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Analysis of building frames with viscoelastic dampers under base excitation

A.K. Shukla† and T.K. Datta‡

Civil Engineering Department, Indian Institute of Technology, New Delhi 110 016, India

Abstract. A frequency domain response analysis is presented for building frames passively controlled by viscoelastic dampers, under harmonic ground excitation. Three different models are used to represent the linear dynamic force-deformation characteristics of viscoelastic dampers namely, Kelvin model, Linear hysteretic model and Maxwell model. The frequency domain solution is obtained by (i) an iterative pseudo-force method, which uses undamped mode shapes and frequencies of the system, (ii) an approximate modal strain energy method, which uses an equivalent modal damping of the system in each mode of vibration, and (iii) an exact method which uses complex frequency response function of the system. The responses obtained by three different methods are compared for different combinations of viscoelastic dampers giving rise to both classically and non-classically damped cases. In addition, the effect of the modelling of viscoelastic dampers on the response is investigated for a certain frequency range of interest. The results of the study are useful in appropriate modelling of viscoelastic dampers and in understanding the implication of using modal analysis procedure for building frames which are passively controlled by viscoelastic dampers against base excitation.

Key words: viscoelastic dampers; base excitation; storage and loss moduli; pseudo-force; modal strain energy.

1. Introduction

Building frames subjected to earthquake excitations are conventionally designed for lateral forces smaller than those required for strong or even moderate earthquakes. Design procedures allow reduction of design forces according to ductility capacity of the structure. A ductile structure is capable of dissipating energy in joints and connections designed to withstand plastic deformations. This ductility demand on the structure implies damage of the structural system, and often, damage of non structural components such as partitions and walls. By incorporating energy dissipating devices (EDDs) such as viscoelastic dampers to the resistance scheme of the structure, the deformation can be reduced significantly. As a result, the ductility demand can be attenuated. This reduced deformation is a natural consequence of an increase of the resistance, stiffness and energy dissipating capacity provided by the dampers.

The impact of a good design of EDDs added to a structure is two fold. First, it can enhance the performance of a structural system designed according to conventional design procedures by means of an increase in the structural damping, a corresponding reduction in the deformation demand on

† Research Scholar
‡ Professor

the structure and a consequent saving over the life time of the structure. Second, by means of an integrated design of the main structure with supplemental dampers, the resistance scheme of the main structural system can be reduced compared to a structural system without dampers.

Viscoelastic dampers (VEDs) are the devices in which vibrations take place in a fluid media. Zhang, *et al.* (1989) evaluated the effect of added viscoelastic dampers on reducing the earthquake response of multi-storey steel frame structures. They used the properties of viscoelastic materials independent of the frequency and temperature, and followed a modal approach to asses their effect on the structural response control. Inaudi, *et al.* (1993) reported various modelling aspects for the constitutive relations of linear viscoelastic materials with the help of frequency-dependent storage and loss moduli of the viscoelastic materials.

The modal equations of the structural systems with viscoelastic dampers may be uncoupled and coupled depending upon the relationship between the structure's dynamic property and the viscoelastic damper's property. Generally, structures with viscoelastic dampers are non-classically damped and therefore, the modal equations of the system become coupled. The dynamic analysis of nonclassically damped systems, as such, has received considerable attention. Early efforts in solving the problem centered around different ways of approximating the damping matrix by an equivalent proportional damping (Tsai 1974) so that classical mode superposition method can be applied. For structural systems like, nuclear power plants and dams, however, such an approximation leads to unsatisfactory results as pointed out by Warburton and Soni (1977). For such cases, a rigorous modal superposition method utilizing a set of complex non-classical normal modes is available (Singh and Suarez 1987), but is seldom used. Igusa, *et al.* (1984) presented a modal decomposition method, wherein the solution of the resulting uncoupled first order equations is put into a form involving a familiar displacement and velocity impulse response functions and their Duhamel integrals. Ibrahimbegovic, *et al.* (1990) presented `Ritz Method' for the dynamic analysis of large discrete linear systems with non-proportional damping, in which they concluded that the real vector basis approach is more efficient than the complex vector basis approach. However, the complex vector basis has the advantage on the accuracy and this does not require the eigenvalue problem to be solved. Claret and Venancio (1991) presented an iterative technique using classical mode shapes of the structure to solve the non-classically damped systems. Jangid and Datta (1993) extended the work of Claret and Venancio (1991) for spectral analysis of structural systems with non-classical damping. Chang, *et al.* (1993) proposed the use of an approximate modal strain energy technique in the analysis of structures with added viscoelastic dampers in the context of earthquake engineering.

Although there have been some studies on the use of VEDs in building frames for the reduction of seismic response, the relative uncertainty in the prediction of responses by different models used to characterize VEDs is not thoroughly investigated. This evaluation is important in relation to the appropriate modeling of VEDs. There is also a lack of studies regarding applicability of approximate analysis techniques used for finding the seismic response of frames with VEDs. In this paper, shear frame model of building frames with viscoelastic bracings is analyzed to obtain the storey displacements and absolute accelerations under harmonic base excitations. Three different mathematical models to represent the behavior of the viscoelastic dampers (VEDs) are considered namely, Kelvin model, Linear hysteretic model and Maxwell model. An example problem of two storey shear frame with VED resistance scheme is solved by two widely used approximate methods, namely, the pseudo force method and the modal strain energy method for two cases of damping. The results are compared with those obtained by the exact method of analysis in order to investigate the applicability of the two approximate methods for the dynamic analysis of the frames with VEDs under base excitation. The relative uncertainty in the prediction of the responses of the structure with VEDs is assessed by comparing the storey displacements and absolute accelerations obtained by analyzing the same two storey frame and a 12-storey frame with the three models used to characterize the VEDs.

2. A linear model for viscoelastic damper

The linear model uses a linear spring which models a potential energy, quadratic in deformation, and linear dashpot which models a dissipative force, proportional to deformation rate. Viscoelastic damper elements are modelled as parallel and series combinations of those linear springs and dashpots. The rate dependence is introduced by using the time derivative of the deformation and/or force.

In general, series and parallel combinations of linear springs and dashpots provide a force deformation relation of the form

$$C[f(t)]=D[\delta(t)] \tag{1}$$

where C [.] and D [.] are linear differential operators with constant coefficients. Using Laplace transformation, Eq. (1) provides a transfer function $\boldsymbol{H(s)}$ in the form

$$F(s)=\frac{D(s)}{C(s)}\delta(s)=H(s)\delta(s) \tag{2}$$

which relates force $F(s)$ to the deformation $\delta(s)$.

The frequency response function of the viscoelastic element is then obtained by substituting $s=j\omega$ in Eq. (2), where $j=\sqrt{-1}$ to yield

$$H(j\omega)=E_s(\omega)+jE_l(\omega) \tag{3}$$

$E_s(\omega)$ is referred to as the storage modulus, and $E_l(\omega)$ is called the loss modulus (Fig. 1). These moduli provide a physical understanding of the element resistance as composed of a frequency-dependent spring $k_d(\omega)$, and a frequency-dependent dashpot $c_d(\omega)$ given by

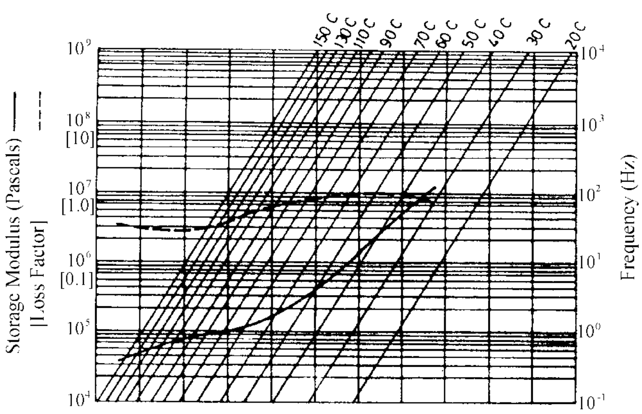


Fig. 1 Storage and loss moduli of viscoelastic damper

$$k_d(\omega)=E_s(\omega) \qquad c_d(\omega)=\frac{E_l(\omega)}{\omega} \tag{4}$$

$E_s(\omega)$ is an even function of frequency, while $E_l(\omega)$ is an odd function of frequency.

2.1. Kelvin element

The Kelvin model of VED (element) consists of a linear spring in parallel with a viscous damper. The force in the element satisfies

$$f(t)=k_d\delta(t)+c_d\dot{\delta}(t) \tag{5}$$

In the frequency domain, Eq. (5) can be written as

$$F(j\omega)=(k_d+jc_d\omega)\delta(j\omega) \tag{6}$$

The dissipation of energy per cycle in harmonic deformation is linearly proportional to the deformation frequency:

$$W_{cycle}=\delta_{\max}^2 c_d\pi\omega \tag{7}$$

The main disadvantage of this model in modelling the viscoelastic material is that it defines a loss modulus linearly dependent on the frequency and a storage modulus independent of frequency which is not an accurate representation for most materials and in particular, for polymers or rubbers.

2.2. Linear hysteretic element

In this model, the force satisfies the following equation in the frequency domain

$$F(j\omega)=k_d(1+j\xi\,\mathrm{sgn}(\omega))\delta(j\omega) \tag{8}$$

The loss factor ξ (ratio of the loss and the storage moduli of the element) is frequency independent for this element. This model has the property of frequency independence of the dissipated energy in a deformation cycle

$$W_{cycle}=\delta_{\max}^2\xi\pi k_d \tag{9}$$

Thus, this model is more versatile than the Kelvin model since many materials exhibit energy dissipation independent of the frequency of the deformation. The fact that frequency-domain techniques must be used to analyze structures with structural damping, constitutes its most significant limitation.

2.3. Maxwell element

A Maxwell element consists of a linear spring with constant α in series with a linear viscous dashpot with constant $\tau\alpha$. This model satisfies the following differential equation

$$\dot{f}(t)+\frac{1}{\tau}f(t)=\alpha\dot{\delta}(t) \tag{10}$$

In frequency domain, it may be represented as

$$F(j\omega)=\frac{\alpha\omega j}{j\omega+1/\tau}\delta(j\omega) \tag{11}$$

From Eq. (11), the storage modulus and the loss modulus for the Maxwell model may easily be obtained. Using a Maxwell model, the mechanical behavior of the viscoelastic damper can be modeled with much more accuracy since both storage modulus and loss modulus are fully dependent on the excitation frequency. The main mechanical characteristic of a Maxwell model is its relaxation time τ. The energy dissipation in one cycle is given by

$$W_{cycle}=\delta_{\max}\alpha\pi\frac{\omega\tau}{1+(\omega\tau)^2} \tag{12}$$

Eq. (12) shows that the energy dissipated in a cycle in this model increases with frequency for frequencies less than $1/\tau$ and monotonically decreases with frequency for frequencies larger than $1/\tau$.

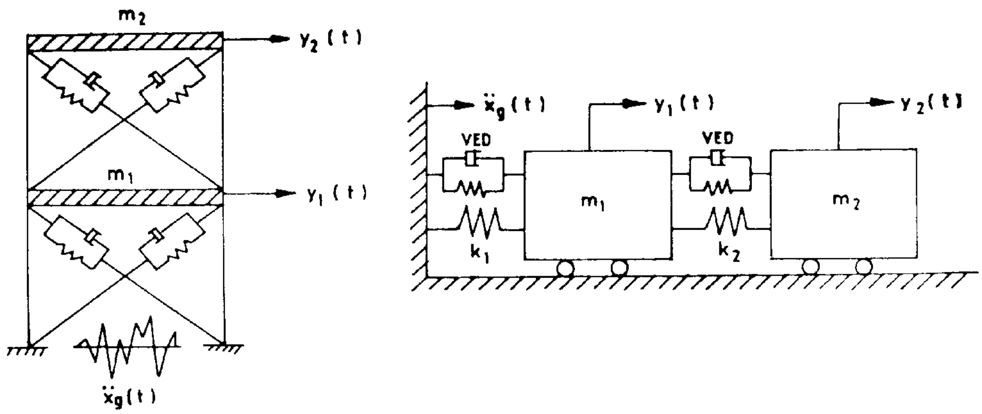
3. MDOF system with viscoelastic damper

Consider a linear, damped N-degree-of-freedom structure containing linear energy dissipation devices as shown in Fig. 2. The system can be described by the following differential equation

$$M\,\ddot{y}(t)+C\,\dot{y}(t)+K\,y(t)+\sum_{i=1}^{N_e}B_i^T f_i(t)=I_0\ddot{x}_g(t),\;\; y(0)=y_0,\;\; \dot{y}(0)=\dot{y}_0 \tag{13a}$$

$$M\,\ddot{y}_a(t)+C\,\dot{y}(t)+K\,y(t)+\sum_{i=1}^{N_e}B_i^T f_i(t)=0 \tag{13b}$$

where $y(t)$ and $\ddot{y}_a(t)$ are vectors of displacements and absolute accelerations, respectively; $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$ represent the mass, the stiffness and the damping matrices, respectively; $\ddot{x}_g(t)$ represents the ground acceleration; $\boldsymbol{I_0}$ is the influence coefficient vector; $f_i(t)$ is the force in the i-th energy dissipation device, and N_e is the number of such devices; $\boldsymbol{B_i}$ is the coefficient matrix which is



(a) (b)

Fig. 2 Model of a 2DOF structure with viscoelastic damper: (a) two storey shear frame with VED; (b) idealized model defined later. The element forces are related to the element deformations by one of the models of the linear constitutive relations described before. The element deformations are related to the coordinates **y(t)** by

$$\delta_i(t)=B_i\, y(t) \tag{14}$$

In frequency domain, Eq. (13) may be written as

$$[(j\omega)^2 M+(j\omega)C+K]Y(j\omega)+\sum_{i=1}^{N_e} B_i^T F_i(j\omega)=I_0\ddot{X}_g(j\omega) \tag{15a}$$

$$M\ddot{Y}_a(j\omega)+[(j\omega)C+K]Y(j\omega)+\sum_{i=1}^{N_e} B_i^T F_i(j\omega)=0 \tag{15b}$$

in which $Y(j\omega)$, $\ddot{Y}_a(j\omega)$, $F_i(j\omega)$ and $\ddot{X}_g(j\omega)$ are the fourier transforms of **y(t)**, $\ddot{y}_a(t)$, $\boldsymbol{f_i(t)}$ and $\ddot{x}_g(t)$ respectively. Writing $F_i(j\omega)$ in terms of storage and loss moduli, Eqs. (15a) and (15b) may also be written in the form

$$S(j\omega)Y(j\omega)=I_0\ddot{X}_g(j\omega) \tag{16a}$$

$$M\ddot{Y}_a(j\omega)+S_1(j\omega)Y(j\omega)=0 \tag{16b}$$

in which

$$S(j\omega)=-\omega^2 M+j\omega C+K+\sum_{i=1}^{N_e} B_i^T E_{S_i}(\omega)B_i+j\sum_{i=1}^{N_e} B_i^T E_{l_i}(\omega)B_i \tag{17a}$$

$$S_1(j\omega)=j\omega C+K+\sum_{i=1}^{N_e} B_i^T E_{S_i}(\omega)B_i+j\sum_{i=1}^{N_e} B_i^T E_{l_i}(\omega)B_i \tag{17b}$$

3.1. Transformation for modal analysis

$Y(j\omega)$ may be written in terms of modal co-ordinates as

$$Y(j\omega)=\Phi Q(j\omega) \tag{18}$$

where $\boldsymbol{\Phi}$ is a real-valued matrix of size Nxm and $Q(j\omega)$ is the Fourier transform of the modal coordinates $q_l(t)$ l=1, 2, ..., m. Using the modal transformation, and premultiplying Eq. (16a) by $\boldsymbol{\Phi}^T$, the following equation is obtained

$$\Gamma(j\omega)Q(j\omega)=\Phi^T I_0\ddot{X}_g(j\omega) \tag{19}$$

where

$$\Gamma(j\omega)=\Phi^T\left(-\omega^2 M+j\omega C+K+\sum_{i=1}^{N_e} B_i^T E_{S_i}(\omega)B_i+j\sum_{i=1}^{N_e} B_i^T E_{l_i}(\omega)B_i\right)\Phi \tag{20}$$

3.2. Conditions for modal decoupling and coupling

$\Gamma(j\omega)$ will not be diagonal for any modal matrix $\boldsymbol{\Phi}$, unless $\boldsymbol{C}$, $\sum_{i=1}^{N_e} B_i^T E_{S_i}(\omega)B_i$ and $\sum_{i=1}^{N_e} B_i^T E_{l_i}(\omega)B_i$

are classical for all ω. When these terms are classical $\Gamma(j\omega)$ would be diagonal and the following equations could be written in modal coordinates:

$$Q_l(j\omega)=\frac{\phi_l^T I_0 \ddot{X}_g(j\omega)}{\phi_l^T S(j\omega)\phi_l}=\frac{\phi_l^T I_0 \ddot{X}_g(j\omega)}{\Gamma_{ll}(j\omega)} \qquad l=1,2,\dots,m \tag{21}$$

$$\Gamma_{ll}(j\omega)=\phi_l^T\left(-\omega^2 M+j\omega C+K+\sum_{i=1}^{N_e} B_i^T E_{S_i}(\omega)B_i+j\sum_{i=1}^{N_e} B_i^T E_{l_i}(\omega)B_i\right)\phi_l$$

The solution of Eqs. (16a) and (16b) can then be obtained by solving m independent modal equations in the frequency domain. Using FFT techniques and modal combination ($y(t)=\Phi q(t)$), the time history of responses $y(t)$ and $\ddot{y}_a(t)$ can be obtained. The conditions under which the modal equations can be decoupled is illustrated by the 2-DOF model of the building frame shown in Fig. 2. The mass matrix is characterized by the masses $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ associated to the degrees of freedom $\boldsymbol{y_1(t)}$ and $\boldsymbol{y_2(t)}$, respectively. The stiffness matrix is characterized by the parameters $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$. Two viscoelastic dampers modeled by Maxwell elements are located in the model, $B_1=[1\ 0]$ and $B_2=[-1\ 1]$. The dampers have the same relaxation time τ and stiffnesses α_1 and α_2 at high frequency ($\alpha_i=E_{S_i}(\infty)$). Defining $\theta(\omega)=\omega^2/\omega^2+(1/\tau)^2$ and $\vartheta(\omega)=\omega/\tau/(\omega^2+(1/\tau)^2)$, the following equation in frequency domain may be written:

$$[(j\omega)^2 M+(j\omega)C+K+\begin{bmatrix}\alpha_1+\alpha_2 & -\alpha_2\\ -\alpha_2 & \alpha_2\end{bmatrix}\theta(\omega)+j\begin{bmatrix}\alpha_1+\alpha_2 & -\alpha_2\\ -\alpha_2 & \alpha_2\end{bmatrix}\vartheta(\omega)]Y(j\omega)=MI\ddot{X}_g(j\omega) \tag{22a}$$

$$M\ddot{Y}_a(j\omega)+[(j\omega)C+K+\begin{bmatrix}\alpha_1+\alpha_2 & -\alpha_2\\ -\alpha_2 & \alpha_2\end{bmatrix}\theta(\omega)+j\begin{bmatrix}\alpha_1+\alpha_2 & -\alpha_2\\ -\alpha_2 & \alpha_2\end{bmatrix}\vartheta(\omega)]Y(j\omega)=0 \tag{22b}$$

in which

If $\alpha_2/\alpha_1=k_2/k_1$ and $\boldsymbol{C}$ is mass **($\boldsymbol{M}$)** and stiffness **($\boldsymbol{K}$)** proportional, the system

$$M=\begin{bmatrix}m_1 & 0\\ 0 & m_2\end{bmatrix} \quad C=\begin{bmatrix}c_1+c_2 & -c_2\\ -c_2 & c_2\end{bmatrix} \quad K=\begin{bmatrix}k_1+k_2 & -k_2\\ -k_2 & k_2\end{bmatrix} \quad I=\begin{bmatrix}-1\\ -1\end{bmatrix}$$

can be uncoupled into two modal equations. Let ϕ_1 and ϕ_2 be normalized with respect to the mass matrix, then for $\alpha_2/\alpha_1=k_2/k_1$ the following equations hold:

$$Q_i(j\omega)=\frac{\phi_i^T MI}{\Gamma_{ii}(j\omega)}\ddot{X}_g(j\omega),\ \ i=1,\ 2 \tag{23}$$

$$\Gamma_{ii}(j\omega)=-\omega^2+\omega_i^2+\omega_i^2\frac{\alpha_1}{k_1}\theta(\omega)+j\omega_i^2\frac{\alpha_1}{k_1}\vartheta(\omega) \quad i=1,2 \tag{24}$$

When the damping matrix or the resistance scheme of the viscoelastic elements of the structure is non-classical, $\Gamma(j\omega)$ in Eq. (20) is not diagonal and the mxm coupled Eq. (19) can be solved to directly obtain the responses $y(t)$ by some approximate methods. There are various approximate methods (Claret *et al.* 1991, Ibrahimgovic *et al.* 1990, Igusa *et al.* 1984, Inaudi *et al.* 1993, Jangid *et al.* 1983, Singh *et al.* 1987, Tsai 1974, Warburton *et al.* 1977) to solve the coupled equations of motion as given by Eq. (19). The simplest solution for Eq. (19) can be obtained by ignoring the off-diagonal terms of the coupled equations of motion and solving a set of uncoupled equations, equal to the number of modes considered in the analysis. The accuracy of the response thus obtained depends upon the relative magnitude of the off-diagonal terms in the coupled equations of the motion. Here-in the solutions of Eqs. (15a) and (15b) are obtained by two approximate methods namely, Iterative Pseudo-Force method and Modal Strain Energy method, which are supposed to provide a good estimate of the response. The reason for choosing these two methods is that both use undamped mode shapes and frequencies and solve a set of uncoupled set of equations equal to the number of modes. The applicability of the methods for the case of structures with VEDs is investigated by comparing the responses obtained by these methods with those obtained by the exact method of analysis.

4. Iterative pseudo force (P-F) method

In this technique of solution, the modal coupling introduced by the off-diagonal terms are treated as Pseudo-Force and are transferred to the right hand side of Eq. (25). The equations of motion are solved iteratively, each time solving a set of uncoupled equations of motion. At the n-th iteration, the following equations of motion are solved

$$Q^{(n)}(j\omega)=Q^{(0)}(j\omega)+A(j\omega)Q^{(n-1)}(j\omega) \qquad n=1,2,3,.... \tag{25}$$

where

$$\boldsymbol{Q}^{(n)}(j\omega)=\begin{bmatrix} Q_1^{(n)}(j\omega) \\ Q_2^{(n)}(j\omega) \\ \cdots \\ Q_m^{(n)}(j\omega) \end{bmatrix} \tag{26}$$

The elements of the matrix $A(j\omega)$ are

$$A_{ij}=\frac{\Gamma_{ij}(j\omega)}{\Gamma_{ii}(j\omega)} \text{ if } \quad i \neq j \tag{27}$$

$$A_{ij}=0 \quad \text{if} \quad i=j$$

and $Q_i^{(0)}(j\omega)$ is given by

$$Q_i^{(0)}=\frac{\phi_i^T I_0}{\Gamma_{ii}(j\omega)}\ddot{X}_g(j\omega), \quad i=1, 2,\ldots,m \tag{28}$$

It can be shown that the necessary and sufficient condition for the algorithm to converge to the exact solution for any excitation $\ddot{x}_g(t)$ is that the eigen values of $A(j\omega)$ be in the unit circle for $(-\infty < \omega < \infty)$ (Claret and Venanci 1991). A sufficient condition for the convergence of the algorithm is given by

$$\sum_{i=1}^{m} |A_{li}(j\omega)| < 1 \qquad l=1, 2,\ldots, m \tag{29}$$

Furthermore, this condition can be stated in terms of the matrix $\Gamma(j\omega)$, since from Eq. (27) it is clear that the condition given by Eq. (29) holds if and only if the matrix $\Gamma(j\omega)$ is diagonal dominant, that is if

$$\left|\phi_i^T S(j\omega)\phi_i\right| > \sum_{l=1, l\neq i}^{m} \left|\phi_i^T S(j\omega)\phi_l\right|, \quad i=1, 2, \ldots, m \quad -\infty < \omega < \infty \tag{30}$$

5. The modal strain energy method

The modal strain energy (MSE) method is a procedure to determine a set of real-valued mode shapes, natural frequencies and damping ratios for linear structures with frequency-dependent stiffness and damping matrices to approximate the dynamics of those structures. In this approach, the mode shapes and natural frequencies of the approximate system are obtained by solving an eigen value problem that neglects the loss moduli of the viscoelastic elements of the structure. Once a set of mode shapes and natural frequencies are obtained, the modal damping ratios of the approximate system are computed equating the loss moduli of the viscoelastic elements of the structure at the natural frequencies of the modes, to that of the modal equations. As a consequence, the MSE method seeks a set of uncoupled modal equations to approximate the response of the system described by Eq. (16). The method is briefly described below:

Consider the vectors, $\hat{\phi}_l$, and natural frequencies, $\hat{\omega}_l$, that solve the following eigen value problem:

$$\hat{\omega}_l^2 M\hat{\phi}_l = (K + \sum_{i=1}^{N_e} B_i^T E_{S_i}(\hat{\omega}_l)B_i)\hat{\phi}_l \quad l=1, 2, \ldots, m \tag{31}$$

Here the functions $E_{S_i}(\omega)$ are necessarily nonnegative, nondecreasing, and bounded as in the case of viscoelastic materials.

Let $\hat{\Phi}=[\hat{\phi}_1 \ldots \hat{\phi}_m]$. With these $\hat{\phi}$, the modal transformation of Eq. (16) will be same as that given by Eq. (20) except that Φ is replaced by $\hat{\Phi}$. By neglecting the off-diagonal terms of the transformed equation, taking $\omega=\hat{\omega}_l$ in $E_{S_i}(\omega)$ in the l-th equation, and dividing equation l by $\hat{\phi}_l^T M\hat{\phi}_l$, the following equation is obtained:

$$[-\omega^2 + j\omega\frac{1}{\hat{\phi}_l^T M\hat{\phi}_l}\hat{\phi}_l^T(C + \sum_{i=1}^{N_e} B_i^T\frac{E_{l_i}(\omega)}{\omega}B_i)\hat{\phi}_l + \hat{\omega}_l^2]\hat{Q}_l(j\omega) = \frac{\hat{\phi}_l^T I_0 \ddot{X}_g(j\omega)}{\hat{\phi}_l^T M\hat{\phi}_l} \quad l=1, 2, \ldots, m \tag{32}$$

By taking $\omega=\hat{\omega}_l$ in the term $E_{l_i}(\omega)/\omega$ of Eq. (32), the l-th modal equation can then be transformed back to the time domain in the form of a second-order differential equation to yield

$$\ddot{q}_l(t) + 2\hat{\omega}_l\hat{\xi}_l\dot{q}_l(t) + \hat{\omega}_l^2 q_l(t) = \frac{\hat{\phi}_l^T I_0 \ddot{x}_g(t)}{\hat{\phi}_l^T M\hat{\phi}_l} \quad l=1, 2, \ldots, m \tag{33}$$

where the modal frequencies $\hat{\omega}_l$ and damping ratios $\hat{\xi}_l$ can be expressed as:

$$\hat{\omega}_l^2 = \frac{\hat{\phi}_l(K + \sum_{i=1}^{N_e} B_i^T E_{S_i}(\hat{\omega}_l)B_i)\hat{\phi}_l}{\hat{\phi}^T M\hat{\phi}_l}, \quad \hat{\xi}_l = \frac{\hat{\phi}_l^T(\hat{\omega}_l C + \sum_{i=1}^{N_e} B_i^T E_{l_i}(\hat{\omega}_l)B_i)\hat{\phi}_l}{2\hat{\phi}_l^T(K + \sum_{i=1}^{N_e} B_i^T E_{S_i}(\hat{\omega}_l)B_i)\hat{\phi}_l} \tag{34}$$

Once $q_l(t)$ is obtained, the response $y(t)$ may be determined as before by modal superposition.

6. Exact method

In this method, direct inversion of the complex matrix $S(j\omega)$ of Eq. (16) is done to obtain the frequency components of the response:

$$Y(j\omega)=S^{-1}(j\omega)I_0\ddot{X}_g(j\omega) \tag{35}$$

$y(t)$ is then obtained by inverse fourier transform.

Once $Y(j\omega)$ is determined by any of the above methods, $\ddot{Y}_a(j\omega)$ can be obtained from Eq. (16b). $\ddot{y}_a(t)$ is then determined by inverse fourier transform of $\ddot{Y}_a(j\omega)$.

7. Numerical study

Consider the 2-DOF structure (Fig. 2) subjected to unit harmonic support excitation. The mass matrix is characterized by the masses m_1 and m_2 associated to the degrees of freedom $y_1(t)$ and $y_2(t)$ respectively. The stiffness matrix is characterized by the parameters k_1 and k_2. For the numerical study, following parameters of the structure are selected:

$$M=m\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad K=k_s\begin{bmatrix}3 & -1\\ -1 & 3\end{bmatrix} \quad C=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \quad k_s/m=100$$

The viscoelastic damper parameters are so selected that both uncoupled and coupled cases are covered. The following two cases are considered for the numerical study.

Case 1: when damper properties are classical (uncoupled case) and represented by Kelvin model, the stiffness and damping matrices added on to the structure take the form:

$$k_d=\begin{bmatrix}3 & -1\\ -1 & 3\end{bmatrix} \quad \text{and} \quad c_d=\begin{bmatrix}3 & -1\\ -1 & 3\end{bmatrix}$$

Case 2: when damper properties are non-classical (coupled case) and represented by Kelvin model, the stiffness and damping matrices added on to the structure take the form:

$$k_d=\begin{bmatrix}3 & 0\\ 0 & 0\end{bmatrix} \quad \text{and} \quad c_d=\begin{bmatrix}3 & 0\\ 0 & 0\end{bmatrix}$$

For other VED models, the form of the stiffness and damping matrices remain the same as above with suitable parameters replacing k_d and c_d. The damper parameters for the three VED models are listed in Table 1.

For the structural system selected, the undamped natural frequencies and mode shapes (without viscoelastic dampers) are:

$$\omega_1 = 7.65 \text{ rad/s} \qquad \phi_1^T = [0.383\ 0.924]$$

Table 1 Properties of viscoelastic dampers

Maxwell Model		Kelvin Model		Linear Hyst. Model	
α/m	τ	k_d/m	c_d/m	k_d/m	ξ
100	0.1	50	5	50	0.9

$$\omega_2 = 18.48 \text{ rad/s} \qquad \phi_2^T = [0.924\ -0.383]$$

In Figs. 3-8, variations of the $|Y(j\omega)|$ and $|\ddot{Y}_a(j\omega)|/5g$ with frequency ω are compared between those obtained by the P-F method, the MSE method and the exact method, for all the three models and for both classically and non-classically damped conditions. For classically damped condition (Figs. 3-5), the responses obtained by the three methods compare well over the entire frequency range of interest, except for the Maxwell model. For this model (Fig. 3), the responses obtained by the MSE and exact methods differ by varying degrees at lower frequency range. Between the frequency range of 0-6 rad/s, the difference is quite significant; the MSE method underestimates the

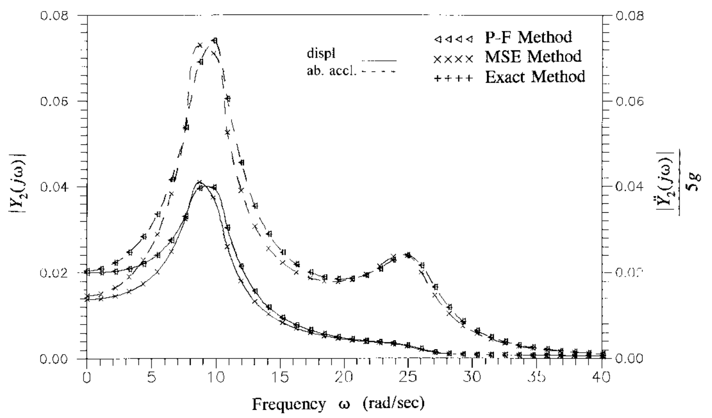


Fig. 3 Frequency variation of responses (Maxwell model, uncoupled case)

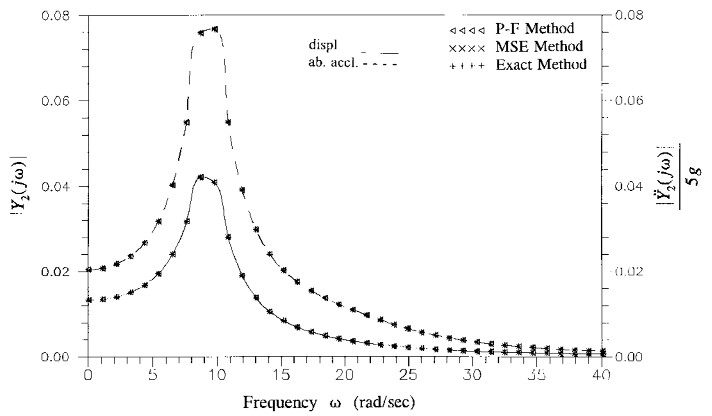


Fig. 4 Frequency variation of responses (Kelvin model, uncoupled case)

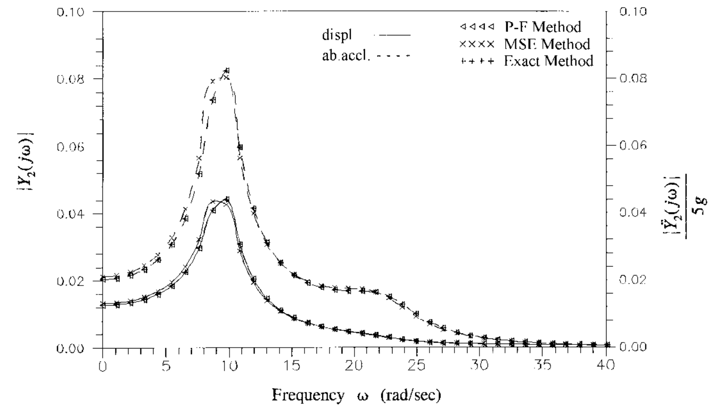


Fig. 5 Frequency variation of responses (Linear hysteretic model, uncoupled case)

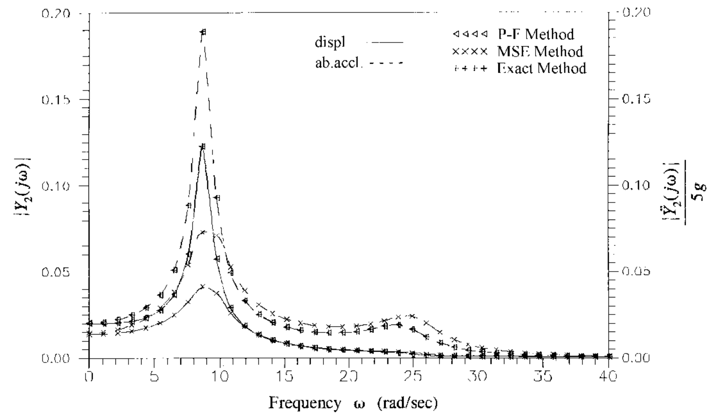


Fig. 6 Frequency variation of responses (Maxwell model, coupled case)

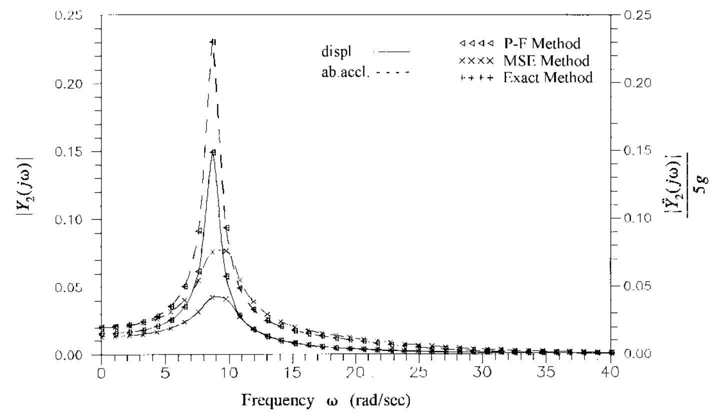


Fig. 7 Frequency variation of responses (Kelvin model, coupled case)

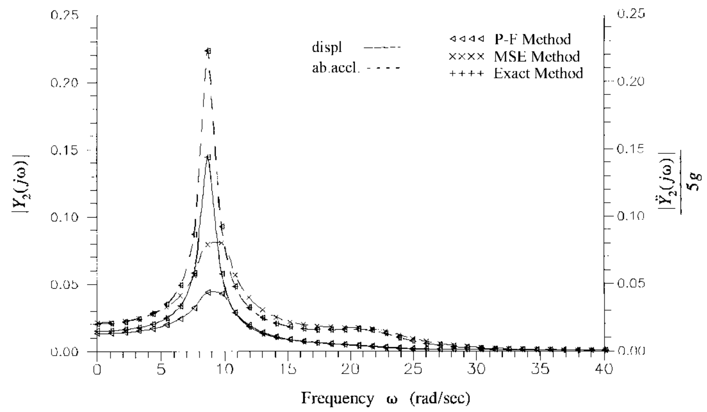


Fig. 8 Frequency variation of responses (Linear hysteretic model, coupled case)

responses.

For the non-classically damped condition, the responses obtained by the three methods are shown in Figs. 6-8. It is seen that the P-F method provides same results as those obtained by the exact method. The MSE method underestimates the response significantly under resonating condition (at the first natural frequency). For frequencies greater than the resonating frequency, the responses predicted by the MSE method are quite comparable with those predicted by the exact method. For frequencies lower than the resonating frequency, the MSE method gives less response than that obtained by the exact method. However, the difference between the two is not large.

8. Comparison between the responses obtained by different models

The three models for VEDs are designated as: (I) the Linear hysteretic model; (II) the Kelvin model; and (III) the Maxwell element. In order to make the models comparable, the parameters of models (II) and (III) are selected as functions of those of model (I) such that all models exhibit the same hysteresis loop in a cycle of harmonic deformation of frequency $\omega_0=\omega_1\sqrt{1+k_d^{(I)}/k_s}$, where ω_1 is the first undamped natural frequency of the structure. This implies that for model (II)

$$k_{d_i}^{(II)}=k_{d_i}^{(I)} \quad c_{d_i}^{(II)}=\frac{k_{d_i}^{(I)}\xi}{\omega_0} \tag{36}$$

while for model III

$$\tau=\frac{1}{\xi\omega_0} k_{d_i}^{(III)}=k_{d_i}^{(I)}\frac{1+(\tau\omega_0)^2}{(\tau\omega_0)^2}=k_{d_i}^{(i)}(1+\xi^2) \tag{37}$$

The responses $Y_1(j\omega)$, $Y_2(j\omega)$, $\ddot{Y}_{a_1}(j\omega)$ and $\ddot{Y}_{a_2}(j\omega)$ are computed for comparable models designated by I, II, III. The following symbols are used to compare the responses:

$$\begin{aligned} g_1&=\left|Y_1^{II}(j\omega)\right|/\left|Y_1^{I}(j\omega)\right| \quad & f_1&=\left|Y_1^{III}(j\omega)\right|/\left|Y_1^{I}(j\omega)\right| \\ g_2&=\left|Y_2^{II}(j\omega)\right|/\left|Y_2^{I}(j\omega)\right| \quad & f_2&=\left|Y_2^{III}(j\omega)\right|/\left|Y_2^{I}(j\omega)\right| \end{aligned} \tag{38}$$

Similarly

$$g_{a_1}=\left|\ddot{Y}^{II}_{a_1}(j\omega)\right|/\left|\ddot{Y}^{I}_{a_1}(j\omega)\right| \qquad f_{a_1}=\left|\ddot{Y}^{III}_{a_1}(j\omega)\right|/\left|\ddot{Y}^{I}_{a_1}(j\omega)\right|$$
$$g_{a_2}=\left|\ddot{Y}^{II}_{a_2}(j\omega)\right|/\left|\ddot{Y}^{I}_{a_2}(j\omega)\right| \qquad f_{a_2}=\left|\ddot{Y}^{III}_{a_2}(j\omega)\right|/\left|\ddot{Y}^{I}_{a_2}(j\omega)\right| \tag{39}$$

The above quantities are determined for (ξ_1=ξ_2=0.8), $0 < k_d/k_s < 2$, and for excitation frequencies near the first natural frequency of the undamped structural system. The responses are obtained for three excitation frequencies $\omega = 0.75\omega_1$, ω_1 and $1.25\omega_1$. Figs. 9 and 10 show the variations of the g_1, g_2, g_{a1} and g_{a2} with k_d/k_s for non-classically damped (case 2) condition. It is seen that the ratios g_1, g_2 etc. remain almost insensitive to the variation of k_d/k_s and is equal to almost unity for the three frequencies of excitation.

Figs. 11 and 12 show the variations of f_1, f_2, f_{a1} and f_{a2} with k_d/k_s for the same cases considered before. It is seen that f_1 increases linearly with k_d/k_s for the frequency of excitation equal to $0.75\omega_1$ and ω_1. For other cases, the ratios f_1, f_{a1}, f_2 etc remain almost insensitive to the variation of response for certain cases of excitation.

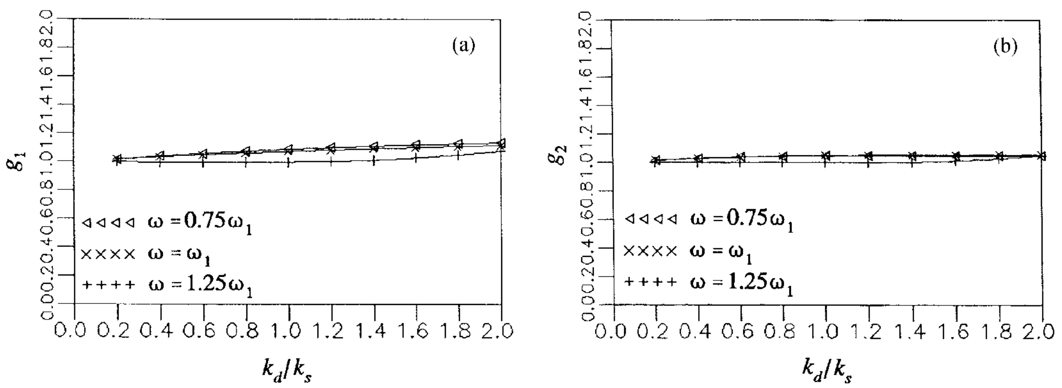


Fig. 9 Comparison of displacements for Kelvin and Linear hysteretic models: (a) For displacement ratio g_1; (b) For displacement ratio g_2

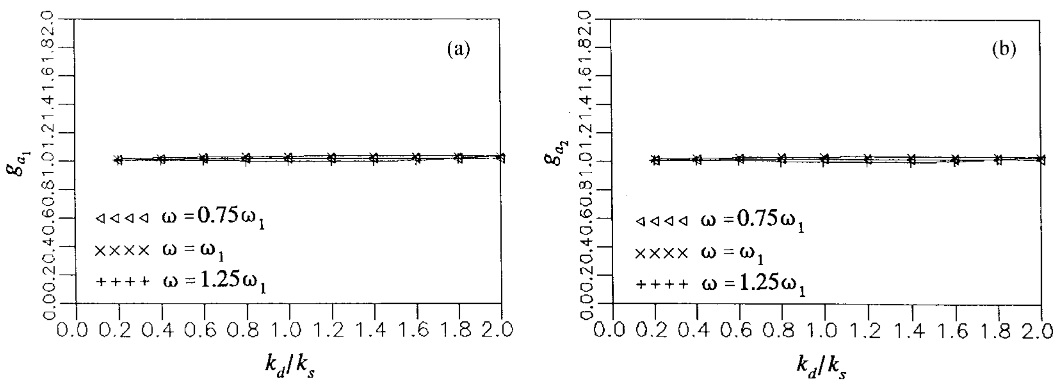


Fig. 10 Comparison of absolute accelerations for Kelvin and Linear hysteretic models: (a) For acceleration ratio g_{a_1}; (b) For acceleration ratio g_{a_2}

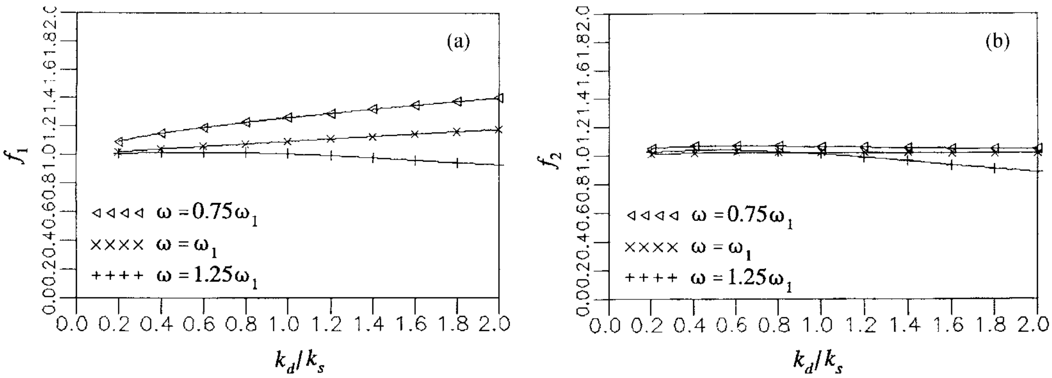


Fig. 11 Comparison of displacements for Maxwell and Linear hysteretic models: (a) For displacement ratio f_1; (b) For displacement ratio f_2

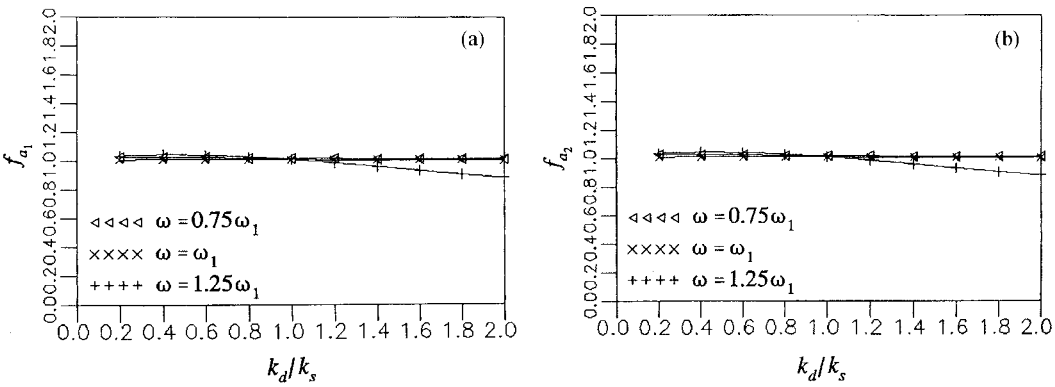


Fig. 12 Comparison of absolute accelerations for Maxwell and Linear hysteretic models; (a) For acceleration ratio f_{a_1}; (b) For acceleration ratio f_{a_2}

In order to investigate the relative performance of different models in actual practice, a 12 storeyed shear building frame with VED bracings in all storeys is analysed for harmonic ground excitation. The mass and stiffness properties are shown in Table 2. For the unbraced frame 2% modal damping is assumed for all modes. VEDs are uniformly distributed in all stories with stiffness property of the ith storey VEDs given as $K_{d_i}=\Sigma K_i/8N$, in which N=12, ΣK_i is the stiffness of the ith storey. K_{d_i} is defined for the linear hysteretic model. For other models, comparable visco elastic damper properties are obtained like those for the 2 storey building frame.

The responses of the 12 storey building frame provided with VEDs for harmonic ground motion are shown in Table 3. The excitation frequency is equal to the undamped natural frequency of the

Table 2 Properties of 12 storey shear building frame

Storey	1	2	3	4	5	6	7	8	9	10	11	12
Mass 10^3 Kg	115	110	108	108	100	91	91	85	85	79	79	70
Stiffness 10^6 N/m	205.7	192.5	164	139.7	102.3	89	71.5	67.7	56	49.5	44	40.5

Table 3 Comparison of responses for 12 storey shear frame (Top storey responses)

Response	VED Model	P-F Method	MSE Method	Exact Method
Maximum Displacement (cm)	Linear Hysteretic Model	0.279E+01	0.293E+01	0.266E+01
	Kelvin Model	0.280E+01	0.332E+01	0.264E+01
	Maxwell Model	0.263E+01	0.239E+01	0.293E+01
Maximum Absolute Accln. (cm/s^2)	Linear Hysteretic Model	0.958E+02	0.927E+02	0.966E+02
	Kelvin Model	0.695E+02	0.668E+02	0.714E+02
	Maxwell Model	0.118E+03	0.158E+03	0.129E+03

Maxm. Displacement (uncontrolled) = 0.536 E+01 cm
Maxm. Absolute Acceleration (uncontrolled) = 0.208 E+03 cm/s^2

frame, and the amplitude of ground acceleration is taken as 0.8 m/sec^2. It is seen that the displacement responses obtained by linear hysteretic model and the Kelvin model are practically the same. Maxwell model provides slightly higher value of the response. Same trend of results is observed for the absolute acceleration response.

9. Conclusions

The response of building frames passively controlled by VEDs is obtained for harmonic ground excitation. The VEDs are modeled using three different models namely Kelvin model, Linear hysteretic model and Maxwell model and the responses are obtained by three different methods ie. an iterative pseudo-force method, an approximate modal strain energy method and the exact method. The relative displacement and absolute acceleration obtained by different methods using different models for VEDs, are compared. The results of the numerical study lead to the following conclusions:

1. The Pseudo-Force method provides exact responses for all the three models being used to describe the force-deformation characteristics of VED.
2. The MSE method provides exact responses when properties of viscoelastic dampers are such that the total system is classically damped.
3. For the general case of non-classically damped system, the MSE method under-estimates the responses significantly near resonating condition. For frequencies greater than the resonating frequency, the MSE method predicts fairly good responses.
4. The Linear-hysteretic and Kelvin models of VEDs provide almost the same responses.
5. The Maxwell model generally tends to provide higher responses than the other two models.

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