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Seismic detailing of reinforced concrete beam-column connections

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Abstract. A simplified analysis procedure utilizing the strut-tie modeling technique is developed to take a close look into the post-elastic deformation capacity of beam-column connections in ductile reinforced concrete frame structures. Particular emphasis is given to the effect of concrete strength decay and quantity and arrangement of joint shear steel. For this a fan-shaped crack pattern is postulated through the joints. A series of hypothetical rigid nodes are assumed through which struts, ties and boundaries are connected to each other. The equilibrium consideration enables all forces in struts, ties and boundaries to be related through the nodes. The boundary condition surrounding the joints is obtained by the mechanism analysis of the frame structures. In order to avoid a complexity from the indeterminacy of the truss model, it is assumed that all shear steel yielded. It is noted from the previous research that the capacity of struts is limited by the principal tensile strain of the joint panel for which the strain of the transverse diagonal is taken. The post-yield deformation of joint steel is taken to be the only source of the joint shear deformation beyond the elastic range. Both deformations are related by the energy consideration. The analysis is then performed by iteration for a given shear strain. The analysis results indicate that concentrating most of the joint steel near the center of the joint along with higher strength concrete may enhance the post-elastic joint performance.

Key words: beam-column connections; concrete strength; cracks; joint shear steel; post-elastic deformation; strut-tie model.

1. Introduction

The importance of robust beam-column connections in reinforced concrete frame structures cannot be over-emphasized, since joints provide the required structural integrity for seismic resistance. In this regard, the most desirable design objective for new construction is that the joints remain mostly elastic in the event of an earthquake. This can be met by providing adequate transverse joint reinforcement along with appropriate bond and anchorage of longitudinal beam and column reinforcement. This may result in highly congested joint steel details. However, in most existing frame structures, concrete beam-column connections are usually insufficiently reinforced with

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transverse hoops. In an earthquake this can lead to unwanted inelastic response in the joint due to the large magnitude of shear forces being transferred. The behavior of frame structures is thus limited by the inelasticity of those joints.

ATC-11 (1983) reviewed about fifty technical papers concerning the behavior of concrete beam-column connections and despite the lack of consensus between researchers, summarized joint shear resisting mechanisms in terms of three mechanism models: beam shear; truss; and compression strut. In the beam shear mechanism, a 45° diagonal crack is assumed through the joint and the joint shear capacity is computed as the sum of contributions from the concrete and the transverse joint steel. In this model the concrete contribution is determined by empirical formulas expressed in terms of $\sqrt{f_c'}$ which implies the involvement of concrete tensile strength. In the joint truss mechanism, concrete segments between the diagonal cracks are in compression parallel to the cracks and horizontal ties act as truss members in tension. In the compression strut mechanism, the joint shear capacity is determined by the axial compression strength of a concrete strut formed diagonally across the joint. Note that all of these models describe the level of *forces* at the formation of failure mechanism but not the corresponding *deformation*. Due to the lack of well-defined design procedures, the vulnerability of beam-column joints in frame structures has been demonstrated experimentally (Mander *et al*. 1996a, b) and in the field due to damaging earthquakes (Seible and Priestley 1990).

In the present study, a simplified analysis procedure utilizing the strut-tie modeling technique is developed to take a look into the post-elastic deformation capacity of beam-column connections in ductile reinforced concrete frame structures. Particularly a fan-shaped crack pattern is postulated through the joint to take the complicated stress distribution in concrete segments into account. The proposed model is like the one in combination of joint truss and compression strut mechanisms described in ATC-11 but with the different crack pattern. The major variables in the proposed model are the concrete compression strength and the plastic deformation of joint shear steel. It is assumed for analysis that the concrete strength in compression degrades as the diagonal tensile strain of the joint panel increases and the post-yield deformation of joint steel is the only source of the joint shear deformation beyond the elastic range. The energy balance is considered to relate these deformations. The analysis is then performed by iteration for a series of given joint shear strains, resulting in a joint shear capacity envelope. The proposed analysis procedure is compared to the experimental observations. From the analysis, some design recommendations and future research directions for joints are suggested.

2. Strut-tie model

It is now a century since Ritter (1899) first introduced the plastic truss concept to assign the shear strength of a cracked reinforced concrete beam. Later, Dilger (1966) performed an extensive study and formulated the cracked shear stiffness using a constant angle continuum truss model. Paulay (1971) also adopted the approach by adding the deformation component to the truss model to assess the seismic shear stiffness of coupling beams. It is noted that Paulay was the first to utilize a variable angle truss model. More recently, Vecchio and Collins (1986) introduced the utilization of concrete tensile strength in assessing the shear resistance, referred to as the Modified Compression Field Theory (MCFT) for structural concrete. In parallel, Hsu (1993) introduced the Softened Truss Model (STM). Both MCFT and STM have been developed to capture the strength-deformation relationship of a differential portion of a membrane-type element. The force-deformation performance of a differential panel (and hence section) can be predicted based on equilibrium and compatibility requirements using appropriate constitutive relations for cracked (softened) concrete and steel (Hsu 1996). Recently, Pang and Hsu (1996) modified the STM for cracks to incline at a fixed angle following the principal stresses due to the applied loading.

In contrast to the MCFT and STM (continuum) truss models that really only address the performance of one critical differential panel element, the strut-tie modeling technique has been the most appropriate tool to consider the complicated flow of stresses in structural concrete components (Schlaich *et al.* 1987, MacGregor 1992). Parts of beams and columns near concentrated loads, corners, openings, beam-column connections and other discontinuities are included in this category. In the strut-tie model, it is assumed that a series of potential cracks exist in structural concrete elements in a specific pattern reflecting the state of stress distribution. A strut-tie model consists of a set of struts for concrete in compression and ties for steel in tension that are connected through hypothetical rigid nodes. The force equilibrium will hold through struts, ties and the boundary condition.

A new approach utilizing struts and ties to model both the strength and deformation of reinforced concrete beam-column joints is introduced in what follows. The major contribution of the suggested strut-tie modeling technique is to consider an entire joint element for strength-deformation analysis. This is in contrast with the MCFT and SAT methods that only deal with a single differential element and are thus unable to explain the flow of stress within the joint. Note that a conventional strut-tie analysis can model the flow of forces within a joint, but being based on plasticity alone is unable to predict the connection deformations.

2.1. Joint crack pattern

Schlaich, *et al*. (1987) defined two standard regions in structural concrete elements: B- and D-regions. In B-region the Bernoulli's hypothesis is assumed valid, while in D-region the strain distribution over a section is disturbed and may be significantly nonlinear. Based on this definition, Kim and Mander (1999) extensively investigated two truss models: a constant crack angle truss and a variable crack angle truss. The constant angle truss is considered appropriate for the undisturbed region of sufficiently "long" beam-column elements, while the variable angle truss for squat elements and the disturbed end-regions of "long" elements.

Beam-column joints can be regarded as squat elements where the entire portion belongs to the disturbed region. Accordingly, a fan-shaped variable angle crack pattern can be postulated for typical joints as shown in Fig. 1. It is noted that the identical crack pattern can be postulated for whatever the joint type is. The joint type will affect the boundary condition surrounding the joint panel that will finally affect the level of stresses.

2.2. Model description

Based on the postulated crack pattern, strut-tie models for cracked concrete beam-column connections can be constructed. For this it is assumed that the crack pattern is determined by the number of transverse joint steel that are evenly spaced through the joint as shown in Fig. 2. Concrete struts represent the intensity of compressive stresses parallel to the direction of cracks, while tensile ties represent the transverse joint steel. Forces in struts, ties and boundaries are

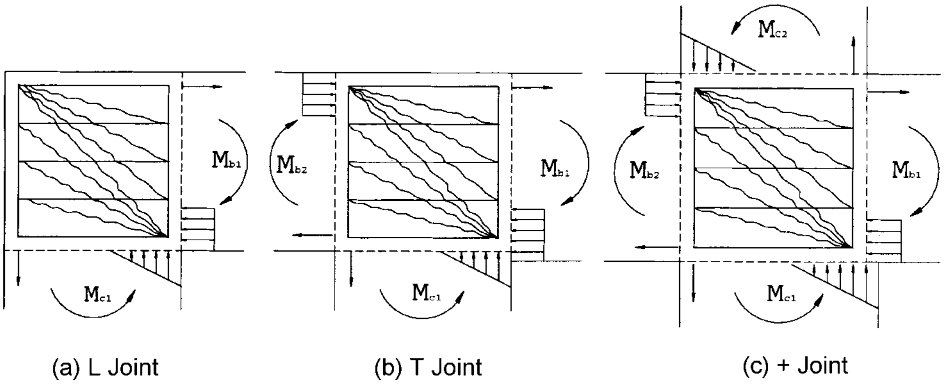


Fig. 1 Postulated crack pattern for beam-column connections

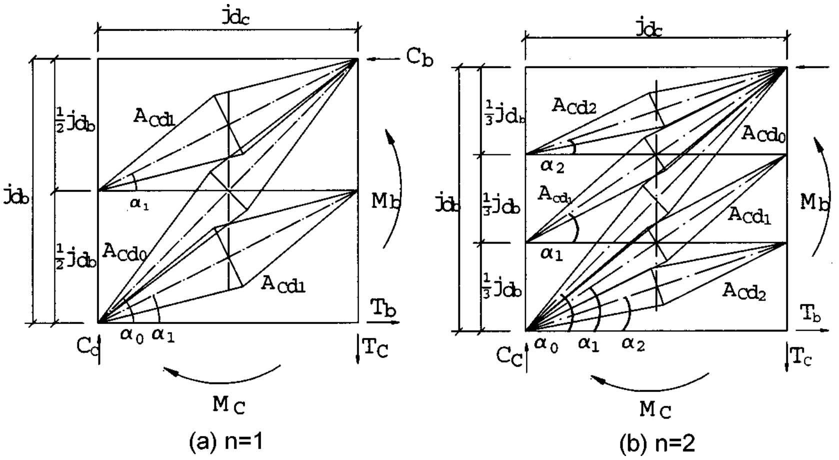


Fig. 2 Strut-tie models for cracked concrete joints

balanced through the nodes. In addition to the struts defined by transverse joint steel, the corner-to-corner diagonal strut is assumed to directly transfer the fraction of forces.

It is important to determine the axial rigidity of struts and ties to relate forces to joint deformations. Unlike the section areas of ties, the dimensions of struts cannot be explicitly determined. However, strut sizes can be measured from a scaled sketch as indicated in Fig. 2. This approach was verified analytically by Kim and Mander (1999). It is assumed that the area of struts in the shape of thin rhombus is in maximum along the center line of the joint panel and minimum at the corner of the joint panel. Taking the average for analysis purpose, the section area of *i*th strut can be obtained by

$$A_{cdi} = \frac{\cos\alpha_i}{2(1+n)(1+\cos^2\alpha_i)} b j d_b \tag{1}$$

where α_i = angle measured from the axis of *i*th strut to the horizontal line, that is

$$\alpha_i = \tan^{-1}\left[\left(1 - \frac{i}{1+n}\right)\frac{jd_b}{jd_c}\right] \tag{2}$$

in which n = number of transverse joint steel and b = thickness of joint panel. jd_b and jd_c are respectively the distance between internal couples of beam and column and taken as the distance between two farthest longitudinal steel layers. It is noted that i=0 denotes the quantity for the corner-to-corner diagonal strut.

2.3. Force equilibrium

The force in ith strut is stabilized by transverse hoop steel except the one in the corner-to-corner diagonal strut that is for i=0. In order to avoid a complexity from the indeterminacy within the strut-tie model, it is assumed that all joint shear steel yields. The force in ith strut is thus given by

$$F_{cdi} = f_{cdi} A_{cdi} \tag{3}$$

where f_{cdi} = compressive stress in ith strut. Based on the experimental observation on various concrete panel elements, Vecchio and Collins (1986) suggested an upper limit of compressive concrete stress in the direction of diagonal cracks when subjected to shear. The limit is employed here for compressive stress in struts and expressed as

$$\frac{f_{cd}^{\max}}{f_c'} = \frac{1}{0.8 + 170\varepsilon_1} \leq 1 \tag{4}$$

where f_c' = compressive strength of concrete cylinder, ε_1 = principal tensile strain. Note that ε_1 is taken as the diagonal tensile strain of the joint panel for simplicity. Eq. (4) denotes that as ε_1 increases, concrete strength in struts degrades. In accordance to the assumption of steel yielding, the force in ith strut given in Eq. (3) is also limited by

$$F_{cdi} \leq \frac{A_{sh} f_y}{\cos\alpha_i} \tag{5}$$

where $i \neq 0$, A_{sh}=cross section area of transverse joint steel evenly spaced and f_y=yield strength of transverse joint steel. The sum of strut forces to the longitudinal direction of the adjacent element to the connection is balanced with the flexural compression force C_c for columns or C_b for beams at the boundary. That is,

(i) when a plastic hinge forms in column,

$$C = C_c$$

$$C_c = F_{cdo}\sin\alpha_o + 2\sum_{i=1}^{n} F_{cdi}\sin\alpha_i \tag{6}$$

(ii) when a plastic hinge forms in beam,

$$C = C_b$$

$$C_b = \sum_{i=0}^{n} F_{cdi}\cos\alpha_i \tag{7}$$

where C=compressive force in concrete due to flexure. Since the flexural strength, C_c or C_b, is proportional to the magnitude of the concrete strut capacity, Eqs. (6) and (7) denote that the mechanism strength will degrade as the strength of the struts degrades. Therefore, the initial strength, $C_{c,initial}$ or $C_{b,initial}$, determined by mechanism analysis based on the assumption that the beam-column connection is within the elastic range, will degrade as the diagonal tensile strain of the joint panel increases. From this notion, the force in the corner-to-corner diagonal strut given in Eq. (3) is limited by

(i) when a plastic hinge forms in column,

$$F_{cdo} \le \left(C_{c,\,initial} - 2\sum_{i=1}^{n} F_{cdi} \sin\alpha_i \right) \Big/ \sin\alpha_o \tag{8}$$

(ii) when a plastic hinge forms in beam,

$$F_{cdo} \le \left(C_{b,\,initial} - \sum_{i=1}^{n} F_{cdi} \cos\alpha_i \right) \Big/ \cos\alpha_o \tag{9}$$

The corresponding capacity index resulting from the degradation of mechanism strength due to the inelasticity of joints can be expressed as

$$D(\gamma_j) = \frac{C}{C_{initial}} \tag{10}$$

where γ_j=joint shear strain. It is necessary to relate the joint shear strain to the diagonal tensile strain for the evaluation of joint shear capacity envelope.

Since the strut force is governed by the yield strength of joint shear steel as indicated in Eq. (5), the corresponding strut stress can also be expressed in terms of steel properties by dividing Eq. (5) by the strut area given in Eq. (1), thus

$$f_{cdi} = \frac{F_{cdi}}{A_{cdi}} = 2\rho_v \left(1 + \frac{1}{\cos^2\alpha_i} \right) f_y \tag{11}$$

in which ρ_v=the volumetric ratio of joint shear steel to concrete and given by

$$\rho_v = \frac{A_{sh}}{bs} \tag{12}$$

where b=width of section and $b \approx d_c$ for circular column, d_c=center-to-center diameter of circular hoop steel and s=transverse joint steel spacing given by

$$s = \frac{jd_b}{1+n} \tag{13}$$

Dividing Eq. (11) by f_c' gives the level of strut stress due to joint steel yielding, that is

$$\frac{f_{cdi}}{f_c'} = 2\rho_v \left(1 + \frac{1}{\cos^2\alpha_i} \right) \left(\frac{f_y}{f_c'} \right) \le 1 \tag{14}$$

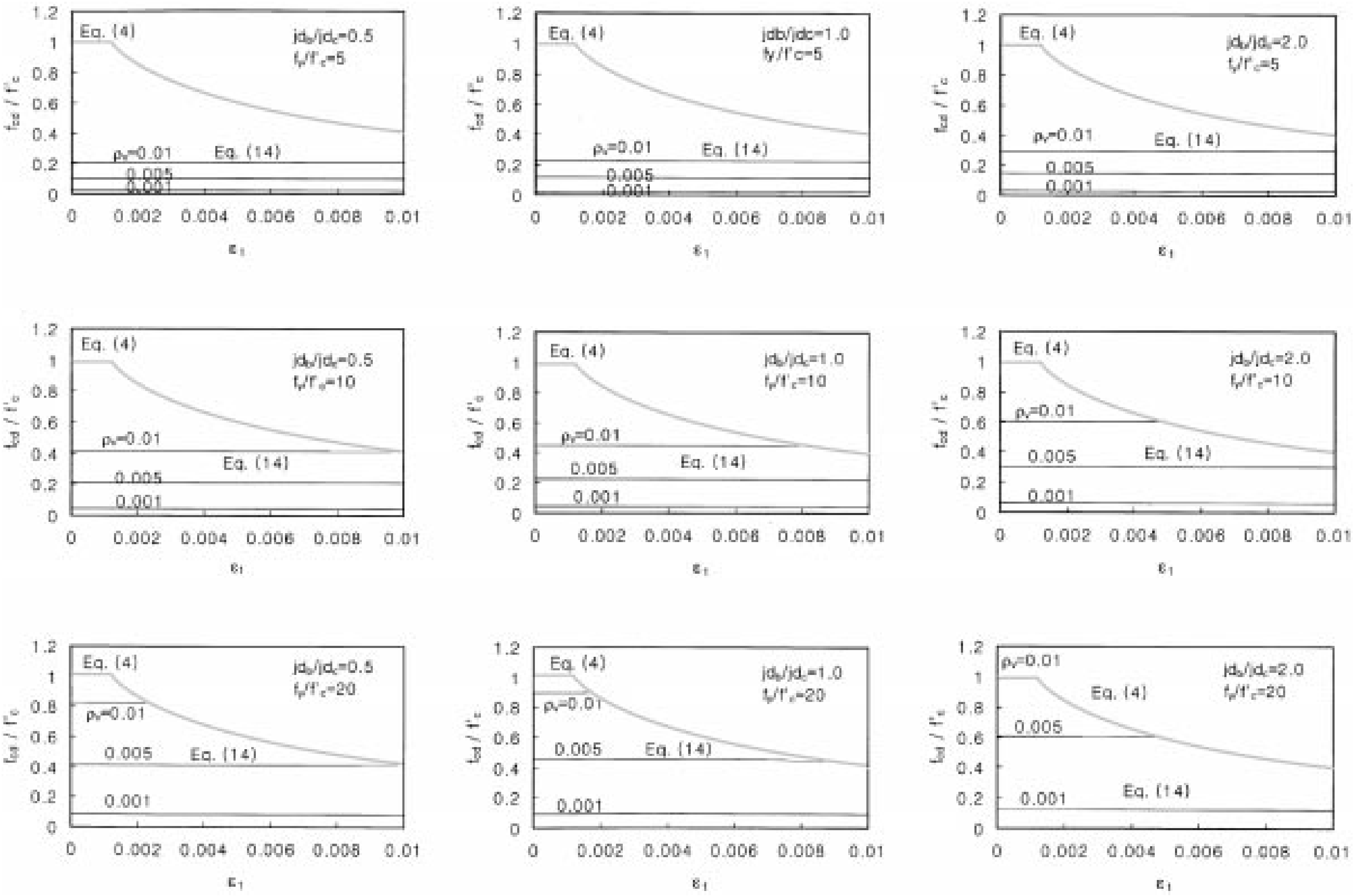


Fig. 3 Compressive stresses in struts over the diagonal tensile strain

The level of strut stresses given in Eq. (14) is also governed by the upper limit in Eq. (4). Fig. 3 presents the level of strut stress given in Eqs. (4) and (14) over the diagonal tensile strain ε_1 for various values of ρ_v, jd_b/jd_c and f_y/f_c'. Note that larger amount of joint shear steel and larger values of f_y/f_c' and jd_b/jd_c ratios result in higher levels of strut stresses. Not surprisingly, therefore, it should be recognized that the use of higher strength concrete will be beneficial to the behavior of beam-column connections for which high strength reinforcement is used.

2.4. Plastic deformation of joints

It is assumed that the plastic deformation of joint shear steel is the only source of post-elastic joint shear deformation as indicated in Fig. 4. The actual deformed shape of the joint in Fig. 4(a) can be idealized to the one in Fig. 4(b). If Δ, the elongation of joint hoop steel, is imposed as a result of joint deformation, the diagonal tensile strain can be obtained from Fig. 4(c) by

$$\varepsilon_1 = \frac{\delta}{L_d} = \frac{\Delta \cos\alpha_o}{\sqrt{jd_b^2 + jd_c^2}} \tag{15}$$

where δ=diagonal elongation of joint and L_d=diagonal length of joint. Assuming at this stage that the energy for the external work done (*EWD*) by joint shear force V_j is consumed by only the plastic deformation of joint steel, that is

$$EWD = IWD \tag{16a}$$

$$V_j \gamma_j jd = (\Sigma A_{sh} f_y)\Delta \tag{16b}$$

where $V_j = C$ and $\gamma_j jd = (jd_b + jd_c)\gamma_j/2$ for average joint deformation. Then the plastic deformation of joint shear steel is given from Eq. (16) by

$$\Delta = \frac{C(jd_b + jd_c)}{2\Sigma(A_{sh} f_y)}\gamma_j \tag{17}$$

Substituting Eq. (17) into Eq. (15) gives

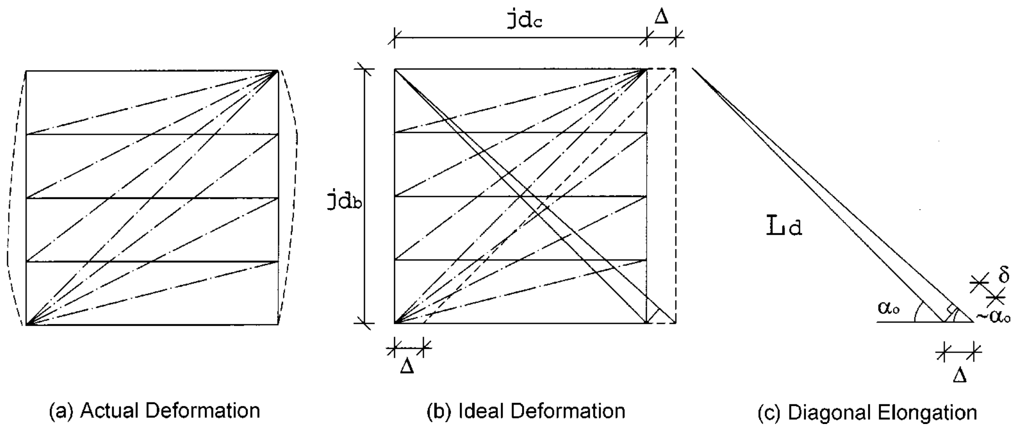
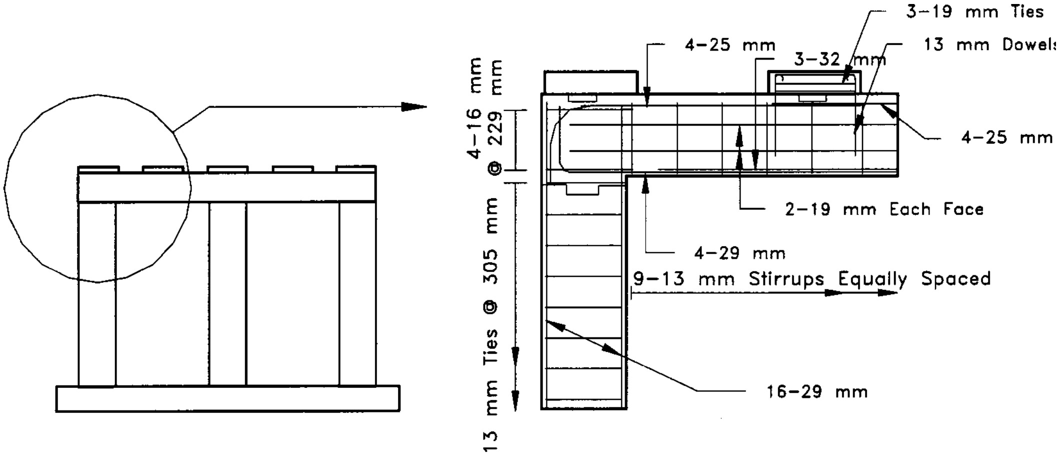


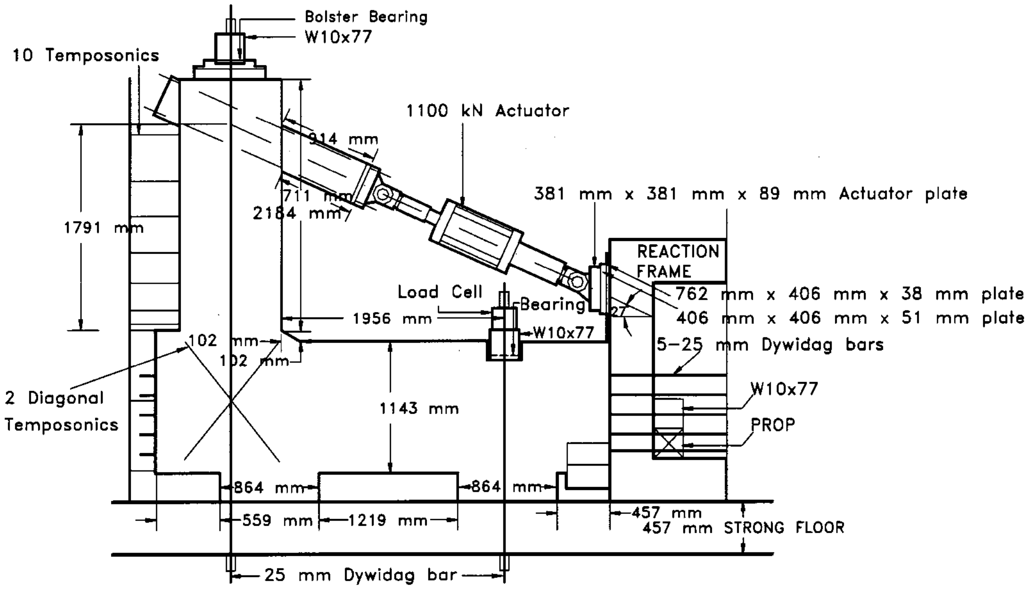
Fig. 4 Inelastic deformation of beam-column joint panel

$$\varepsilon_1 = \frac{C\cos\alpha_o(\cos\alpha_o + \sin\alpha_o)}{2\Sigma A_{sh} f_y}\gamma_j \tag{18}$$

The joint shear strain in above equations includes the elastic range as well as the post-elastic range. The diagonal tensile strain calculated by Eq. (18) will be used for the calculation of upper limit of compression stress in struts given in Eq. (4). Note that C is included in Eq. (18) for the calculation of ε_1 which is also a variable of formula for C. Therefore, the whole analysis procedure should be performed by iteration.



(a) Details of pier subassemblage



(b) Test setup

Fig. 5 Prototype pier subassemblage tested by Mander *et al.* (1996a)

3. Analysis procedure

The analysis procedure for the joint capacity envelope using the previously derived equations can be summarized as follows:

Step 1. Perform the plastic analysis of the frame structure under consideration and determine the boundary condition of joints.
Step 2. Calculate the initial value $C_{initial}$.
Step 3. Given the value of γ_j, calculate ε_1, F_{cdi} and C by iteration (Eqs. 3 to 14 and 18).
Step 4. Calculate the capacity index $D(\gamma_j)$ resulting from the strength degradation (Eq. 10).
Step 5. Increase γ_j a step and perform steps 3 and 4.
Step 6. Repeat steps 3 to 5 until prescribed value of γ_j is reached.

4. Worked example

A prototype frame subassemblage tested by Mander, *et al.* (1996a) is adopted herein as an illustrative example. This example was selected because it is one of a few full-scale specimens where an experimental relationship between lateral force and joint shear strain is reported. The L-shaped specimen, presented in Fig. 5 with the test setup, consists of a part of an exterior circular column and a part of beam. The dimensions are 838 mm diameter for column and 838 mm $\times$ 838 mm for beam and jd_b=jd_c=682 mm. Material strengths are 44.8 MPa for column concrete and 40.7 MPa for beam and joint concrete. Axial loading in column was P=343 kN ($0.0139 f_c' A_g$) with gravity loading only. The column section is reinforced with 16-D29 (#9) with f_y=269 MPa. Two D19 (#6) U-shaped bars with f_y=476 MPa are placed in the joint transversely surrounding the column steel anchored in the joint. These U-bars are regarded as the joint shear steel in this example. Assuming that the plastic hinge forms at the critical section of the column, the force in the flexural compression concrete stress block is initially $C_{c,initial}$=1966 kN for pull direction with $P_{\max}$=944 kN and $C_{c,initial}$=1229 kN for push direction with $P_{\min}$=13 kN. However, since the column axial loading in this example varied between maximum and minimum because of the frame action

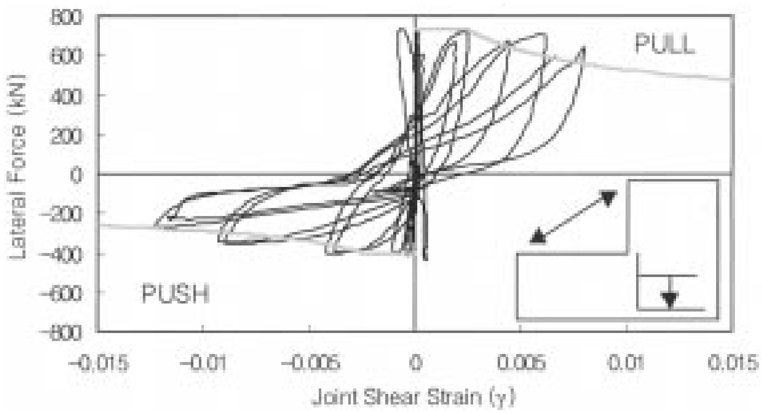


Fig. 6 Comparison between theory and experiment

due to lateral cyclic loading, the constant loading with the gravity was applied for analysis.

The analysis was performed in accordance with the procedure stated above and the results are compared with the experimental observations in Fig. 6. Very good agreement in the push (joint opening) direction is evident. In the pull direction (joint closing), although the agreement is not perfect, the trend is correct albeit somewhat conservative for large inelastic joint deformation. Evidently, the additional axial load that exists when loading in the "pull" direction provides some additional capacity-presumably due to the larger concrete stress blocks that occur in the adjoining members.

It is of interest to investigate if there is any optimal arrangement of joint steel. For this the prototype frame subassemblage is again considered. If the 2-D19 U-shaped bars can be replaced by 4-D13 (#4) or 1-D25 (#8), keeping the volume of joint steel approximately the same, then these three cases of joint steel placement can be compared to each other. All other design parameters are the same with the exception of giving the variation of concrete as 25 MPa, 35 MPa and 45 MPa

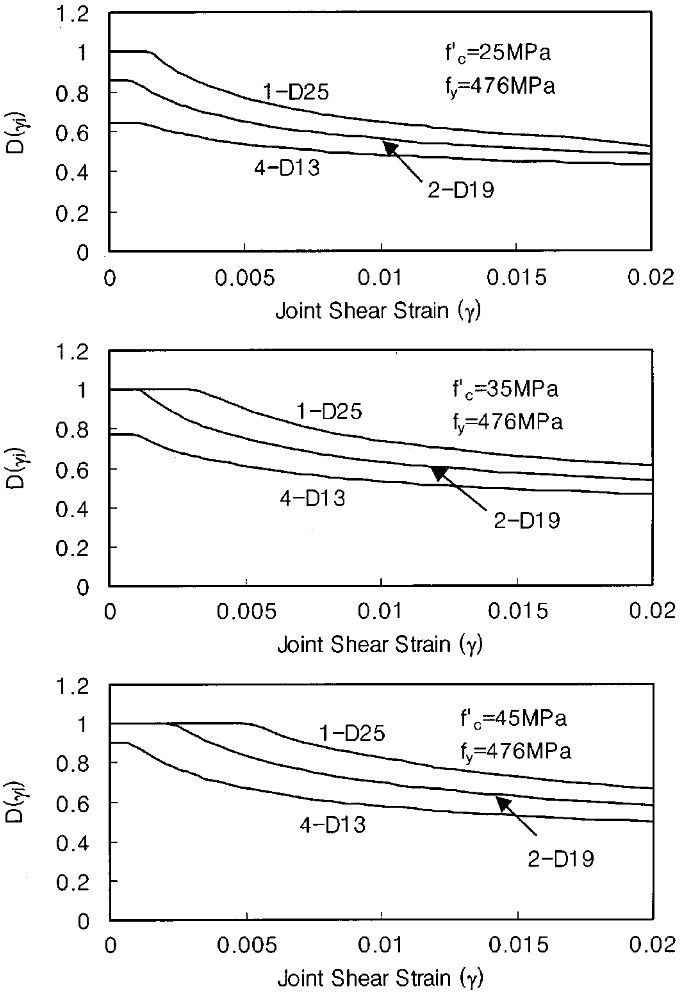


Fig. 7 Effect of concrete strength and transverse steel layout on joint performance

within the joint. The comparison is made in Fig. 7 for various joint steel configuration and concrete compression strengths. The joint shear capacity envelopes indicate that the strength degradation is delayed as the less number of joint shear steel is placed for the same amount of joint steel and higher strength concrete is used. In the analysis, it is assumed that the bond strength is maintained.

5. Conclusions

This paper sets forth the theoretical framework around a strut and tie modeling approach that predicts the inelastic performance of beam-column connections. Based on the investigation presented herein it is concluded that the post-elastic behavior of beam-column connections can be effectively modeled using the strut-tie technique with a fan-shaped crack pattern. The theory shows that following initial concrete cracking and the subsequent yielding of the transverse shear reinforcement, the compressive strength of the concrete struts degrades as the diagonal tensile strain within the joint panel grows.

Unfortunately, there is a paucity of experimental results available to validate the approach. Evidently, this is because few researchers take detailed measurements of shear force-deformation (shear strain) behavior of connections. Notwithstanding, the strut-tie analysis approach shows promise against the experimental results of one full-scale test presented herein. Good agreement between the predicted and observed joint shear force-deformation results is demonstrated. Further validation remains the subject of future and ongoing research.

It has also been the intent of this research to use the results as part of design studies as a means to explore alternative ways of reinforcing beam-column joints. The preliminary study presented in this paper suggests that a large concentration of joint steel clustered near the mid-height of the joint may be more effective in providing post-cracking ductility and delaying the strength degradation of the diagonal concrete struts. It is recommended that new near-full scale carefully instrumented experiments be conducted to substantiate this finding.

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