Structural Engineering and Mechanics, Vol. 10, No. 2 (2000) 165-180

DOI: http://dx.doi.org/10.12989/sem.2000.10.2.165

Strength of prestressed concrete beams in torsion

Chris G. Karayannis† and Constantin E. Chalioris‡

Department of Civil Engineering, Democritus University of Thrace, Xanthi, 67100 Greece

Abstract. An analytical model with tension softening for the prediction of the capacity of prestressed concrete beams under pure torsion and under torsion combined with shear and flexure is introduced. The proposed approach employs bilinear stress-strain relationship with post cracking tension softening branch for the concrete in tension and special failure criteria for biaxial stress states. Further, for the solution of the governing equations a special numerical scheme is adopted which can be applied to elements with practically any cross-section since it utilizes a numerical mapping. The proposed method is mainly applied to plain prestressed concrete elements, but is also applicable to prestressed concrete beams with light transverse reinforcement. The aim of the present work is twofold; first, the validation of the approach by comparison between experimental results and analytical predictions and second, a parametrical study of the influence of concentric and eccentric prestressing on the torsional capacity of concrete elements and the interaction between torsion and shear for various levels of prestressing. The results of this investigation presented in the form of interaction curves, are compared to experimental results and code provisions.

Key words: torsional strength; prestressed concrete; nonlinear analysis; torsion and shear; torsion with shear and flexure.

1. Introduction

Although prestressed concrete beams are very practical and frequently used elements in actual structures, the research effort devoted to the torsional strength of these members under pure torsion or combined torsion with bending is very limited (Mukherjee and Warwaruk 1971, Mattock and Wyss 1978, Wafa *et al.* 1992). Furthermore, for T-beams under predominant torsion, although the flange contribution to resistance against torsion is very significant, there is no commonly accepted method for the quantification of this influence (Hsu 1984).

The classical elastic approach by Saint Venant to the torsion problem, although properly describes the elastic behavior of concrete, fails to predict the ultimate torsional strength even in the case of rectangular plain concrete elements. This theory is based on the assumption that brittle failure of the element occurs when the maximum tensile stress reaches the concrete maximum tensile strength. This approach, however, is not consistent with the fact that even brittle materials such as concrete, exhibit post cracking resistance in tension (Gopalaratnam and Shah 1985) and in shear (Vecchio and Collins 1986). Thus, ignoring this post cracking tension softening phenomenon, the elastic theory consistently underestimates the ultimate torsional strength, which for plain concrete elements has been experimentally observed to be roughly up to 50% greater than the predicted one (Hsu 1984,

† Associate Professor
‡ Civil Engineer, Ph.D

Karayannis 1995a).

Further, two more approaches, the plastic theory and the skew bending theory (Hsu 1984), have been proposed for the estimation of the torsional strength of concrete elements. However, while the former is not theoretically quite satisfactory, the skew-bending theory (although describes the failure mode of concrete elements of rectangular cross-section very well) is rendered useless in practice because of the mathematical complexity involved for usual flanged section (Cowan 1965, Hsu 1984).

For prestressed concrete in torsion, Hsu (1984) has proposed that from the failure criteria it is possible to derive a simple prestress factor, which is defined as the strength ratio of a prestressed element to a nonprestressed element. He has shown that this prestress factor can be used complementary to the elastic, plastic and skew bending theories.

Recently, an efficient numerical approach for the prediction of the torsional behavior of concrete elements has been proposed (Karayannis and Soulis 1990). This algorithm, initially based on the classical elastic theory, uses a special numeric of technique for the solution of the governing equation of torsion. Recently the technique includes a finite difference scheme resulting from a second-order finite element shape function for the solution of the equation of torsion and it can be applied to elements with practically any cross-section since it utilizes isoparametric numerical mapping. The efficiency and the accuracy of this approach in predicting the ultimate torsional strength of concrete beams not reinforced in the transverse direction has been dramatically improved after a major modification by Karayannis (1995a). The modification takes into account a bilinear stress-strain relationship for the concrete in tension to cater for the tension softening phenomenon, and allows for a realistic prediction of the entire response of a concrete element subjected to monotonically increasing torque (Karayannis 1995a). Initial efforts for the application of the modified method in prestressed girders (Karayannis 1994) yielded promising results.

A further use of this analytical method, properly adapted in order to include tri-linear and exponential stress-strain relationships for the material in tension, has already been proposed for the prediction of the torsional behavior of steel-fiber concrete (Karayannis 1995b). Verification of this approach based on results of experimental work conducted for this purpose, has also been successfully attempted (Karayannis 2000).

The aim of the present work is twofold; first, the proper modification of the developed analytical method and its validation in order to describe the torsional behavior of prestressed concrete beams, and second, the use of the method as an analytical tool for the investigation of the influence of the prestressing force on the torsional strength of these elements.

The proposed method is mainly applied to plain prestressed concrete elements, but is also applicable to prestressed concrete beams without heavy transverse reinforcement. This is justified because in these cases the highly compressive action of the prestressing force improves the elastic behavior and the peak strength of the specimen but simultaneously increases significantly the possibility for a short brittle post peak response. Thus, the observed behavior of concrete elements with high prestressing as a whole is very alike to the behavior of the same elements subjected to the same combination of loading (torsion with axial force) without taking into account the small influence of the existing reinforcement.

The validation of the developed analysis is achieved by providing comparisons between experimental results and analytical predictions. These comparisons comprise prestressed concrete beams in pure torsion as well as in torsion combined with shear and bending, compiled from works around the world in an attempt to establish the validity based on a broad range of studies. The second aim of the present work is approached by a parametrical study of the torsional responses of rectangular and flanged prestressed concrete beams. The interaction of torsion and bending due to eccentric prestressing and the influence of the combined loading on the ultimate strength of these elements are also studied. The results of this investigation and the produced interaction curves are compared with the results of previous works (both experimental and analytical) and with the provisions of design codes.

2. Governing equations

2.1. Torsion

The theory of Saint Venant and the complimentary approach by Prandtl (Hsu 1984) describe the response in torsion of a homogeneous structural element based on the assumptions that (i) the element has a constant cross-section along its length and its axis coincides with the axis of torsion, (ii) the angle of twist per unit length is constant and (iii) there is no skew restraint. The above-mentioned theories yield the equation

$$\nabla^2 F = -2G\vartheta \tag{1}$$

where F=stress function which satisfies all boundary conditions; ϑ=angle of twist per unit length and G=shear modulus of rigidity.

In cases where the material's properties vary over the cross-section, Eq. (1) can be written in the form

$$\frac{\partial}{\partial x}\left(\frac{1}{G}\frac{\partial F}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{G}\frac{\partial F}{\partial y}\right)=-2\vartheta \tag{2}$$

The relationships of the developing shear stress components τ_{zx}, τ_{zy}, with the function F, are

$$\tau_{zx}=\partial F/\partial y \quad \text{and} \quad \tau_{zy}=-\partial F/\partial x \tag{3}$$

and the shear stress at a point is

$$\tau = (\tau_{zx}^2 + \tau_{zy}^2)^{1/2} \tag{4}$$

Since, only shear stresses develop on the cross-section of an element subjected to pure torsion without skew restraint, an infinitesimal element on this cross-section is in pure shear stress state. In Fig. 1 an infinitesimal element in pure shear stress state is displayed. From this figure it is deduced that in the case of pure torsion the response can be characterized by the behavior of the material in direct tension where the tensile stress is equal to the developing shear stress. This is in full compliance with the conclusions drawn from early experimental efforts for the study of the behavior of plain concrete subjected to pure torsion, which revealed that the material fails in tension rather than shear (Anderson 1935, Cowan 1965).

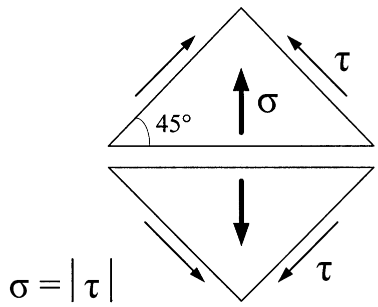


Fig. 1 An infinitesimal element in pure shear characterized by the material behavior in direct tension

2.2. Prestressing

The normal stresses σ_i at a point i in the cross-section which develop due to the prestressing force P, are calculated as

$$\sigma_i = -\frac{P}{A_c} + \frac{Pe}{I} y_i \tag{5}$$

where e is the distance of the point where the prestressing force is applied from the centroidal axis (eccentricity), A_c is the cross-section area, I is the moment of inertia and y_i is the distance of point i from the centroidal axis.

3. Numerical formulation

An efficient numerical algorithm for the analysis of concrete elements in torsion, recently proposed by Karayannis (1995a), is employed in the present work. The element section is discretized by 8-node isoparametric elements. That is, all physical elements (x, y) regardless of their configuration are mapped to the (ξ, η) coordinates (isoparametric mapping) such that $-1 \le \xi \le 1$ and $-1 \le \eta \le 1$, where the nodes of the mapped elements are located at $\xi = \pm 1$ and $\eta = \pm 1$.

Let J be the Jacobian transformation matrix and T denotes the transpose matrix; then $H = J^T J$. It follows that if H^{-1} is the inverse of matrix H, the shear stress components in the computational field are derived as

$$\begin{bmatrix} \tau_{z\eta} \\ \tau_{z\xi} \end{bmatrix} = H^{-1} \begin{bmatrix} \partial F / \partial \xi \\ \partial F / \partial \eta \end{bmatrix} \quad \text{such that} \quad \begin{bmatrix} -\tau_{zy} \\ \tau_{zx} \end{bmatrix} = J \begin{bmatrix} \tau_{z\eta} \\ \tau_{z\xi} \end{bmatrix} \tag{6}$$

If Δ denotes the determinant of matrix J and G is the shear modulus of elasticity then the governing equation of torsion (Eq. 2) can be transformed into the local (ξ, η) domain as

$$\frac{\partial}{\partial \xi}\left(\frac{\Delta}{G}\tau_{z\eta}\right) + \frac{\partial}{\partial \eta}\left(\frac{\Delta}{G}\tau_{z}\xi\right) = -2\vartheta\Delta \tag{7}$$

At the center (i, j) of an element in the computational grid Eq. 7 may be discretized as

$$\left(\frac{\Delta}{G}\tau_{z\eta}\right)_{i,j+1}-\left(\frac{\Delta}{G}\tau_{z\eta}\right)_{i,j-1}+\left(\frac{\Delta}{G}\tau_{z\xi}\right)_{i+1,j}-\left(\frac{\Delta}{G}\tau_{z\xi}\right)_{i-1,j}=-2\vartheta\Delta_{i,j} \tag{8}$$

Thus, the governing equation of torsion can be approximated in a finite difference form (Karayannis 1995a).

4. Material model

The formulation is intended to describe the response of concrete elements under combined loadings with the predominant and critical action being the torsional moment. With this in mind, special care has been taken for the material model in tension since in these cases the behavior of the material in tension characterizes the response of the element.

Thus, in the present work, a bilinear stress-strain model for the behavior of concrete in direct tension which includes post-cracking tension softening branch is adopted (Fig. 2). According to this model the stress and strain are shown to increase proportionally up to the point of ultimate tensile strength of concrete, f_{ct}. At this point (u in Fig. 2) the strain value ε_{cr} is equal to f_{ct}/E_{cs}, where E_{cs} is the secant modulus of elasticity of concrete in tension at peak. After that point only the strain increases while the stress decreases linearly to zero at the point (f in Fig. 2) where the strain is equal to the ultimate experimentally observed tensile strain of concrete ε_{ctu}. If the stress is relaxed at a point in the softening branch (say point a in Fig. 2) then the unloading occurs along the line 0a (small cracks close) and the new updated stress-strain relationship follows the path 0af. Thus, irreversibility is invoked due to the stiffness degradation. The slope of the line 0a decreases as the point a approaches the ultimate strain ε_{ctu}. This way the post-cracking part of the adopted model is characterized by the maximum acceptable strain ε_{ctu}, which can be expressed as

$$\varepsilon_{ctu}=\alpha\varepsilon_{cr} \tag{9}$$

where α is a coefficient which represents the level of the involved tension softening. Coefficient α

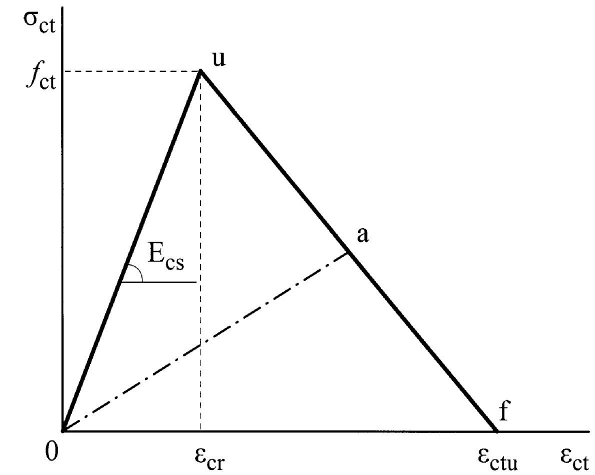


Fig. 2 Concrete response in direct tension with tension softening branch

can be considered as a material property which, except for the influence of the stress distribution, mainly depends on the nature and the size of the aggregates and the other ingredients of concrete. The influencing role of size effect on this coefficient should also be considered. Realistic values for the coefficient α can be obtained either directly from experimental works on the tensile behavior of concrete (Gopalaratnam and Shah 1985) or from comparisons between experimental curves for concrete beams in torsion and their analytical predictions using the proposed analysis (Karayannis 1995a). Thus, for the commonly used concrete mixtures, the values α=5 to 8 can be considered as realistic ones (Karayannis 1994, 1995a, 1995b).

5. Material failure criteria

As pointed out earlier, concrete is expected to fail in tension due to torsion since in the examined cases the torsion is the predominant and critical action. It is obvious, though, that the strength of concrete at each point of the cross-section is greatly influenced by the developing stress state at this point. Thus, in the case of prestressed concrete beams under the simultaneous action of torsion, bending and shear, it has to be considered that at each point of the element's cross-section a biaxial state of stress is developed (Fig. 3). For this reason the dual failure criteria by Cowan for the concrete under biaxial loading (Cowan 1965, Hsu 1984) are employed. According to these criteria, Mohr's failure envelope has been simplified into two straight lines AB and BD as shown in Fig. 4.

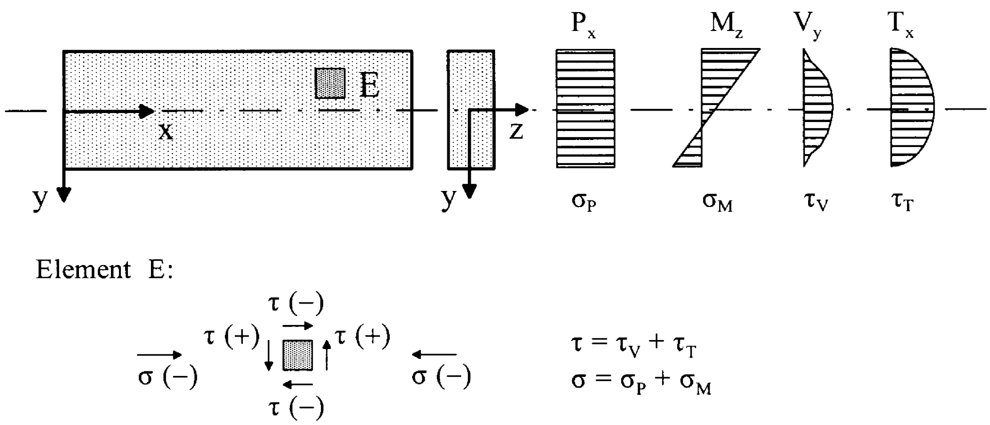


Fig. 3 Biaxial stress state under combined actions of prestressing, flexure, shear and torsion

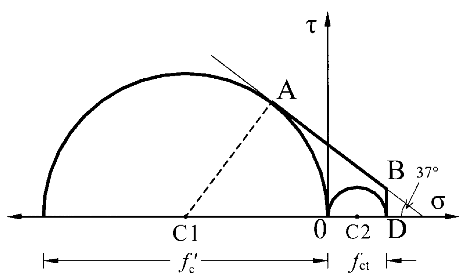


Fig. 4 Mohr's failure envelope for concrete in biaxial stress-state, as simplified by Cowan

The inclined line AB is assumed to be tangential to the circle C1 for the uniaxial compression and has an angle to the horizontal equal to 37° which represents the internal friction angle. This line determines the failure state when concrete fails primarily in compression. The line BD is tangential to circle C2 for uniaxial tension and determines the failure state when concrete fails primarily in tension. This criterion is directly applicable by the present approach since the procedure uses the shear stress obtained by the analysis at each mesh point. For a given stress state (σ, τ) the failure criterion is given by the expressions

$$\frac{\tau}{f_{ct}} = \frac{f_c'}{f_{ct}}\sqrt{0.0396 + 0.120\frac{\sigma}{f_c'} - 0.1594\left(\frac{\sigma}{f_c'}\right)^2} \tag{10}$$

$$\frac{\tau}{f_{ct}} = \sqrt{1 + \frac{f_c'}{f_{ct}}\left(\frac{\sigma}{f_c'}\right)} \tag{11}$$

Where f_c' is the cylinder concrete strength in compression; f_{ct} is the tensile strength of concrete as it is obtained from the stress-strain model (Fig. 2) and σ is the normal stress.

6. Influence of concentric prestressing on the torsional strength

The validation of the described analysis by providing comparisons between results from experimental studies and analytical predictions is first attempted. Experimental results concerning the influence of concentric prestressing on the ultimate torsional strength of rectangular beams have been compiled from studies by Wafa *et al.* (1992), Allos and Rashid (1989) and Mukherjee and Warwaruk (1971). A group of rectangular beams which includes 8 specimens with f_c'/f_{ct}=12.39 to

Table 1. Comparison of experimental data and predicted values for rectangular beams with concentric prestressing subjected to pure torsion (f_c'/f_{ct}=12.39 to 17.68 with mean value of 13.40)

Code name	b/h (cm/cm)	f_c' (MPa)	f_r (MPa)	f_{ct} (MPa)	$\frac{f_c'}{f_{ct}}$	P (kN)	T_{exp} (kNcm)	T_{pred} (kNcm)	$\frac{T_{exp}}{T_{pred}}$	$T_{pred,P=0}$ (kNcm)	$\frac{P}{bhf_c'}$	$\frac{T_P}{T_{P=0}}$
Wafa *et al.* (1992)												
B0.0-0		37.93	4.00	2.83	13.42	0.00	268.0	250.3	1.07	250.3	0.00	1.07
B0.0-2a		40.72	4.19	2.84	14.36	187.72	444.0	478.8	0.93	251.1	0.18	1.77
B0.0-2b	10/25	36.59	4.13	2.90	12.63	182.09	489.0	479.7	1.02	255.8	0.20	1.91
B0.0-4a		37.07	3.81	2.10	17.68	332.44	457.0	502.2	0.91	185.4	0.36	2.46
B0.0-4b		35.56	4.70	2.87	12.39	276.65	558.0	558.6	1.00	253.1	0.31	2.20
Allos and Rashid (1989)												
A3	10/17.5	41.04	4.08	3.20	12.81	161.60	416.5	371.8	1.12	188.5	0.23	2.21
A4		41.04	4.08	3.20	12.81	323.20	514.1	490.7	1.05	188.5	0.45	2.73
Mukherjee and Warwaruk (1971)												
C206	15.24/30.48	39.21	4.38*	3.13	12.52	375.47	1653.9	1445.0	1.14	762.6	0.21	2.17
Mean value:		38.65	4.17	2.88	13.40				1.030			

*$f_r = 0.7\sqrt{f_c'}$

Table 2. Comparison of experimental and predicted data for rectangular beams with concentric prestressing subjected to pure torsion (f_c'/f_{ct}=8.46 to 9.63 with a mean value of 9.05, concrete with steel-fiber volume fraction 0.5 to 1.0%)

Specimens b = 10 cm h = 25 cm	f_c' (MPa)	f_r (MPa)	f_{ct} (MPa)	$\frac{f_c'}{f_{ct}}$	P (kN)	v_P $\frac{P}{bhf_c'}$	r_{exp} $\frac{T_{exp}}{T_{exp,P=0}}$	r_{pred} $\frac{T_{pred}}{T_{pred,P=0}}$	$\frac{r_{exp}}{r_{pred}}$
Wafa *et al.* (1992)									
B0.5-0	35.10	4.94	3.65	9.63	0.00	0.00	1.00	1.00	1.00
B1.0-0	39.08	6.94	4.62	8.46	0.00	0.00	1.00	1.00	1.00
B0.5-2	34.16	5.72	3.55	9.63	184.95	0.22	1.65	1.76	0.94
B1.0-2	42.89	6.92	4.97	8.63	183.60	0.17	1.69	1.58	1.07
B1.0-4	39.14	6.95	4.26	9.19	328.80	0.34	2.06	2.02	1.02
Mean value:	38.07	6.29	4.21	9.05					1.006

17.68 and concentric prestressing, tested in pure torsion, are presented in Table 1. Geometrical data, test results, analytical predictions are also presented in Table 1. Furthermore, comparisons of the analytical results to the experimental ones (T_{exp}/T_{pred}) and the ratio $T_p/T_{P=0}$ of the increased ultimate torsional strength due to the influence of the prestressing (T_P) to the ultimate torsional strength without the prestressing ($T_{P=0}$), are also included in Table 1. A second group of experimental data for prestressed rectangular beams with f_c'/f_{ct}=8.46 to 9.63 tested in pure torsion (Wafa *et al.* 1992), are presented in Table 2.

Further, Fig. 5 presented the influence of the prestressing level, v_P=$P(bh\,f_c')$, on the ultimate torsional strength T_P of rectangular elements in terms of the ratio $T_P/T_{P=0}$. Thus, curves 1 and 2 in Fig. 5 represent the influence of v_P on the torsional strength of concrete for strength ratios f_c'/f_{ct}=13.40 and 9.05, respectively, and are compared with the experimental results of Tables 1 and 2, respectively. The strength ratios of the concrete (compressive strength to tensile strength), for the tests in Tables 1 and 2, were f_c'/f_{ct}=12.39 to 17.68 with mean value of 13.40, and f_c'/f_{ct}=8.46 to 9.63 with mean value of 9.05, respectively, whereas the analytical curves 1 and 2, shown in Fig. 5, were calculated based on the proposed analysis for f_c'/f_{ct}=13.40 and 9.05, respectively. From these comparisons it can be deduced that the analytical results are in very good compliance with the measured ones.

Also shown in Fig. 5 are the analytical predictions deduced by the approach of the prestress factor as it has been introduced by Hsu (1984) to account for the influence of the prestress in the elastic, plastic and skew bending theories. According to this approach

$$T_n=T_{np}\,\gamma \tag{12}$$

where T_n is the ultimate torsional strength of the prestressed member, T_{np} the ultimate torsional strength of the nonprestressed member deduced by the skew bending theory and γ the prestress factor which in this case is given as (Hsu 1984)

$$\gamma=\sqrt{1+\frac{\sigma}{0.85f_r}} \tag{13}$$

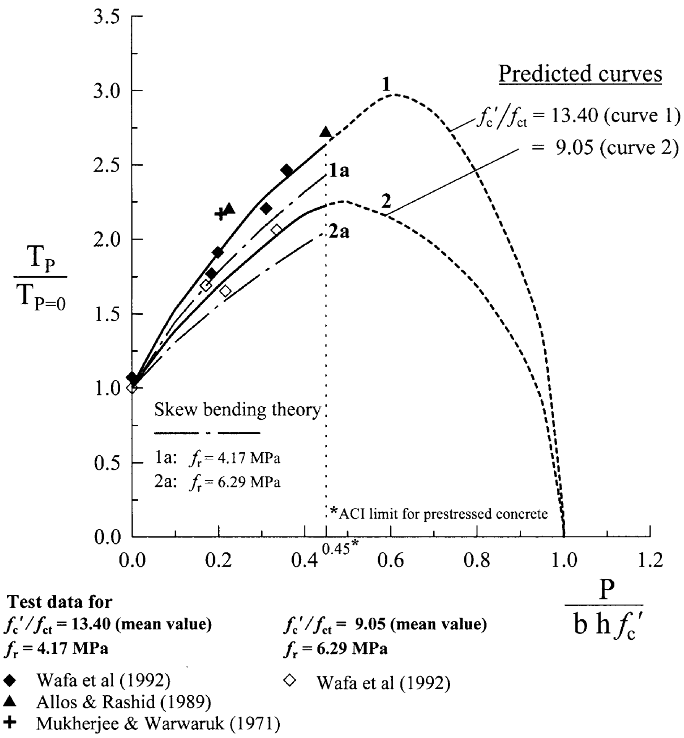


Fig. 5 Predicted interaction curves of concentric prestressing force and ultimate torsional strength of concrete elements and comparison with experimental values

where σ is the prestressing stress and f_r the modulus of rupture.

Thus, based on this approach, analytical curves 1a and 2a for the strength values of the tests in Tables 1 and 2, respectively, are also presented in Fig. 5.

Further, comparisons of the entire experimental response curves of torque moment T versus rotation ϑ, for prestressed concrete beams in torsion (Wafa *et al.* 1992), with predicted behavior curves yielded by the proposed analysis, are presented in Fig. 6. These comparisons comprise of 5 cases (see also Table 1) with prestressing levels $v_P=P(bh\,f_c')=0$, 0.18, 0.20, 0.36 and 0.31. The mean value of the ratio of the measured ultimate torsional capacity T_{exp} to the predicted one T_{pred} for these cases is $T_{\text{exp}}/T_{\text{pred}}=1.03$ (Table 1) with a standard deviation of 0.084. From these comparisons it can be shown that for the examined cases the proposed analytical method describes the entire response of the prestressed beams very well. Moreover, some differences in behavior between experimental and theoretical results observed near the ultimate torque are attributed to the load control conditions under which the beams (Wafa *et al.* 1992) were tested.

In Fig. 6 the analytical behavior curves of the beams considering stress-strain relationship for concrete in tension without post-cracking part are also shown. From these results it can be seen that for the examined cases the ratio $T_{u,soft}/T_{u,\alpha=1} \cong 1.37$, where $T_{u,soft}$ is the ultimate torque taking into account the tension softening phenomenon (post-cracking part of the stress-strain relationship) and $\alpha=8$, see also Eq. (9), and $T_{u,\alpha=1}$ is the ultimate torque considering $\alpha=1$ in Eq. (9) (elastic stress-strain relationship without post-cracking part).

Furthermore, experimental results concerning the influence of concentric prestressing on the

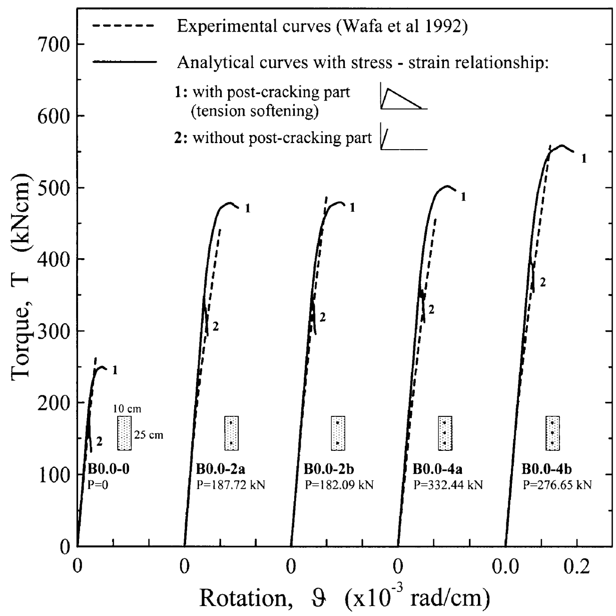


Fig. 6 Comparison of experimental response curves of prestressed concrete beams in torsion with predicted behavior curves obtained by the proposed analysis

Table 3 Comparison of experimental and predicted values for flanged beams with concentric prestressing subjected to pure torsion (f_c'/f_{ct} =11.3 to 16.7 with a mean value of 14.0)

Specimens $b_f/h_f/b/h$ = 30/5/10/25 (cm)	$\frac{f_c'}{f_{ct}}$	P (kN)	v_P $\frac{P}{A_c f_c'}$	r_{exp} $\frac{T_{exp}}{T_{exp,P=0}}$	r_{pred} $\frac{T_{pred}}{T_{pred,P=0}}$	$\frac{r_{exp}}{r_{pred}}$
Victor and Aravindan (1978) – T-beams						
TC1	14.2	0.00	0.00	1.00	1.00	1.00
TC4	14.0	0.00	0.00	1.00	1.00	1.00
TB1	13.0	74.16	0.05	1.17	1.21	0.96
TB2	14.3	74.16	0.05	1.24	1.26	0.98
TB3	15.1	74.16	0.05	1.33	1.26	1.05
TB4	12.2	74.16	0.05	1.36	1.26	1.07
TB6	14.0	74.16	0.04	1.25	1.26	0.99
TB11	13.7	148.33	0.09	1.46	1.51	0.97
TB16	11.3	148.33	0.12	1.51	1.54	0.98
TB17	16.7	148.33	0.08	1.41	1.49	0.95
TB21	14.8	148.33	0.09	1.45	1.51	0.96
TB24	13.0	296.65	0.20	1.81	1.94	0.94
TB25	13.5	296.65	0.18	1.81	1.94	0.94
TB26	14.2	296.70	0.18	1.97	1.94	1.02
Mattock and Wyss (1978) – I-beam						
A1	15.8	966.22	0.15	1.53	1.54	1.00
Mean value:	14.0					0.986

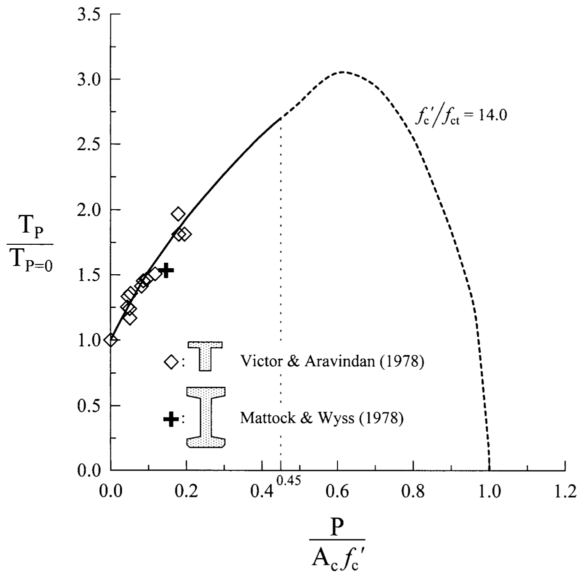


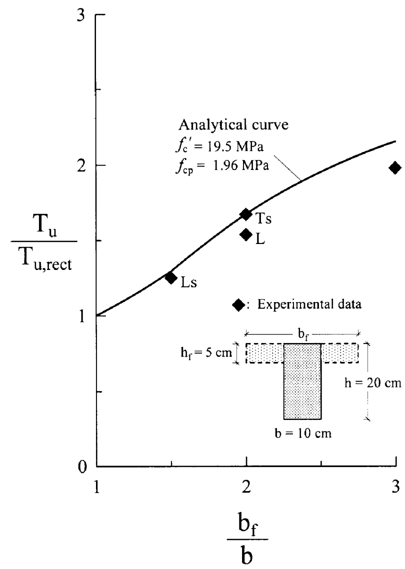
Fig. 7 Predicted interaction curve of concentric prestressing force and ultimate torsional strength of flanged beams and comparison with experimental values

ultimate strength of flanged beams compiled from the work of Victor and Aravindan (1978) and Mattock and Wyss (1978), are presented in Table 3 and Fig. 7 and compared with the analytical predictions by the proposed approach.

Comparisons of analytical predictions of the ultimate torque and experimental data compiled from Mattock and Wyss (1978) for flanged girders (with I-shape cross-section) tested in torsion combined with flexure, shear and prestressing are presented in Table 4. From these comparisons it can be concluded that the proposed analytical model predicts well the ultimate torque of flanged beams subjected in torsion combined with flexure, shear and prestressing (Karayannis 1994).

Table 4 Comparison of experimental (Mattock and Wyss 1978) and predicted values for flanged girders (I-shape) tested in torsion combined with flexure, shear and prestressing (In the analysis the softening coefficient is taken α=7.0)

Code name	f_c' (MPa)	f_{sp} (MPa)	P (kN)	M (kNcm)	V (kN)	T_{exp} (kNcm)	T_{cal} (kNcm)	$\frac{T_{exp}}{T_{cal}}$
A1	52.41	3.68	966.22	–	11.56	4451	4760	0.94
A3	49.38	3.70	956.44	19941	163.56	5100	4952	1.03
A4	49.17	3.81	962.22	39068	320.44	4447	4526	0.98
A5	51.52	3.82	955.56	59659	489.33	1534	1565	0.98
A7	53.24	4.03	987.56	26767	92.44	4677	4920	0.95
A8	50.83	3.72	963.56	43886	151.56	3781	3972	0.95
A9	46.97	4.23	976.44	59971	207.11	2429	2026	1.20
						Mean value:		0.998
						Standard deviation:		0.115



Experimental data:

Beam	f'_c	f_{cp}	T_{exp}
Ls	19.24	1.93	175.00
L	18.38	2.00	215.37
Ts	20.70	2.00	234.27
T	19.52	1.90	277.10

(tests from Karayannis & Chalioris 2000)

Fig. 8 Contribution of T-beam flanges to the torsional capacity

The lack of knowledge of the contribution of the flanges of T-beams to the shear resistance is emphasized (Zararis and Penelis 1986). In this work, the contribution of the flange of T-beams to the torsional capacity based on the proposed method is also examined. Thus, in Fig. 8 the curve of the ratio $T_u/T_{u,rect}$ versus b_f/b is presented. $T_{u,rect.}$ is the ultimate torque of the beam without flanges (rectangular cross-section) and b_f is the width of the flanges.

Finally, interaction curves of the torsional strength increase $T_p/T_{P=0}$ versus the applied prestressing

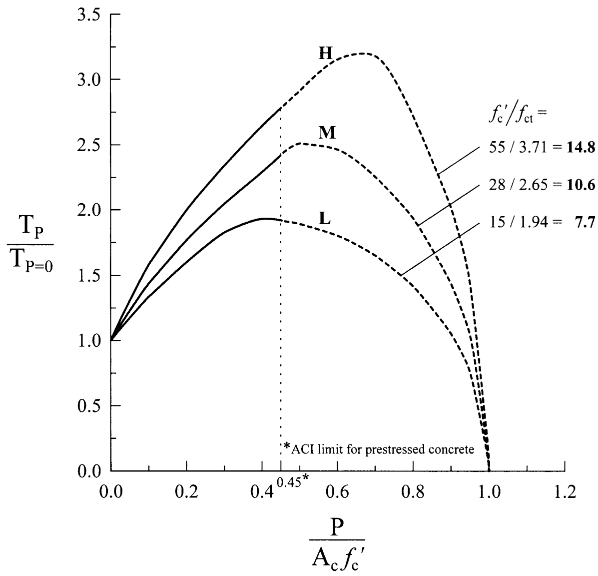


Fig. 9 Interaction curves of concentric prestressing force and ultimate torsional strength for f_c'=55, 28 and 15 MPa

level $v_P = P(A_c f_c')$ obtained using the proposed analysis, for high strength concrete f_c' =55 MPa (curve H), for commonly used concrete f_c' =28 MPa (curve M) and for low strength concrete f_c' =15 MPa (curve L), are presented in Fig. 9. It is noted that the effect of the concrete strength on the torsional capacity of prestressed concrete elements needs more discussion since the validation comparisons included in the present work were mainly for concrete strength between 35 to 45 MPa.

7. Interaction of torsion and eccentric prestressing

The influence of eccentric prestressing on the ultimate torsional strength of concrete beams is also studied. In this case, a constant bending moment without shear force along the entire length of the beam is developed.

Interaction curves of $T_p/T_{P=0}$ versus the applied prestressing level $v_P = P/(A_c f_c')$, for three different prestressing eccentricities expressed in terms of the developed flexural moment, are presented in Fig. 10. The level of the applied flexural moment is expressed in terms of the ratio v_M of the applied moment M to the ultimate flexural moment M_u of the element without prestressing (P=0) and torsion (T=0), which is easily calculated as

$$M_u = \frac{I}{y_c} f_r \quad \text{which for rectangular sections becomes} \quad M_u = \frac{bh^2}{6} f_r \tag{15}$$

where f_r is the modulus of rupture and y_c the distance of the centroidal axis from bottom.

Interaction curves of Fig. 10 were calculated for concrete compression strength f_c' =28 MPa while the tensile strength f_{ct} and the modulus of rupture f_r were estimated (ACI 318) as $f_{ct}=0.5\sqrt{f_c'}$ and $f_r=0.7\sqrt{f_c'}$ (in SI units), respectively.

From Fig. 10 it can be concluded that increasing the eccentricity of the applied prestressing force

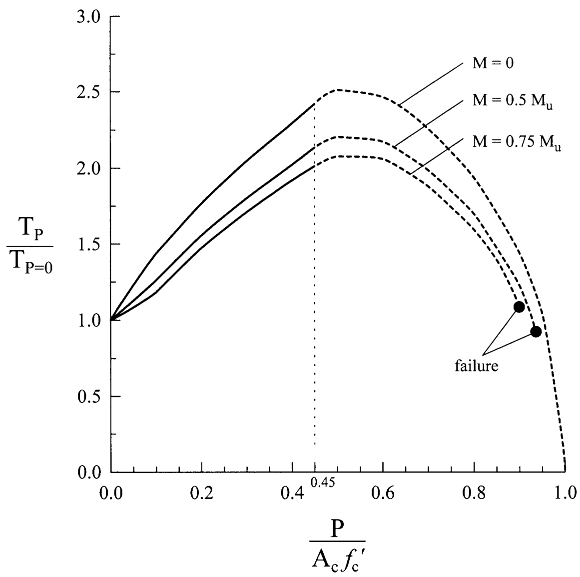


Fig. 10 Interaction curves of prestressing force and ultimate torsional strength combined with flexure (eccentric prestressing)

decreases the influence of the prestressing on the ultimate torsional strength of the element. For the examined cases, this decrease ranges from 0 to 17.2% for applied flexural moment level v_M=0 to 0.75, respectively.

8. Interaction of torsion and prestressing combined with shear and flexure

The influence of the prestressing force on the ultimate strength of concrete beams subjected to torsion combined with shear and flexure, is also studied in this work.

Interaction curves of $T_p/T_{P=0}$ versus the applied prestressing level v_P, for concrete compression strength f_c'=28 MPa and loadings which include combined shear and flexure, are presented in Fig. 11. The level of the applied shear force is expressed as the ratio v_V of the acting shear V to the ultimate shear strength V_u of the section in pure shear state without prestressing and torsion (P=0 and T=0). The maximum shear stress for a rectangular cross-section is given as $\tau_{\max}=(3/2)(V/bh)$ and considering that $\tau_{\max} \cong f_{ct}$ it concluded that $V_u \cong (2/3)bhf_{ct}$.

Thus, from Fig. 11 it is concluded that in elements under combined torsion and shear, the ultimate torsional strength can be significantly improved by the influence of the acting prestressing force. Further, in the Fig. 11 shown is that for the case of an element under shear level v_V=0.95 without

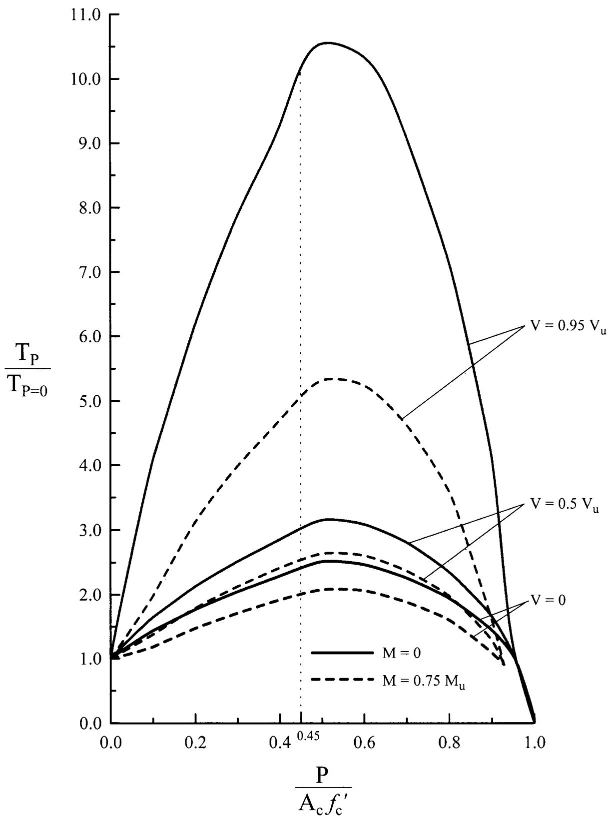


Fig. 11 Interaction curves of prestressing force and ultimate torsional strength for combined loading with shear and flexure

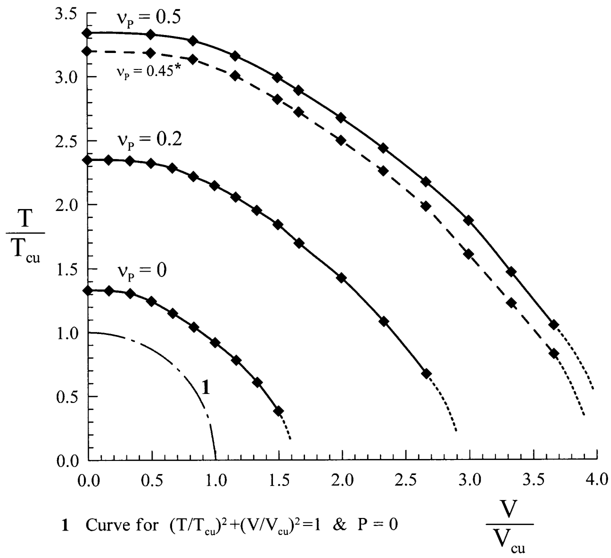


Fig. 12 Interaction curves of T/T_{cu} versus V/V_{cu} for various levels of prestressing (ν_P=0, 0.2, 0.45 and 0.5)

flexure, the ultimate torsional strength T_P can be increased up to 10.5 times for an applied prestressing level of ν_P=0.5, compared with the torsional strength of the element without the influence of the prestressing force $T_{P=0}$. The favorable influence of the prestressing force on the ultimate torque decreases as the flexural load increases. Thus, for the previous case, if a flexural moment equal to $0.75M_u$ is also applied on the element the ultimate torsional strength becomes only 5.3 times greater than the torsional strength without the influence of the prestressing force $T_{P=0}$.

Furthermore, aiming to a better understanding of the influence of the prestressing force on the capacity of elements under combined torsion and shear, interaction curves of torsion versus shear for different levels of prestressing (ν_P=0, 0.2, 0.45, 0.5) are presented in Fig. 12. The torsion and the shear are expressed in terms of the ratio T/T_{cu} and V/V_{cu}, respectively, where T_{cu} and V_{cu} are the ultimate torque and the shear strength of plain concrete, respectively. In Fig. 12 the values of T_{cu} and V_{cu} are taken as (ACI 318):

$$T_{cu}=1.6\sqrt{f_c'}\,b^2h \text{ (psi) (or } T_{cu}=0.13\sqrt{f_c'}\,b^2h \text{ in SI units)} \tag{16}$$

$$V_{cu}=2.68\sqrt{f_c'}\,bd \text{ (psi) (or } V_{cu}=2.22\sqrt{f_c'}\,bd \text{ in SI units)} \tag{17}$$

In Fig. 12 the predicted interaction curves are also compared to the curve produced by the relation

$$(T/T_{cu})^2+(V/V_{cu})^2=1 \tag{18}$$

which holds for elements without prestressing ν_P=0 (ACI 318 1971 through 1989).

9. Concluding remarks

A theoretical tool for the analysis and the prediction of the ultimate strength of prestressed concrete beams under torsion and torsion combined with shear and flexure, has been introduced.

The approach takes into account the post cracking softening response of concrete in tension and it can be applied to elements with practically any cross-section since it utilizes numerical mapping. Predictions by the proposed method have been compared with the experimental results compiled from works around the world in an attempt to establish the validity of the approach based on a broad range of studies. These comparisons were made for prestressed concrete rectangular and flanged beams in pure torsion as well as in torsion combined with shear and flexure. From these comparisons it can be concluded that for all the cases examined, the proposed analytical method predicts well the ultimate torsional strength and the influence of the prestress on it. Further, using the proposed analysis as a tool, strength interaction curves between ultimate torsion and prestress level have been produced. The influence of flexure on these interaction curves has been examined too. Finally, the influence of the applied prestress level on the strength of beams under torsion combined with shear has also been examined and presented in the form of interaction curves.

References

ACI Committee 318, (1995), "Building code requirements for structural concrete (ACI 318-95) and commentary (ACI 318R-95)", American Concrete Institute, Detroit.

Allos, A.E. and Rashid, A.H. (1989), "Prestressed concrete rectangular beams subjected to sustained torque", *ACI Structural Journal*, **86**, 469-472.

Anderson, P. (1935), "Experiments with concrete in torsion", *Trans. ASCE,* **100**, 949-983.

Cowan, H.J. (1965), *Reinforced and Prestressed Concrete in Torsion*, Edward Arnold (Publishers) Ltd., London.

Gopalaratnam, V.S. and Shah, S.P. (1985), "Softening response of plain concrete in direct tension", *ACI Journal*, **82**, 310-323.

Hsu, T.T.C. (1984), *Torsion of Reinforced Concrete,* Van Nostrand Reinhold, New York.

Karayannis, C.G. and Soulis, J.V. (1990), "A numerical method for torsional analysis of structural elements", *J. Computers & Structures,* **36**(4), 755-768.

Karayannis, C.G. (1994), "A model with tension softening for concrete beams under combined torsion, flexure, shear and prestressing", *2nd Conf. on Computational Structures Technology,* Proc. Vol. "*Advances in Finite Element Techniques*", CIVIL-COMP Ltd., 161-167.

Karayannis, C.G. (1995a), "Torsional analysis of flanged concrete elements with tension softening", *J. Computers & Structures,* **54**(1), 97-110.

Karayannis, C.G. (1995b), "A numerical approach to steel fiber reinforced concrete under torsion softening", *J. Structural Engineering Review,* **7**(2), 83-91.

Karayannis, C.G. (2000), "Nonlinear analysis and tests of steel-fiber concrete beams in torsion", *J. Structural Engineering and Mechanics,* **9**(4), 323-338.

Karayannis, C.G. and Chalioris, C.E. (2000), "Experimental validation of smeared analysis for plain concrete in torsion", *J. of Structural Engineering, ASCE,* **126**(6).

Mattock, A.H. and Wyss, A.N. (1978), "Full-scale torsion, shear and bending tests of prestressed I-girders", *J. of P.C.I.,* **23**(2), 22-41.

Mukherjee, P.R. and Warwaruk, J. (1971), "Torsion, bending, and shear in prestressed concrete", *J. of the Structural Division, ASCE*, **97**, 1063-1079.

Vecchio, E.J. and Collins, M.P. (1986), "The modified compression field theory for reinforced concrete elements subjected to shear", *ACI Journal*, **83**, 219-231.

Victor, D.J. and Aravindan, P.K. (1978), "Prestressed and reinforced concrete T-beams under combined bending and torsion", *ACI Journal*, **75**, 526-532.

Wafa, F.F., Hasnat, A. and Tarabolsi, O.F. (1992), "Prestressed fiber reinforced concrete beams subjected to torsion", *ACI Structural Journal*, **89**, 272-283.

Zararis, P.D. and Penelis, G.Gr. (1986), "Reinforced concrete T-beams in torsion and bending", *ACI Journal*, **83**, 145-155.