

## Structural optimization in practice: Potential applications of genetic algorithms

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**Abstract.** With increasing competition, the engineering industry is in need of optimization of designs that would lead to minimum cost or weight. Recent developments in Genetic Algorithms (GAs) makes it possible to model and obtain optimal solutions in structural design that can be put to use in industry. The main objective of this paper is to illustrate typical applications of GAs to practical design of structural systems such as steel trusses, towers, bridges, reinforced concrete frames, bridge decks, shells and layout planning of buildings. Hence, instead of details of GA process, which can be found in the reported literature, attention is focussed on the description of the various applications and the practical aspects that are considered in Genetic Modeling. The paper highlights scope and future directions for wider applications of GA based methodologies for optimal design in practice.

**Key words:** optimization; structures; genetic algorithms; steel trusses; reinforced concrete frames; bridge structures; multi-storey buildings; shells; structural reliability; layout planning; knowledge modules.

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### 1. Introduction

Optimal design methods assist engineers to evolve the best possible design in terms of cost, weight, reliability or a combination of these parameters. Problems of optimization can be formulated as non-linear programming problems and mathematical programming techniques were used to solve these problems (Schmit 1960). Structural optimization using mathematical programming was prohibitively expensive in the early stages of its development and hence applications to practical problems were limited in scope. Recent advances in computer hardware have encouraged researchers to give a new thrust to structural optimization as can be seen by the large number of publications especially in the last ten years (Kamat 1993). However, practical problems which involve consideration of discrete sizes, configuration and topology optimization of structures are quite complex and mathematical programming techniques offer only limited scope for obtaining satisfactory solution (Betts 1985). The focus of recent research has been to adapt the development in evolutionary computation for realistic formulation providing solutions for large practical problems (Goldberg and Samtani 1986, Jenkins 1991).

Evolutionary computation is a common term accepted in all computational methods that model natural phenomena: genetic inheritance and Darwinian theory of survival of the fittest (Foget 1995). Genetic algorithms are search algorithms based on the principles of natural evolution (Goldberg 1989). Advantages of genetic algorithms are the parallel approach to search space and their broad

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applicability. Research in the last seven years has shown that it can be successfully used in structural optimization for size and shape optimization with consideration of constructibility aspects.

The purpose of this paper is to show the potential of genetic algorithms for optimal design of structural systems. With this objective, illustrative examples are drawn from different types of structures: trusses and towers, bridges, reinforced concrete frames, shells and buildings. In all these examples, attention is focussed on the design variables and practical aspects considered in the GA modeling so that they can be adopted in practical design of these types of structures. Additional and future research needs are brought out at the end of the paper for wider application of GA-based optimization in practice.

## 2. Optimal design of trusses and towers

Design optimization of trusses can be carried out at three different levels (Jenkins 1991, Adeli and Cheng 1993, Rajeev 1993). In the first, topology and configuration are kept constant and only member sizes (cross sectional properties) are varied (Size Optimization). In second, positions of joints are also varied along with the member sizes (Configuration Optimization). In the third level, all the three classes of variables are considered, viz., topology, configuration and member sizes (Topology Optimization). Genetic Algorithms-based methodologies are successfully used for obtaining optimal design solutions simultaneously considering topology, configuration and discrete values for cross-sectional parameters in a unified manner. A brief description of the formulations and typical examples are given in the following sub-sections. More details can be obtained from Rajeev and Krishnamoorthy (1992, 1997).

### 2.1 Size optimization

General formulation of size optimization problems for truss structures with discrete member sizes is given below:

$$\text{Min } f(x) = \sum_{i=1}^{\text{members}} A_i L_i \rho \quad (1)$$

subject to

$$u_j^L \leq u_j \leq u_j^U \quad j=1, \dots, n_d \quad (2)$$

$$\sigma_i \leq \sigma_{all} \quad i=1 \text{ to number of members} \quad (3)$$

$$\begin{aligned} A_i^L &\leq A_i \leq A_i^U \\ A_i &\in S \end{aligned} \quad (4)$$

where  $A_i$  and  $L_i$  are respectively the area and length of member  $i$ ,  $\rho$  is the weight density of material and  $u_j$  is the displacement of  $j$ th degree of freedom,  $n_d$  is the number of degrees of freedom where displacements are constrained,  $\sigma_i$  is the stress in member  $i$  and  $S$  is a set containing discrete values to be assigned to size variables. The superscripts  $L$  and  $U$  in the above equations refer to lower and upper bound values, respectively.

## 2.2 Configuration optimization

In addition to size variables, configuration variables are also added as design variables. The following two additional constraints are added to the set of Eqs. (1)-(4) to define the optimization problem:

$$\sigma_i \leq \sigma_{ik} \quad (5)$$

$$x_q^L \leq x_q \leq x_q^U \quad q=1, \dots, n_c \quad (6)$$

where  $\sigma_{ik}$  is the Euler buckling compressive stress for member  $i$ ,  $x_q$  is the coordinate variable, and  $n_c$  is the number of coordinates which are allowed to change.

Euler buckling compressive limit is computed as,

$$\sigma_{ik} = \frac{-K_i E_i A_i}{L_i^2} \quad (7)$$

where  $K_i$  is the buckling coefficient of member  $i$ , which depends on the shape of cross section and end conditions and  $E_i$  is the Young's modulus of member  $i$ .

One important difficulty in practical applications is to fix the lower bound values  $A_i^L$  of cross sectional members. A two-phase method has been proposed (Rajeev and Krishnamoorthy 1997) to reduce the size of the search space by automatically arriving at lower bound values for design variables, which results in considerable improvement in the efficiency of the optimization process.

In all the problems, the constrained optimization problem is solved as an unconstrained problem using penalty function method. Accordingly the fitness function for GA optimization is calculated as,

$$F = f(x)(1 + C) \quad (8)$$

where the constraint violation coefficient is defined as,

$$C = \sum g_i \quad \text{for } g_i > 0 \quad (9)$$

where  $g_i$  represents constraints that are expressed in normalised form, for example,

$$g_i = \frac{\sigma_i}{\sigma_{all}} - 1 \leq 0 \quad (10)$$

## 2.3 Topology optimization

It is a known fact that modification of topology and position or layout of structural members can considerably improve the design. There are very few methods reported in the literature, which can address problems with topology variation during the process of optimization. GA based methodology has been shown to be effective for obtaining optimal design solution of framed structures, by varying simultaneously topology, joint positions and member cross sectional properties (Rajeev 1993, Ohsaki 1995, Rajan 1995).

In topology optimization, the number of members and joints defining the truss vary; implying that the number of design variables itself is a variable. Hence, the binary strings (genes) representing trusses with different topology will have different lengths and Simple Genetic Algorithms (SGA) cannot handle such cases. A Variable-length Genetic Algorithm (VGA) has been proposed to handle

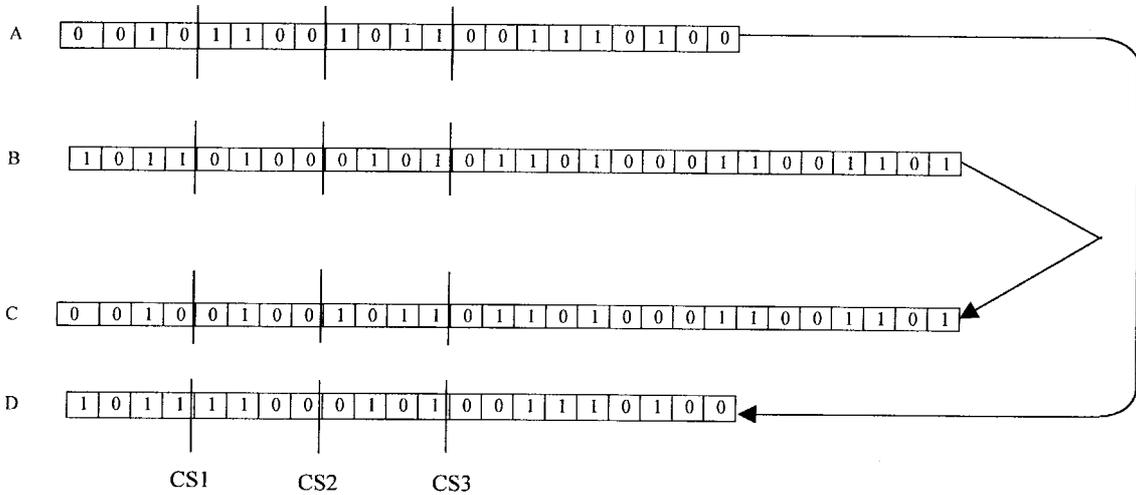


Fig. 1 Crossover and cut and splice operation in VGA

such situations.

Let A and B be two strings selected for recombination and their lengths are different as they may represent two different topologies. Crossover sites are selected randomly as shown in Fig. 1. The strings in between the crossover sites are interchanged as in SGA and the strings to the right side of the crossover site are cut from the strings and are spliced to form offspring C and D as shown in Fig.1. The GA process is then continued using two phase algorithms with adaptive move limits until convergence is reached.

### 2.4 Optimal design of microwave antenna tower

In order to illustrate the application of GA to solve large practical problems, the design of a 41m height microwave antenna tower has been chosen. A change in topology of the tower is due to a difference in the number of panels and also due to different bracing patterns adopted in different panels.

The variables that affect the design of the tower are: (i) number of panels; (ii) top width of panel; (iii) height of each panel; (iv) type of bracing in each panel; (v) number of subpanels; (vi) secondary bracing configuration; (vii) cross sectional properties of members.

A typical individual string consists of substrings representing size variable, configuration variables and topology variables. The composition of a sub-string representing a panel is shown in Fig. 2 and explained in Table 1.

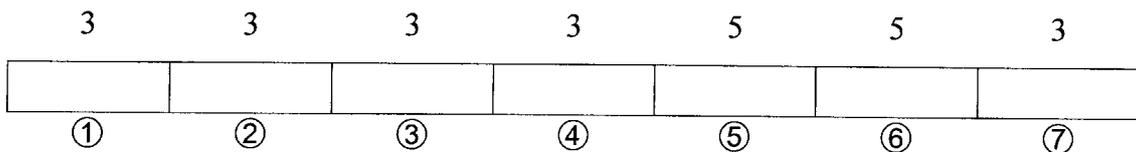


Fig. 2 Composition of individual string for each panel - Microwave antenna tower

Table 1 Composition of individual string panel-Microwave antenna tower

Ref No. (Fig 2)	Design variable	Substring length
	Size of leg member	
1	Size of leg member	3
2	Size of diagonal member	3
3	Size of horizontal member	3
4	Size of horizontal bracing member	3
5	$x$ -coordinate of panel top joint	5
6	$y$ -coordinate of panel top joint	5
7	type of bracing, number of subpanels and type of secondary bracing	4

The seventh variable shown in Table 2 represents the bracing type, number of subpanels and type of secondary bracing. Four characters are assigned to this variable, which results in sixteen different variations. Sixteen different possibilities of X-bracing with and without secondary members are considered for this variable and are shown in Table 2. Few typical X and K braced panels considered are shown in Fig. 3.

From Table 1, it can be seen that the individual string representing the genetic model of each panel has 26 characters. Thus, depending on the number of panels considered lengths of individual strings representing different topology can be computed. For example, the tower with 6 panels will be represented by an individual string of length 156. The control variable for the present problem is the number of main panels in the tower, as this decides the individual string length. It may be noted that, in the present genetic modeling scheme used, strings with the same length can represent different topologies. This is due to the coding scheme adopted for the bracing patterns. This type of genetic modeling with combination of variations in bracing patterns and number of panels allows

Table 2 Decoding scheme for type of bracing in panels Microwave antenna tower

Decimal Equivalent	Pattern considered
0	X bracing with 3 subpanels (no secondary bracing)
1	X bracing with 4 subpanels (no secondary bracing)
2	X bracing with 5 subpanels (no secondary bracing)
3	X bracing with 6 subpanels (no secondary bracing)
4	X bracing with 3 subpanels (with secondary bracing)
5	X bracing with 4 subpanels (with secondary bracing)
6	X bracing with 5 subpanels (with secondary bracing)
7	X bracing with 6 subpanels (with secondary bracing)
8	K bracings with 2 sub divisions
9	K bracings with 3 sub divisions
10	K bracings with 4 sub divisions
11	K bracings with 5 sub divisions
12	K bracings with 6 sub divisions
13	K bracings with 7 sub divisions
14	K bracings with 8 sub divisions
15	K bracings with 9 sub divisions

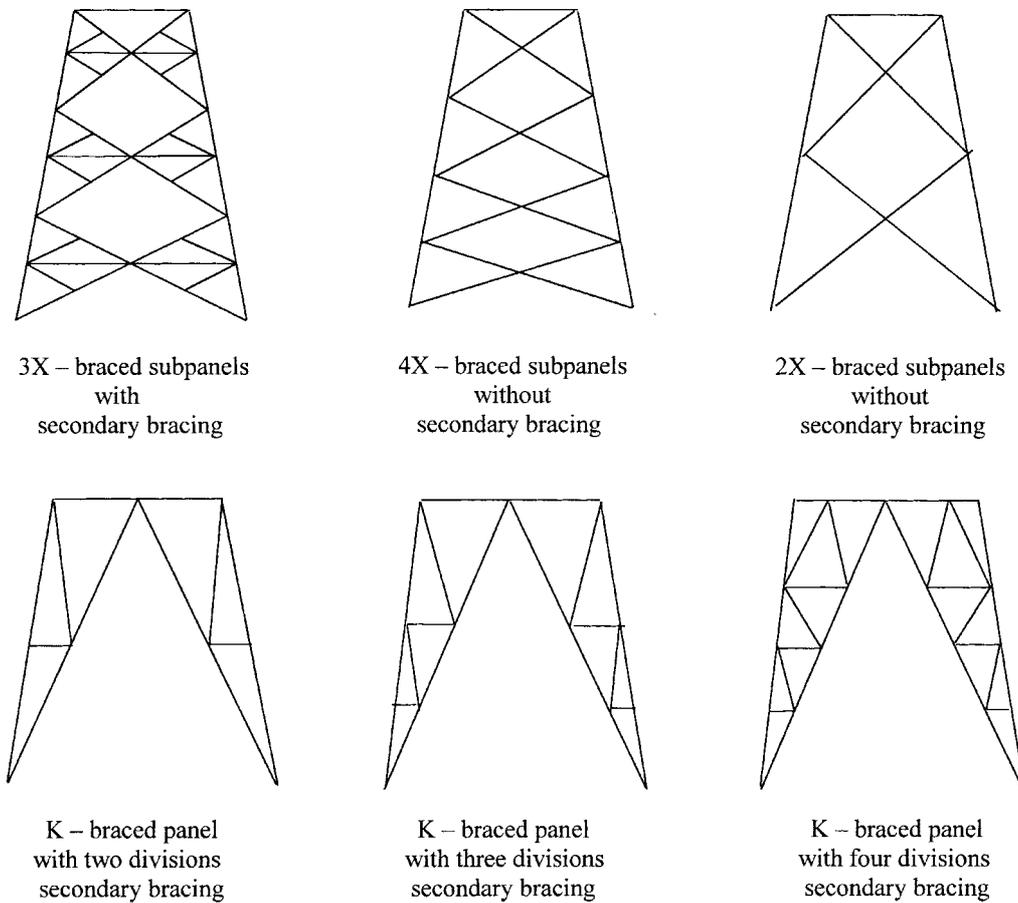


Fig. 3 Typical bracing patterns considered - Microwave antenna tower

one to consider many different possible topologies for the tower.

In the present exercise the control variable, i.e., the number of panels is allowed to take one value between 5 and 8. The tower with 5 panels is represented by a string length 130 and one with 8 panels is represented by a string of length 208.

The generation history during the course of optimization is presented in Fig. 4. The optimal solution obtained has a weight of 48.88 N which is 5.5% lower than that obtained from fixed topology-fixed geometry optimization and 4.1% lower than that obtained from configuration optimization. It is seen that the savings obtained in configuration optimization is not much compared to that obtained in discrete size optimization. But a considerable saving in weight is achieved when the topology of the tower is modified. This shows the influence of topology on weight of the optimal design solution.

One of the significant aspects of GA is that it is possible to obtain solutions of multi modal optimization and it is of practical importance in engineering design. The sharing operations illustrated by Goldberg and Richardson (1987) has been used and the solutions obtained for the microwave antenna tower are shown in Fig.5 and the results are summarised in Table 3.

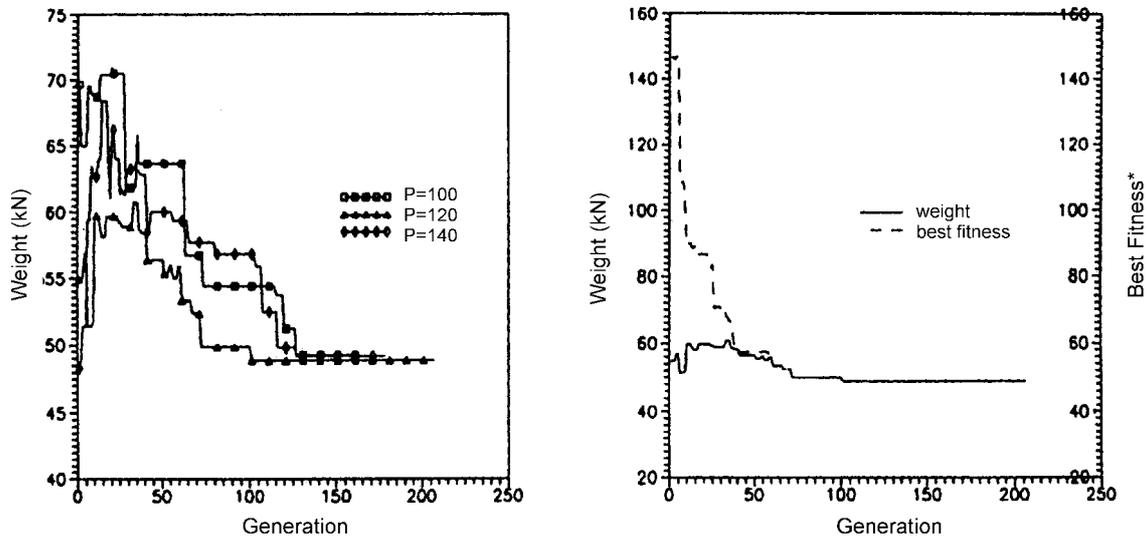


Fig. 4 Generation history - microwave antenna tower

Table 3 Topology details of different solutions for microwave tower

Solution	Bracing Patterns							Topology		Weight (kN)
	P1	P2	P3	P4	P5	P6	P7	joints	members	
1	K	K	X2	X2	X3	X4	X5	96	300	48.86
2	K	K	K	X2	X3	X4	X5	84	264	48.98
3	K	K	X2	X3	X4	X4	X5	92	288	49.33

K-K bracing; X3-X bracing with 3 subpanels; P1-Panel 1 from bottom

A two-stage GA-based design optimization has been carried out by Manoj (1996) for optimization of transmission line towers using cold-formed sections. The optimal topology has been obtained using the methodology discussed in this section. Subsequently, the member level optimization has been carried out to obtain dimensions of cold-formed sections as these sections can be manufactured to any practically constructible shape.

### 3. Bridge structures

There are two types of bridge structures taken up for illustration to show the application of GA to solve problems of practical interest. First we consider a steel railway bridge for a span of 124.4 m and a typical configuration is shown in Fig. 6(a).

The objective of the design problem is to arrive at appropriate cross sectional parameters, configuration and topology for the bridge so that its weight is a minimum. All the members are built-up sections using plates of different thicknesses. These plates used for making the members are manufactured with a length of 12 m. This constraint has implicitly fixed the number of panels in the superstructure as 11 including two half panels at the end as shown in Fig. 5. Different

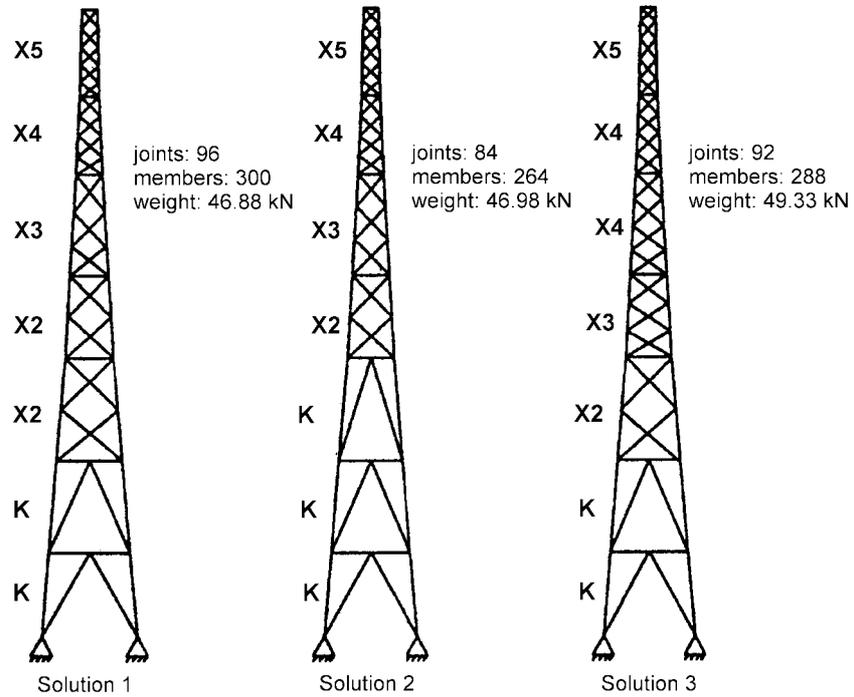


Fig. 5 Configuration of multiple solutions obtained - Microwave antenna tower

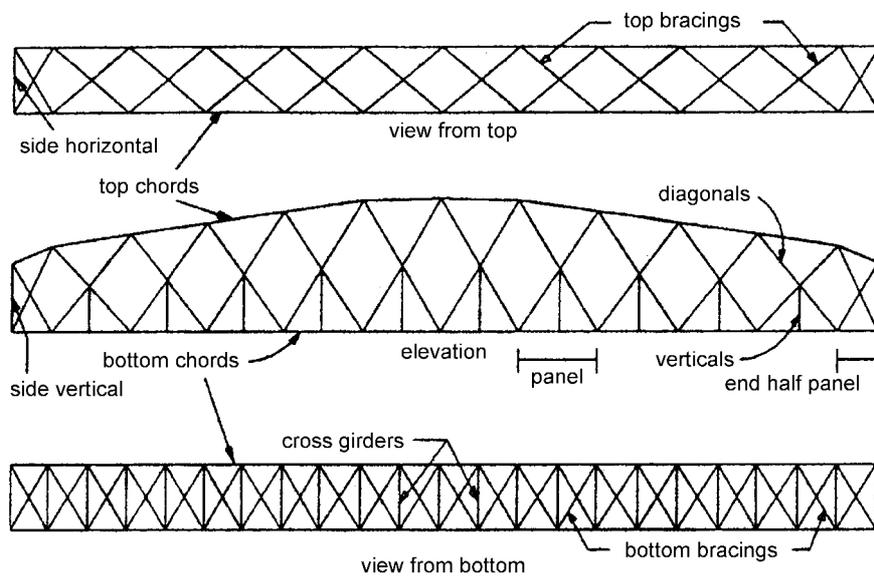


Fig. 6(a) Typical configuration of bridge superstructure

topologies are obtained by varying the number of panels and using different bracing patterns as shown in Fig. 6(b).

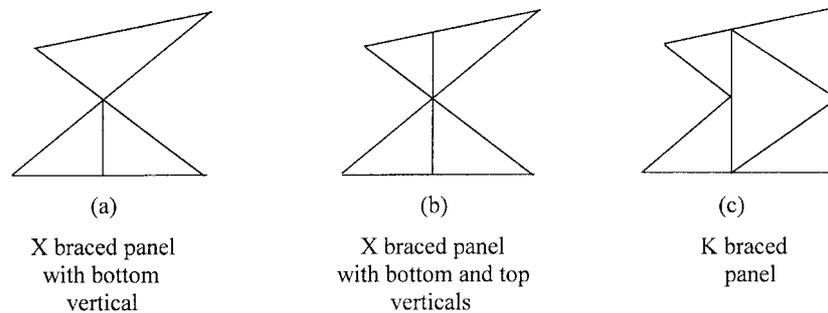


Fig. 6(b) Different bracing patterns considered for bridge structure

### 3.1 Genetic modelling

Genetic modeling consists of defining an individual string to represent a design and a corresponding decoding scheme. For this, members of the structure are to be grouped first. Eleven different types of members are identified as:

(1) top chords; (2) bottom chords; (3) diagonals (main bracing members); (4) bottom vertical members in panel; (5) top vertical members in panel; (6) vertical and top horizontal members at two ends; (7) cross girders; (8) bottom bracing (bracing the bottom chords); (9) top bracing (bracing the top chords); (10) sway bracing (connecting the diagonals); (11) side bracing (at two ends).

As it is not economical to have the same cross sectional parameters for all members belonging to the same type, those members falling in each panel are considered as separate groups. Thus there are six top chord groups, six bottom chord groups, six diagonal groups and six vertical member groups. But the shape of the member is one group for each type of bracing members, cross girders and side members. Thus 32 different member groups are identified.

### 3.2 Optimal solution

The bridge structure is analysed as a three dimensional frame for constraint evaluation. The following load combinations are considered:

- dead load + live load in full span
- dead load + six cases of partial live load
- dead load + live load in full span + wind load
- dead load + six cases of partial live load + wind load

The optimal solution obtained is shown in Fig. 7.

### 3.3 Reinforced concrete bridge decks

GAs have been used successfully to optimize the cost of concrete bridge decks. A typical example of T-beam bridge (Fig. 8) that is commonly used in the span range of 10 to 25 m is discussed very briefly here.

The variables selected for optimization are: (1) The depth of slab; (2) The depth of the T-beam; (3) The web thickness of the girder; (4) The number of cross beams; (5) The number of longitudinal girders; (6) The type of steel used for slab and beam; (7) The type of concrete used;

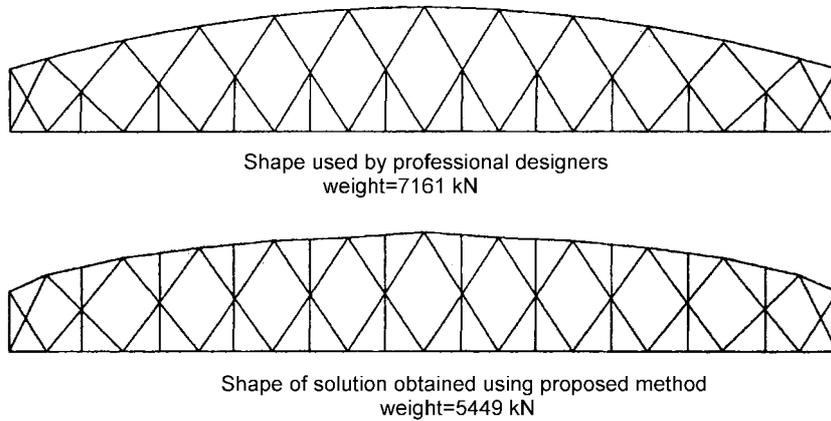


Fig. 7 Optimal shape of bridge superstructure

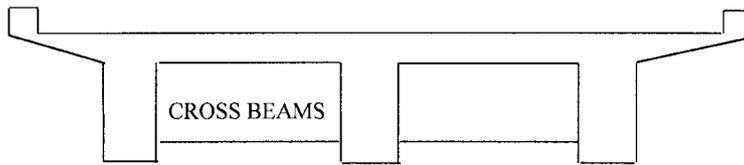


Fig. 8 Typical cross section of a T-beam type bridge

(8) The type of steel used for shear reinforcement; (9) The type of shear reinforcement (two legged stirrup or four legged stirrup); (10) Type of shear steel reinforcement; (11) Diameter of shear steel reinforcement; (12) Diameter of main steel reinforcement; (13) Diameter of distribution reinforcement.

The objective function represents the cost of the bridge including cost of concrete, steel and

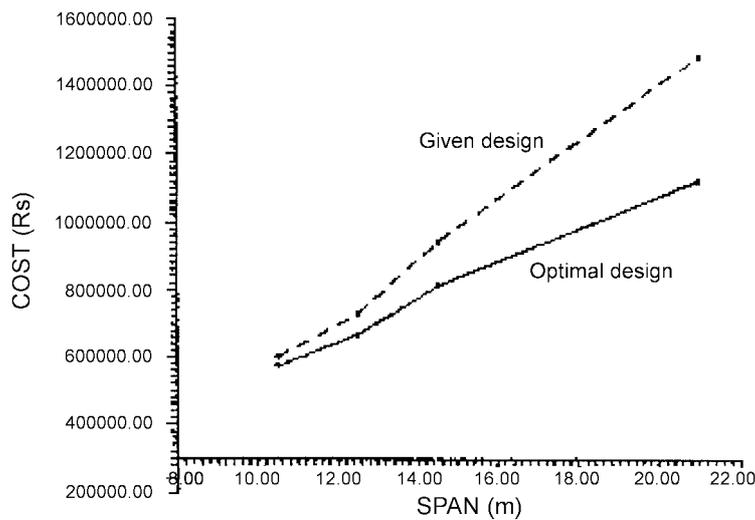


Fig. 9 Plot of span versus cost for a T-beam

formwork. The design conditions and practical considerations are specified as constraints to the problem. To appreciate the advantage of optimization in design, the plot of cost of the bridge versus span in given in Fig. 9.

#### 4. Concrete framed structures

The problem of design optimization of concrete framed structures is more complex compared to steel structures (Huanchan and Zheng 1993). GA-based methodology provides a viable tool to handle the difficult and practical problem of reinforcement detailing.

Members of frames are first grouped into beams and columns. Then different beam and column groups are identified based on same cross-sectional properties, member loading, etc. The design variables for columns are: breadth and depth of cross section, and area of reinforcement. In the case of beams, the area of reinforcement is not kept the same throughout the beam member. Hence, three typical sections, viz., one near the left support, one at mid span and one near the right support are considered in the optimal design formulation. Thus seven design variables are considered for each span of a beam and they are: breadth and depth of cross section, area of top continuous bars, area of bottom continuous bars, area of top additional bars at left support, area of bottom additional bars at mid span and area of top additional bars at right support.

Objective function and constraints: The objective function for beam and column groups are specified considering cost of concrete, steel and framework. The constraints for columns and beams include satisfaction of design specifications of code of practice and practical consideration. The details of genetic modelling are given in Rajeev and Krishnamoorthy (1998).

##### 4.1 Illustrative example

Fig. 10 shows the geometry and loading of a two-bay two storey frame. Member grouping is done

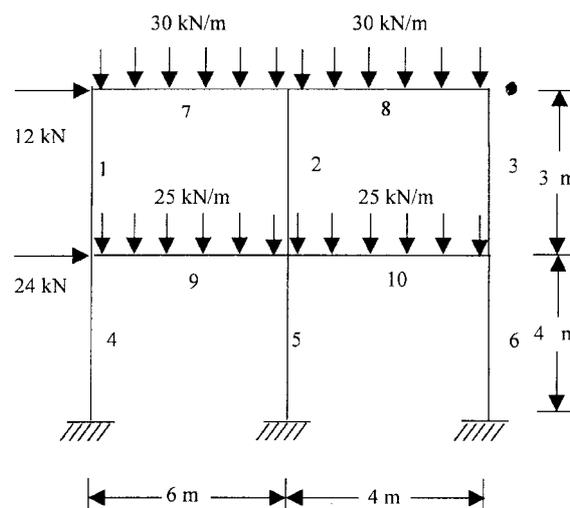
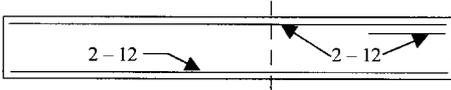
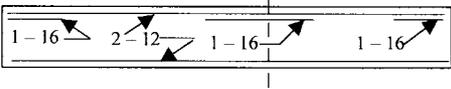


Fig. 10 Geometry and loading: Two-bay two-storey frame

Table 4 Results – Two-bay two-storey frame

group	breadth (mm)	depth (mm)	$A_s$ (1) (mm <sup>2</sup> )
1	200	200	4 × 12
2	200	200	6 × 12
3	200	250	4 × 16
4	250	350	
5	200	450	

such that columns 1 and 4 are in group 1, columns 2 and 5 in group 2 and columns 3 and 6 in group 3. Beams 7 and 8 form group 4 while beams 9 and 10 form group 5. This grouping results in an individual string length of 80. The results of the optimal design are given in Table 4.

## 5. Long span reinforced concrete roofs

In many of the public and industrial buildings, and auditoriums, the major concern is to have roofing with large unobstructed area free of columns. Reinforced concrete shells and folded plates are used for long span roofs. GA-based methodology has been used for optimal design of simply supported circular cylindrical shells of various configuration such as single barrel shells and V-type folded plates (Latheswari 1994). A brief description of one of the examples is given below. For the circular cylindrical shell shown in Fig. 11, the design variables are: thickness of the shell, semi central angle, width of the edge beam and depth of the edge beam.

The reinforcement provided for the shell can be divided into 3 groups:

- (i) Longitudinal reinforcement to resist the longitudinal force,
- (ii) Transverse reinforcement to resist the transverse moment, and

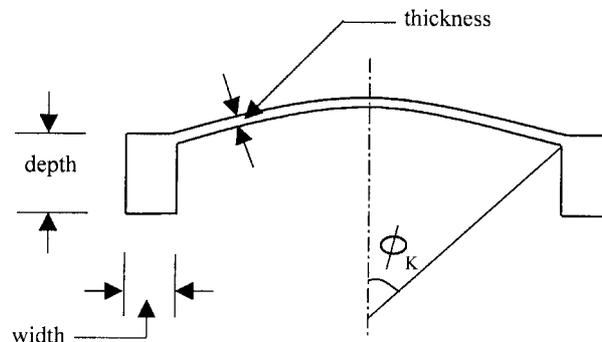


Fig. 11 Design variables for a single shell

(iii) Diagonal tension reinforcement to resist the combined effect of longitudinal force, transverse force and shear force.

The edge beam is designed to resist the longitudinal force, bending moment and shear.

The objective function specifies the cost of the structure that includes costs of concrete, steel and framework. The constraints are imposed to ensure safe and satisfactory performance without violating any code provisions. The compressive stress at any point in the shell should not exceed the permissible compressive stress in concrete, expressed as,

$$\frac{\sigma}{\sigma_{cbc}} - 1.0 \leq 0 \quad (11)$$

where  $\sigma$  is the maximum compressive stress calculated at the crown due to longitudinal force and  $\sigma_{cbc}$  is the permissible bending stress in concrete. The buckling stress at any point in the shell should not exceed the permissible buckling stress, expressed as,

$$\frac{f}{f_{ac}} - 1.0 \leq 0 \quad (12)$$

where  $f$  is the maximum buckling stress calculated and  $f_{ac}$  is the allowable buckling stress in cylindrical shells. Code provisions are used to calculate  $f_{ac}$ .

In addition to the above constraints, there are four side constraints specifying the bounds on the variables, which are taken care of in the genetic coding. The genetic model has a string length of 16. Design tables are generated using GA-based optimization giving optimal values of cross section and the quantity of reinforcing steel required for various spans and chord widths (Latheswari 1994).

## 6. Planning and design of buildings: Multi-criteria optimization

Layout planning is one of the most difficult problems in architectural design. The decisions to be made during the layout planning are:

- (a) shape of the building and its dimensions in plan,
- (b) allocation of major functional spaces,
- (c) the number of floors and floor height,
- (d) the number of service cores and their location in building plan and
- (e) the configuration of various service units inside the core(s).

The layout planning of multistorey buildings is further complicated by the numerous design alternatives and the decision taken at this stage in turn affects the selection of the structural system, usability, functional performance (natural lighting, thermal and acoustic performance), construction and operating costs and thus the overall economy. Thus the involvement of experts representing different domains is most intensive in the layout planning stage. The complexity increases when objectives are conflicting in nature, which is true in most of the practical cases and hence requires to be treated as an optimization problem. The process of decision making based on an isolated design objective while ignoring the rest has very limited usefulness. Hence it is essentially a multi-objective or multi-criteria optimization problem (Grierson and Park 1998).

In recent research by Mashood (2000), the layout optimization problem is formulated as a multi-objective optimization problem, having three different criteria, viz.,

- (i) maximize the rental area in the building

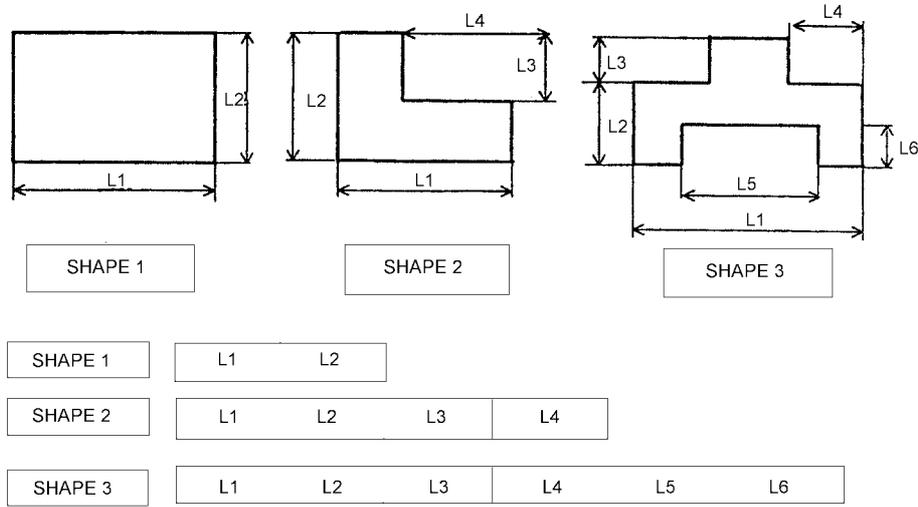


Fig. 12 Shape and variable representation in genetic modeling

(ii) maximize the grade of circulation by means of a compact and efficient vertical transportation system and good circulation, and

(iii) minimize the heat gain on the building, aiming at (a) improved comfort in the non air-conditioned buildings and (b) reduced cooling cost in air-conditioned buildings.

Genetic algorithms have been recognized to be well suited for multi-objective optimization. The ability to handle complex problems involving features such as discontinuities, multi-modality and noisy function evaluation, reinforces the potential effectiveness of genetic algorithms. Fig. 12 shows a schematic representation of design variables and their string representation for typical shapes of layouts.

Since the number of variables differs, some control information is required to decode an individual string from genetic space to parametric space. Here, the shape of the layout is considered as the control variable since it determines the number of variables and hence the string length. Each shape is identified by a unique number representing a member of the discrete set of shapes provided.

### 6.1 Multi-objective genetic algorithm

Mathematically, the multi-objective optimization problem can be represented as:

$$\begin{aligned} &\text{Minimize/Maximize} && f_i(\mathbf{x}); i=1, 2, \dots, N && (13) \\ &\text{Subject to} && && \end{aligned}$$

$$g_j(\mathbf{x}) \leq 0; j=1, 2, \dots, m \tag{14}$$

$$h_k(\mathbf{x})=0; k=1, 2, \dots, l \tag{15}$$

The parameter  $\mathbf{x}$  is a p-dimensional vector having p design variables,  $g_j(\mathbf{x})$  and  $h_k(\mathbf{x})$  represents the set of inequality and equality constraints respectively. The vector  $f_i(\mathbf{x})$  represents an N-dimensional hyperspace of objective functions. Every point in the design variable space represents a solution and gives a certain point in the objective function space. The position of this point is determined by the

values of the objective functions and depicts the quality of the solution.

Generally, in single objective optimization, it is aimed at determining a particular set  $x^*$ , from the feasible set of solutions, which yields the optimum value of the objective function. However, in a typical multi-objective optimization problem, there exists a set of solutions that are superior to the rest of the solutions in the search space when all objectives are considered, but are inferior to the other solutions in space in one or more objectives. These solutions are known as *Pareto-optimal* solutions or *non-dominated* solutions (Hans 1988). The rest of the solutions are known as *dominated* solutions.

Pareto optimality concept has been used in conjunction with GA and a procedure known as Non-Dominated Sorting GA (NSGA) has been used to solve the multi-objective optimization problem (Srinivas and Deb 1994). A software GALOP has been developed and its essential feature is that it provides a hybrid framework integrating knowledge modules (KMs) which contain the knowledge base and processes related to Local Authority Regulation, Functional space estimation, Layout shapes, Thermal performance evaluator, Elevator module and the GA based optimizer. The hybrid approach provides an efficient model for interaction of KMs at well-defined levels with the optimizer while (i) defining the feasible solution space; (ii) constraint generation, and (iii) fitness evaluation.

In order to highlight how a practical problem is solved using GA, the following example is briefly described here. For more details, reference can be found in Mashood (2000).

Fig. 13 shows the layout of an existing building. The data for this building are given in Table 5.

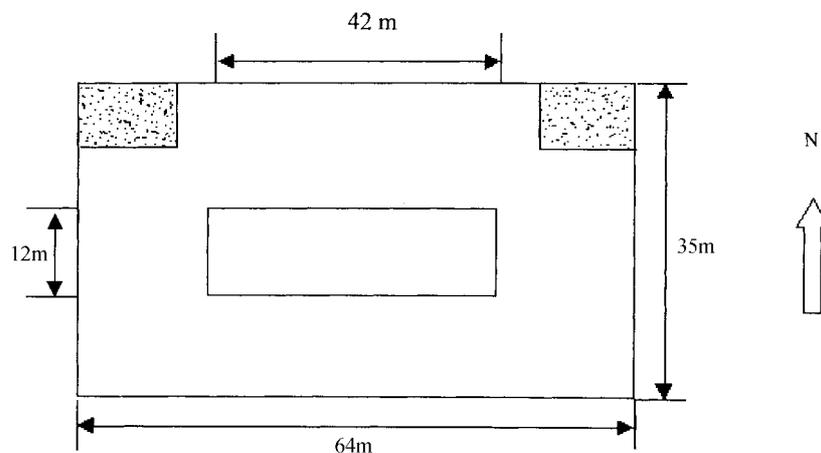


Fig. 13 Layout details of existing building – Example II (SHAPE 1)

Table 5 Input details – Example II

Office area required	12,800 sq.m
Possible orientations	ALL
Maximum no. of storeys	13
Location	Chennai
Core type	Single
Shapes	12 Shapes (Fig. 14)

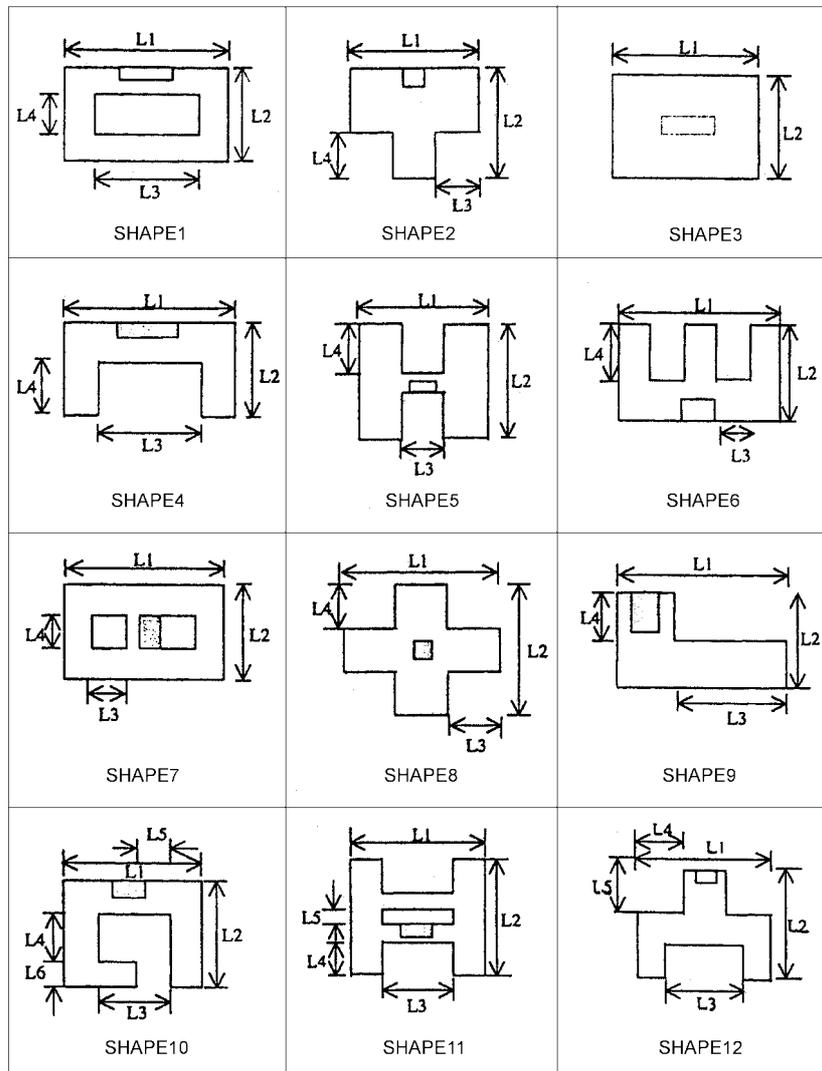


Fig. 14 Layout shapes considered for optimization

In order to restrict the population size and computing time, only twelve different shapes as shown in Fig. 14 are considered in the GA process. However, each shape can have different variations with respect to positioning of service cores.

It has been found that the solutions obtained by GA for the three objective functions, viz., grade of service, service area and heat gain are better than that of the existing building (Mashood 2000).

## 7. Reliability based structural optimization

Even though the partial safety factors recommended by various codes can be used in deterministic design optimization to take care of the probabilistic nature of loads and resistance parameters, the

resulting designs may not be optimal. The need for reliability based optimization is emphasized by Moses (1997).

The RBSO problem can be formulated in the most general form as:

$$\text{Minimize the total cost } C = C_i + P_f C_f \quad (16)$$

where  $C_i$  is initial cost which may be approximated in the case of pin jointed structures as product of unit cost factor,  $K$  and structural weight,  $W$  (i.e.  $C_i = KW$ ),  $P_f$  is probability of failure of the structure, and  $C_f$  is cost of structural failure.

In reality, it is very difficult to assess the actual cost of structural failure accurately, since various possible damage/accident scenarios including loss of human lives have to be evaluated. Hence, instead of a multi-criteria optimization, target reliability approach is adopted. Hence the problem is stated as:

Minimize the structural weight:

$$\text{Subjected to} \quad w = \sum_{i=1}^n \rho A_i L_i \quad (17)$$

$$\beta_{sys} \geq \beta_{sys\_target}$$

$$\beta_i \geq \beta_{i\_target} \quad \text{for } i=1, 2, \dots, n \text{ (if specified)}$$

$$\beta_j \geq \beta_{\Delta j\_target} \quad \text{for } j=1, 2, \dots, m \text{ (if specified)} \quad (18)$$

and

where,

- $\rho$  : weight density
- $A_i$  : area of cross section of  $i$ th member
- $L_i$  : length of  $i$ th member and is expressed as a function of joint coordinates
- $\beta_i$  : Hasofer and Lind reliability index of  $i$ th member
- $\beta_{\Delta j}$  : Hasofer and Lind reliability index pertaining to displacement limit at  $j$ th node
- $\beta_{sys}$  : reliability index for the system
- $\beta_{sys\_target}, \beta_{i\_target}, \beta_{\Delta j\_target}$  : corresponding target values for reliability indices
- $n$  : number of members present
- $m$  : number of nodes at which displacement limits are imposed

The design variable set consists of member cross sectional areas and the coordinates of those joints (hereafter called 'moving joints') which are allowed to change their positions to alter the configuration of the truss. The variables representing member cross sectional areas can take values from a discrete set of values in the database. The coordinates of all the moving joints are treated as continuous variables. They are allowed to vary within specified ranges at specified step sizes.

The solution vector space is mapped to the genetic space by binary coding of design variables. In the case of variables representing the discrete member sizes, their indices pertaining to their relative position in the database are used for mapping (Thampan *et al.* 1998).

The Fitness Function:

$$F=w\left(1.0+K_1\sum_{i=1}^ng_i+K_2\sum_{j=1}^mh_j+K_3s\right) \quad (19)$$

where

$$\begin{aligned} g_i &= 1 - \frac{\beta_i}{\beta_{i-target}} \text{ if } \beta_{i-target} \text{ is specified, and } \beta_i < \beta_{i-target} \\ &\text{else } g_i = 0.0 \quad \text{for } i=1, 2, \dots, n \\ h_j &= 1 - \frac{\beta_{\Delta j}}{\beta_{\Delta j-target}} \text{ if } \beta_{\Delta j-target} \text{ is specified, and } \beta_{\Delta j} < \beta_{\Delta j-target} \\ &\text{else } h_j = 0.0 \quad \text{for } j=1, 2, \dots, m \\ s &= 1 - \frac{\beta_{sys}}{\beta_{sys-target}} \text{ if } \beta_{sys} \text{ is specified, and } \beta_{sys} < \beta_{sys-target} \\ &\text{else } = 0.0 \end{aligned} \quad (20)$$

$K_1, K_2, K_3$  are penalty factors whose values are problem dependent and  $\geq 1.0$ .

The member strength values and applied loads are random variables of known probability distributions. The Hasofer and Lind reliability indices for members are calculated using Advanced First order Second Moment (AFOSM) method with Hasofer and Lind – (HL – RF) Rackwitz-Fiessler algorithm. The system level reliability assessment is performed with a modified branch-and-bound algorithm based on failure mode approach. The GA based formulation requires the exact value of probability of system failure for the evaluation of fitness function, only when a violation of the corresponding constraint is present. The bounds on probability of failure can be used to detect violations.

It has been established that GA-based methodology is well suited for RBSO and important structures can be optimized considering constraints pertaining to total or partial collapse, functional requirements, residual reliability with respect to expected types of damages and member failures. The main advantage of RBSO is that the designer has the flexibility of specifying the target reliability indices pertaining to different types of failures and obtain optimal solutions.

## 8. Future directions

In this paper, we have demonstrated the scope and potential of GA for structural optimization in practice. There is wide scope to carry out research and development and major areas are highlighted below:

1. Integration with FEA packages for industrial application: A stage has been reached now when GA-based methodologies can be integrated with FEA packages and special purpose optimization modules can be developed for optimization of different types of structural systems, machine components etc.

2. Application of Multi-objective optimization to practical systems: Engineering design is a multi-disciplinary activity. The major thrust of future research will be on development of GA-based methodologies for multi-criteria optimization to satisfy the objectives from different disciplines. This

development will help to build integrated systems with multi-criteria optimization processor and knowledge-bases leading to a collaborative engineering design environment.

3. Reliability-based optimization: With the development of new materials and also application of optimization techniques, there is growing concern on the reliability of the systems we build. Reliability-based design needs further research for development of computationally efficient algorithms for reliability analysis in optimization environment.

4. High performance computing for applications: All the above developments would require high performance computing. The inherent parallelism in GA process is an added advantage for its implementation on distributed computing environment leading to cost effective solutions of practical problems.

5. Innovations in engineering design: Genetic algorithm is a part of evolutionary computation and multiple solutions are evolved in the stochastic search process. More work is needed in developing a hybrid framework integrating GA processes with Artificial Intelligence and expert systems, and Artificial Neural Networks that would lead to innovations in engineering design.

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