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Stress analysis with arbitrary body force by triple-reciprocity BEM

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Abstract. Linear stress analysis without body force can be easily solved by means of the boundary element method. Some cases of linear stress analysis with body force can also be solved without a domain integral. However, domain integrals are generally necessary to solve the linear stress problem with arbitrary body forces. This paper shows that the linear stress problem with arbitrary body forces can be solved approximately without a domain integral by the triple-reciprocity boundary element method. In this method, the distribution of arbitrary body forces can be interpolated by the integral equation. A new computer program is developed and applied to several problems.

Key words: elasticity; body force; boundary element method; computational mechanics; numerical analysis.

1. Introduction

The domain integral becomes necessary in the boundary element method under the presence of body forces. However, the domain integrals for the cases of uniform gravitational force and centrifugal force can be easily transformed into the boundary integrals by means of the Galerkin tensors (Danson 1983).

The case of non-uniform body force, in which the density is a variable, has not yet been clarified. On the other hand, Neves, Nowak and Brebbia (1989, 1991, 1993, and 1994) have proposed the conventional multiple-reciprocity method, which uses a boundary-only formulation. In the conventional multiple-reciprocity method, the body force must be given analytically, and infinite numbers of fundamental solutions are necessary to make the solution converge in the case of complicated body force. Ochiai, Sekiya and Ishida (1994, 1996) have proposed a method using the boundary-type cells for coarse approximation.

In this paper, the distribution of body force is interpolated by using the integral equation. Using these interpolated values, the linear elasticity problem with arbitrary body force can be solved

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without internal cells by the triple-reciprocity boundary element method. The conventional boundary element method needs internal cells for the domain integral. The internal cells decrease the merit of the boundary element method, in which the arrangement of data is simple. In the presented method, the fundamental solution of lower order is used. This paper deals with the extension of the improved multiple-reciprocity boundary element method, which has been applied for heat conduction analysis and thermal stress analysis (Ochiai and Sekiya 1994, 1995, 1996).

2. Theory

2.1. Fundamental equation

In order to carry out the stress analyses for the case in which the body force b_j is present, the following boundary integral equation must be solved (Danson 1983, Brebbia *et al.* 1984):

$$c_{ij}u_j = \int_\Gamma \{u^{[1]}_{ij}p_j - p^{[1]}_{ij}u_j\}d\Gamma + \int_\Omega u^{[1]}_{ij}b_j d\Omega, \tag{1}$$

where c_{ij} is the free coefficient, u_j and p_j are the j-th component of the displacement and the surface traction, and the notations Γ and Ω denote the boundary and the domain, respectively. As shown in Eq. (1), when there exits an arbitrary body force, the domain integral becomes necessary. Using r to denote the distance between the observation point and the loading point, Kelvin's solutions $u^{[1]}_{ij}$ and $p^{[1]}_{ij}$ are given by

$$u^{[1]}_{ij} = \frac{1}{8\pi(1-\nu)G}\left[(3-4\nu)\delta_{ij}\ln\left(\frac{1}{r}\right) + r_{,i}r_{,j}\right], \tag{2}$$

$$p^{[1]}_{ij} = \frac{-1}{4\pi(1-\nu)r}\left\{[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}]\frac{\partial r}{\partial n} - (1-2\nu)(r_{,i}n_j - r_{,j}n_i)\right\}, \tag{3}$$

where, the shearing modulus G and Poisson's ratio ν are assumed to be constant. Moreover, let us denote $\nu' = \nu/(1+\nu)$ in the case of plane stress, $r_{,i} = \partial r/\partial x_i$. In the case of arbitrary body force, it is generally difficult to transform the domain integral of Eq. (1) into the boundary integral.

2.2. Interpolation of body force

The distribution of body force b_j is interpolated by using the integral equation in order to transform the domain integral into the boundary one. The interpolation must be considered separately for the individual directions of body force b_j (j=x, y). Deformation of a thin plate is utilized in order to interpolate the distribution of body force $b_j^{s[1]}$, where superscript S indicates a surface distribution. The following equations can be used for interpolation:

$$\nabla^2 b_j^{S[1]} = -b_j^{S[2]} \tag{4}$$

$$\nabla^2 b_j^{S[2]} = -b_j^{P[3]} \tag{5}$$

where $b_j^{P[2]}$ is a Dirac-type function, which has a value only at a point. The term $b_j^{S[2]}$ of Eq. (4) corresponds to the sum of curvatures $\partial^2 b_j^{S[1]}/\partial x^2$ and $\partial^2 b_j^{S[1]}/\partial y^2$. From Eqs. (4) and (5), the following equation can be obtained:

$$\nabla^4 b_j^{S[1]} = b_j^{P[3]} \tag{6}$$

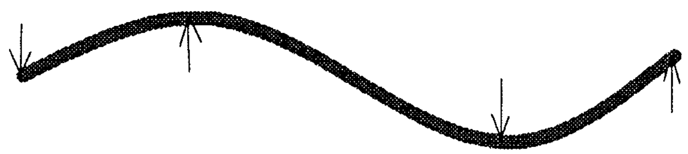
The equation is the same type of equation as that for deformation $w^S{}_1$ of a thin plate with point load P as

$$\nabla^4 w^{S[1]} = \frac{P}{D}, \tag{7}$$

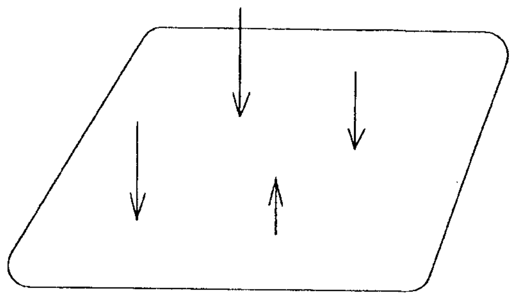
where Poisson's ratio ν=0 and the flexural rigidity D=1. The natural spline comes from a deformation of a thin beam in order to interpolate the one-dimensional distribution, as shown in Fig. 1(a). In the paper, deformation of a thin plate is utilized in order to interpolate the two-dimensional distribution $b_j^{S[1]}$. The deformation $w^{S[1]}$ is given, and the force of point load P is unknown. The force of point load P is obtained inversely from the deformation of the fictitious thin plate, as shown in Fig. 1(b). The term $w^{S[2]}$ corresponds to the moment of the beam. The moment $w^{S[2]}$ on the boundary is assumed to be 0, the same as in the case of the natural spline. Thin means that the thin plate is simply supported. In this method, the distribution of body force is assumed as a free-form surface (Ochiai 1995, 1996, Micchelli 1986, Nira 1987). Eqs. (4) and (5) are similar to the equation which is used to generate the free-form surface using an integral equation.

2.3. Representation of body force by integral equation

Distribution of body force is represented by an integral equation. The harmonic function T_1 and biharmonic function T_2 are used for interpolation.



(a) Thin beam with unknown point load



(b) Fictitious thin plate with unknown point load

Fig. 1 Interpolation using fictitious thin plate

$$T_i = \frac{1}{2\pi}\ln\left(\frac{1}{r}\right) \tag{8}$$

$$T_2 = \frac{r^2}{8\pi}\left[\ln\left(\frac{1}{r}\right) + 1\right] \tag{9}$$

The harmonic function T_1 and biharmonic function T_2 have the next relation:

$$\nabla^2 T_2 = T_1 \tag{10}$$

The body force $b_j^{S[1]}$ is given by Green's theorem and Eqs. (4)~(10) as

$$cb_j^{S[1]}(P) = -\sum_{f=1}^{2}(-1)^f\int_\Gamma\left\{T_f(P,Q)\frac{\partial b_j^{S[f]}(Q)}{\partial n} - \frac{\partial T_f(P,Q)}{\partial n}b_j^{S[f]}(Q)\right\}d\Gamma - \sum_{m=1}^{M} T_2 b_j^{P[3]}{}_m \tag{11}$$

where on the smooth boundary c=0.5 and in the domain c=1. Moreover, $b_j^{S[2]}$ in Eq. (11) is similarly given by

$$cb_j^{S[2]}(P) = \int_\Gamma\left\{T_1(P,Q)\frac{\partial b_j^{S[2]}(Q)}{\partial n} - \frac{\partial T_1(P,Q)}{\partial n}b_j^{S[2]}(Q)\right\}d\Gamma + \sum_{m=1}^{M} T_1 b_j^{P[3]}{}_m \tag{12}$$

Integral Eqs. (11) and (12) are used in order to interpolate the distribution. The thin plate spline $F(p,q)$, which is used to make a free-form surface, is defined as

$$F(p,q) = r^2\ln(r) \tag{13}$$

Eqs. (9) and (13) denote the same type of function.

2.4. Improved multiple-reciprocity boundary element method

Function $u^{[2]}{}_{ij}$ is assumed to be

$$\nabla^2 u^{[2]}{}_{ij} = u^{[1]}{}_{ij} \tag{14}$$

Generally, function $u^{[f]}{}_{ij}$ is defined as

$$\nabla^2 u^{[f+1]}{}_{ij} = u^{[f]}{}_{ij} \tag{15}$$

Let the number of the point body force $b_j^{P[3]}$ be M and the shape of the line body force $b_j^{L[f]}$ be Γ_L. Using Eqs. (4), (5), (10) and Green's theorem, Eq. (1) becomes

$$c_{ij}u_j = \int_\Gamma\{u^{[1]}{}_{ij}p_j - p^{[1]}{}_{ij}u_j\}d\Gamma + \int_\Gamma\sum_{f=1}^{2}(-1)^f\left\{u^{[f+1]}{}_{ij}\frac{\partial b_j^{S[f]}}{\partial n} - \frac{\partial u^{[f+1]}{}_{ij}}{\partial n}b_j^{S[f]}\right\}d\Gamma - \sum_{m=1}^{M} u^{[3]}{}_{ij}b_j^{P[3]}{}_m \tag{16}$$

Next, the function $u^{[f]}{}_{ij}$ is obtained and Kelvin's solution $u^{[1]}{}_{ij}$ is obtained as

$$u^{[1]}{}_{ij} = \frac{-1}{2(1-\nu)G}A^{[1]}{}_{,ij} + \frac{\delta_{ij}A^{[1]}{}_{,kk}}{G}, \tag{17}$$

where $A^{[1]}$ is the biharmonic function, which is given as

$$A^{[1]} = \frac{r^2}{8\pi}\left[\ln\left(\frac{1}{r}\right) + Q\right], \tag{18}$$

$$Q = \frac{7-8\nu}{2(3-4\nu)} \tag{19}$$

Conventional Kelvin's solutions u_{ij} given by Eqs. (2) and (3) can be obtained.
Next, the function $A^{[f]}$ which satisfies

$$\nabla^2 A^{[f+1]} = A^{[f]}, \tag{20}$$

is considered. In the two-dimensional case, function $A^{[f]}$ can be obtained by

$$A^{[f]} = \int \frac{1}{r}\left[\int r A^{[f-1]} dr\right] dr. \tag{21}$$

Function $A^{[f]}$ is generally given by (Ochiai and Nisitani 1996)

$$A^{[f]} = \frac{r^{2f}}{2\pi\{(2f)!!\}^2}\left[\ln\left(\frac{1}{r}\right) + Q + \sum_{e=2}^{f+1}\frac{1}{e}\right], \tag{22}$$

where $(2f)!!=2f(2f-2)\ldots2$ (Abramowitz, *et al.* 1970). Accordingly, Eqs. (16), (18), and (22) yield $u^{[f]}_{ij}$

$$u^{[f]}_{ij} = \frac{-1}{2(1-\nu)G}A^{[f]}_{,ij} + \frac{\delta_{ij}A^{[f]}_{,kk}}{G} \tag{23}$$

Substituting Eq. (22) into Eq. (23), the next functions $u^{[2]}_{ij}$ and $u^{[3]}_{ij}$ and their normal derivatives, which are used in this paper, are obtained as follows:

$$u^{[2]}_{ij} = \frac{r^2}{256(1-\nu)G\pi}\langle\langle \delta_{ij}[4(7-8\nu)\{Q-\ln(r)\}-1] - 2r_{,i}r_{,j}[4\{Q-\ln(r)\}-1]\rangle\rangle \tag{24}$$

$$\frac{\partial u^{[2]}_{ij}}{\partial n} = \frac{r}{128(1-\nu)G\pi}\langle\langle \delta_{ij}r_{,k}n_k[4(7-8\nu)\{Q-\ln(r)\}-(15-16\nu)] - (n_i r_{,j} + n_j r_{,i})[4\{Q-\ln(r)\}-1] + 4r_{,i}r_{,j}r_{,k}n_k\rangle\rangle, \tag{25}$$

$$u^{[3]}_{ij} = \frac{-r^4}{4608(1-\nu)G\pi}\langle\langle[3(11-12\nu)\{\ln(r)-Q\} - 16 + 18\nu]\delta_{ij} + r_{,i}r_{,j}[-12\ln(r) + 5 + 12Q]\rangle\rangle, \tag{26}$$

$$\frac{\partial u^{[3]}_{ij}}{\partial n} = \frac{-r^3}{4608(1-\nu)G\pi}\langle\langle \delta_{ij}r_{,k}n_k[12(11-12\nu)\{\ln(r)-Q\} - 31 + 36\nu] + 2r_{,i}r_{,j}r_{,k}n_k[-12\ln(r) - 1 + 12Q] + (n_i r_{,j} + n_j r_{,i})[-12\ln(r) + 5 + 12Q]\rangle\rangle \tag{27}$$

The i-th component of a unit normal vector is denoted by n_i.

Next, the internal stress is obtained. The relation between stress and displacement is given by

$$\sigma_{ij} = \lambda\delta_{ij}u_{k,k} + \mu(u_{i,j} + u_{j,i}) \tag{28}$$

where Lame's constants λ and μ are defined by

$$\lambda = \frac{2\nu G}{(1-2\nu)}, \tag{29}$$

$$\mu = G. \tag{30}$$

From Eqs. (16) and (28), internal stress is given by

$$\sigma_{ij} = \int_\Gamma [D_{ijk}p_k - S_{ijk}u_k]d\Gamma + \sum_{f=1}^{2}\int_\Gamma (-1)^f\left\{\sigma^{[f+1]}{}_{ijk}\frac{\partial b_k{}^{S[f]}}{\partial n} - \frac{\partial \sigma^{[f+1]}{}_{ijk}}{\partial n}b_k{}^{S[f]}\right\}d\Gamma$$

$$-\sum_{m=1}^{M}\sigma^{[3]}{}_{ijk}b_k{}^{P[3]}{}_m, \tag{31}$$

where functions D_{ijk} and S_{ijk} are given by

$$D_{ijk} = [(1-2\nu)\{\delta_{ki}r_{,j} + \delta_{kj}r_{,i} - \delta_{ij}r_k\} + 2r_{,i}r_{,j}r_{,k}]\frac{1}{4\pi(1-\nu)r}, \tag{32}$$

$$S_{ijk} = \left\{2\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij}r_{,k} + \nu(\delta_{ik}r_{,j} + \delta_{jk}r_{,i}) - 4r_{,i}r_{,j}r_{,k}]\right.$$

$$\left. + 2\nu(n_i r_{,j}r_{,k} + n_j r_{,i}r_{,k}) + (1-2\nu)(2n_k r_{,i}r_{,j} + n_i\delta_{jk} + n_j\delta_{ik}) - (1-4\nu)n_k\delta_{ij}\right\}\frac{G}{2\pi(1-\nu)r^2} \tag{33}$$

Functions $\sigma^{[f]}{}_{ijk}$ and its normal derivatives are given by

$$\sigma^{[2]}{}_{ijk} = \frac{r}{64(1-\nu)\pi}\langle\langle (\delta_{ik}r_{,j} + \delta_{jk}r_{,i})[4(3-4\nu)\{\ln(r) - Q\}$$

$$+ (7-8\nu)] - \delta_{ij}r_{,k}[4(1-\nu)\{ln(r) - Q\} - (1-8\nu)] - 4r_{,i}r_{,j}r_{,k}\rangle\rangle, \tag{34}$$

$$\frac{\partial\sigma^{[2]}{}_{ijk}}{\partial n} = \frac{1}{128(1-\nu)\pi}\langle\langle (\delta_{ik}n_j + \delta_{jk}n_i)[4(3-4\nu)\{\ln(r) - Q\} + (7-8\nu)]$$

$$+ 4(3-4\nu)(\delta_{ik}r_{,j} + \delta_{jk}r_{,i})r_{,m}n_m + \delta_{ij}n_k[-4(1-4\nu)\{\ln(r) - Q\} - 1 + 8\nu]$$

$$- 4(1-4\nu)\delta_{ij}r_k r_{,m}n_{,m} + 8r_{,i}r_{,j}r_{,k}r_{,m}n_m - 4(n_i r_{,j}r_{,k} + n_k r_{,i}r_{,j} + n_j r_{,i}r_{,k})\rangle\rangle, \tag{35}$$

$$\sigma^{[3]}{}_{ijk} = \frac{-r^3}{2304(1-\nu)\pi}\langle\langle \delta_{ij}r_{,k}[-12(1-6\nu)\{\ln(r) - Q\} + (5-18\nu)] + 12(5-6\nu)$$

$$\{\ln(r) - Q\} - (13-18\nu)](\delta_{ik}r_{,j} + \delta_{jk}r_{,i}) + 2r_{,i}r_{,j}r_{,k}[-12\ln(r) - 1 + 12Q]\rangle\rangle, \tag{36}$$

$$\frac{\partial\sigma^{[3]}{}_{ijk}}{\partial n} = \frac{-r^2}{2304(1-\nu)\pi}\langle\langle 2\delta_{ij}r_{,k}r_{,m}n_m[-12(1-6\nu)\{\ln(r) - Q\}$$

$$+ (-1+18\nu) + 2r_{,m}n_m(\delta_{ik}r_{,j} + \delta_{jk}r_{,i})[12(5-6\nu)\{\ln(r) - Q\}$$

$$+ 17 - 18\nu] - 24r_{,i}r_{,j}r_{,k}r_{,m}n_{,m} + \delta_{ij}n_k[12(1-6\nu)\{\ln(r) - Q\}$$
$$+ (5 - 18\nu)] + (\delta_{ik}n_j + \delta_{jk}n_i)[12(5-6\nu)\{\ln(r) - Q\} - (13 - 18\nu)]$$
$$+ 2(n_i r_{,j} r_{,k} + n_j r_{,i} r_{,k} + n_k r_{,i} r_{,j})[-12\ln(r) - 1 + 12Q] \rangle\rangle . \tag{37}$$

Function D_{ijk} in Eq. (32), $\sigma^{[2]}{}_{ijk}$ and $\sigma^{[3]}{}_{ijk}$ have the following relationship:

$$\nabla^4 \sigma^{[3]}{}_{ijk} = \nabla^2 \sigma^{[2]}{}_{ijk} = D_{ijk}. \tag{38}$$

2.5. Numerical procedure for integral equation

In practice, the values of $b_j^{[1]}$ are given, but $\partial b_j^{S[1]}/\partial n$, $b_j^{S[2]}$, $\partial b_j^{S[2]}/\partial n$, $b_j^{P[3]}/$ in Eq. (16) are not given. In the conventional multiple-reciprocity method, the fundamental solution of higher order is used in some problems, but when the higher order (infinite order) fundamental solution and the higher order analytical derivations are used, the merit of the BEM is lost. Therefore, a method interpolating the distribution of body force by the fundamental solution of lower order will be shown. The interpolation using contour lines has already been shown. In this paper, the interpolation using the internal points is used. For an easy understanding, constant elements are used for the boundary. Superscript P denotes $b_j^{P[3]}$. Replacing $b_j^{S[f]}$ and $\partial b_j^{S[f]}/\partial n$ by vectors $\boldsymbol{W}_f$ and $\boldsymbol{V}_f$, respectively, and discretizing Eq. (11), we obtain

$$\boldsymbol{H}_1 \boldsymbol{W}_1 = \boldsymbol{G}_1 \boldsymbol{V}_1 + \boldsymbol{H}_2 \boldsymbol{W}_2 - \boldsymbol{G}_2 \boldsymbol{V}_2 - \boldsymbol{G}^P{}_2 \boldsymbol{W}^P{}_3, \tag{39}$$

where $\boldsymbol{H}_1$, $\boldsymbol{G}_1$, $\boldsymbol{H}_2$, $\boldsymbol{G}_2$, and $\boldsymbol{G}^P{}_2$ are the matrices with the following element for a given boundary point 'i':

$$H_{1ij} = \frac{1}{2}\delta_{ij} + \int_{\Gamma_j} \frac{\partial T_1(P,Q)}{\partial n} d\Gamma_j, \tag{40}$$

$$G_{1ij} = \int_{\Gamma_j} T_1(P,Q) d\Gamma_j, \tag{41}$$

$$H_{2ij} = \int_{\Gamma_j} \frac{\partial T_2(P,Q)}{\partial n} d\Gamma_j, \tag{42}$$

$$G_{2ij} = \int_{\Gamma_j} T_2(P,Q) d\Gamma_j, \tag{43}$$

$$G^P{}_{2ij} = T_2(P, q^P). \tag{44}$$

As the case F=2 is considered, $b_j^{[2]}$ is obtained using Eq. (12) as follows:

$$\boldsymbol{H}_1 \boldsymbol{W}_2 = \boldsymbol{G}_1 \boldsymbol{V}_2 + \boldsymbol{G}^P{}_1 \boldsymbol{W}^P{}_3, \tag{45}$$

where $\boldsymbol{G}^P{}_1$ is a matrix with the following elements:

$$G^P{}_{1ij} = T_1(P, q^P). \tag{46}$$

Moreover, using the value $b_j(p^P)$ at internal points, we obtain

$$\boldsymbol{W}(p^P) = -\boldsymbol{H}_3\boldsymbol{W}_1 + \boldsymbol{G}_3\boldsymbol{V}_1 + \boldsymbol{H}_4\boldsymbol{W}_2 - \boldsymbol{G}_4\boldsymbol{V}_2 - \boldsymbol{G}^P{}_3\boldsymbol{W}^P{}_3, \tag{47}$$

where $\boldsymbol{H}_3$, $\boldsymbol{G}_3$, $\boldsymbol{H}_4$, $\boldsymbol{G}_4$, and $\boldsymbol{G}^P{}_3$ are the matrices with the following elements:

$$H_{3ij} = \int_{\Gamma_j} \frac{\partial T_1(p^P, Q)}{\partial n} d\Gamma_j, \tag{48}$$

$$G_{3ij} = \int_{\Gamma_j} T_1(p^P, Q) d\Gamma_j, \tag{49}$$

$$H_{4ij} = \int_{\Gamma_j} \frac{\partial T_2(p^P, Q)}{\partial n} d\Gamma_j, \tag{50}$$

$$G_{4ij} = \int_{\Gamma_j} T_2(p^P, Q) d\Gamma_j, \tag{51}$$

$$G^P{}_{3ij} = T_2(p^P, q^P). \tag{52}$$

Assuming $\boldsymbol{W}_2 = \boldsymbol{0}$, the following equation is obtained using Eqs. (39), (45), and (47):

$$\begin{bmatrix} \boldsymbol{G}_1 & -\boldsymbol{G}_2 & -\boldsymbol{G}^P{}_2 \\ \boldsymbol{0} & -\boldsymbol{G}_1 & -\boldsymbol{G}^P{}_1 \\ \boldsymbol{G}_3 & -\boldsymbol{G}_4 & -\boldsymbol{G}^P{}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \\ \boldsymbol{W}^P{}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_1\boldsymbol{W}_1 \\ \boldsymbol{0} \\ \boldsymbol{H}_3\boldsymbol{W}_1 + W(P^P) \end{bmatrix} \tag{53}$$

From Eq. (53), we obtain $\boldsymbol{V}_1$, $\boldsymbol{V}_2$, and $\boldsymbol{W}^P{}_3$. If the boundary is divided into N_0 constant elements, and N_1 internal points are used, the simultaneous linear algebraic equations with $(2N_0+N_1)$ unknowns must be solved.

The solution procedure is as follows. First, in order to interpolate the distribution of body force, the boundary is divided into elements, and the values of body force on the boundary and at the internal points are given. Using Eq. (53), the unknown values $\boldsymbol{V}_1$, $\boldsymbol{V}_2$ and $\boldsymbol{W}^P{}_3$ are obtained in order to interpolate the distribution of body force. Using these values and Eq. (16), the displacement and the surface traction on the boundary are obtained.

3. Numerical examples

In order to ensure the accuracy of the present method, the stresses in a square domain are obtained for the case in which the gravitational force $-g$ acts in the y-direction. It is assumed that the length of a side of the square domain is 1m and that no other external force is acting, as shown in Fig. 2. Denoting the standard vertical length as L, the density is considered as a function of vertical coordinate y, i.e.,

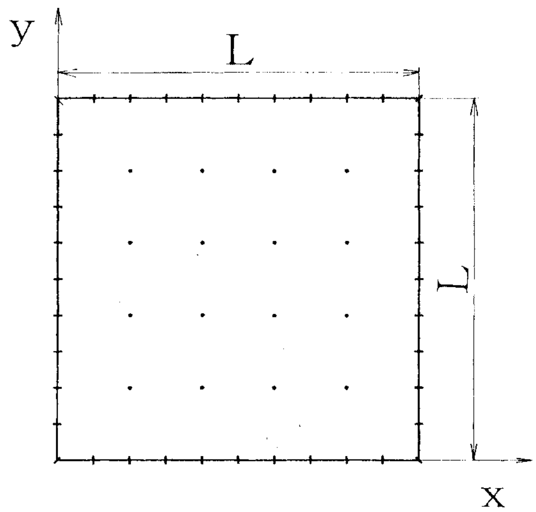


Fig. 2 Square domain with variable density

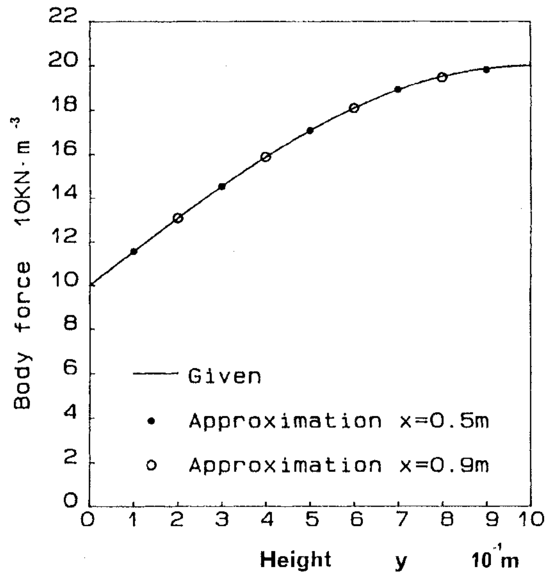


Fig. 3 Interpolation of body force distribution

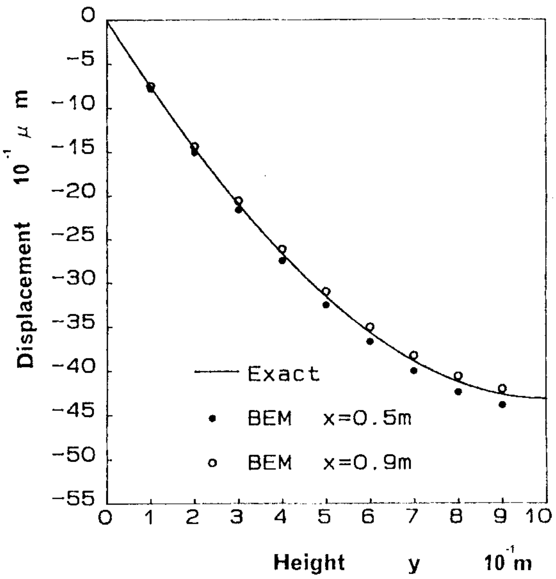


Fig. 4 Distribution of displacement in the case of variable density

$$\rho(y) = \rho_0\left[1 + \sin\left(\frac{\pi y}{2L}\right)\right] \tag{54}$$

The problem is treated as the plane stress problem, where Young's modulus is E=210 GPa and Poisson's ratio is 0.3. Denoting the density as ρ, we assume that $\rho=1\times10^5$ N·m^{-3}. Fig. 3 shows the comparison between the value given by Eq. (54) and the value interpolated by Eq. (53) using

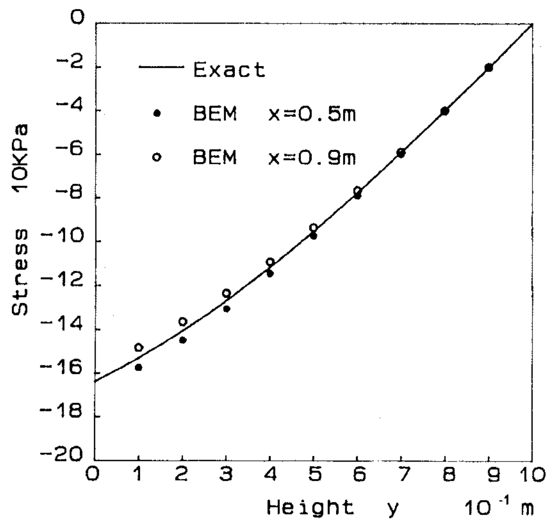


Fig. 5 Distribution of stress in the case of variable density (σ_{yy})

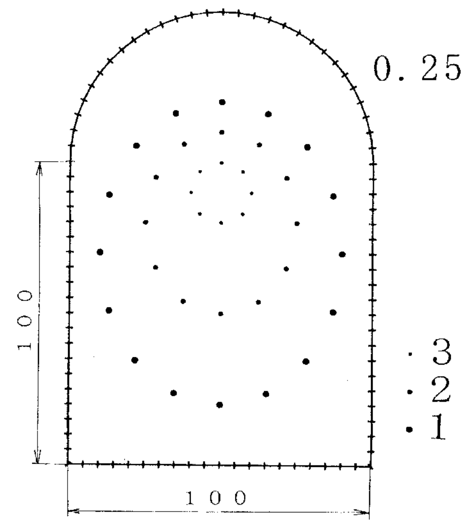


Fig. 6 Region with variable density ($g\rho\times10^5$ N·m^{-3})

internal points as shown in Fig. 2. In Figs. 4 and 5, the displacements and stresses are shown for x=0.5 m and x=0.9 m. The solid lines in these figures are the accurate solutions considered as a one-dimensional problem given by

$$Dis = \frac{-g\rho_0}{E}\left[Ly - \frac{y^2}{2} + \frac{4L^2}{\pi^2}\sin\left(\frac{\pi y}{2L}\right)\right] \tag{55}$$

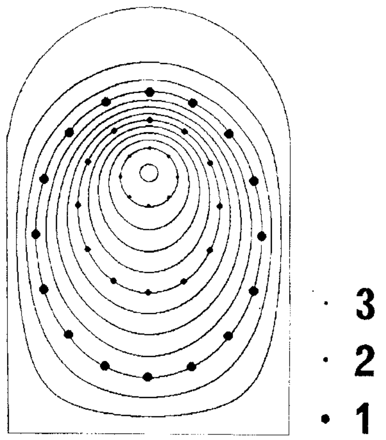


Fig. 7 Interpolation of body force distribution ($g\rho\times10^5$ N·m^{-3})

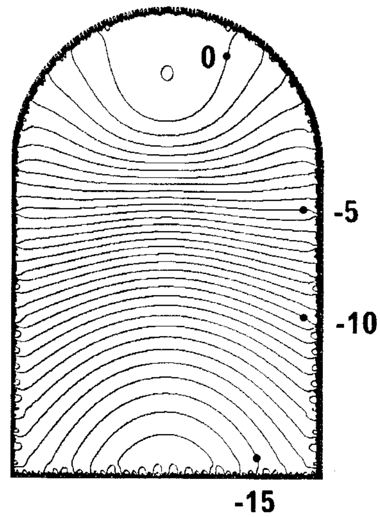


Fig. 8 Distribution of stress in the case of variable density (σ_{yy} kPa)

$$\sigma(y) = -g\rho_0\left[L - y + \frac{2L}{\pi}\cos\left(\frac{\pi y}{2L}\right)\right] \tag{56}$$

As an example of a case in which it is difficult to obtain an accurate solution, the stress distribution was obtained for the domain with variable density, the width of which is 100mm, as shown in Fig. 6. Denoting the acceleration due to gravity as g, $g\rho_0$=0.25×10^5 N·m^{-3} on the boundary is assumed. The distribution of body force is given by points ($g\rho$=1, 2, 3×10^5 N·m^{-3}) as shown in Fig. 6. Let Young's modulus be 210 GPa, and Poisson's ratio be 0.3. The calculation is carried out for a plane stress problem. The distribution of the given specific weight and its approximation obtained by Eq. (53) are compared to each other in Fig. 7. The distribution of stress is shown in Fig. 8. For this example, many hours are required to prepare the distributed data for conventional BEM.

4. Conclusions

By improving the conventional multiple-reciprocity boundary element method, it was shown that it is possible to interpolate the arbitrary distribution of body force only by using the fundamental solution of lower order. It was shown that stress analysis with an arbitrary body force can be carried out without internal cells solely by adding the data for the values of body force on the boundary and at internal points. The fundamental solutions for this analysis were shown.

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