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A computer program for the analysis of reinforced concrete frames with cracked beam elements

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Abstract. An iterative procedure for the analysis of reinforced concrete frames with beams in cracked state is presented. ACI and CEB model equations are used for the effective moment of inertia of the cracked members. In the analysis, shear deformations are taken into account and reduced shear stiffness is considered by using effective shear modulus models available in the literature. Based on the aforementioned procedure, a computer program has been developed. The results of the computer program have been compared with the experimental results available in the literature and found to be in good agreement. Finally, a parametric study is carried out on a two story reinforced concrete frame.

Key words: reinforced concrete frames; analysis; effective moment of inertia; effective shear modulus; deflections; beam in cracked state.

1. Introduction

In practice, the analysis of reinforced concrete frames is usually carried out by using a linear elastic model, which consists of uncracked beam elements. In fact, if cracking occurs in some members due to excessive load, aforementioned analysis will not be valid. Because, the flexural rigidity of these members will decrease resulting in additional deflections and a redistribution of the internal forces. In reinforced concrete construction, a designer must satisfy not only the strength requirements, but also serviceability requirements, and therefore the control of the deformations becomes more important. On the other hand, the comparison between theoretical and experimental results of the member deflections can represent a valuable verification of theoretical model studies. Hence, for accurate determination of the member deflections, the prediction of flexural and shear rigidities of reinforced concrete members after cracking becomes important. The concrete in the tensile regions of the members, even after cracking, has residual flexural and shear resistances due to the bond action between steel and concrete which contribute to flexural and shear rigidities of the members. However, it is difficult to accurately estimate their contribution due to complexities in the actual behaviour of reinforced concrete members.

Cracked state in reinforced concrete elements can be considered by several methods, which are available in the literature (Ngo and Scordelis 1967, Nilson 1968, Channakeshava and Sundara 1988). These methods consider the constitutive relationships of both steel and concrete together

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with the bond-slip relationship. Due to the complexities of the actual behaviour of the reinforced concrete frame; cumbersome computations are necessarily carried out. Hence, these methods can not be easily adopted by the design engineers.

A simplified method for the finite element analysis of reinforced concrete beam elements in cracked state was introduced by Cosenza (1990). In Cosenza's study, the contribution of tensile resistance of concrete to flexural rigidities were considered by moment-curvature relationship models, such as constant tension stiffening, linear tension stiffening, ACI (ACI, Committee 435 1966) and CEB (1985) models. In the evaluation of the flexibility influence coefficients, simply supported beam element with uniformly distributed load was used. In fact, the structures are usually subjected to point span loads as well as uniformly distributed loads. In addition, beams are usually tested under combinations of point loads in the laboratory. Also, the shear deformations were not considered in the formulation. Whereas, after the development of cracks, shear deformation can be large and should be included in the analysis. Therefore, reduction of shear rigidity due to cracking should also be considered for improving the results of the analysis.

In practice, it is always possible to come across reinforced concrete framed buildings with some regions in a cracked state. Therefore, it may be important to analyze an existing structure of this type. On the other hand, for the ductile behavior of frames some cracking is desired at the beams rather than columns under excessive gravity and lateral loads. The ductile behavior of frames can be achieved if it is designed so that the sum of the ultimate resistance moment of columns framing into the beam-column joint is at least twenty percent more than the sum of the ultimate moment resistance of the beams framing into the same joint. This condition complies with the ACI-318-89 "Strong Column-Weak Beam" requirement for frames. Therefore, in the present study cracking is considered only for beam elements. Hence, the linear elastic stiffness equation is used for columns. The cracked member stiffness equation for the beams is evaluated, including a point span load as well as uniformly distributed one, taking shear deformation effect into consideration. In obtaining the flexibility influence coefficients a cantilever beam is used which greatly simplifies the integral equations for the case of point load.

In the present study, a computer program is developed for the analysis of reinforced concrete frames with craked beam elements. In the program, the variation of the shear rigidity due to cracking is considered by reduced shear stiffness models (Al-Mahaidi 1978, Cedolin and dei Poli 1977, Yuzugullu and Schnobrich 1973), and the effective flexural rigidity is evaluated by ACI and CEB model equations. The results of the program are verified with the experimental results available in the literature. Finally, a parametric study is carried out on a two-story reinforced concrete frame.

2. Models for the effective flexural rigidity of the member in cracked state

The effective moment of inertia which includes the effect of cracking and the participation of tensile concrete between cracks is given by ACI and CEB in the following forms:

ACI Model

$$I_{eff}=\left(\frac{M_{cr}}{M}\right)^m I_g+\left[1-\left(\frac{M_{cr}}{M}\right)^m\right]I_{cr}, \qquad \text{for } M \geq M_{cr} \tag{1a}$$

$$I_{eff} = I_g, \qquad \text{for } M<M_{cr} \tag{1b}$$

where m=3. This equation was first presented by Branson (1963) with m=4 when I_{eff} is required for the calculation of curvature in an individual section.

CEB Model

$$I_{eff}=\left[\beta_1\beta_2\left(\frac{M_{cr}}{M}\right)^2\frac{1}{I_g}+\left(1-\beta_1\beta_2\left(\frac{M_{cr}}{M}\right)^2\right)\frac{1}{I_{cr}}\right]^{-1}, \quad \text{for } M\geq M_{cr} \tag{2a}$$

$$I_{eff}=I_g, \text{ for } M<M_{cr} \tag{2b}$$

in which, β_1=1 for high bond reinforcement and 0.5 for plain bars; β_2=1 for the first loading and 0.5 for the loads applied in a sustained manner or in a large number of load cycles (Ghali and Favre 1986).

In Eqs. (1) and (2), M_{cr} is the moment corresponding to flexural cracking, M is the bending moment considered, I_g and I_{cr} are the moments of inertia of the gross section and the cracked transformed section, respectively.

In the literature (Cosenza 1990, Sakai and Kakuta 1980, Al-Shaikh and Al-Zaid 1993), many authors have shown that, for the estimation of the instantaneous deflection, the effective moment of inertia procedure given by ACI and CEB is the best among the commonly accepted simplified methods. Hence, these models are used in the present study.

3. Models for reduced shear stiffness of concrete

Effective shear modulus of concrete due to cracking is given by several models in the literature.

Al-Mahaidi (1978) suggested the following hyperbolic expression for the reduced shear stiffness $\overline{G}_c$ to be employed in the constitutive relation of cracked concrete

$$\overline{G}_c=\frac{0.4G_c}{\varepsilon_1/\varepsilon_{cr}}, \quad \text{for } \varepsilon_1\geq\varepsilon_{cr} \tag{3a}$$

$$\overline{G}_c=G_c \quad \text{for } \varepsilon_1<\varepsilon_{cr} \tag{3b}$$

in which, G_c is the elastic shear modulus of uncracked concrete, ε_1 is the principal tensile strain normal to the crack and ε_{cr} is the cracking tensile strain.

Cedolin and dei Poli (1977) observed that a value of $\overline{G}_c$ linearly decreasing with the fictitious strain normal to the crack would give better predictions for beams failing in shear, and recommended the following equation

$$\overline{G}_c=0.24\,G_c\,(1\text{-}250\,\varepsilon_1). \tag{4}$$

Yuzugullu and Schnobrich (1973) used a constant value for reduced shear modulus

$\overline{G}_c$=0.25 G_c, for deep beams

$\overline{G}_c$=0.125 G_c, for shear wall and shear wall-frame systems. (5)

In the computer program developed in the present study, aforementioned models are used for the effective shear modulus of cracked concrete.

4. Formulation of the problem

In this section, the flexibility influence coefficients of a beam element will first be evaluated, then using compatibility conditions and equilibrium equations, stiffness matrix and the load vector of a beam element with some regions in cracked state will be obtained.

A typical member subjected to a point and a uniformly distributed loads, and positive end forces with corresponding displacements are shown in Fig. 1. For computing the relations between nodal actions and basic deformation parameters of a general planar element, a cantilever model is used (Fig. 2). The basic deformation parameters of a general planar element may be established by applying unit loads in turn in the direction of 1-3. Then, the compatibility conditions give the following equations

$$f_{11}\, P_1 = d_1 \tag{6a}$$

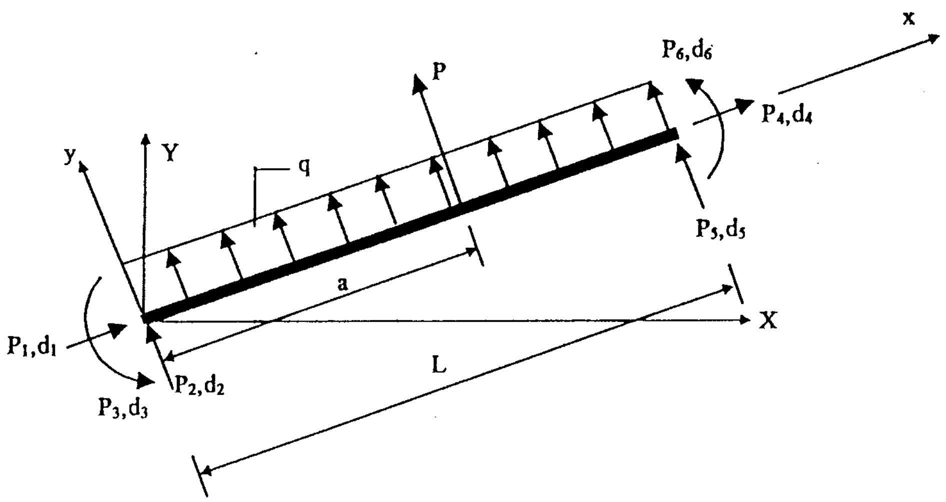


Fig. 1 A typical beam element

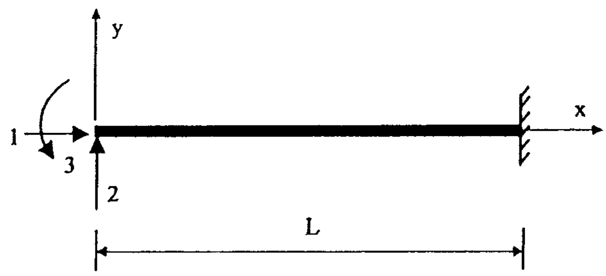


Fig. 2 Model for computing the relations between the basic nodal actions and the basic deformation parameters

$$f_{22}\,P_2 + f_{23}\,P_3 = d_2 \tag{6b}$$

$$f_{32}\,P_2 + f_{33}\,P_3 = d_3 \tag{6c}$$

or in matrix form

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & f_{23} \\ 0 & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \tag{7}$$

where f_{ij} is the displacement in i-th direction due to the application of unit loads in j-th direction, and can be obtained by using the principal of virtual work as follows

$$f_{ij} = \int_0^L \frac{M_i M_j}{E_c I_{eff}} dx + \int_0^L \frac{V_i V_j}{\bar{G}_c A} s\,dx. \tag{8}$$

In Eq. (8), (M_i, V_i) and (M_j, V_j) are the bending moments and shear forces due to the application of unit loads in i-th and j-th directions, respectively, E_c denotes the modulus of elasticity of concrete, A and s are the cross sectional area and the shape factor, respectively.

Inverting the flexibility matrix in Eq. (7) gives the following stiffness influence coefficients

$$K_{11} = 1/f_{11} = E_c\,A/L \tag{9a}$$

$$K_{22} = f_{33}/(f_{22}f_{33} - f_{32}f_{23}) \tag{9b}$$

$$K_{23} = K_{32} = -f_{23}/(f_{22}f_{33} - f_{32}f_{23}) \tag{9c}$$

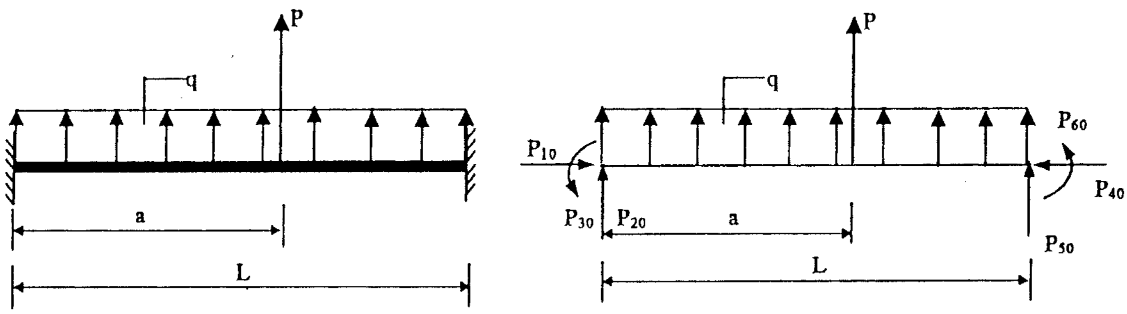
$$K_{33} = f_{22}/(f_{22}f_{33} - f_{32}f_{23}) \tag{9d}$$

$$K_{12} = K_{13} = K_{21} = K_{31} = 0. \tag{9e}$$

Using equilibrium conditions, the following mixed terms are obtained

$$K_{52} = -K_{55} = -K_{22} \tag{10a}$$

$$K_{53} = -K_{23} \tag{10b}$$



(a) a member with fixed ends (b) fixed end forces of the member

Fig. 3 Fixed end forces of the member subject to a point and a uniformly distributed loads

$$K_{62} = -K_{65} = K_{22}\,L - K_{32} \tag{10c}$$

$$K_{66} = K_{33} + K_{22}\,L^2 - 2\,K_{32}\,L. \tag{10d}$$

The member fixed-end forces for the case of a point and a uniformly distributed loads can be obtained by means of compatibility and equilibrium conditions as follows (Fig. 3).

$$P_{10} = P_{40} = 0 \tag{11a}$$

$$P_{20} = -(f_{33}f_{20} - f_{23}\,f_{30})/(f_{22}f_{33} - f_{23}\,f_{32}) \tag{11b}$$

$$P_{30} = -(f_{22}f_{30} - f_{23}\,f_{20})/(f_{22}f_{33} - f_{23}\,f_{32}) \tag{11c}$$

$$P_{50} = -(q\,L + P + P_{20}) \tag{11d}$$

$$P_{60} = -[-q\,L^2/2 - P\,(L-a) - P_{20}\,L + P_{30}] \tag{11e}$$

in which f_{i0} (i=2, 3) is the displacement in i-th direction due to the application of span loads which can be evaluated by means of the principal of virtual work in the following form

$$f_{i0} = \int_0^L \frac{M_i M_0}{E_c I_{eff}}dx + \int_0^L \frac{V_i V_0}{\bar{G}_c A}s\,dx \tag{12}$$

where M_0 and V_0 are the bending moment and shear force due to the span loads. Finally, the member stiffness equation can be obtained as

$$\underline{k}\;\underline{d} + \underline{P}_0 = \underline{P} \tag{13}$$

where $\underline{k}$ (6×6) is the stiffness matrix, $\underline{d}$ (6×1) is the displacement vector, $\underline{P}_0$ (6×1) is the fixed end force vector and $\underline{P}$ (6×1) is the total end force vector of the member. Eq. (13) is given in the member coordinate system (x, y). Therefore, it should be transformed to the structure coordinate system (X, Y).

The flexibility influence coefficients can now be obtained by using Eqs. (8) and (12) with the following procedure.

The bending moments and shear forces as seen in Eqs. (8) and (12) are given in terms of the x coordinate as follows

$$M_2(x) = x; \qquad V_2(x) = 1 \tag{14a}$$

$$M_3(x) = -1; \qquad V_3(x) = 0 \tag{14b}$$

$$M_0(x) = \begin{cases} \dfrac{qx^2}{2}, & 0 \le x \le a \\[2ex] \dfrac{qx^2}{2} + P(x-a), & a < x \le L \end{cases} \tag{14c}$$

$$V_0(x) = \begin{cases} qx, & 0 \le x \le a \\ qx + P, & a < x \le L \end{cases} \tag{14d}$$

Using Eqs. (8), (12) and (14) the flexibility influence coefficients can be obtained as

$$f_{22}=\frac{1}{E_c}\int_0^L \frac{x^2}{I_{eff}}dx+\frac{k}{A}\int_0^L \frac{1}{\bar{G}_c}dx \tag{15a}$$

$$f_{23}=\frac{1}{E_c}\int_0^L \frac{x}{I_{eff}}dx \tag{15b}$$

$$f_{33}=\frac{1}{E_c}\int_0^L \frac{1}{I_{eff}}dx \tag{15c}$$

$$f_{20}=\frac{q}{2E_c}\int_0^L \frac{x^3}{I_{eff}}dx+\frac{qk}{A}\int_0^a \frac{x}{\bar{G}_c}dx+\frac{P}{E_c}\int_a^L \frac{x(x-a)}{I_{eff}}dx+\frac{k}{A}\int_a^L \frac{(qx+P)}{\bar{G}_c}dx \tag{15d}$$

$$f_{30}=-\frac{q}{2E_c}\int_0^L \frac{x^2}{I_{eff}}dx-\frac{P}{E_c}\int_a^L \frac{(x-a)}{I_{eff}}dx \tag{15e}$$

It should be noted that, since the member has cracked and uncracked regions, integral operations in Eq. (15) will be carried out at each region individually. In general, the member has three cracked regions and two uncracked regions as seen in Fig. 4.

In the cracked regions where $M>M_{cr}$, I_{eff} and $\bar{G}_c$ vary with M along the region. Therefore the integral values in these regions should be determined by a numerical integration technique. The

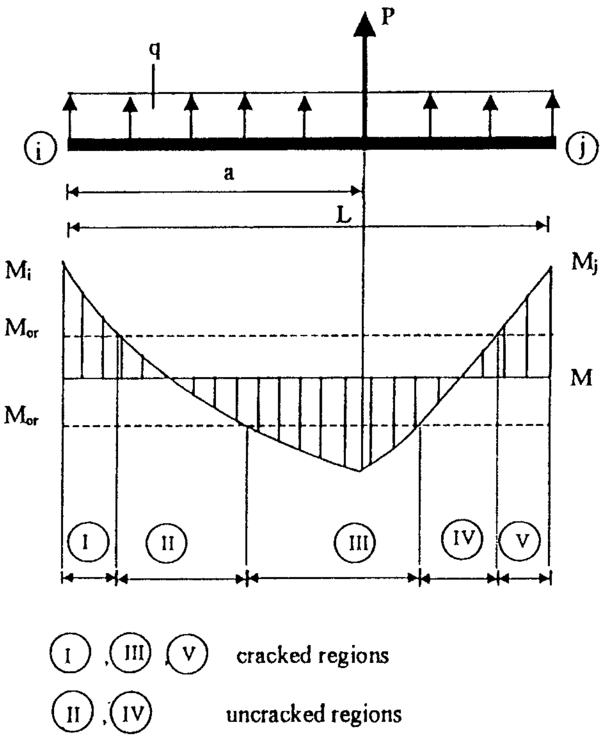


Fig. 4 Cracked and uncracked regions of the member

variation of effective moment of inertia and effective shear modulus of concrete in the cracked regions necessitate the redistribution of the moments in the structure. Hence, iterative procedure should be applied to obtain the final deflections and internal forces of the structure.

5. Computer program

In this section, a general purpose computer program, called 'CRACK', developed for the analysis of reinforced concrete frames with beam elements in the cracked state, is introduced.

The program is coded in *FORTRAN 77* language for *MS-DOS* operating system. The flow chart

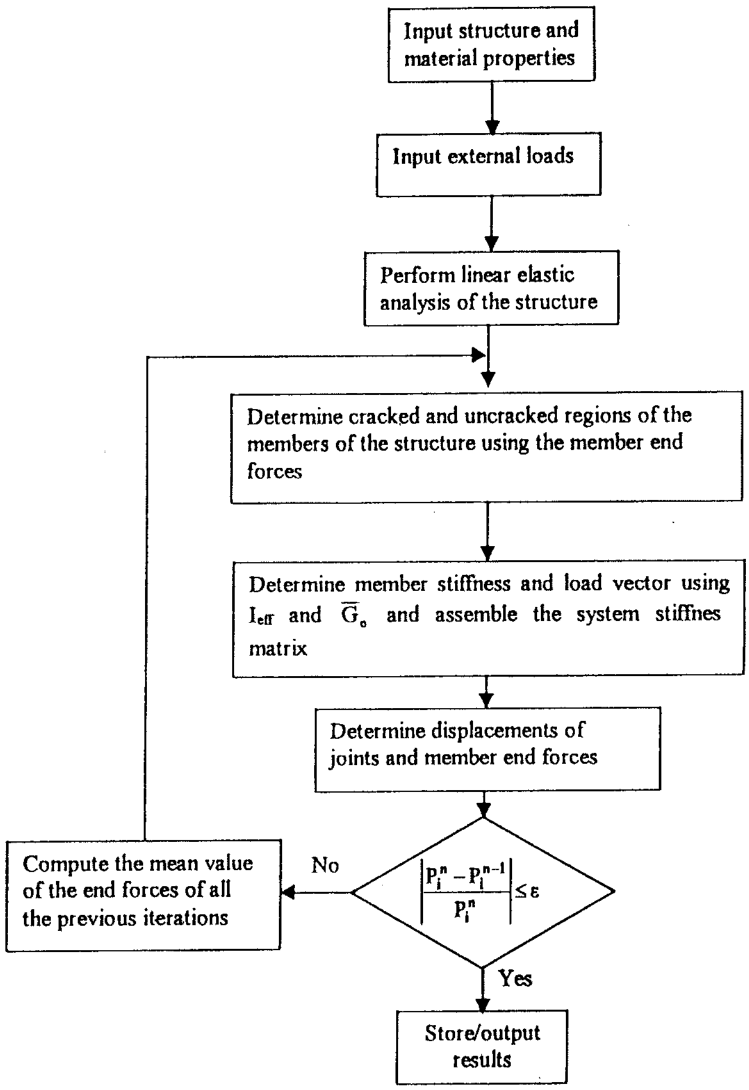


Fig. 5 Solution procedure of the program

of the solution procedure of the program is given in Fig. 5. In the solution procedure, the member end forces used at each iteration step are taken as the mean value of the end forces of all previous iterations. This procedure accelerates the convergence of the algorithm. In the program,

$$\left|\frac{P_i^n - P_i^{n-1}}{P_i^n}\right| \leq \varepsilon \tag{16}$$

is used as the convergence criterion. Here, ε is the convergence factor, n is the iteration number and P_i^n (i=1-6) is the end forces of each member of the structure for the n-th iteration.

The input data file has five data blocks, which are named 'General Information', 'Coordinates', 'Member Properties', 'Restraints', and 'Loads'. The description of a data file will be illustrated in the next section.

Due to space limitation, the listing of the computer program is not given in the paper. A PC version and the manual of the program can be obtained free of charge from the authors upon request.

6. Verification of theoretical results and a frame example

In this section, three examples are presented. The first two examples are taken from the literature to verify the results of the computer program. The third example introduces the parametric study of a two story single bay reinforced concrete frame.

6.1. Example 1

In this example, the test results given by Al-Shaikh and Al-Zaid (1993) for a simple beam with a mid-span load (Fig. 6) are compared with the results of the program 'CRACK'. The test beam is modeled by two beam elements in order to compute mid-span deflection. In the analysis, ACI and Al-Mahaidi (1978) models are used for the effective moment of inertia and shear modulus of concrete in cracked region, respectively.

The comparison between the test and theoretical results for the mid-span deflection is given in Table 1. As seen in Table 1, the theoretical results are in good agreement with the test results.

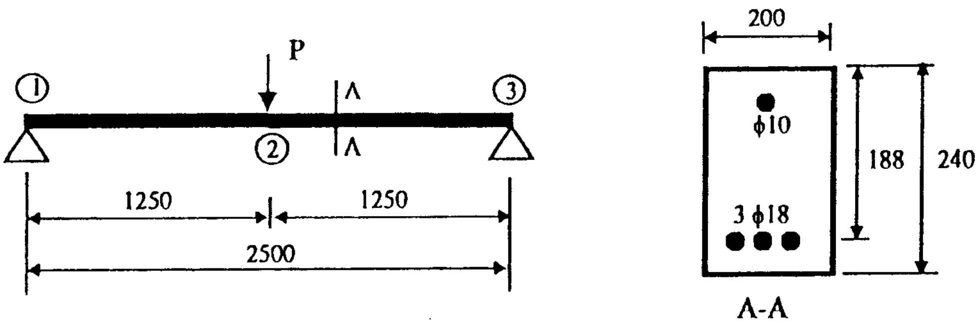


Fig. 6 Simply supported beam with midspan load Al-Shaikh and Al-Zaid (1993)(dimensions in mm)

Table 1 Measured and predicted mid-span deflection of simply supported beam

Total load P (kN)	Midspan deflection (mm)		Ratio (B/A)
	Test results of Al-Shaikh and Al-Zaid (1993) (A)	Present study (B)	
16.43	0.96	0.86	0.90
19.62	1.25	1.22	0.98
23.81	1.70	1.76	1.04
29.18	2.35	2.49	1.06
36.06	3.27	3.43	1.05
40.55	3.85	4.04	1.05
44.70	4.31	4.60	1.07
50.68	5.04	5.39	1.07
55.74	5.68	6.06	1.07
60.60	6.32	6.70	1.06
		mean ratio	1.04

6.2. Example 2

In this example, the two span continuous beam tested by Washa and Fluck (1956) are considered. The continuous beam is modelled by six beam elements as shown in Fig. 7. The cross-sections of the beams, the uniform loads and the spans are also shown in the figure. In the analysis, I_{eff} is predicted using ACI and CEB models and $\overline{G}_c$ is evaluated using Al-Mahaidi's model. The input data file for (X1, X4) beam is presented in Table 2. In computing tensile strength and modulus of elasticity of concrete, the following equations (TS500 1984) are used.

$$f_{ct}=0.35\sqrt{f_c} \quad (\text{N/mm}^2) \tag{17a}$$

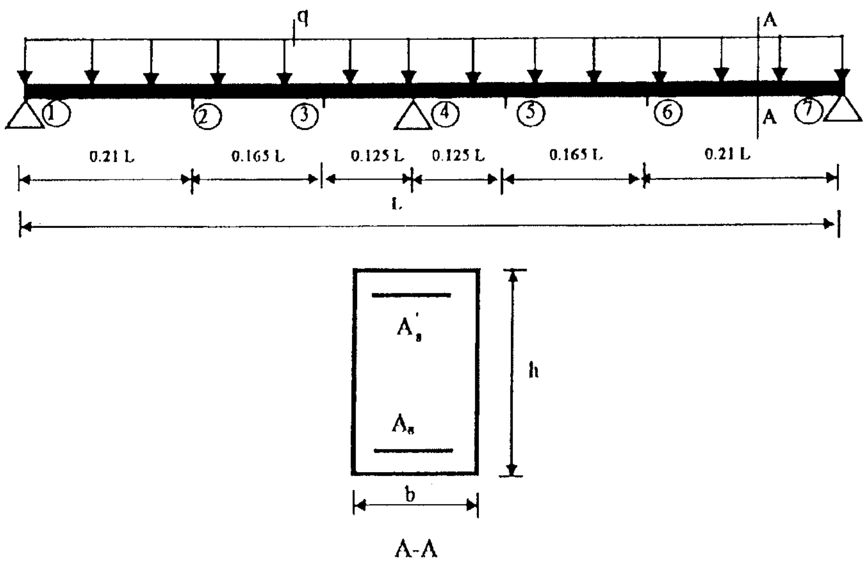
$$E_c=3250\sqrt{f_c}+14000 \quad (\text{N/mm}^2) \tag{17b}$$

where f_c and f_{ct} are the design characteristic compressive and tensile strengths of concrete, respectively. The cracking moment, M_{cr} is calculated by the program using the following equations

$$M_{cr}=f_r\, I_g\, /y_t \tag{18}$$

where f_r is the flexural tensile strength of concrete which is taken as two times of the design characteristic tensile strength of the concrete (TS500 1984), y_t is the distance from centroid of gross section to the extreme fiber in tension.

The comparison between the experimental and the theoretical deflections at joint 2 obtained by the program 'CRACK' and Cosenza (1990) are given in Table 3. It is seen that, the deflections calculated by 'CRACK' using ACI method (Eq. 1a with m=4) and CEB method (Eq. 2a with $\beta_1\beta_2$=0.8) are in good agreement with the experimental results.



(a) two-spans continuous beam model

Beam	q (kN/m)	L (mm)	b (mm)	h (mm)	Positive Moment Region		Negative Moment Region	
					A_s (mm²)	A'_s (mm²)	A_s (mm²)	A'_s (mm²)
X1,X4	2.77	12192.0	152.4	203.2	400	400	684	600
X2,X5	2.77	12192.0	152.4	203.2	400	200	684	600
X3,X6	2.77	12192.0	152.4	203.2	400	-	684	600
Y1,Y4	2.13	12679.7	304.8	127.0	516	516	1000	1000
Y2,Y5	2.13	12679.7	304.8	127.0	516	258	1000	1000
Y3,Y6	2.13	12679.7	304.8	127.0	516	-	1000	1000
Z1,Z4	0.99	10668.0	304.8	76.2	284	284	516	645
Z2,Z5	0.99	10668.0	304.8	76.2	284	142	516	645
Z3,Z6	0.99	10668.0	304.8	76.2	284	-	516	645

(b) beam cross-sections

Fig. 7 Two span continuous beams of Washa and Fluck

6.3. Example 3

In this example, the two-story reinforced concrete frame shown in Fig. 8 is analysed by the program 'CRACK'. The frame is subjected to two lateral point loads at the floor levels, and uniformly distributed and point span loads on the two beams. The intensity of the uniform load is

Table 2. Description of input data

ACI STRUCTURAL JOURNAL, JANUARY 1956 (WASHA & FLUCK): *Title*

***GENERAL INFORMATION:** Block name*

Number of joints, number of elements, cracking parameter (1: considered, 0: not considered), shear deformation parameter (1: considered, 0: not considered), Young's modulus of steel.

7 6 1 1 2.0E2

Model and its parameters for effective moment of inertia

'ACI' 4.0 0.0

Model for effective shear modulus

'ALM'

COORDINATES: Block name

Joint number, *X* coordinate and *Y* coordinate

1	0.0	0.0
2	2560.3	0.0
3	4572.0	0.0
4	6096.0	0.0
5	7620.0	0.0
6	9144.0	0.0
7	12192.0	0.0

MEMBER PROPERTIES: *Block name*

Element number, topology (I and J), cross-sectional dimensions (b, h), concrete cover, the modulus of elasticity and design characteristic tensil strength of concrete, areas of tension and conpression steel

1	1	2	152.4	203.2	50.0	29.964	17.19E-4	400.00	400.00
2	2	3	152.4	203.2	50.0	29.964	17.19E-4	400.00	400.00
3	3	4	152.4	203.2	50.0	29.964	17.19E-4	684.00	600.00
4	4	5	152.4	203.2	50.0	29.964	17.19E-4	684.00	600.00
5	5	6	152.4	203.2	50.0	29.964	17.19E-4	400.00	400.00
6	6	7	152.4	203.2	50.0	29.964	17.19E-4	400.00	400.00

RESTRAINTS: *Block name*

Joint number, restraint conditions for translational and rotational displacements (1: inactive, 0: active)

1	1	1	0
2	1	0	0
3	1	0	0
4	1	1	0
5	1	0	0
6	1	0	0
7	1	1	0

LOADS: *Block name*

Number of loaded element, number of joints subjected to direct loads

6 0

Element number, uniform load (force/length), point load intensity and its distance from end I

1	-2.77E-3	0.0	0.0
2	-2.77E-3	0.0	0.0
3	-2.77E-3	0.0	0.0
4	-2.77E-3	0.0	0.0
5	-2.77E-3	0.0	0.0
6	-2.77E-3	0.0	0.0

Joint number, joint loads in *X*, *Y* and rotational directions

Table 3 Comparison of experimental (Washa and Fluck 1956) and predicted deflections at joint 2

Beam No	Deflection at joint 2 (mm)					Ratio			
	Experimental (A)	Eq. (1a) m=4		Eq. (2a) β_1 β_2=0.8		B/A	C/A	D/A	E/A
		Present study (B)	Cosenza (1990) (C)	Present study (D)	Cosenza (1990) (E)				
X1,X4	14.2	14.2	16.5	14.1	16.0	1.00	1.16	0.99	1.13
X2,X5	14.4	14.2	16.8	14.1	16.3	0.98	1.16	0.98	1.12
X3,X6	13.2	14.3	17.1	14.2	16.6	1.08	1.09	1.08	1.05
Y1,Y4	22.6	22.5	25.7	22.1	25.1	1.00	1.14	0.98	1.11
Y2,Y5	23.6	22.7	26.2	22.2	25.7	0.96	1.11	0.94	1.09
Y3,Y6	25.4	22.8	26.7	22.4	26.2	0.90	1.05	0.88	1.03
Z1,Z4	26.4	28.0	33.0	28.0	32.3	1.06	1.25	1.06	1.22
Z2,Z5	28.7	28.1	33.4	28.1	32.6	0.98	1.16	0.98	1.13
Z3,Z6	30.5	28.2	33.8	28.3	33.0	0.92	1.11	0.93	1.08
				Mean ratio		0.98	1.14	0.98	1.11

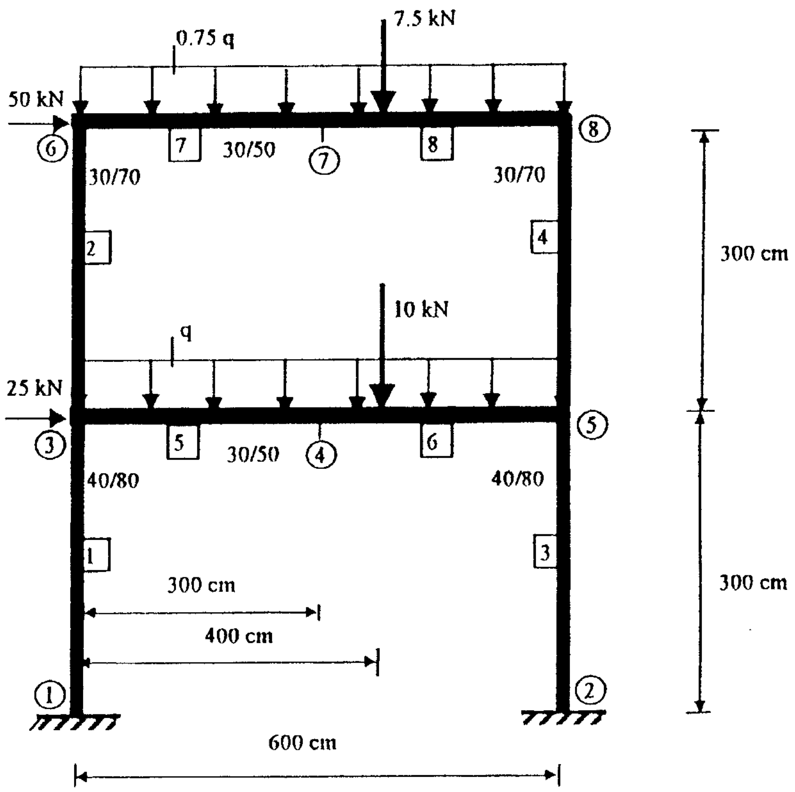


Fig. 8 Two story reinforced concrete frame

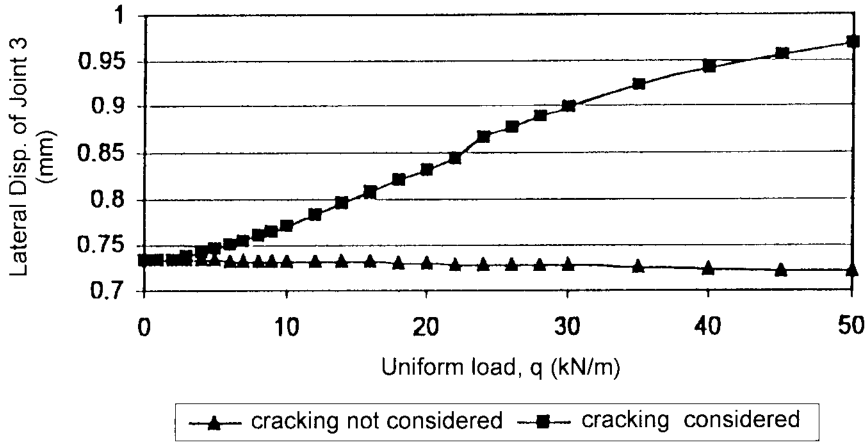


Fig. 9 The variation of the lateral displacement of joint 3 with uniform load

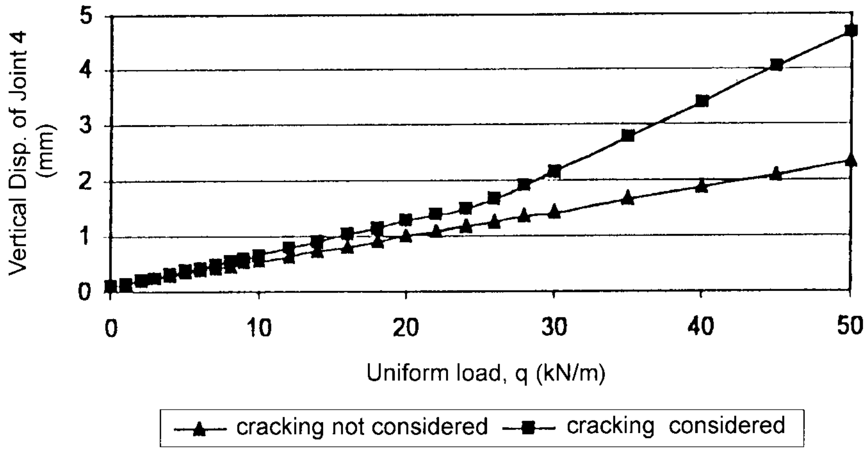


Fig. 10 The variation of the vertical displacement of Joint 4 with uniform load

varied from 0 to 50 kN/m while others remain constant. The variation of the lateral displacement of joint 3 and the vertical displacement of joint 4 with the uniform load, when cracking is considered and not considered for beams, are shown in Figs. 9 and 10. As seen from the figures, the differences, in the lateral displacement of joint 3 and the vertical displacement of joint 4 between the two cases increase with the increase in the loads. The difference becomes significant at higher loads, such as, 34% for the lateral displacement and 102% for the vertical displacement for q=50 kN/m.

7. Conclusions

An iterative procedure has been developed to analyze reinforced concrete frames with elements in a cracked state. In the procedure, cracking is considered only for beam elements, hence, for columns, the linear elastic stiffness equation is used. This assumption is compatible with the "Strong Column-Weak Beam" frame requirement. Therefore, the proposed procedure should not be used for the reinforced concrete frames in which the aforementioned requirement is not satisfied.

The variation of shear rigidity in the craked regions of beams has been considered by employing various reduced shear stiffness models and the variation of the flexural rigidity of a cracked beam element has been evaluated by using ACI and CEB model equations.

The capability and the reliability of the procedure have been tested by means of comparisons with the theoretical and experimental results available in the literature. The theoretical results of the procedure have been found to be in good agreement with the experimental results. This procedure can be used to predict the deflections of statically determinate and indeterminate beams. The procedure can also be used to analyze reinforced concrete frames with some beams in a craked state for the purpose of repair and strengthening.

The proposed procedure is efficient from the viewpoints of computational effort and convergence rate. Since the procedure is an iterative one, a computer program has been developed for rapid application. Although the procedure and the computer program are evaluated for two-dimentional frames, they can be extended easily to cover three-dimensional structures.

In the present study, the time dependent effects in concrete have not been considered. However, these effects can be included in the analysis by employing any of the available methods, such as effective modulus method (Faber 1927) and age adjusted effective modulus method (Bazant 1972).

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