a small neighboorod $(x_0-\Delta x/2,\ x_0+\Delta x/2)$ of the cross-section of abscissa x_0, then the first order variation of the (squared) frequency with $\Delta(EJ)\cdot\Delta x$ is:

$$\omega_r^2\left(EJ+\Delta(EJ)\right)-\omega_r^2(EJ)=\left(u^{(r)''}(x_0,\,EJ)\right)^2\cdot\Delta(EJ)\cdot\Delta x \tag{8}$$

Eq. (8) states that the effect of a localized stiffness variation on a (squared) frequency is proportional to the squared curvature of the related mode shape of the unperturbed system, evaluated at the cross-section where the variation occurs. That is, $\left(u^{(r)}(x_0,\,EJ)\right)^2$ gives an indication of the frequency sensitivity of the r-th mode to stiffness variations that occur near the cross-section of abscissa x_0. Moreover, Eq. (8) shows that frequency change is positive for increase in stiffness and negative in the opposite case.

Fig. 8 shows frequency sensitivity of the cell without brace as evaluated on the basis of the analytical model MSMP. Generally speaking, first and third modes have high sensitivity on regions close to beam-column joints, while middle points of members denote low sensitivity. The opposite

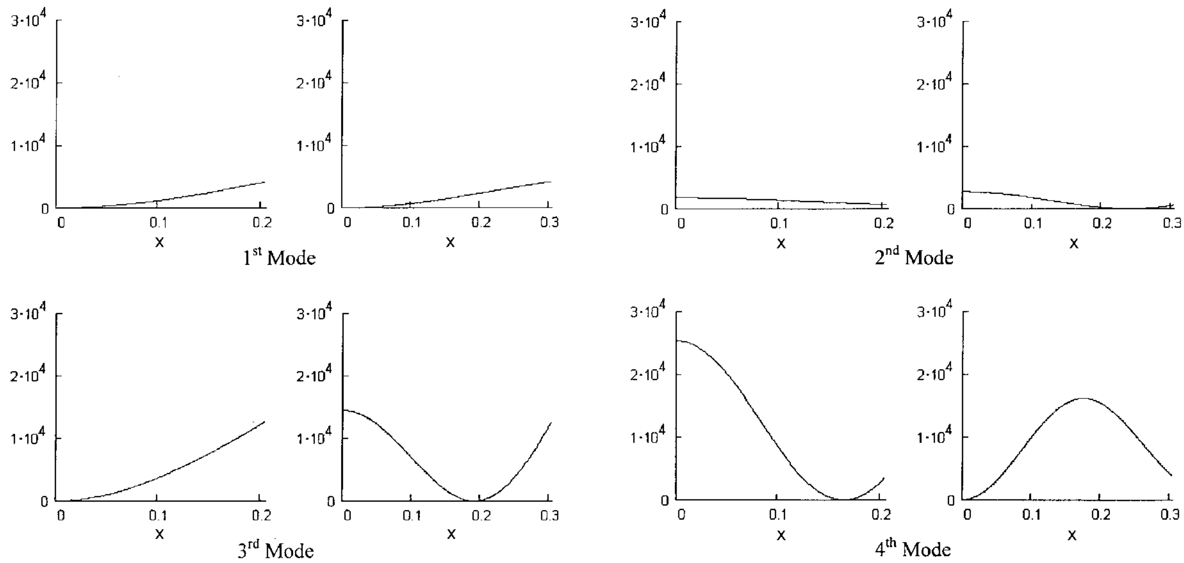


Fig. 8 Behavior of the square of curvature of first four normalized mode shapes of the cell without brace evaluated on half of vertical beam elements (left) and on half of horizontal beam elements (right) (Abscissa x meters increases from the center of each element toward the beam-column joint)

situation happens for two remaining vibrating modes. These simple observations will be useful in the next section when we will try to improve the accuracy of the analytical model of the cell.

4. Identification of an accurate analytical model of the cell

4.1 Structural identification from dynamic test-data

Structural identification of the cell was worked out in two main steps. First we determined an accurate analytical model of the cell without the brace. Subsequently, we considered the presence of the brace element. In this section we consider in detail the first situation.

A comparison between experimental frequencies and corresponding theoretical values of the model MSMP shows that MSMP underestimates first and third frequencies (–12.05% and −17.49%, respectively), while it overestimates the others (+6.41% for the second and +7.14% for the fourth), see Table 3. Assuming that the class of models used to describe the dynamic behavior of the cell is sufficiently rich and that inertia distribution is accurate enough, one can reasonably suspect that modelization errors are mainly due to an inadequate representation of the real stiffness of the elements and possibly to an inadequate description of the beam-column joints. Previous studies on steel frame systems showed that some structural details, like the finite size of the joints, may have a significant effect on the determination of an accurate analytical model of the structure (Morassi 1990, Morassi and Rovere 1997). In those cases the structure was arranged by welding or bolting both ends of the beams to columns and, generally speaking, the analysis pointed out that adequate modeling of joints implies a significant increase in stiffness in the regions of the frame close to structural joints. Yet, the structure being investigated here has different features, as the member ends are connected to friction joints and it is accordingly decisive to assess their subsequent efficiency in connecting the horizontal beams to the columns. The remaining part of the present section will be devoted to identifying the origin of modeling errors of the preliminary analytical model of the cell MSMP. At this stage we shall make use of both qualitative-analytical considerations allowed by the sensitivity analysis in section 3 and an identification technique specially employed for similar structure problems, as well as, eventually, the interpretation of some static tests performed on the column-horizontal beam joint (see section 4.2).

A parametrical analysis of the cell dynamic behavior was preliminarily carried on in order to evaluate the effect of the possible insertion of compliance on the horizontal beam-column joints. Simulations in structural schemes involving elastic hinges with adjustable stiffness located within the joints showed that the fundamental vibration mode and the third mode of the cell are rather sensitive to the insertion of such a compliance. In such cases, a remarkable part of the deformation energy is stored in the joint hinges and the angle between the horizontal beam and the column is substantially unlike a right angle. The measured modes detected no situations of this kind, see Fig. 7, which let us assume that the joints are meant to fully restore the continuity between column and horizontal beam and, indeed, to bring about a further degree of stiffness within the cell. The latter effect may be partly due to the finite size of the joints as well as to the presence of the connection plate. As a matter of fact, the length of the contact area between member and plate is fairly broad with reference to the length of the member and, additionally, the plate has greater stiffness than the sections the horizontal beams and the columns are made of.

Firstly it is possible to verify to what extent the hypothesis of a stiffening effect induced by the joint is valid by working with the sensitivity of frequencies. It can be deduced from the considerations at the end of section 3 that an increase in stiffness in the sections close to the joints is supposed to significantly enhance frequencies f_1 and f_3 (the latter in particular, for equal stiffness variation, see Fig. 8), whereas f_2 and f_4 are expected to increase by little, being thus consistently overestimated by the model. The very sensitivity analyses also proves that by provisional approximation of a decrease in the second and fourth frequencies, while keeping the first and third ones unchanged, may be caused by a stiffness reduction in the middle region of the members. As a conclusion, despite the heuristic approach of such analysis, we are prone to believe that in the preliminary model the stiffness of the joints and the stiffness of the single beams have been undervalued and overestimated respectively.

Taking the above aspects into account and with a view to update MSMP model of the cell without brace, we used the structural identification technique presented by Davini *et al.* (1993) and then adopted by Davini *et al.* (1995) and Morassi and Rovere (1997) in the study of some structural diagnostic problems.

Following is a brief outline of the identification technique. A finite element model, whose free undamped vibrations are governed by the differential equation

$$\boldsymbol{M}\frac{d^2\boldsymbol{u}(t)}{dt^2}+\boldsymbol{K}\boldsymbol{u}(t)=\mathbf{0}, \tag{9}$$

substitutes the continuous Euler-Bernoulli model of the cell. In Eq. (9) $\boldsymbol{M}$ and $\boldsymbol{K}$ are the global mass and stiffness matrices, respectively, and $\boldsymbol{u}(t)$ is the vector of nodal displacements. As usual, $\boldsymbol{M}$ and $\boldsymbol{K}$ are obtained by assembling the contributions of all the Ne structural elements of the discrete model. In particular

$$\boldsymbol{K}=\sum_{e=1}^{N_e}\alpha_e\boldsymbol{K}_e, \tag{10}$$

where $\boldsymbol{K}_e$ is the stiffness matrix (in the reference of the nodal displacements) of the eth finite element for an initial ("guess") model of the frame, and α_e is the collection of the "stiffness multipliers" normalized to the unit value for the reference configuration. Then, one can consider the $\alpha_e s$ as descriptive of the stiffness distribution of the system.

The present approach to identification is of the variational type. We try to determine the stiffness distribution of the frame looking for those optimal values $(\alpha_e)_{opt}$ of the $\alpha_e s$ that minimize the distance

$$F(\alpha_e)=\sum_{j=1}^{M}\left[\frac{\tilde{f}_j^{\,2}-f_j^{\,2}(\alpha_e)}{\tilde{f}_j^{\,2}}\right]^2 \tag{11}$$

between the first M experimental $\tilde{f}_j$ and analytical f_j eigenfrequencies. As discussed in Capecchi and Vestroni (1993) and in Antonacci *et al.* (1999), the choice of using only the first few frequencies in identification implies various sources of indeterminacy, like the nonuniqueness of the optimal solution as a consequence of the nonconvexity of the objective function F. In this sense, the adoption of suitable a priori assumptions, such as structural symmetries or information about stiffness values within some regions of the structure, is crucial in order to get around the intrinsic difficulties of the identification problem. The working hypothesis hereafter is that the model to be identified is a perturbation of the guess model, so that it may be sensible to use the latter as the initial point of the updating procedure. The minimization algorithm is based on an iterate improved gradient method that updates the stiffness distribution at each step according to the sensitivity of the first M eigenfrequencies to stiffness changes (see identity (2)).

For identification we use the finite-element model shown in Fig. 6(a). With the chosen mesh, the spectral properties of the discrete model are practically indistinguishable from those of the continuous one, at least for the vibration modes we are considering. In applying the identification technique, we construct the objective function using the first four measured frequencies and we take the configuration corresponding to MSMP model as the initial point in minimization. Here we present the main results of the analysis and we refer to Mitri (1997) (Ch. 4) for a complete account of the structural identification of the cell.

A first optimal model, MSM1, is obtained confining the identification parameters to the (two) beam elements close to all structural joints only and by fixing the remaining to their reference value (e.g., α_e=1). Considering the symmetry of the frame, stiffnesses to be identified are those of elements 1 (=10=11=20), 2 (=9=12=19), 21 (=30=31=40) and 22 (=29=32=39). The results presented in Fig. 9 show that inaccuracy of the guess MSMP model was due to an underestimate of the stiffness in elements close to the beam-column joints and to a overestimate of the stiffness in adjacent elements. This seems to suggest that, on one hand there is an increase of overall stiffness due to the finite size of the beam-column joints, and on the other hand there is a localized compliance at both ends of the members possibly because of the flexibility of friction joints. As a result of the comparison between experimental and theoretical frequencies reported in Table 3, updating proves to substantially enhance the accuracy of the analytical model for odd frequencies f_1 and f_3, with percentage errors of −2.64% e −4.15%, respectively. As regards the even modes, on the contrary, optimal model MSM1 is still quite rough: percentage deviations between experimental and theoretical data shift from +6.41% to +8.59% for the second frequency and from +7.14% to +3.44% for the fourth frequency. In fact, the analysis in section 3 allows to learn that the regions in

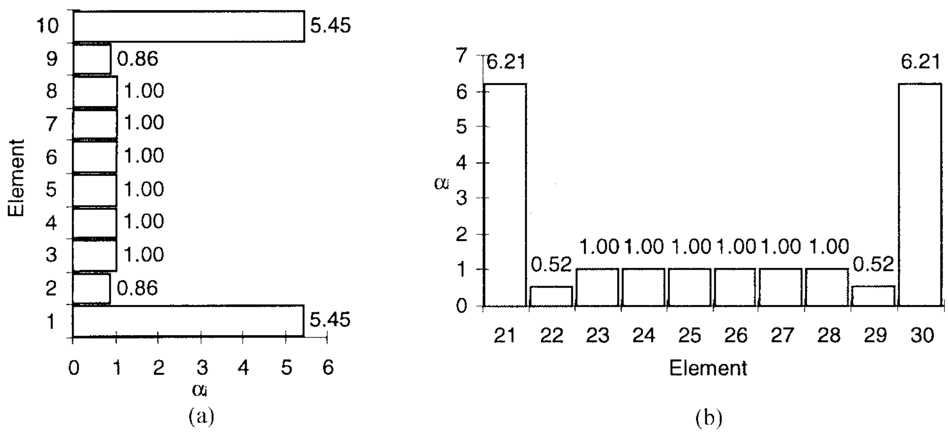
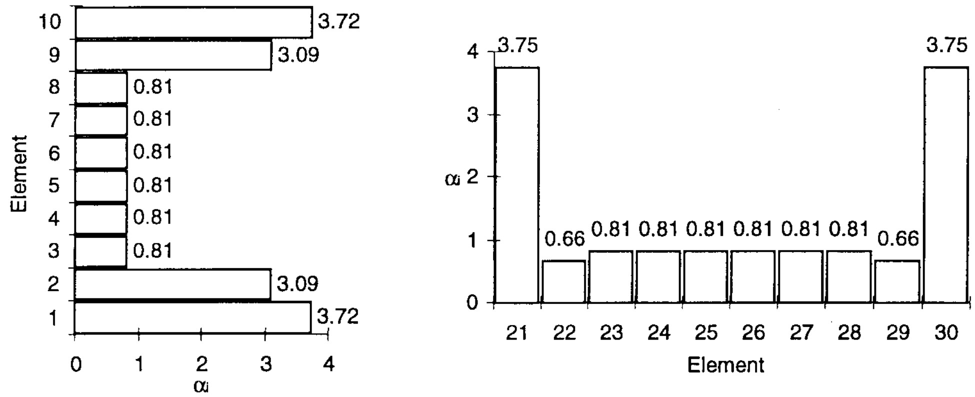


Fig. 9 Identification of the cell without brace: optimal stiffness multipliers α_i for identified model MSM1 (α_i =1 for MSMP): (a) column elements; (b) beam elements



(a) (b)

Fig. 10 Identification of the cell without brace: optimal stiffness multipliers α_i for identified model MSM2 (α_i =0 for MSMP): (a) column elements; (b) beam elements

proximity to the joints have low sensitiveness in relation to frequencies f_2 and f_4, and therefore the serious errors affecting the frequencies mentioned cannot be reduced if only the two elements in proximity to the joints are regarded as actual identification parameters. Such a conclusion called for the drawing up of a new optimal model, which can be called MSM2, and in it the (uniform) stiffness of the beam elements forming the remaining sections of the horizontal beam and the column is a parameter to be identified. The optimal stiffness distribution achieved through MSM2 confirms that the presence of the joint does have a stiffening effect and, at the same time, it indicates that the stiffness of the beam elements had been overestimated by roughly 20% in the preliminary model; see Fig. 10. In the meantime there seems to be no compliance effect whatsoever caused by the friction joint. Model MSM2 thoroughly accounts for the vibration modes in the field of frequencies in question, being affected by errors concerning the first four frequencies which range from 1% to 4%; see Table 3 (columns 7 and 8).

In what follows we present an interpretation of the reduced bending stiffness for the mid-sections of the column and beam elements of the cell. According to classical kinematic hypotheses on small vibration theory for beam-like systems, the analytical model which governs the infinitesimal in-plane bending vibrations of the cell assumes that i) the behavior of the composite cross-section is that of a "one-piece" cross-section; and that ii) transversal cross-sections rotate around the principal direction orthogonal to the vertical plane, let say the axis *x-x*, when beam elements are subjected to bending vibrations. Real situation is different and more complicated. Because of the extremity constraint, each beam element is subjected to end bending couples with direction parallel to the axis *x-x*. Axis *x-x* does not coincide with a principal direction of inertia of the cross-section of a single beam and then bending vibrations not necessarily confined on the vertical plane may occur in each beam element. It follows that admissible kinematical configurations expected by the analytical model of the frame are a set smaller than the set of configurations reachable by the real system. Then, as with any constrained system, it is reasonable to expect an overestimate of the effective stiffness of the beam elements of the frame.

It is worth noticing that the elucidated case consisting of two cell models, which both are on many accounts "optimal" and physically "reasonable", is not anomalous; it is actually recurrent in the identification problems via dynamic analysis. For concrete applications the data at disposal are in fact few and not exhaustive to guarantee the uniqueness of the inverse problem (Gladwell 1986). Hence, there can be for instance a number of different distributions of stiffness corresponding to the same first four frequencies of the system. As a result, only by introducing further information dependent on the system, for example, tests of a different kind can be useful as selection criteria of an optimal model. In this case, it is to be immediately made clear that the MSM2 model shall be considered fit for describing the dynamics of the cell. That can be accounted for not merely in terms of more restricted variations between theoretical and experimental frequencies, but also because the outcomes obtained via dynamical analysis have been borne out by a number of static characterization tests on the friction joints, as shown in the following section.

4.2 Considerations on static characterization tests on the horizontal beam-column joint

It is to be reminded that the horizontal beam-column joint consists of a rectangular plate and by means of a clamped bolt the two horizontal beam and column members are connected to it. The parameter peculiar to the connection is the “moment of resistance” ensuing from the friction forces that arise on the contact surfaces between the members and the joint plate. More in detail, owing to the clamping of the bolt the contact area between members and joint-plate is concerned with standard pressures which, on trying to carry out a relative rotation in between the member and the plate, cause friction forces on the contact surface itself. The efficiency of the constraint basically depends on the “moment of resistance” rate and the maximum moment engaging the joint: if the latter is lower than the former a global restoration of the continuity is reasonably expected (*i.e.*, an ideal node). In the opposite case the joint introduces a local flexibility on the cell.

To characterize the behavior of the friction joint, some beam samples similar to those used for achieving the cell horizontal beams and columns have been subjected to a number of static tests. More in detail, the constraint at the left end has re-created the connection between beam and plate as present in the structural joint of the cell, see Fig. 11(a), whereas the right end is sulyected to an increasing concentrated load. This way the transversal displacement in some points of the beam shaft as well as the displacement in proximity to the constraint section have been measured with the aim to detect possible relative rotations between the member and the joint plate. In the tests here described such rotations proved to be virtually non-existant up to loads close to a given “limit” value, which once reached allowed to occur a rigid rotation of the whole beam around the bolt. The average moment of resistance measured is equal to about 50 Nm. The response of the member subjected to impulses ascribable to those employed for exciting the cell in the dynamic characterization tests has highlighted the maximum value of the moment affecting the joint being of about 10 Nm and therefore considerably lower than the moment of resistance within the friction joint. That leads to completely disregard the presence of relative rotations between plate and member, the capability of the friction joint of entirely restoring the continuity between joint and horizontal beam being thus confirmed, as suggested by identification results obtained in model MSM2.

Fig. 11(b) compares the experimental and theoretical end displacement-load relationship in the clamped beam. Theoretical displacement has been obtained by referring to the identified bending stiffness of the optimal model MSM2. The good agreement between experimental and theoretical estimates bears out the data obtained via dynamical analysis in model MSM2, supporting the interpretation of identification results given at the end of the previous section. Throughout static tests, in fact, the particular application of the load at the end of the cantilever induces a type of

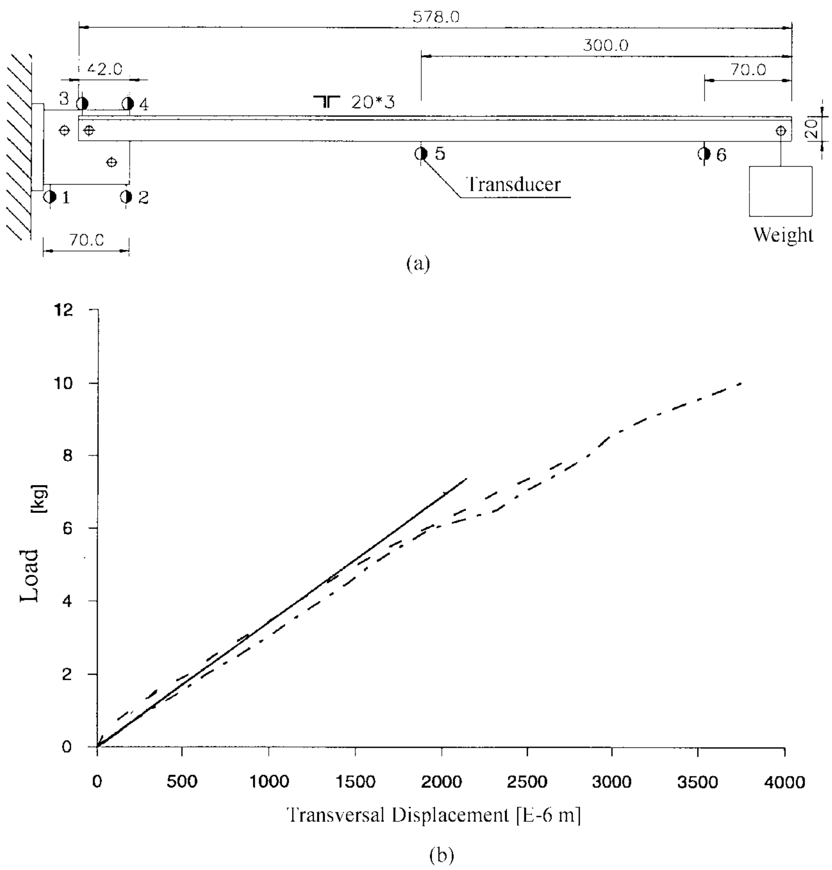


Fig. 11 (a) Experimental set-up for the measurement of node couple (lengths in mm), (b) Examples of load-displacement relationship (transducer 6) (Continuous line: theoretical behavior obtained using the identified bending stiffness of MSM2; dashed line: experimental data)

deformation that the analytical model of the frame is able to describe, i.e. simultaneous bending deflection of the two ∟ beams in the vertical plane and a consequent rotation of the "one-piece" transversal cross-section ⅃∟ around an axis orthogonal to plane of the cell.

4.3 Structural identification of the cell with brace

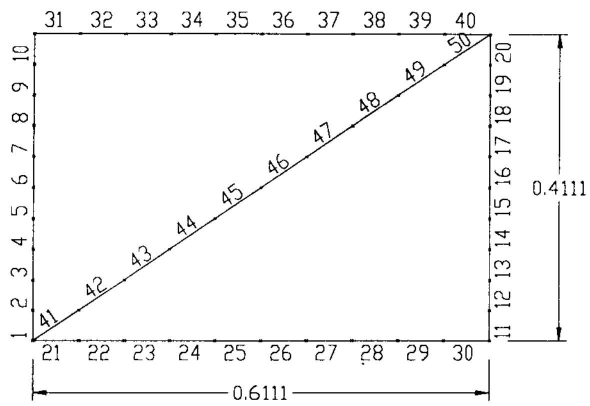
The interpretation of the dynamic tests is based on the identified model MSM2, which a brace has been added to, such brace being fitted at the end and having a nominal bending stiffness determined by dynamical tests carried out on individual rods. The present preliminary analytical model, which shall be called MSDMP, overestimates the first frequency f_1 (+4.54%) and underestimates the other three (–5.72% for f_2, –4.48% for f_3 and –0.51% for f_4), see Table 5. Considering the adequate accuracy of the mechanical description related to the horizontal beams, columns, beam-column joint and the inertial properties of the whole structure, such variations are quite likely to be due to an inexact description of the brace stiffness and, perhaps, to an incorrect modeling of the structural plate-brace joint. When applying the identification technique elucidated in section 4.1, the starting

Table 5 Cell with brace: Comparison between first four experimental f_{exp} and analytical f_{an} frequencies

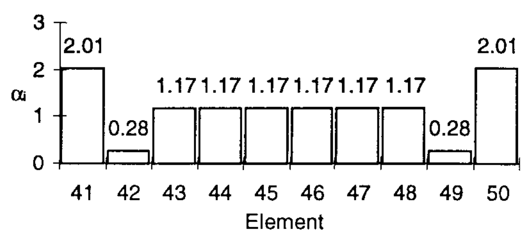
Mode	f_{exp} (Hz)	MSDMP		MSDM1	
		f_{an} (Hz)	$\Delta f\%$	f_{an} (Hz)	$\Delta f\%$
1	167.95	175.58	4.54	170.72	1.65
2	216.68	204.29	−5.72	217.00	0.15
3	345.34	329.87	−4.48	329.78	−4.51
4	417.66	415.51	−0.51	409.75	−1.89

Note: Analytical values are for preliminary model MSDMP and for identified model MSDM1 $\Delta f\%=100\times(f_{an}-f_{exp})/f_{exp}$

reference point has been the preliminary finite element model MSDMP and the first four proper frequencies have served as data. The optimal model, which shall be called MSDM1, has been achieved by taking as parameters the stiffness values both of the two elements of the brace that are contiguous to the joints and of the elements forming the remaining section of the brace. Making use of the system symmetries, the stiffness values to be identified are those related to the elements 41 (=50), 42 (=49) and 43 (=44=45=46=47=48). The results indicate that the inaccuracy of the initial model is caused by the stiffness being respectively underestimated in the brace element located close to the joint, overestimated in the contiguous beam element and underestimated in the central elements; see Fig. 12. What described above suggests the simultaneous presence of a stiffening effect due to the finite size of the joint and a compliance located at the end of the brace that is likely to be ascribed to the peculiarity of the friction joint in question. As a matter of fact, the moment of resistance affecting the joint in between the plate and the brace is smaller than in the column (or horizontal beam)-plate joint, in that the contact section is more limited and the brace consists of one single rod. The MSDM1 model thoroughly accounts for the vibration modes within the field of frequencies in question highlighting errors in the first four frequencies ranging from 0.15% to 5%; see Table 5. As a result, it can be stated that the compatibility of experimental values and analytical ones is good even as regards the vibration modes.



(a)



(b)

Fig. 12 Cell with brace: (a) mesh of the finite element model with element numbers and optimal stiffness multipliers α_i of the brace for identified model MSDM1 (α_i =1 for MSDMP)

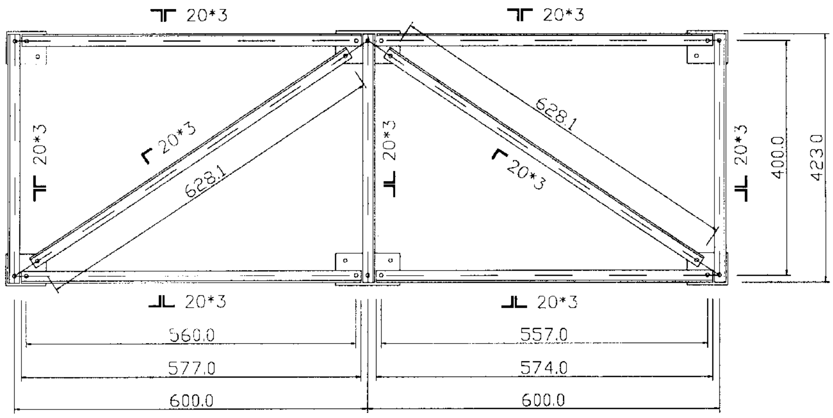


Fig. 13 Experimental model of the truss (lengths in mm)

5. Substructuring identification of a truss

The present section aims at showing how the identified model of the cell can be used for defining an accurate analytical model concerning a complex structure achieved through assembling a number of cells. Such structure is a truss consisting of a double symmetric cell, as illustrated in Fig. 13. The same types of sectionals as in the cell with brace have been employed for horizontal beams, columns and braces.

The dynamic behavior of the truss has been investigated following some dynamical test similar to those formerly described; see Mitri (1997). The results of the experimental modal analysis are reported in Table 6 and in Fig. 14.

The interpretation of the measures has called for a preliminary reference to an analytical model,

Table 6 Experimental frequencies of the truss

Mode Shape	Frequency (Hz)
a	164.18
b	190.81
c	192.58
d	196.95
e	216.78
f	220.71
g	229.70
h	249.54
i	273.50
l	296.47
m	310.11
n	312.05
o	317.95
p	324.06

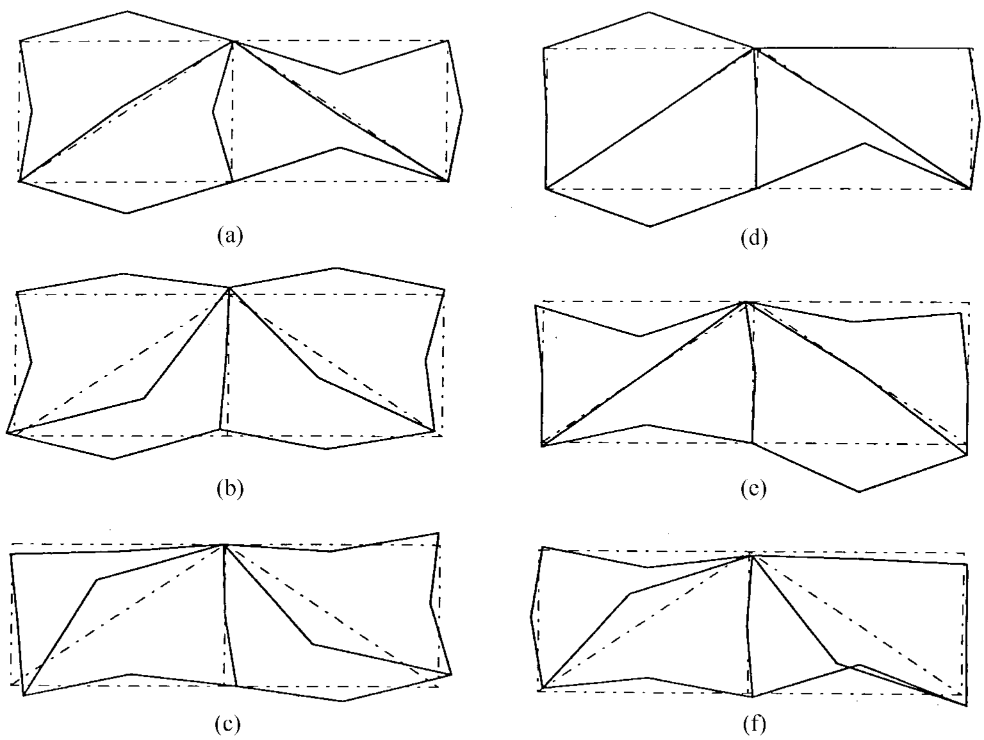


Fig. 14 Experimental normalized mode shapes *a, b, c, d, e, f* of the truss (Dot-dashed line: Reference configuration)

which shall be called DMMP, determined on the ground of the model identified for the single cell with brace. Also in this case the mesh of the discrete model is provided with ten finite elements on every member. The comparison between experimental and analytical frequencies for the first three vibration modes of the system is reported in Table 7. It is to be remarked that the preliminary theoretical model is already fairly accurate and, more specifically, the order set forth via analytical approach is the same as the experimental ones, though the frequencies in the second and third mode are very close. It is worth noticing that the order of the two experimental modes would prove to be the other way round had there been under survey an analytical model with ideal inner joints and nominal stiffness degrees for the members.

In order to further enhance the model in question, the identification technique described above has

Table 7 Truss: Comparison between first three experimental f_{exp} and analytical f_{an} frequencies. Analytical values are for preliminary model DMMP and for identified model DMM1 $\Delta f\%=100\times(f_{an}-f_{exp})/f_{exp}$

Mode	f_{exp} (Hz)	DMMP		DMM1	
		f_{an} (Hz)	$\Delta f\%$	f_{an} (Hz)	$\Delta f\%$
1	164.18	163.92	–0.16	162.85	–0.81
2	190.81	200.92	5.29	182.76	–4.22
3	192.58	207.65	7.83	192.72	0.07

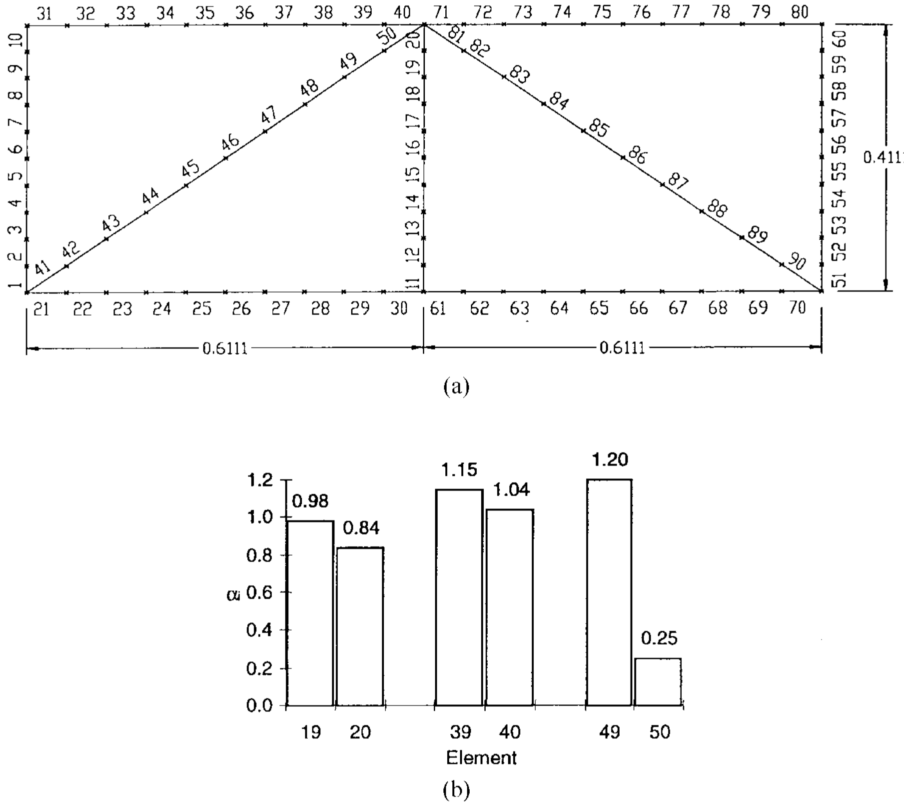


Fig. 15 Truss: (a) mesh of the finite element model with element numbers and (b) optimal stiffness multipliers α_i for identified model DMM1 (α_i =1 for DMMP)

been applied; the DMMP preliminary model has been resorted to as the starting point for the minimization process and the cost function has been obtained by means of the first three proper frequencies. An optimal model, which shall be called DMM1, has been achieved by taking as identification parameters the stiffness both of the two elements of every rod that are contiguous to the joints. Taking advantage of the truss symmetries, the stiffness values to be identified are those of elements 39 (=72=29=62), 40 (=71=30=61), 20 (=11), 19 (=12), 49 (=82) and 50 (=81). The results basically bear out the aptness of the model identified for the individual cell with brace and highlight correction rates approximating the unit as to the stiffness coefficients. According to the identification it is seen fit to introduce one further compliance for the sole joint in between brace and joint; see Fig. 15. The optimal DMM1 model thoroughly accounts for the dynamic behavior of the truss and its frequency errors are marginal indeed.

The analysis carried on in the present section has shown that the substructures identification technique can be used for determining an accurate analytical model of a periodic structure achieved by assembling more structures that are nominally alike. On drawing the conclusions, it is to be reminded that the same procedure has been adopted in Mitri and Morassi (1998) in order to identify a truss that consists of six cells (including the double brace). In spite of the good accuracy of the theoretical model of the substructure, the related analytical model of the full truss was quite rough. Reasons for such disagreement remain unclear and are probably due to a coupling between in-plane and out-of-plane motions, motions that the analytical model is not capable of describing, due to inevitable building eccentricities and assembling imperfections.

6. Conclusions

The paper has been focused both on the capabilities and the indeterminacy characterizing structural identification problems even in quite simple instances as well as on the cautions that should be accordingly adopted. In particular, we have considered a structural identification problem for a steel frame via dynamic data. From the analysis there emerges the importance of the accuracy of the modeling and of the correct choice of the identification parameters in applying a variational approach to structural identification. Moreover, it proved crucial to complete the dynamic investigation with static tests in order to assure a greater reliability of the structural behavior and to reduce the nonuniqueness of the inverse problem. As shown, the result following the analysis may accurate enough to serve practical purposes, but carrying out the analysis requires awareness and careful consideration of the peculiarities of the mechanical problem. Thus, if we may draw a general conclusion, it is that the greatest possible information on the physics of the system ought to be used, in order to overcome the difficulties underlying the identification problem. Furthermore, the application of identification techniques via modal analysis to complex structures is to be regarded, at the state of the knowledge in the field, like an “art” that links up several different disciplines rather than a systematic procedure merely providing information on mechanical systems.

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