*Structural Engineering and Mechanics, Vol. 10, No. 4 (2000) 359-369*
DOI: http://dx.doi.org/10.12989/sem.2000.10.4.359


# Random loading identification of multi-input-multi-output structure

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**Abstract.** Random loading identification has long been a difficult problem for Multi-Input-Multi-Output (MIMO) structure. In this paper, the Pseudo Excitation Method (PEM), which is an exact and efficient method for computing the structural random response, is extended inversely to identify the excitation power spectral densities (PSD). This identified method, named the Inverse Pseudo Excitation Method (IPEM), resembles the general dynamic loading identification in the frequency domain, and can be used to identify the definite or random excitations of complex structures in a similar way. Numerical simulations are used to reveal the the difficulties in such problems, and the results of some numerical analysis are discussed, which may be very useful in the setting up and processing of experimental data so as to obtain reasonable predictions of the input loading from the selected structural responses.

**Key words:** random; loading; identification; simulation; vibration.

## 1. Introduction

For linear structures subjected to multiple stationary random excitations, which may be partially coherent, the direct problem, i.e., the computation of the PSDs of the various responses from the given excitation PSD, has been sloved (Nigam 1983, Bendat and Piersol 1980, Zhang and Wang 1988), in particular, by using the exact and efficient Pseudo Excitation Method (PEM), which also can be called a fast CQC algorithm of PSD matrix (Lin 1992, Zhong 1996, Lin *et al.* 1994). There are two types of inverse problems (also called back anlysis problems) corresponding to this direct problem. The first is the structure identification problem, for which the loading and response PSD functions are all known and are used to identify the properties of the structure. Many publications cover such problems and their success in engineering, see Zhang and Wang (1988). The second is the loading identification problem, for which the response, the information, and structural properties are known and are used to identify the information about the loading. For the second type of inverse problem, if there is only one exciation acting on the structure, the problem is quite straightforward and will not be discussed in this paper. For MIMO problems, however its solution is not easy and there was relatively no research reported in this area, which demonstrates its difficulty. To the authors' knowledge, some such work has been done at high expense (Dobson and Rider 1990), but had nevertheless yielded identification results that are unacceptable because of their very

poor precision. However, such inverse problems are of great importance in practical engineering because the direct measurement of the responses may at times be relatively easy.

For any direct problem for which the excitation PSD matrix has been given, the PSD matrix of any response can be computed quickly and precisely by means of the PEM (Lin 1992, Zhong 1996, Lin *et al.* 1994). In this paper, the PEM is inversely modified to generate an inverse Pseudo Excitation Method (IPEM), which deals with this identification method as the general dynamic loading identification in the frequency domain, so that the excitation PSD matrix can be obtained from a set of known response PSD function. In addition, the IPEM is suited to the identification of arbitrarily coherent and stationary multiple random excitations of proportionally or non-proportionally damped, linear systems. Computer simulations show that this approach is not only simple and efficient, but also has good noise-resistance properties. The investigation also shows that it is very important to optimize the selection of measured quantities in order to achieve identification precision.

## 2. Pseudo excitation method (PEM) for solving the stationary random response of structures

The conventional formula for solving the stationary random response of linear structures in the frequency domain is

$$[S_{yy}]=[H]^{*}[S_{xx}][H]^{T} \tag{1}$$

in which $[S_{xx}]$ is the known excitaion PSD matrix, $[H]$ is the transfer function matrix, $[S_{yy}]$ is the response PSD matrix to be computed, and the superscripts * and $T$ represent complex conjugate and transposition, respectively.

For the case in which all excitations are fully coherent or partially coherent, it is assumed that $[S_{xx}]$ is a $p\times p$ matrix with rank $r$ ( $\leq p$). Since $[S_{xx}]$ must be a Hermitic matrix (Zhang and Wang 1988, Lin 1992), it can be decomposed into

$$[S_{xx}]=\sum_{j=1}^{r}\{a\}_j^{*}\{a\}_j^{T} \tag{2}$$

Introducing the pseudo excitation (Lin 1992, Zhong 1996, Lin *et al.* 1994)

$$\{x\}_j=\{a\}_j\cdot e^{i\omega t} \qquad (j=1, 2, ..., r) \tag{3}$$

leads to the harmonic response $\{y\}_j$ given by (Zhong 1996)

$$\{y\}_j=[H]\{x\}_j \tag{4}$$

Substituting Eq. (3) into Eq.(4), yields

$$\{y\}_j=\{b\}_j\cdot e^{i\omega t} \tag{5}$$

where

$$\{b\}_j=[H]\{a\}_j \tag{6}$$

Using Eqs. (1), (2), and (6), the response PSD matrix is given in terms of the $\{b\}_j$ as follows:

$$[S_{yy}]=[H]^*[S_{xx}][H]^T=[H]^*\sum_{j=1}^{r}\{a\}_j^*\{a\}_j^T[H]^T=\sum_{j=1}^{r}\{b\}_j^*\{b\}_j^T \tag{7}$$

## 3. The inverse pseudo excitation method (IPEM) for structural random loading identification

Let Eq. (1) be written in the form (Bendat and Piersol 1980)

$$[S_{xx}]=[H]^{+*}[S_{yy}][H]^{+T} \tag{8}$$

in which the superscript + represents generalized inversion (performed as described below, at the end of section 4) since $[H]$ is generally not a square matrix. Provided that the responses of the struture are caused entirely by the random loading to be identified, then Eq. (8) gives the algorithm for computing $[S_{xx}]$ from the known PSD matrix $[S_{yy}]$. Clearly, Eq. (8) is the backwards extension of the PEM described above. Therefore, it is called the inverse Pseudo Excitation Method (IPEM). For complicated engineering problems, direct use of Eq. (8) is quite inefficient, so it should be dealt wih analogously to the last sections, as follows.

For the case in which all excitations are fully coherent or partially coherent, by using Eq. (7), $[S_{yy}]$ can be decomposed into $\sum_{j=1}^{r}\{b\}_j^*\{b\}_j^T$. The pseudo response $\{y\}_j=\{b\}_j\cdot e^{i\omega t}$ then follows from Eq. (5) and applying Eq. (6) inversely gives the pseudo excitaion $\{x\}_j$ of Eq. (3) as

$$\{x\}_j=\{a\}_j\cdot e^{i\omega t}=[H]^{+}\{b\}_j\cdot e^{i\omega t} \tag{9}$$

Hence, Eqs. (7), (8) and (9) enable the excitation PSD matrix $[S_{xx}]$ to be given in terms of the $\{a\}_j$ as follows:

$$[S_{xx}]=[H]^{+*}[S_{yy}][H]^{+T}=[H]^{+*}\sum_{j=1}^{r}\{b\}_j^*\{b\}_j^T[H]^{+T}=\sum_{j=1}^{r}\{a\}_j^*\{a\}_j^T \tag{10}$$

For large systems, the direct use of Eq. (10) is still computationally very expensive. Therefore, the following equation reduction scheme should be used, which is based on the mode-superposition method.

## 4. Excitation power spectral density identification for large complex systems

The equation of motion of a large system can be expressed as

$$[M]\{\ddot{Y}\}+[C]\{\dot{Y}\}+[K]\{Y\}=\{F\} \tag{11}$$

in which $[M]$, $[C]$, and $[K]$ are, respectively, the mass, damping, and stiffness matrix, dots denote differentiation with respect to time, and $\{Y\}$ and $\{F\}$ are, respectively, the displacement and applied force vectors. Even though the order $n$ of $\{Y\}$ and $\{F\}$ is very high, the number, $l$, of non-zero excitations in $\{F\}$ and the number, $m$, of measured responses in $\{Y\}$ are both $<<n$. Therefore, let

$$\{F\}=[E_f]\{f\} \tag{12}$$
$$\{y\}=[E_y]\{Y\} \tag{13}$$

where $\{f\}$ is the $l$ dimensional vector consisting of the non-zero elements in $\{F\}$, $\{y\}$ is the $m$ dimensional vector consisting of the $m$ measured quantities in $\{Y\}$, and $[E_f]$ and $[E_y]$ are matrices which contain only zero and unity. Assuming that the $m\times m$ matrix $[S_{yy}(\omega)]$ is known, usually being specified in the form of a series of values at discrete frequency points, the requirement is to find the $l\times l$ excitation PSD matrix $[S_{ff}(\omega)]$ by back analysis.

The mode-superposition scheme is now used. Firstly, the lowest $q$ modes, denoted by the $n\times q$ matrix $[\Phi]$, and the corresponding diagonal $q\times q$ eigenvalue matrix $[\Omega^2]$ must be found by solving

$$[K][\Phi]=[M][\Phi][\Omega^2] \tag{14}$$

subject to

$$[\Phi]^T[M][\Phi]=[I] \quad (q\times q \text{ unit matrix}) \tag{15}$$

Reducing Eq. (11) by means of the modal matrix $[\Phi]$ gives

$$\{Y\}=[\Phi]\{U\} \tag{16}$$

and

$$\{\ddot{U}\}+[C]^0\{\dot{U}\}+[\Omega^2]\{U\}=[\Phi]^T[E_f]\{f\} \tag{17}$$

in which

$$[C]^0=[\Phi]^T[C][\Phi] \tag{18}$$

For proportionally damping and non-proportionally damping systems, the following derivation is applied in which partial coherency is assumed for generality.

Substituting Eq. (16) into Eq. (13) gives

$$\{y\}=[E_y][\Phi]\{U\} \tag{19}$$

Then, decomposing $[S_{yy}]$ by using Eq. (7) and forming the pseudo responses according to Eq. (5) gives $\{y\}_j=\{b\}_j\cdot e^{i\omega t}$, which from Eq. (19) can be regarded as being produced by the pseudo "excitation"

$$\{U\}_j=\{c\}_j\cdot e^{i\omega t} \tag{20}$$

This $\{U\}_j$ can be considered to be generated by the pseudo excitation $\{f\}_j=\{a\}_j\cdot e^{i\omega t}$, so that Eq. (17) gives

$$\{U\}_j=[H][\Phi]^T[E_f]\{f\}_j \tag{21}$$

in which

$$[H]=(-\omega^2[I]+[\Omega^2]+i\omega[C]^0)^{-1} \tag{22}$$

where $i=\sqrt{-1}$. Substituting Eq. (21) into Eq. (19) produces

$$\{y\}_j=[R]\{f\}_j \tag{23}$$

in which

$$[R]=[E_y][\Phi][H][\Phi]^T[E_f] \tag{24}$$

Therefore,

$$\{f\}_j=[R]^+\{y\}_j \tag{25}$$

From Eq. (25), we can find that the IPEM resembles the definite dynamic loading identification in the frequency domain (Zhang and Wang 1988), and this method can be used to identify the definite or random excitaions of complex structures in a similar way. Finally, using the IPEM gives (see Eq. 10)

$$[S_{ff}]=[R]^{+*}[S_{yy}][R]^{+T}=\sum_{j=1}^{r}\{f\}_j^*\{f\}_j^T \tag{26}$$

When the number of excitations $l$ exceeds the numbers of measured responses $m$, the $m\times l$ generalized inverse matrix of $[R]$, i.e.,

$$[R]^+=([R]^T[R]^*)^{-1}[R]^T \tag{27}$$

does not exist. Therefore, $[S_{ff}]$ cannot be found from $[S_{yy}]$.

When $l<m$, the inversion procedure is used to solve for $l$ unknowns from the $m$ equations by means of the least square method, so that only approximate solutions can be expected.

Clearly, when $l=m$, the equation set can be solved exactly in theory. Therefore, in general, the inverse problem is solvable under the conditions $l \leq m$ and $l \leq q$. However, even though these conditions are met, there is no guarantee that a satisfactory excitation PSD matrix will be obtained. Practical situations are rather complex, as discussed via the following simulation example.

## 5. Computer simulation and discussion

The stiffness and mass distributions of structure with nine degrees of freedom are shown in Fig. 1. For simplicity, dimensionless quantities ($m$=1 and $k$=1) are assumed. The complete set of natural angular frequencies is

1.00000, 1.03736, 1.13705, 1.27174, 1.41421, 1.54359, 1.64533, 1.70994, and 1.73205,

and the damping ratios are all equal to a value of 0.03. Two horizontal forces are applied at nodes 3 and 6, and the corresponding PSD matrix is

$$[S_{ff}]=\begin{bmatrix}3.0 & 2.0\\ 2.0 & 4.0\end{bmatrix}+\begin{bmatrix}0.0 & 1.0\\ -1.0 & 0.0\end{bmatrix}\cdot i \tag{28}$$

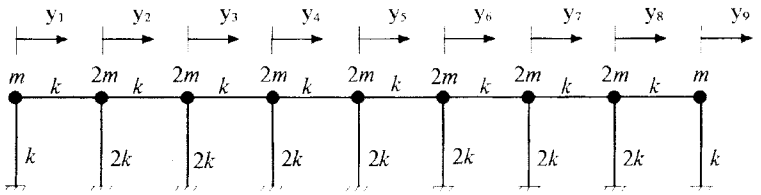



Fig. 1 Nine DOF structure

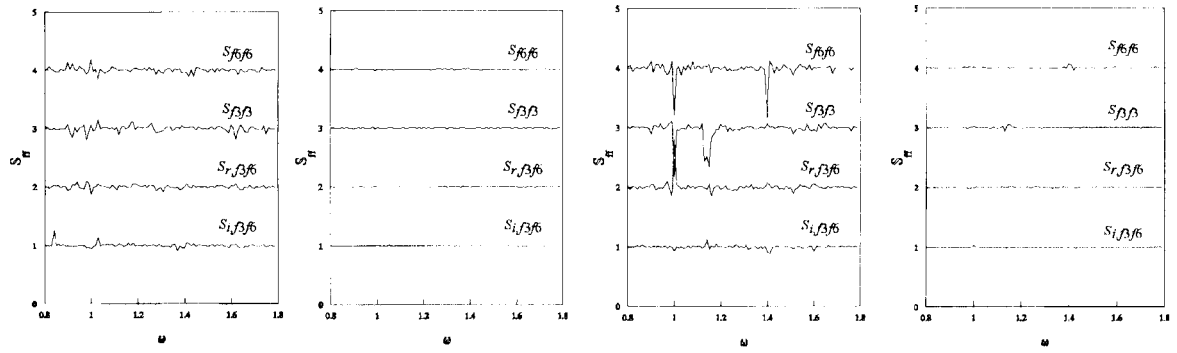
within the significant frequency region $\omega \in [0.8, 1.8]$. This region is dispersed with an interval of $\Delta\omega$=0.01. Only 2 or 3 digits of the response PSD results generated from the direct random vibation analysis (Lin 1992) were taken as the "measured values" for the computer simulation. Some interesting observations are reported in the following six subsections. All results presented were obtained by the IPEM and are curves of the auto-PSD of the excitations $f_3$ and $f_6$, i.e. $S_{f3f3}$ and $S_{f6f6}$, and their cross-PSD $S_{f3f6}$, which has real and imaginary parts $S_{r,f3f6}$ and $S_{i,f3f6}$ respectively.

### 5.1. The effect of the precision of measurement of the response PSD matrix on the precision with wihich the excitation PSD matrix is identified

Because noise always exists in the measurements, only a limited number of digits of the measured response values are reliable for use in the loading identification. When only the two effective digits of all measured values were used, the results obtained for all damping ratios=0.03 are plotted in Fig. 2(a). It can be seen that the identification values of the PSD functions fluctuate around their exact values, see Eq. (28), of 3.0, 4.0, 2.0, and 1.0. The maximum error (taken directly from the computed results) is 6.5% for $S_{r,f3f6}$=1.869 when $\omega$=1.00. In contrast, if the measurement precision is so high that the first three effective digits of all the measured quantities are reliable, Fig. 2(b) replaces Fig. 2(a) and obviously the four curves obtained are very close to their theoretical (i.e. exact) values.

### 5.2. The effect of structual damping on the identification precision

When the damping ratios were all changed from 0.03 to 0.01, Figs. 2(a) and (b) became Figs. 2(c) and (d). Hence, it can be seen that higher identification precision is achieved for heavier damping. This is because lightly damped structures can experience serious ill conditioning in the vicinity of their natural frequencies. This problem might be overcome when the displacement PSDs were assumed to have an accuracy of three effective digits. Unfortunately, in normal circumstances it would be virtually impossible to measure PSDs to such high accuracy. Therefore, it is practically impossible to avoid the abnormal fluctuation (or "jumping") shown in Figs. 2(a) and (c) under




(a) 0.03, 2 (b) 0.03, 3 (c) 0.01, 2 (d) 0.01, 3

Fig. 2 Loading PSD functions of points 3 and 6 identified from the displacement PSD functions of points 2, 5, and 7 with all nine modes participating in the back analysis (The two numbers beneath each figure are the values of all damping ratios and the number of digits accuracy of the displacement PSD values)

normal experimental conditions, as is now discussed further.

### 5.3. Further analysis on the abnormal jumping of identification results in the vicinity of natural frequencies

For the present example, $[H]$ is a diagonal matrix with given damping ratios, so that Eq. (24) can be converted into

$$[R]=[E_y]\left(\sum_{j=1}^{q} H_j\{\phi\}_j\{\phi\}_j^T\right)[E_f]=\sum_{j=1}^{q} H_j[E_y]\{\phi\}_j\{\phi\}_j^T[E_f] \tag{29}$$

where

$$H_j(\omega)=(\omega_j^2-\omega^2+2\zeta_j\omega_j\omega)^{-1} \tag{30}$$

It is known that $[R]$ will be singular if its rank is less than $q$ (Wilkinson and Reinsch 1971). If $q'$ of the $q$ terms summed in Eq. (29) is zero, $[R]$ must be singular and of rank $q$-$q'$. Furthermore, Eq. (30) shows that $H_j(\omega)\approx(2\zeta_j\omega_i\omega)^{-1}$ when $\omega\approx\omega_j$, so that if the $j$-th damping ratio $\zeta_j$ is very small, $H_j(\omega)$ will be very large, particularly when $\omega_j$ is small. Hence, one (or more) of the terms summed in Eq. (29) becomes almost negligible, which causes the pathological jumping of the identified curves, i.e., it causes the high sensitivity of the identification results to the measured values.

This phenomenon can be elucidated further through examing intermediate computation data. For example, when $\omega$=1.00, abnormal jumping occurrs in the curves of Fig. 2(c). The displacement responses PSD matrix of the points 2, 5, and 7, which was used to identify the excitation PSD matrix, was (taking only the first two effective digits)

$$[S_{yy}]=\begin{bmatrix}1.2 & 1.1 & 1.1\\ 1.1 & 1.1 & 1.0\\ 1.1 & 1.0 & 0.99\end{bmatrix}\times 10^2+\begin{bmatrix}0.0 & 2.8 & -3.2\\ -2.8 & 0.0 & -5.7\\ 3.2 & 5.7 & 0.0\end{bmatrix}\cdot i \tag{31}$$

The identified excitation PSD values for nodes 3 and 6 were

$$\{S_{f_3f_3},\ S_{f_6f_6},\ S_{r,f_3f_6},\ S_{i,f_3f_6}\}=\{2.187,\ 3.210,\ 2.795,\ 0.932\} \tag{32}$$

where the first four digits of the IPEM results are given for possible reference, although two digits would normally be trusted. The exact theorectical values, see Eq. (28), were {3.004, 4.000, 2.000, 1.000}. Now suppose that a measurement error in the third digit results at the value 1.20 of the (1.1) element of $[S_{yy}]$ being replaced by 1.25, then the IPEM back analysis would give

$$\{S_{f_3f_3},\ S_{f_6f_6},\ S_{r,f_3f_6},\ S_{i,f_3f_6}\}=\{2.911,\ 3.860,\ 2.135,\ 1.120\} \tag{33}$$

Hence, an error of 4.2% in only one of the measured values results in differences in the loading identification of {24.9%, 16.8%, −30.9%, 11.4%}. This demonstrates the high sensitivity of the loading identification to the response measurement accuracy.

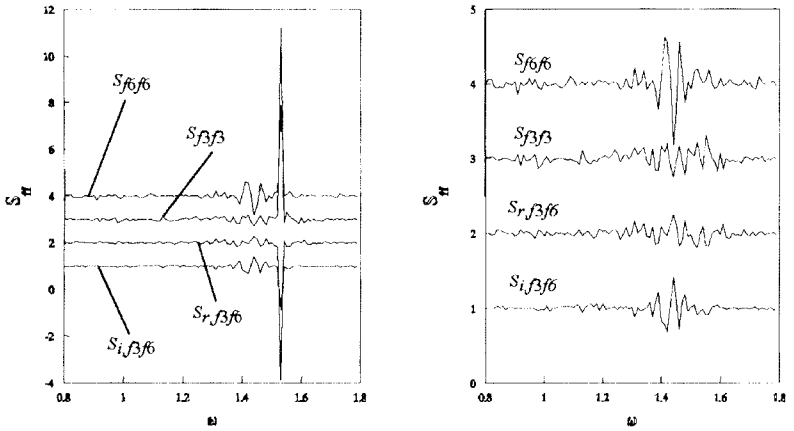
Because this problem is highly pathological, the corresponding ill-conditioned Eq. (25) was sloved by four methods to ensure that the results are correct. These were the Singular Value Decomposition (SVD) and Cholesky triangularizartion decomposition methods, plus two other methods for solving

pathological equations (Dobson and Rider 1990, Wilkinson and Reinsch 1971). All four methods gave practically identical results, i.e., at least the first five effective digits agreed on an IBM/586 PC computer. It is certain, then, that Eq. (25) was solved correctly. Therefore, the poor identification accuracy was mainly due to the poor measurement precision and not the inaccurate solution of pathological equations.

Since it is very difficult to measure to a precision of three reliable digits, jumping is inevitable in the back analysis. However, this does not mean that such abnormal results must be accepted. If such jumping happens only near the natural frequencies, it can be smoothed out by using adjacent points on the curves, for example, in Fig. 2(c).

### 5.4. Selection of measured points by computer simulation

Improper selection of measurement points is another possible cause of unfavorable results. When the measurement points of the above example were changed from 2, 5, and 7 to 2, 4, and 7, with only the first two digits of the displacement response PSD values used for loading identification, Fig. 2(a) was replaced by Fig. 3(a), in which considerable jumping is observed near the natural frequency $\omega=1.53$. Even if this point was modified by taking the average of two measured values on either side of it, the remainder of the identification curves are still inadequate, see Fig. 3(b). Clearly, such unfavorable combinations of measurement points should be avoided. But the problem is that most engineers would find it very difficult to judge which is the best combination of measurement points to use, i.e., to choose 2, 4, and 7 or 2, 5, and 7. Therefore, selection of measurement points by means of computer simulation, as above, must be a good suggestion since, in current engineering practice, such selection is usually based on engineers experience or intuitive feelings, which may be an important reason why relativey little successful identification of structural random loading has so far been reported.




(a) unaltered (b) revised

Fig. 3 Loading PSD function of points 3 and 6 identified from the displacement PSD functions of points 2, 4, and 7 with all nine modes participating in the back analysis (All damping ratios were 0.03 and two digits of accuracy were retained for the displacement PSD values. The two cases are for when maximum jumping point was unaltered or was revised.)

### *5.5. Influence of the accuracy of structural parameters on the identification precision*

Inaccurate evaluation of structual stiffness and mass parameters can exert a remarkable effect on loading identification precision. For example, changing the stiffness of the middle (i.e., 5th) element of the structure of Fig. 1 from $2k$ to $2.04k$ altered the structural natural frequencies to

1.00122, 1.03736, 1.13924, 1.27174, 1.41598, 1.54359, 1.64685, 1.70994, 1.73279

and replaced the identification results of Fig. 2(a) by those of Fig. 4(a), for which each curve deviates from the theoretical one more considerably, with a maximum of 12.6% for $S_{r,f3f6}$ (=2.252) at $\omega$=0.98. As a second example, when the middle (i.e., 5th) mass of the structure of Fig. 1 was changed from $2m$ to $2.04m$, the structural natural frequencies became

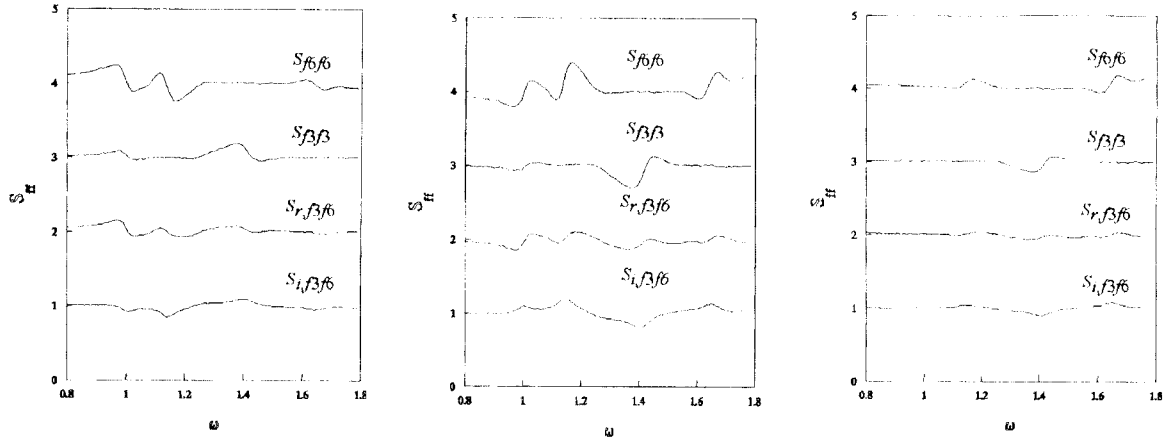
0.99937, 1.03736, 1.13563, 1.27174, 1.41245, 1.54359, 1.64329, 1.70994, 1.73101

and the identification results of Fig. 2(a) were replaced by those of Fig. 4(b), for which the maximum deviation of each curve from the theoretical value is 9.7% for $S_{r,f6f6}$ (=4.386) at $\omega$=1.16. As a third example, the changes of the first two examples were made simultaneously. Then Fig. 2(a) was replaced by Fig. 4(c), for which every curve deviated from its theoretical value by much smaller amounts than for the first two examples, the maximum error being 6.4% for $S_{f3f3}$ (=2.807) at $\omega$=0.98. The amounts are smaller because the structual natual frequencies were

1.00062, 1.03736, 1.13783, 1.27174, 1.41421, 1.53459, 1.64479, 1.70994, 1.73170

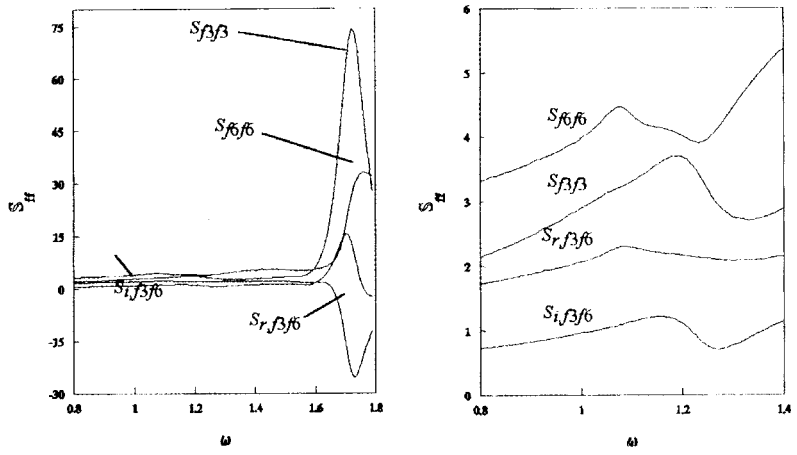
which are, as a whole, closer to the natural frequencies of the original structure than those of the first two examples, In fact, 2nd, 4th, 6th, and 8th natural frequencies are identical, while the difference of the other five frequencies is small.

The results of Figs. 4(a-c) show that even if the structural stiffness and mass parameters are not




(a) stiffness of 5th element from $2k$ to $2.04k$ (b) mass of 5th node from $2m$ to $2.04m$ (c) changes of (a) and (b) both made

Fig. 4 Loading PSD function of points 3 and 6 identified from the displacement PSD functions of points 2, 5, and 7 with all nine modes participating in the back analysis (All damping ratios were 0.03 and two digits of accuracy were retained for the displacement PSD values.)




(a) $\omega \in [0.8, 1.8]$ (b) $\omega \in [0.8, 1.4]$

Fig. 5 Loading PSD function of points 3 and 6 identified from the displacement PSD functions of points 2, 5, and 7 with the first seven modes participating in the back analysis, All damping ratios were retained for the displacement PSD values. The two alternative frequency regions were shown.)

evaluated very precisely, so long as the frequency spectrum of the structure can be measured fairly precisely, the identification can be acceptably accurate.

### 5.6. The influence of the number of participant modes on the identification precision

In all the above discussions, all nine modes of the structure were used in the direct analyses, i.e., when computing the response PSD from the excitaiton PSD, as well as in the back analysis, and when computing the excitation PSD from the response PSD by the mode-superposition method. Now suppose that all nine modes are still used for the direct analyses to obtain the simulated "measured values". But that rest modes are used for the back analyses. For instance, taking $q$=7 (modes) and using the first two digits in all the measured values gave the identified excitation PSD curves of Fig. 5(a), which deviate considerably from the curves of Fig. 2(a) ($q$=9), articularly in the high frequency region ($\omega$>1.4) in which the differece can be hundredths of a percent. Even within the region $\omega \in$ [0.8, 1.4], the maximum deviation of the identified values from the theoretical ones is about 30%, see Fig. 5(b). It should be noted that all the natural frequencies of this structures are closely spaced, the ratio of the highest to the lowest being only 1.732. This is why neglecting only the two highest modes caused considerable influence over the whole frequency region. However, many computer simulations performed by the authors for structures with widely spaced natural frequency regions show that neglecting higher modes affects the identification precision relatively little in the lower frequency region.

## 6. Conclusions

A random loading identification approach has been presented. Based on the Inverse Pseudo Excitation Method (IPEM), see Eq. (25), this problem can be disposed of in a similar fashion to

general dynamic loading identification in a the frequency domain. Previously, such a problem has not been solved well. From the primary investigation in this paper, some important phenomena have been found which may contribute to solving the problem. It shows that the computer simulation combined with the IPEM is an efficient method for selecting measured responses points and a promising approach to identifying the random loading.

The IPEM applies to the identification of arbitrarily coherent and stationary multiple random excitations of proportionally or non-proportionally damping, linear systems. In fact, it can use not only displacement but also velocity, acceleration, stress, or their mixed responses to identify the excitation PSD matrix. This method has been used to identify the random ice-forces of a marine platform in the Bohai sea in Northern China, from the measured acceleration responses. This will be introduced in another paper.

## Acknowledgements

The authors are grateful for the support of the National Natural Science Foundation of China.

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