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Retrofitting reinforced concrete beams by bolting steel plates to their sides. Part 1: Behaviour and experiments

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Abstract. A procedure has been developed for bolting steel plates to the sides of existing reinforced concrete beams which can be used to increase the shear strength of beams, increase the flexural strength of beams with enhanced ductility or with only a small loss of ductility, and increase the stiffness of beams in order to reduce deflections and crack widths. It will be shown in this paper, through a qualitative analysis and through the results of testing eight large scale beams, that standard rigid plastic analysis techniques which are commonly used in the design of reinforced-concrete, steel, and composite steel and concrete beams cannot be used directly to design composite bolted-plated reinforced-concrete beams. In the companion paper, quantitative procedures will be used to adapt the standard rigid plastic analysis techniques for this relatively new form of retrofitting.

Key words: retrofitting; rehabilitation; composite; reinforced concrete; plated beams.

1. Introduction

The behaviour of composite bolted side plated reinforced concrete beams as shown in Fig. 1(a) differs substantially from the behaviour of standard composite steel and concrete beams (Oehlers and Bradford 1995, Oehlers and Bradford 1999). This is because the latter is only subject to longitudinal slip, whereas, the former is subject to both longitudinal and vertical slip (Oehlers, Nguyen, Ahmed and Bradford 2000). It will be shown through qualitative analyses and then experimentally through eight large beam tests, how standard rigid plastic analysis techniques that are commonly used in the design of reinforced-concrete, steel, and composite steel and concrete beams cannot be used directly for bolted side plated beams. The standard rigid plastic analysis technique will then be adapted in the companion paper (Oehlers, Ahmed, and Bradford 1998) using quantitative techniques to cope with the unique properties of longitudinal and transverse slip in bolted side plated beams. The associated problems of avoiding local buckling failure of the plate

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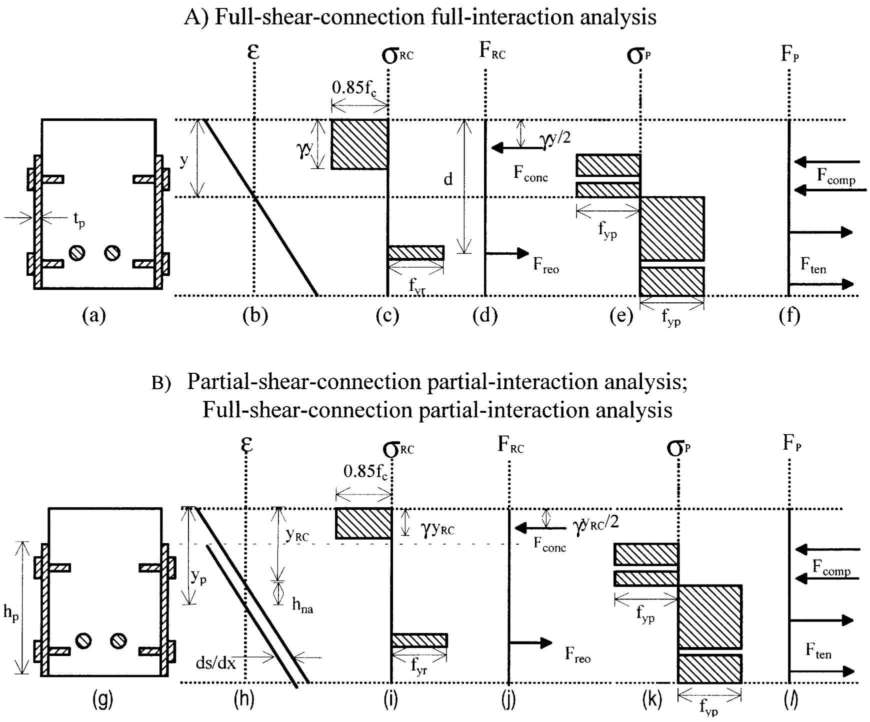


Fig. 1 Rigid plastic analysis technique

and premature failure of the bolt shear connections are dealt with elsewhere (Smith, Bradford and Oehlers 1999a, b, Ahmed 1996).

2. Standard rigid-plastic analysis techniques applied to bolted side plated beams

2.1. Full-shear-connection full-interaction analysis

Let us define the term the *full-shear-connection strength* as the strength P_{fsc} of the shear connection in a shear span that is required to achieve the maximum theoretical flexural capacity of the composite beam. If the strength of the shear connection is P, then we can define the *degree of shear connection* as $\eta = P/P_{fsc}$ so that when $\eta \geq 1$ there is *full-shear-connection* and when $\eta < 1$ then there is *partial-shear-connection.* Let us also define *full-interaction* as occurring when the strain profile through the concrete element is parallel and coincident with the strain profile through the steel element as shown in Fig. 1(b), and *partial-interaction* as when the strain profiles are not coincident as in Fig. 1(h).

The standard procedure used in the full-shear-connection rigid-plastic-analysis of a composite steel and concrete T-beam, are based purely on equilibrium because the concrete element lies above the steel element (Oehlers and Bradford 1995). Hence in a standard full-shear-connection analysis of a composite steel and concrete beam there can be partial-interaction. In contrast in a side plated beam as in Fig. 1(a), it is necessary to assume that the strain profiles are coincident as shown in

Fig. 1(b), that is there is full-interaction as well as full-shear-connection in a composite side plated beam. This is the difference between the standard rigid plastic analysis of a composite steel and concrete beam and the rigid plastic analysis of a composite side plate reinforced concrete beam.

The full-shear-connection full-interaction analysis procedure is illustrated in Fig. 1(A) where the neutral axis depth factor γ in Fig. 1(c) is defined as the depth of the rectangular stress block as a proportion of the depth to the neutral axis y. The analysis procedure is simply to move the coincident neutral axes in Fig. 1(b) up or down until there is equilibrium of all the longitudinal forces in Figs. 1(d) and (f), after which moments can be taken about any convenient point to determine the full-shear-connection full-interaction moment capacity $(M_{fi})_{fsc}$. It can be seen in Fig. 1(e) that allowance needs to be made for the position of the bolt holes. The strength of the shear connection required for full-shear-connection with full-interaction $(P_{fi})_{fsc}$ is the resultant of the forces in either of the elements in Figs. 1(d) or (f).

2.2. Partial-shear-connection partial-interaction analysis

When the strength of the shear connection $P < (P_{fi})_{fsc}$, then there is partial-shear-connection of strength P_{psc} and the analysis procedure is illustrated in Fig. 1(B). In this case there are two neutral axes as in Fig. 1(h) which are shown separated by a vertical distance h_{na}. The individual strain profiles in Fig. 1(h) are now moved independently of the other until the resultant force in each element, as given in Figs. 1(j) and (*l*) which were derived from the stress distributions in Figs. 1(i) and (k), are equal to P_{psc}, after which moments can be taken to derive the partial-interaction partial-shear-connection moment capacity $(M_{pi})_{psc}$. It can be seen in Fig. 1(h) where the strain profiles are not coincident that partial-shear-connection is synonymous with partial-interaction.

3. Reduction in rigid plastic strength

3.1. Neutral axis separation h_{na}

The horizontal distance between the strain profiles in Fig. 1(h) is referred to as the slip strain ds/dx (Newmark, Siess and Viest 1952, Oehlers and Bradford 1995), which for a symmetrically loaded beam is at a maximum at mid-span where the moment is also a maximum. The integration of the slip-strain in Fig. 1(h) along the length of the beam gives the the longitudinal slip s_l. As the bolts resist the interface shear by mechanical action, they must slip and, furthermore, the maximum slip strain occurs at the position of maximum moment. Therefore, even when the strength of the shear connection is such that there is full-shear-connection, as defined in Fig 1(A), there must be partial-interaction as shown in Fig. 1(h). Another way of visualising the problem is that if the plates were adhesively bonded to the beam then a state of full interaction would exist in which case the plated beam could achieve its maximum theoretical flexural capacity. However bolt shear connectors are mechanical shear connectors that require slip to resist shear. This slip requires the slip strain to be at its maximum at the position of the maximum applied moment, hence, the bolted plated beam may not achieve its maximum theoretical flexural capacity.

Let us assume that a composite beam has an abundance of shear connectors, that is $P >> P_{fsc}$. Therefore, the strength of the shear connectors does not control the force distribution within the beam so that we have full-shear-connection. Let us also assume that the slip-strain, ds/dx in Fig.

1(h) that is required to induce the appropriate slips in the connectors, is known at the position of maximum moment. Hence, the neutral axis separation h_{na} is now fixed at the position of maximum moment. Then, Fig. 1(B) can also be used for a full-shear-connection partial-interaction analysis by simply moving the strain profiles in Fig. 1(h), which have a fixed separation h_{na}, up or down until there is longitudinal equilibrium of all the forces in Figs. 1(j) and (l). The resulting force in either of the elements as given in Figs. 1(j) or (l) is the strength of the shear connection required for full-shear-connection when there is partial-interaction $(P_{pi})_{fsc}$, and the moment of all of these forces is the flexural capacity for a beam with full-shear-connection and partial-interaction $(M_{pi})_{fsc}$. Comparing the full-shear-connection full-interaction analysis technique in Fig. 1(A) with the full-shear-connection partial-interaction technique in Fig. 1(B), it can be seen that $(M_{pi})_{fsc}$ is always less than or equal to $(M_{fi})_{fsc}$. It can, therefore, be seen that partial-interaction can reduce the flexural capacity even when there is an abundance of shear connectors.

The slip-strain ds/dx in Fig. 1(h) is proportional to the neutral axis separation h_{na}. The more flexible the shear connectors, the greater the slip required to achieve a specific shear load in the connector and, therefore, the greater the slip-strain and neutral axis separation h_{na} at the position of maximum moment, which means that composite plated beams with flexible connectors will probably be weaker than those with stiff connectors. The effect of the neutral axis separation on the flexural capacity of the side plated beam is illustrated in Fig. 2. The full-interaction full-shear-connection rigid plastic analysis procedure in Fig. 1(A) can be used to predict point C, that is $(M_{fi})_{fsc}$ and $(P_{fi})_{fsc}$ and, hence, the plateau C-D. The partial-interaction partial-shear-connection analysis in Fig. 1(B) can be used to predict the variation in flexural capacity $(M_{pi})_{psc}$ along A-C. If now we have a beam in which $P > (P_{fi})_{fsc}$ but slip requires a neutral axis separation of h_{na} as shown in Fig. 1(h), then the moment capacity $(M_{pi})_{fsc}$ and the shear connector strength $(P_{pi})_{fsc}$ can be determined also from Fig. 1(B) with h_{na} fixed. The results are shown in Fig. 2 at point B and, hence, the upper bound to the strength of a beam with partial interaction is now given by the plateau B-E.

It can be seen in Fig. 2 that partial-interaction only limits the maximum strength that can be achieved, that is the full-shear-connection strength, and does not affect the partial-shear-connection strengths along A-B. Furthermore if we define $\eta = P/(P_{fi})_{fsc}$ then partial-interaction limits the degree of shear connection to $(\eta_{\max})_{pi} = (P_{pi})_{fsc}/(P_{fi})_{fsc} \leq 1$.

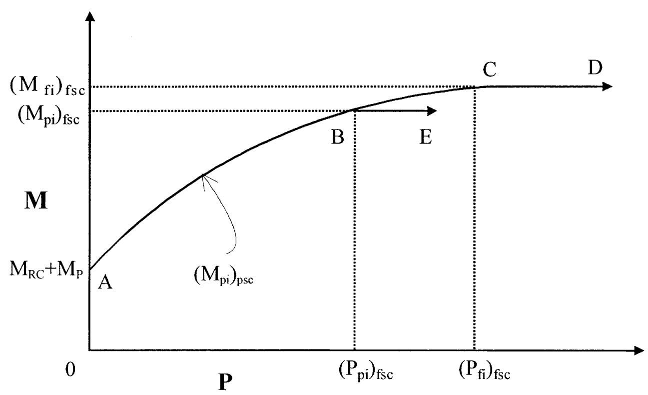


Fig. 2 Variation of flexural capacity in side plated beam

3.2. Neutral axis parameter γ

Unlike the standard analysis of an under-reinforced concrete beam where the neutral axis parameter γ does not affect the flexural capacity, this parameter does affect the flexural capacity of a side plated beam. The neutral axis in side plated beams can occur within the side plate as shown in Fig. 1(b) and, hence, affects the relative distributions of the stresses in Figs. 1(c) and (e). For example if $\gamma = 1$ in Fig. 1(A), then the base of the rectangular stress block in the concrete in Fig. 1(c) would now be in line with the discontinuity in the stresses in Fig. 1(e), so that the relative distributions of the stresses in (c) and (e) would change from those shown.

Ahmed (1996) studied the effects of the neutral axis parameter γ on both the flexural capacity M and on the bond force in the shear connectors P at M. In his parametric study: he varied the concrete strengths from 28 to 55 MPa; he analysed two composite plated sections in which the plate height h_p in Fig. 1(g) was either 40% of the depth of the beam (shallow plate) and positioned as shown, or the same height as the beam (deep plate); and he analysed each section using both a full-interaction non-linear computer model (fi-nla) and the full-interaction rigid-plastic analysis technique (fi-rpa) shown in Fig. 1(A) in which $\gamma = 0.85 - 0.007(f_c - 28)$ and where the units are in N and mm.

For the shallow plated beams, the neutral axes always lay above the top of the plates. The flexural capacities from the rigid plastic analyses $M_{fi\text{-}rpa}$ varied from between 96% to 98% of the flexural capacities from the non-linear computer simulations $M_{fi\text{-}nla}$. This close correlation between the two analysis techniques suggests that the concrete stress/strain relationship used in the non-linear analysis was appropriate. The bond forces $P_{fi\text{-}rpa}$ and $P_{fi\text{-}nla}$ were identical as the plates had yielded in tension in both analysis techniques. For the deep plated beams in which the neutral axes now always lay within the depth of the plate, $M_{fi\text{-}rpa}$ ranged from 98% to 99% of $M_{fi\text{-}nla}$, which once again shows a very good correlation. However, $P_{fi\text{-}rpa}$ ranged from 75% to 82% of $P_{fi\text{-}nla}$. This parametric study would suggest that standard techniques of rigid-plastic analysis can predict the full-interaction flexural strength accurately but severely underestimate the bond forces when the neutral axis lies within the depth of the plate.

The reason for the large discrepancies in the bond forces P is illustrated in Fig. 3 where a typical result from a non-linear analysis in Fig. 3(A) is compared with that for a rigid-plastic analysis in Fig. 3(B) for a deep plated beam. The flexural capacities are almost the same as $M_{fi\text{-}nla} = 249$ kNm and $M_{fi\text{-}rpa} = 245$ kNm. However, the bond forces, which are the resultant force in an element, differ widely and are $P_{fi\text{-}nla} = 310$ kN and $P_{fi\text{-}rpa} = 254$ kN; the rigid plastic analysis underestimating the strength required by 22%.

The stress distributions in the plate elements in Figs. 3(b) and (d) vary because they have slightly different neutral axes and because one is fully plastic and the other partly elastic. As the changes in stress from plastic to elastic, that is zones A-B-C and C-D-E in Fig. 3(b) have identical magnitudes but different signs, this change does not affect the bond force which is the resultant force in Fig. 3(b). Hence, the large discrepancies in the bond forces can only be attributed to the difference in the neutral axis depths which in Ahmed's parametric study diverged by up to 39%. As an example, in Fig. 3 the neutral axis depths vary by 6% but this slight divergence causes a 22% difference in the bond forces. It is, therefore, necessary to find a value for γ which both maintains the accuracy of the moment capacity prediction and gives an accurate prediction of the bond force.

The following function for the *characteristic* value of γ was derived from a parametric study (Ahmed 1996) which was based on minimising the divergence between $P_{fi\text{-}nla}$ and $P_{fi\text{-}rpa}$.

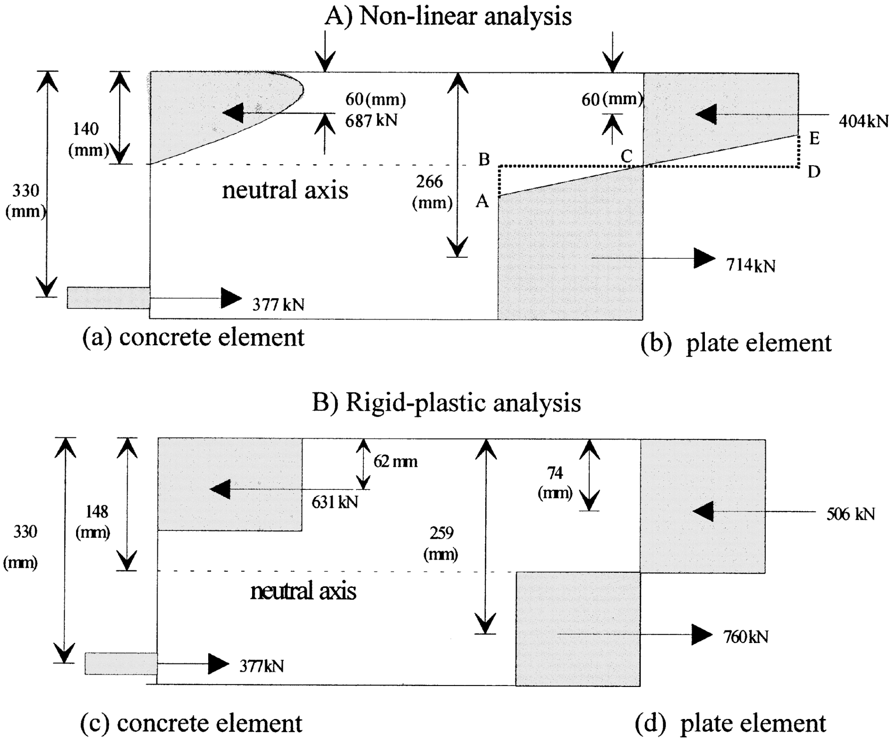


Fig. 3 Comparison of analysis techniques

$$\gamma = 0.997\text{-}0.0019(f_c\text{–}28) \tag{1}$$

where the units are in N and mm and in which the *mean* value can be obtained by replacing the coefficient 0.997 with 0.958. In a parametric study in which Eq. (1) was applied, $M_{fi\text{-}rpa}$ underestimated $M_{fi\text{-}nla}$ by 0.2% to 1.5% which will give a conservative design. Furthermore, $P_{fi\text{-}rpa}$ overestimated $P_{fi\text{-}nla}$ by 1.3% to 4.0% which would mean more connectors would be supplied than required leading also to a conservative design. However as γ is close to unity in Eq. (1), for all intents and purposes $\gamma = 1$ can be used in design.

3.3. *Transverse forces V*

The application of a concentrated load to a composite plated beam is illustrated in Fig. 4(a). A part 2V of the applied load 2F at the mid-span is transferred into the plate through the connectors as illustrated in Fig. 4(b). This load moves outwards through the plate to the ends of the plate where it is transferred by the connectors back into the beam. It can, therefore, be seen in Fig. 4(b) that the bolt shear connectors not only have to transfer the horizontal or longitudinal shear H, as required by rigid plastic analyses, but they are also subjected to transverse or vertical shear forces of magnitude 2V in a shear span. This additional shear load 2V on the connectors will, therefore, reduce the capacity below the theoretical standard rigid-plastic flexural capacities which at present do not allow for transverse forces.

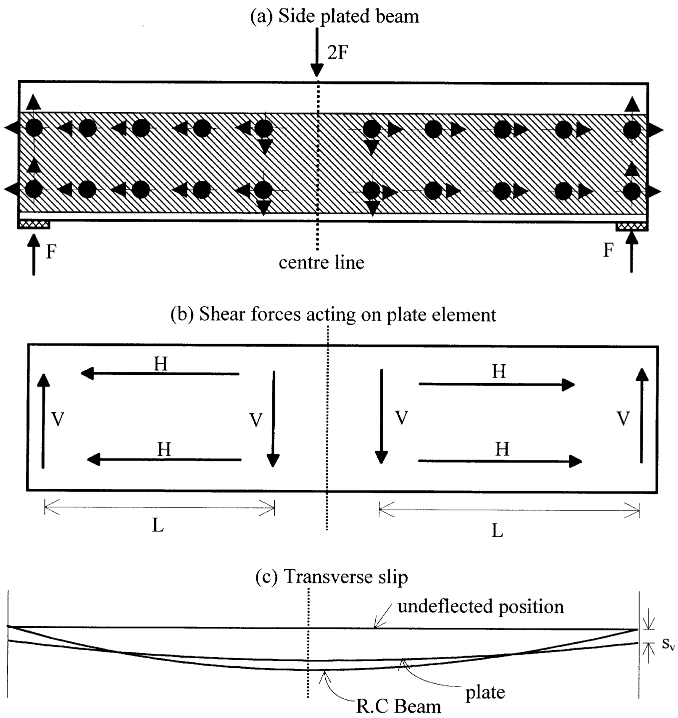


Fig. 4 Shear forces and transverse displacement

3.4. Difference in curvatures $\Delta\kappa$

In order to achieve the vertical forces in the connectors in Fig. 4(b), there must be a transverse or vertical slip s_v between the side plates and the reinforced concrete beam as shown in Fig. 4(c). This can only be achieved by inducing a difference in curvature between the steel element and the reinforced concrete element as shown in Fig. 5(c) where $\Delta\kappa = \kappa_{RC} - \kappa_p$. Integrating the difference in curvature $\Delta\kappa$ twice along the length of the beam will give the transverse slip s_v variation shown in Fig. 4(c).

It can be seen in Fig. 5(c) that the curvature in the plate κ_p will always be less than the curvature κ_{RC} in the concrete, as the load is assumed to be applied to the beam whose deflection then deflects the plate. This reduction in curvature in the plate, due to vertical slip, combined with the fact that

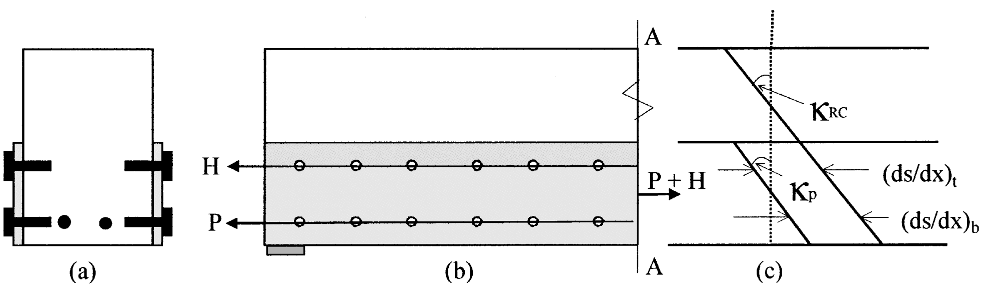


Fig. 5 Difference in curvature $\Delta\kappa$

the curvature in the concrete is limited by the concrete crushing strain, which is often assumed to be 0.003, means that it is unlikely that the plate can strain harden or even yield substantially which will further reduce the flexural capacity of the composite plated beam.

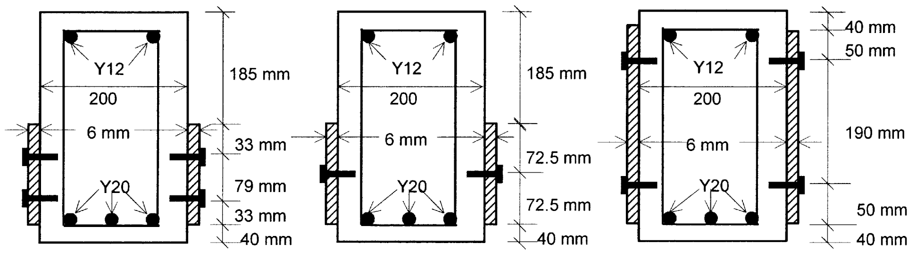
It is also worth noting in Fig. 5(c) that the slip strain at the level of the top bolts $(ds/dx)_t$ will be less than the slip strain at the level of the bottom bolts $(ds/dx)_b$. This difference in slip strain will induce larger maximum slips and, hence, increase the probability of connector fracture due to excessive slip. Furthermore, the difference in slip strain can induce larger connector forces at the bottom level of bolts than in the top level. For example at the maximum flexural strength, the shear connectors at the bottom level may be fully stressed at P as shown in Fig. 5(b) but only partly stressed in the top row at H. This reduction in the overall shear force will reduce the flexural capacity and, furthermore, the eccentricity of loading on the plate this entails can only be resisted by a moment in the plate which will lead to a further reduction in the flexural capacity.

4. Composite bolted plated beam specimens

4.1. Specimens, test rig and instrumentation

The effect of plating beams was determined by testing eight $0.2 \times 0.37 \times 5$ m long simply supported beams (Ahmed 1996). The same reinforced concrete element was used in all the beams and is shown in Fig. 6. The beams are grouped as shown in column 1 of Table 1: Series 1 consisted of two unplated beams which were used as reference strengths to determine directly the increase in strength due to plating; Series 2 consisted of the shallow plated beams shown in Figs. 6(a) and (b) where it can be seen that one or two rows of bolts were used and the longitudinal spacing of these bolts is shown in Figs. 7(a) to (d); and Series 3 comprised the deep plated beams in Fig. 6(c) which had the longitudinal spacings shown in Figs. 7(e) and (f).

The yield strength of the Y20 longitudinal bars in Fig. 6 was 443 MPa, the stirrups were Y12 bars spaced at 185 mm, and the yield strength of the plates varied slightly from 358 to 377 MPa. The cylinder compressive strength of the concrete at the time of the beam test is listed in column 3 of Table 1. A comprehensive series of 24 push-tests reported elsewhere (Ahmed 1996) were used to determine and develop the ideal form of bolt shear connection. It was found in these tests that most bolt connectors are not suitable as they do not possess the dual properties of initial stiffness and then ductility that are essential for plated beam construction, as explained in the previous section on the neutral axis separation h_{na}. The bolt shear connectors used in these beams consisted of internally



(a) Beam B11, B12 and B24 (b) Beam B13 (c) Beam C11 and C12

Fig. 6 Plated beam cross-sections

Table 1 Beam tests

Series # (1)	Beam # (2)	f_c MPa (3)	P_{bolt} kN (4)	M_{rpa} kNm (5)	η (6)	M_{exp} kNm (7)	$(\Delta M)_{rpa}$ % (8)	$(\Delta M)_{exp}$ % (9)	(8)-(9) % (10)
1	A11	49	-	132	-	120	-	-	-
	A21	46	-	128	-	113	-	-	-
2	B11	49	22	234	1.75	188	77	56	21
	B12	49	22	201	0.48	169	52	41	11
	B13	49	22	202	0.43	177	53	47	6
	B24	46	21	226	0.91	181	76	60	16
3	C11	49	22	261	0.70	204	98	70	28
	C12	49	22	249	0.42	204	89	70	19

threaded adhesive anchors which had an outer diameter of 11 mm and sleeve length of 80 mm. The individual dowel strengths of these shear connectors P_{bolt} were determined directly from push specimens that were cast with the beams and they are listed in column 4 of Table 1. Typical load/slip characteristics of these shear connectors (Ahmed 1996) consisted of a relatively stiff portion in which the maximum strength was reached at a slip of 14% of the diameter of the bolt d_{bolt}, a plastic plateau in which the maximum strength was maintained up to a slip of $0.35d_{bolt}$, and then gradual failure which tended to zero strength at a slip of $0.8d_{bolt}$.

The beams were tested with a 1 m length constant moment region and a 4.7 m span as shown in Figs. 7(a) and 8. The plates in the deep plated beams in Figs. 6(c) and Figs. 7(e) and (f) were deliberately restrained against buckling. Rules for spacing connectors to inhibit buckling have been developed elsewhere (Smith, Bradford and Oehlers 1999a, b). Strain gauges were attached to the plates at mid-span, as can be seen in Fig. 8, and Demec gauges were placed above the strain gauges in the compression zone of the beam to give an indication of the curvatures in the reinforced concrete section of the beams. Transducers were used to measure the deflections and both longitudinal and transverse slips.

4.2. Standard rigid plastic properties

The standard rigid plastic properties of the beams were derived from the full-shear-connection full-interaction procedure in Fig. 1(A) and the partial-shear-connection partial-interaction procedure illustrated in Fig. 1(B), using Eq. (1) for the γ factor. The rigid plastic flexural capacities M_{rpa} are listed in column 5 in Table 1, the degree of shear connection η in column 6, and the theoretical increase in the flexural capacity of the reinforced concrete beam due to plating in column 8. It can be seen that there is a wide range of the degree of shear connection from 42% to 175% and the expected increases in strength due to plating ranged from 52% to 98%.

5. Composite plated beam tests

The following three beams have been used to illustrate typical behaviours of plated beams.

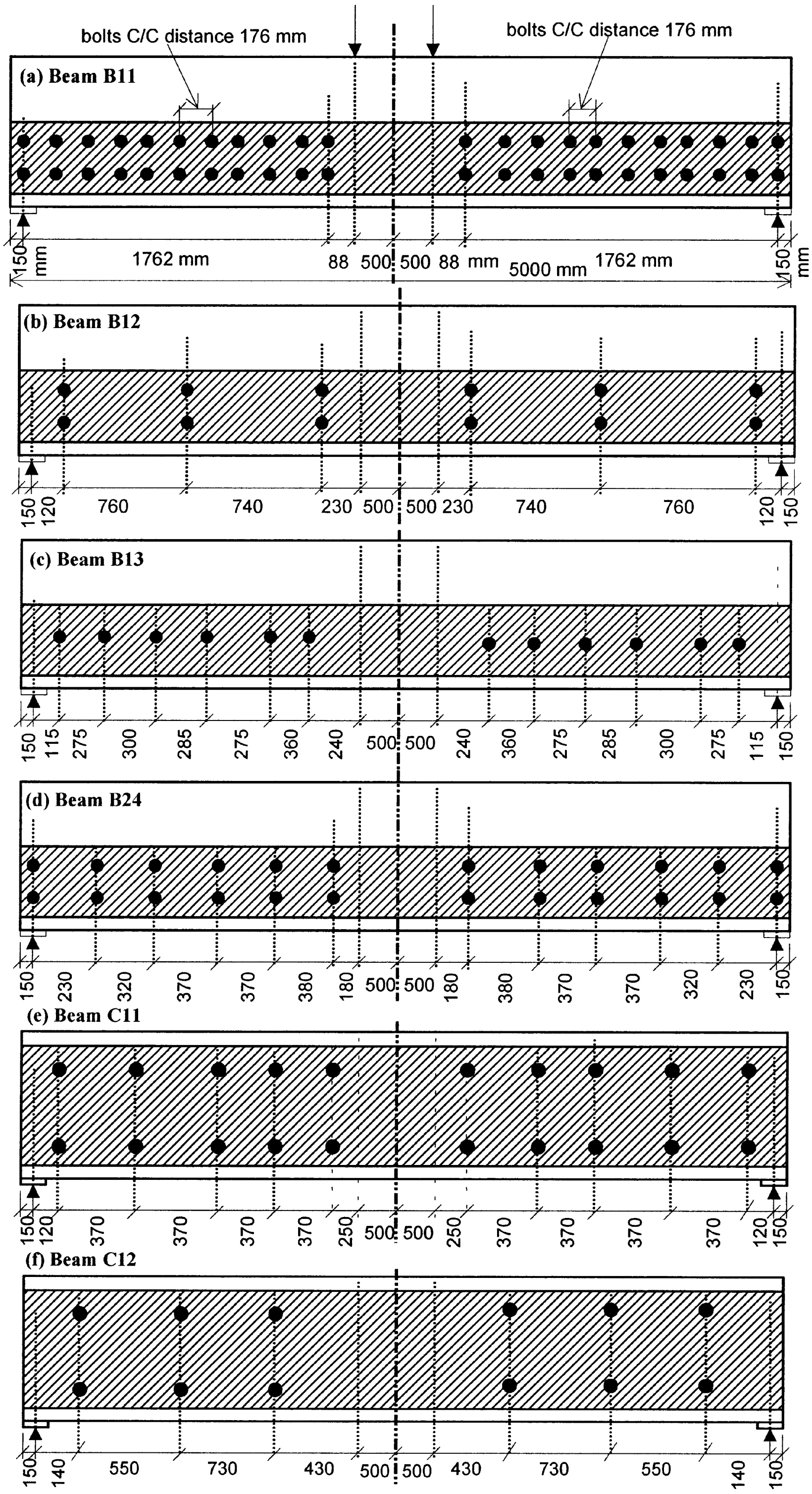


Fig. 7 Distribution of bolt shear connectors

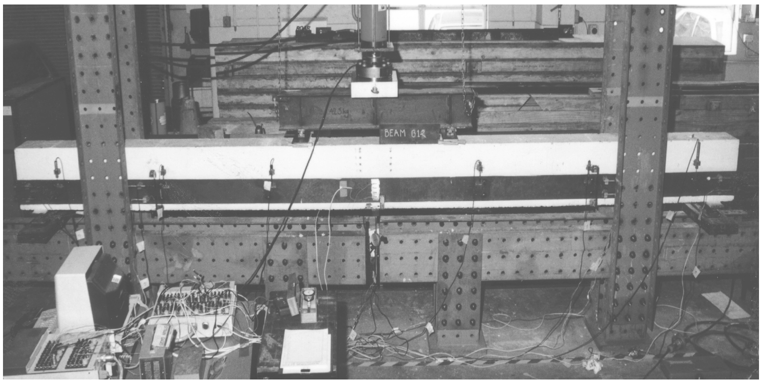


Fig. 8 Test rig

5.1. Beam B11: Shallow plate with a high degree of shear connection (η = 1.75)

The applied-shear-load/mid-span-deflection curve of beam B11 is shown in Fig. 9 and can be compared with the unplated reference beam A11. Beam B11 exhibited the characteristics of a heavily reinforced beam, because soon after the maximum load had been resisted, the concrete crushed and the strength reduced. There is virtually no plastic plateau as compared with beam A11, however, this brittle characteristic is tempered by a gradual reduction in strength as the plate maintains its moment as the reinforced concrete element reduces in strength as the concrete crushes. The experimental strain distributions just prior to failure are shown in Fig. 10 where it can be seen that the curvature in the R.C. beam is much larger than the curvature in the plate due to the $\Delta\kappa$ effect already discussed in this paper. The neutral axis separation h_{na}, also evident in Fig. 10, occurred throughout the cycle of loading even though, because of the large number of shear

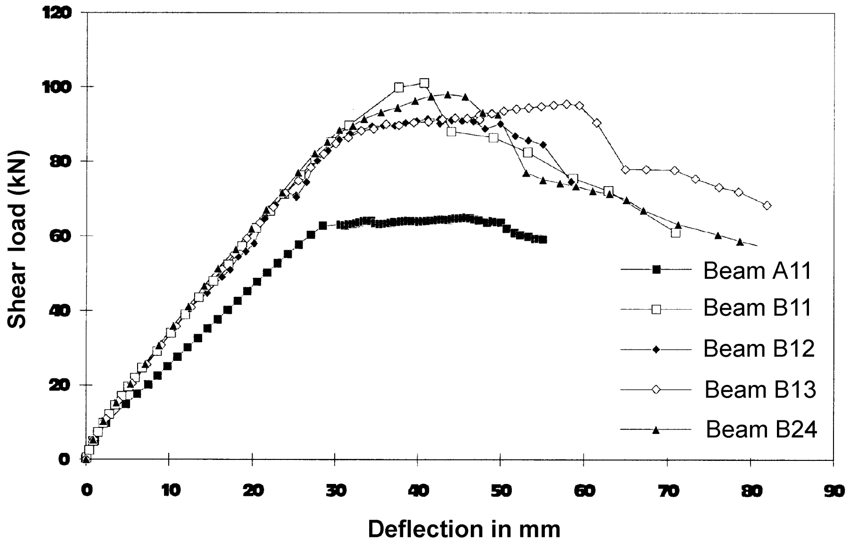


Fig. 9 Shallow plated beams

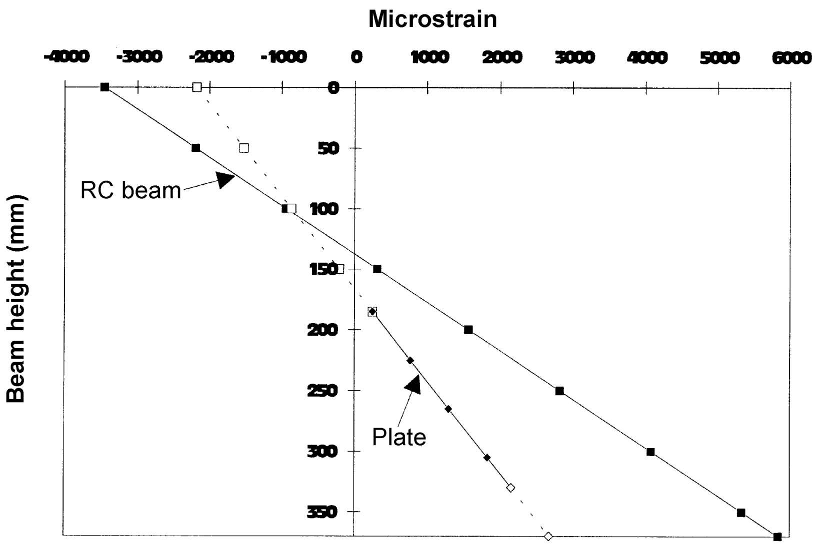
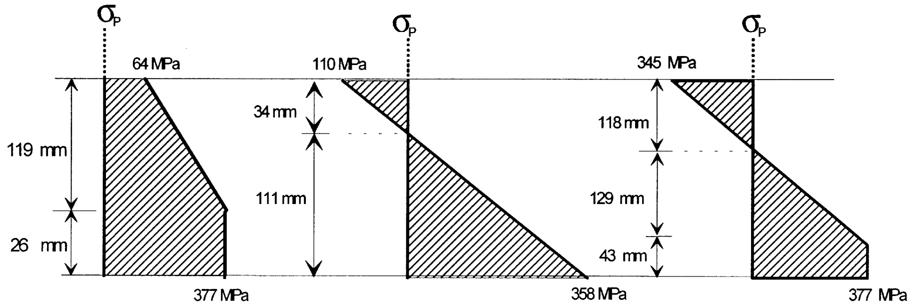


Fig. 10 Strain profile in beam B11 just prior to failure



(a) Beam B11 (b) Beam B12 (c) Beam C11

Fig. 11 Stress profiles in plate prior to failure

connectors, the maximum transverse and longitudinal slips only reached 0.4 mm which is well below that required for the connector to behave in a plastic fashion. The stresses in the plate at the maximum load are shown in Fig. 11(a) where it can be seen that 82% of the plate is still unyielded even though the beam had 75% more shear connectors than is theoretically required for full-shear-connection.

5.2. Beam B12: Shallow plate with a low degree of shear connection (η = 0.48)

The degree of shear connection in this beam was only 48%. As can be seen in Fig. 9, Beam B12 possessed a reasonable ductile plateau after which crushing of the concrete caused a gradual reduction in strength. After the maximum load had been achieved, a connector adjacent to a support fractured. This is a common occurrence in composite beams, where the formation of the plastic hinge after the maximum load has been achieved induces a very rapid build up of the slip-strain around the hinge. There is, therefore, a rapid increase in slip whilst the plastic hinge is rotating which can cause the shear connectors to fracture and, hence, limit the ductility of the composite beam. The variations in the curvature in the reinforced concrete element of the beam and in the plate are shown in Fig. 12, where it can be seen that the difference in curvature $\Delta\kappa$ is fairly constant until the maximum moment is attained, after which the curvature in the plate remains constant whilst that in the reinforced concrete beam increases rapidly. It can be seen in Fig. 11(b) that none of the plate had yielded.

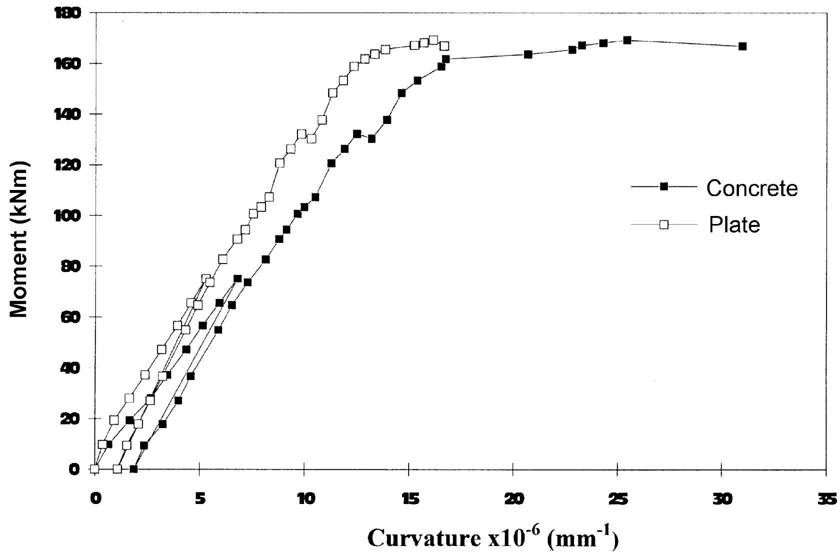


Fig. 12 Moment curvature curve for beam B12

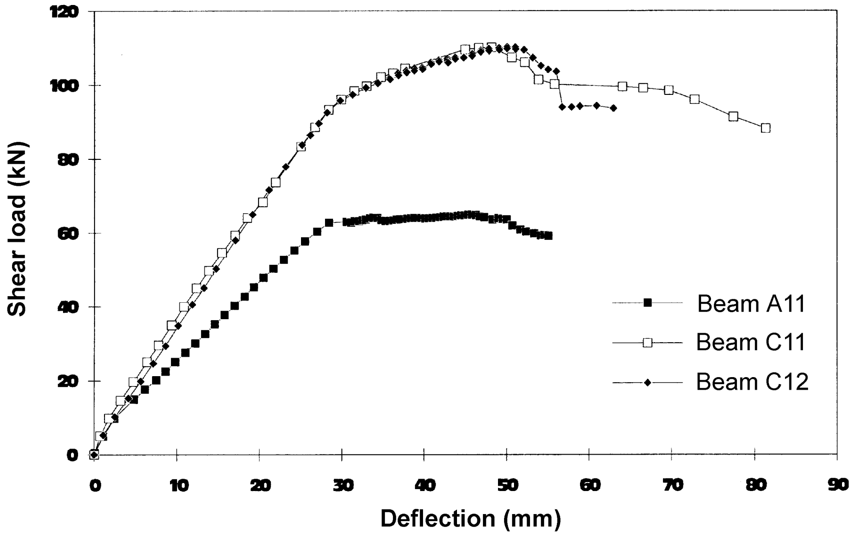


Fig. 13 Deep plated beam

5.3. Beam C11: deep plate (η = 0.70)

The load/deflection characteristics for the deep plated beam C11 is shown in Fig. 13. Even though the plates were restrained against buckling, the beam did eventually fail through buckling. However, it was still able to behave in a ductile fashion, which was followed by a very gradual reduction in strength whilst the plate maintained its relatively large moment capacity during failure of the reinforced concrete element. Although the depth of the plate was almost equal to the depth of the beam, only 15% of the plate attained yield as shown in Fig. 11(c).

6. Comparison of results

The load/deflection curves for the shallow plated beams and the unplated beam are compared in Fig. 9. Only beam B11 which possessed a high degree of shear connection failed in a brittle fashion whilst the remaining shallow plated beams had ductile plateaux. All of the shallow plated beams exhibited a gradual reduction in strength after the concrete crushed, due to the plate maintaining its moment capacity as the reinforced concrete element failed. Both deep plated beams shown in Fig. 13 exhibited the ideal characteristics of a large ductile region followed by a gradual reduction in strength, which emphasises the advantages of extending the plate into the compression region in order to maintain or enhance the ductility and provide a gradual reduction in strength.

The experimentally determined flexural capacities of the beams are listed in column 7 in Table 1 and the experimentally determined percentage increases in the flexural capacities of the R.C. beams due to plating $(\Delta M)_{exp}$ are shown in column 9. It can be seen that plating has increased the strengths from 41% to 70%. However, the experimental increases in column 9 are always less than the theoretical increases derived from standard rigid plastic analyses in column 8. The loss of strength ranges from 6% to 28% as shown in column 10.

7. Conclusions

- A rigid plastic full-shear-connection analysis requires full-interaction and, hence, will always be an upper bound to the flexural capacities of bolted side plated beams where partial-interaction always occurs.
- It is necessary for the bolt shear connection to be both stiff and ductile in order for the beam to achieve the maximum possible flexural strength and not to fail through fracturing of the shear connectors by excessive slip.
- The neutral axis parameter γ has a minor effect on the flexural capacity but a major effect on the bond force. The use of $\gamma = 1$ will give a slightly conservative estimation of both the bond force and flexural capacity.
- In addition to the usual longitudinal forces, the shear connectors resist transverse forces. These transverse forces reduce the curvature in the plate relative to that in the reinforced concrete beam and, hence, prevent the plate from strain hardening and restrict its ability to yield.
- Deep and shallow plated beams were tested with η ranging from 42% to 175%. The plates increased the strengths by 41% to 70% but this was less than the theoretical expected increase by 6% to 28%, because most of the plate cross-sections were unyielded at failure.

• Only one shallow plated beam with a high degree of shear connection behaved in a brittle fashion. The remaining shallow plated beams and deep plated beams had ductile plateaux, and all the beams exhibited a gradual reduction in strength after the concrete started to crush. The deep plated beams exhibited the dual ideal characteristics of a large plastic plateaux followed by a very gradual reduction in strength.

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Notation

d	= diameter; effective depth
ds/dx	= slip strain
F	= force; force profile; applied force
f_c	= compressive cylinder strength of concrete
f_{yp}	= yield strength of plate
f_{yr}	= yield strength of reinforcing steel
H	= horizontal or longitudinal force in shear connectors
h	= height
h_{na}	= neutral axis separation
L	= distance between resultant transverse forces
M	= moment: flexural capacity
P	= strength of shear connectors in a shear span

s_l = longitudinal slip
s_v = vertical or transverse slip
t = thickness
V = vertical or transverse force in shear connection
y = depth to neutral axis
ΔM = increase in flexural capacity due to plating
$\Delta\kappa$ = difference in curvature
ε = strain profile
γ = neutral axis depth factor
η = degree of shear connection; P/P_{fsc}
κ = curvature
σ = stress profile

commonly used suffices:

comp = compression; compressive
conc = concrete
exp = experimentally determined
fi = full-interaction
fsc = full-shear-connection
max = maximum
nla = non-linear full-interaction
p = plate
pi = partial-interaction
psc = partial-shear-connection
RC = reinforced concrete
reo = reinforcing bar
rpa = rigid-plastic analysis
ten = tension