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Free vibration analysis of multiple open-edge cracked beams by component mode synthesis

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Abstract. This study is an investigation of the effect of cracks on the dynamical characteristics of a cantilever beam, having multiple open-edge transverse cracks. The flexibilities due to crack have been identified for several crack depths and locations. In the study the finite element method and component mode synthesis methods are used. Coupling the components is performed by a flexibility matrix taking into account the interaction forces. Each component is modelled by cantilever beam finite elements with two nodes and three degrees of freedom at each node. The results obtained lead to conclusion that, by using the drop in the natural frequencies and the change in the mode shapes, the presence and nature of cracks in a structure can be detected. There is some counter-evidence, however, that the effects due to multiple cracks may interact to make detection more difficult than for isolated cracks.

Key words: structural integrity; damage assessment; defective structures; cracks.

1. Introduction

Vibration analysis, which can be used to detect structural defects such as cracks, of any structure potentially offers an effective, inexpensive and fast means of non-destructive testing. Any accidental or intentional modification in a structure will affect its dynamical behaviour and change its stiffness and damping properties. Several authors (Gudmundson 1982, Gudmundson 1983, Chondors and Dimarogonas 1980) have illustrated that the presence, location and severity of cracks in any structural member can be identified by a decrease in the first few natural frequencies. Gudmundson (1983) and Springer *et al.* (1988) used saw cuts to simulate open cracks and found satisfactory agreement between their experimental results and those predicted by theoretical analysis. Cawley and Adams (1979) used a combination of sensitivity analysis and the finite element method in their influential study to determine crack location. Gouranis and Dimarogonas (1988) presented a finite element model for dynamic analysis of an edge-cracked beam. In this work, in order to consider the discontinuity in both deflection and slope due to the crack, two different shape functions were needed for two segments separated by the crack. Qian *et al.* (1990) developed a finite element model of an edge-cracked beam. They derived the stiffness matrix for a cracked beam element by an energy method. This stiffness was given different values, depending on whether the crack was open or closed. The sign of the stress on the crack faces determined the closure condition. This equation of motion was non-linear requiring a time stepping numerical method. Shen and Taylor (1991) developed an identification procedure to determine the crack characteristics from vibration

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measurements. Shen and Chu (1992) proposed a modified cracked beam theory and simulated numerically the dynamic response of simply supported beams having a fatigue crack. Abraham and Brandon (1995) and Brandon and Abraham (1995) presented a method of utilising substructure normal modes to predict the vibration properties of a cantilever Timoshenko beam with a closing crack. Recently Routolo *et al.* (1996) conducted a harmonic analysis of a cantilever beam with a closing crack using a finite element model of the Euler beam.

By using the component mode method, proposed by Hurty (1965), any complex problem can be replaced by several smaller ones each of more manageable complexity - although the structure must then be re-assembled. In this study the component mode method is used, not only to ease the problem but, to divide a non-linear problem into two linear subsystems. The coupling process utilised in this paper has been presented, for *intact* structures, by Ewins (1984) and Ghlaim (1984).

2. Theoretical model

The chosen model to investigate free vibration of cracked beams is a cantilever beam, of uniform cross section A, having double open-edge transverse cracks of depths r_1 and r_2 at variable positions ξ_1 and ξ_2 (Fig. 1). The cantilever is divided into three components at the crack section leading to a substructure approach, by which global non-linear system is separated into three linear subsystems. The first component has one clamped end and one free end whilst the other components have two free ends. The coupling of the components at the crack section is obtained via compatibility conditions representing the continuity of axial load, shear load and bending moment on each side of the crack. Discontinuities of axial displacement, transverse displacement and slope are permitted. These discontinuities in the displacement field give the opening displacements of the edge cracks. In this study the cracks are assumed to remain open, thus, the stiffness matrices due to cracks do not change. Each component, in Fig. 1, is also divided into finite elements with two nodes and three degrees of freedom at each node as shown in Fig. 2.

2.1. Stiffness and mass matrices of cantilever beam element

The stiffness and mass matrices, for a cantilever beam, have been taken from Petyt (1990) and adapted to three degrees of freedom for each node, $\delta=\{u, v, \psi\}$. The stiffness matrix for the two degrees of freedom (v, ψ) for bending in the xy plane for a two-noded cantilever beam is given by

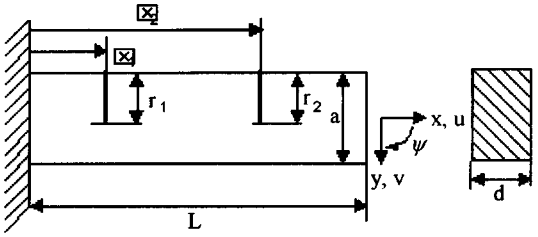

Fig. 1 Geometry of a cracked cantilever beam with double open-edge cracks

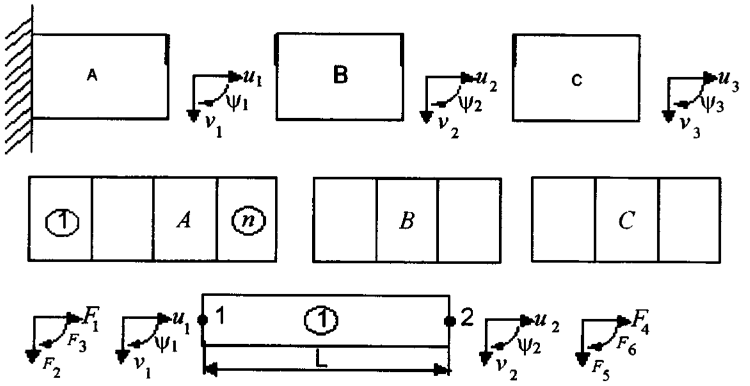

Fig. 2 Components and dividing them into the finite elements

$$[K_I] = \frac{EI_z}{2a^3(1+3\beta)}\begin{bmatrix} 3 & 3a & -3 & 3a \\ 3a & (4+3\beta)a^2 & -3a & (2-3\beta)a^2 \\ -3 & -3a & 3 & -3a \\ 3a & (2-3\beta)a^2 & -3a & (4+3\beta)a^2 \end{bmatrix} \tag{1}$$

where a=L/2 is half length of an element, E is the Young's modulus of elasticity and I_z is the section moment of inertia and

$$\beta = \frac{EI_z}{\kappa AGa^2} \tag{2}$$

in which κ is the shear correction factor, G is the shear modulus and A is the area of the cross section of the element. The stiffness matrix for the one degree of freedom $\{u\}$ local axial displacement in the x direction is (Petyt 1990),

$$K_{II} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{3}$$

Finally, the total stiffness matrix for all the three degrees of freedom for each node is given by:

$$K_{el} = \begin{bmatrix} K_{II}11 & 0 & 0 & K_{II}12 & 0 & 0 \\ 0 & K_I11 & K_I12 & 0 & K_I13 & K_I14 \\ 0 & K_I21 & K_I22 & 0 & K_I23 & K_I24 \\ K_{II}21 & 0 & 0 & K_{II}22 & 0 & 0 \\ 0 & K_I31 & K_I32 & 0 & K_I33 & K_I34 \\ 0 & K_I41 & K_I42 & 0 & K_I43 & K_I44 \end{bmatrix}_{(6\times\ )} \tag{4}$$

(The same structure of equation has been used previously by the authors for a closely related problem (Kisa *et al.* 1998) but with different coefficients).

The mass matrix, for the two degrees of freedom (v, ψ) for bending in the xy plane for a two-noded cantilever beam, is given (Petyt 1990) as

$$[M_I] = \frac{\rho A a}{210(1+3\beta)^2}\begin{bmatrix} m_1 & m_2 & m_3 & m_4 \\ m_2 & m_5 & -m_4 & m_6 \\ m_3 & -m_4 & m_1 & -m_2 \\ m_4 & m_6 & -m_2 & m_5 \end{bmatrix} + \frac{\rho I_z}{30a(1+3\beta)^2}\begin{bmatrix} m_7 & m_8 & -m_7 & m_8 \\ m_8 & m_9 & -m_8 & m_{10} \\ -m_7 & -m_8 & m_7 & -m_8 \\ m_8 & m_{10} & -m_8 & m_9 \end{bmatrix} \tag{5}$$

where

$$\begin{aligned} m_1 &= 156 + 882\beta + 1260\beta^2 & m_1 &= (44 + 231\beta + 315\beta^2)a \\ m_3 &= 54 + 378\beta + 630\beta^2 & m_4 &= (-26-189\beta-315\beta^2)a \\ m_5 &= (16 + 84\beta + 126\beta^2)a^2 & m_6 &= (-12-84\beta-126\beta^2)a \\ m_7 &= 18 & m_8 &= (3-45\beta)a \\ m_5 &= (8 + 30\beta + 180\beta^2)a^2 & m_{10} &= (-12-30\beta + 90\beta^2)a^2 \end{aligned} \tag{6}$$

in which β is given by Eq. (2).

The mass matrix for the one degree of freedom $\{u\}$ local axial displacement in the x direction is (Petyt 1990)

$$[M_{II}] = \rho A a \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \tag{7}$$

Now the total mass matrix for three degrees of freedom at each node can be given as

$$M_{el} = \begin{bmatrix} M_{II}11 & 0 & 0 & M_{II}12 & 0 & 0 \\ 0 & M_I11 & M_I12 & 0 & M_I13 & M_I14 \\ 0 & M_I21 & M_I22 & 0 & M_I23 & M_I24 \\ M_{II}21 & 0 & 0 & M_{II}22 & 0 & 0 \\ 0 & M_I31 & M_I32 & 0 & M_I33 & M_I34 \\ 0 & M_I41 & M_I42 & 0 & M_I43 & M_I44 \end{bmatrix}_{(6\times 6)} \tag{8}$$

2.2. The stiffness matrix due to crack

Considering the cracked node as a cracked element of zero length and zero mass (Gouranis and Dimarogonas 1988), the crack stiffness matrix can be represented by equivalent compliance coefficients. The compliance matrix was taken from Papadopoulos and Dimarogonas (1987) but adapted to the chosen beam element. The compliance coefficients matrix can be written according to the displacement vector $\delta=\{u, v, \psi\}$ as

$$C = \begin{bmatrix} c_{11} & 0 & c_{13} \\ 0 & c_{22} & 0 \\ c_{13} & 0 & c_{33} \end{bmatrix}_{(3\times 3)} \tag{9}$$

The inverse of the compliance matrix C^{-1} is the stiffness matrix of the nodal point. Thus, the stiffness matrix of the cracked nodal element is written as

$$K_{cr} = \begin{bmatrix} [C]^{-1} & -[C]^{-1} \\ -[C]^{-1} & [C]^{-1} \end{bmatrix}_{(6\times\ )} \tag{10}$$

3. Component mode analysis

Consider a component A. The equation of motion for this component is

$$M_A\ddot{q}_A + C_A\dot{q}_A + K_A q_A = f_A(t) \tag{11}$$

where M_A, C_A and K_A are mass, damping and stiffness matrices, respectively, for the component A, q and $f_A(t)$ are the generalised displacement and external force vectors, respectively. For undamped free vibration analysis Eq. (11) becomes

$$M_A\ddot{q}_A + K_A q_A = 0 \tag{12}$$

Assuming that

$$q = \phi\sin(\omega t + \beta),\ \ddot{q} = -\omega^2\phi\sin(\omega t + \beta) \tag{13}$$

and substituting them into Eq. (12), the standard free vibration equation for the component A is obtained as,

$$\omega_A^2 M_A\phi = K_A\phi \tag{14}$$

Making the transformation

$$q_A = \psi_A s_A \tag{15}$$

where ψ_A is the mass normalised modal matrix and s_A is the vector of principal co-ordinates, general co-ordinates are transformed to the principal co-ordinates. By premultiplying ψ_A^T and substituting Eq. (15), Eq. (12) becomes

$$I\ddot{s}_A + \omega_A^2 s_A = \psi_A^T f_A(t) \tag{16}$$

which gives eigenvalues $\omega^2_{Al,...,}\ \omega^2_{An}$ and mass normalised modal matrix ψ_A for the component A.

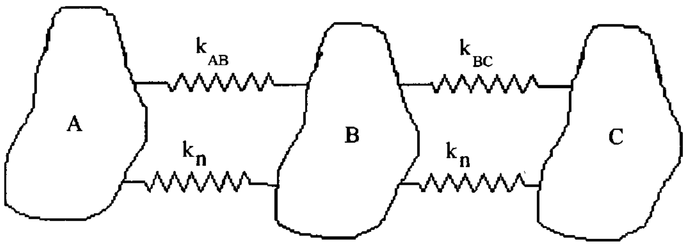

Fig. 3 Three components connected by springs

The same procedure can be followed for each components.

3.1. Coupling of the components

Consider three components *A, B* and *C* connected together via springs (as seen in Fig. 3). The kinetic and strain energy of the components, in terms of principal modal co-ordinates, can be given as

$$
\begin{aligned}
T &= \frac{1}{2}\dot{s}^T M \dot{s} \\
T &= \frac{1}{2}s^T M s
\end{aligned}
\tag{17}
$$

where T and U are kinetic and strain energy, respectively. M and K in Eq. (17) are

$$
M = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \quad K = \begin{bmatrix} \omega_A^2 & 0 & 0 \\ 0 & \omega_B^2 & 0 \\ 0 & 0 & \omega_C^2 \end{bmatrix}
\tag{18}
$$

The strain energy of the connectors, in terms of principal modal co-ordinates, is

$$
U_C = \frac{1}{2}s^T \psi^T K_C \psi s
\tag{19}
$$

where K_C is the connection matrix comprising the stiffness matrices due to cracks. ψ in Eq. (19) can be written as

$$
\psi = \begin{bmatrix} \psi_A & 0 & 0 \\ 0 & \psi_B & 0 \\ 0 & 0 & \psi_C \end{bmatrix}
\tag{20}
$$

The total strain energy of the system is, therefore,

$$
U_T = \frac{1}{2}s^T (K + \psi^T K_C \psi) s
\tag{21}
$$

where K has been given by Eq. (18). The equation of motion of the complete structure is

$$\ddot{s} + (K + \psi^T K_C \psi)s = \psi^T f(t) \tag{22}$$

where ψ has been given by Eq. (20), $f(t)$ is the global force vector for the system. From Eq. (22) the eigenvalues and eigenvectors of the cracked system can be determined. After solving this equation, the displacements for each component are calculated by using Eq. (15).

4. Validation of the method

4.1. Theoretical background

The effects of defects affecting a single degree of freedom can be analysed using a rank-one transformation (Brandon 1990) which computes directly the effect of the modification in structural behaviour using the transformation matrix from spatial to modal co-ordinates. It follows immediately, for example, that the modification will have no effect on a mode - either frequency or shape - if it occurs at a node but will have maximum effect on that mode if it occurs at an antinode (see Brandon 1997).

More general modifications are significantly more difficult to interpret, requiring the computation of the inverse of a matrix of the same rank as the modification. In the present study the effect of two cracks in the structure are simulated. The addition of further defects is straightforward algebraically and their simulation would provide few additional insights.

4.2. Effect on natural frequencies and mode shapes

In the literature the majority of work has analysed single edge cracked beams. Thus, to check the validity of the approach, the model has been first applied to a beam having a single open edge transverse crack. Equations for a single edge crack can be obtained by eliminating expressions for the component C from the given equations. As can be seen from figures there is very good agreement between the present results and those obtained by previous researchers in the field. Fig. 4 shows a plot of the ratio of the first natural frequency of the cracked beam respectively to the first natural frequency of the corresponding intact cantilever beam as a function of the crack depth ratio

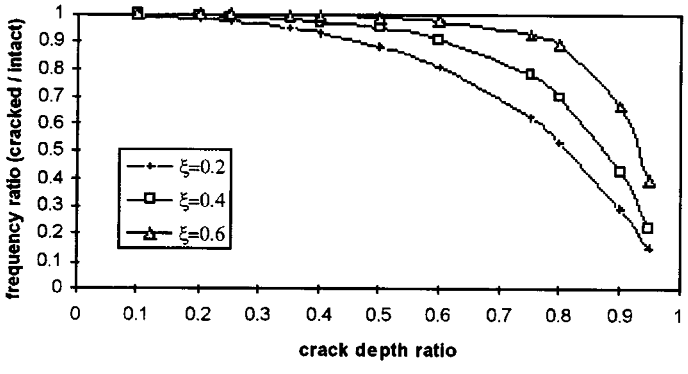

Fig. 4 Frequency ratios for different crack positions, single edge crack

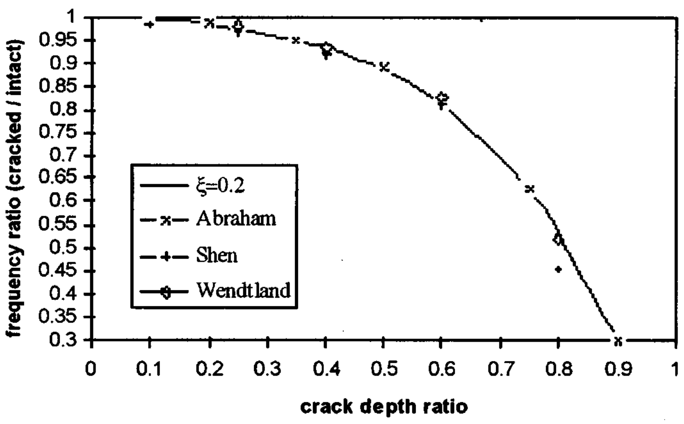

Fig. 5 Change in the fundamental natural frequency in terms of crack ratio, single edge crack

for several crack positions. The natural frequencies of the cracked beam are, as might be expected from simple energy considerations, lower than those of the corresponding intact beam. These differences increase with the depth of the crack. Due to the distribution of bending moment along the beam, which is greatest at the fixed end, a crack near the free end will have a smaller effect on the fundamental frequency than a crack closer to the fixed end. In Fig. 5 the results are compared with the experimental data obtained by Wendtland (1973) and theoretical data obtained by Abraham (1993) and Shen (1990).

After gaining this confidence that the method is consistent with results in the literature, the method is applied to a multi-crack model as in Fig. (1). The five lowest natural frequencies for various crack position pairs (ξ_1/L=0.2; ξ_2/L=0.8 and ξ_1/L=0.4; ξ_2/L=0.6) and crack ratios (r/a=0.2, r/a=0.4, r/a=0.6, r/a=0.8) are given in Table 1 and Table 2. The first, second and third mode shapes are given in Figs. 6-8 for the first case (crack position ξ_1/L=0.2, ξ_2/L=0.8) and crack ratio r/a=0.4. In Figs. 9-11 the first, second and third mode shapes are shown for crack position

Table 1 Natural frequencies of the cracked beam for ξ_1/L=0.20-ξ_2/L=0.80

Natural freqs.	ξ_1/L and ξ_2/L	r/a ratio 0.20	r/a ratio 0.40	r/a ratio 0.60	r/a ratio 0.80	Intact beam
1st mode	0.2-0.8	1020.009	966.4654	841.1242	549.5574	1037.0189
2nd mode	0.2-0.8	6439.084	6371.585	6160.185	5105.507	6458.3438
3rd mode	0.2-0.8	17675.85	16796.51	14841.10	11124.82	17960.564
4th mode	0.2-0.8	33960.71	30927.33	25454.18	18917.30	34995.429
5th mode	0.2-0.8	41096.63	40156.79	38063.39	34406.58	41368.174

Table 2 Natural frequencies of the cracked beam for ξ_1/L=0.40-ξ_2/L=0.60

Natural freqs.	ξ_1/L and ξ_2/L	r/a ratio 0.20	r/a ratio 0.40	r/a ratio 0.60	r/a ratio 0.80	Intact beam
1st mode	0.4-0.6	1028.391	999.6827	922.4313	679.2272	1037.0189
2nd mode	0.4-0.6	6301.214	5854.199	5019.251	3697.001	6458.3438
3rd mode	0.4-0.6	17686.08	16798.65	14631.89	9720.276	17960.564
4th mode	0.4-0.6	34771.10	34053.04	32153.43	28058.85	34995.429
5th mode	0.4-0.6	41080.23	39985.77	37959.20	35869.01	41368.174

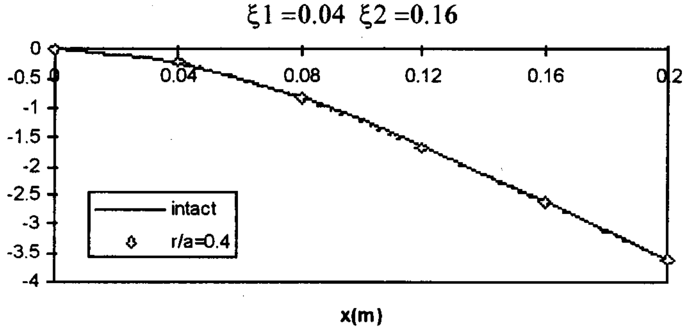

Fig. 6 First mode shape of cracked beam for ξ_1/L=0.20-ξ_2/L=0.80 and r/a=0.4

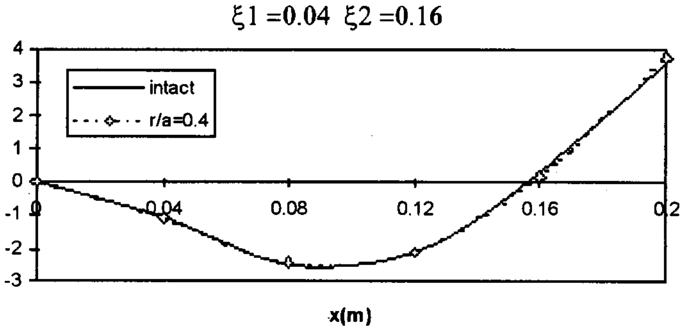

Fig. 7 Second mode shape of cracked beam for ξ_1/L=0.20-ξ_2/L=0.80 and r/a=0.4

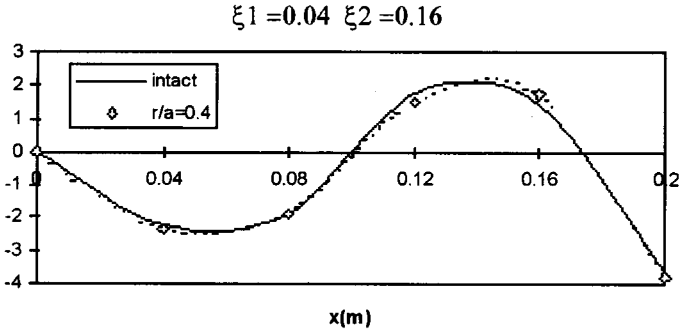

Fig. 8 Third mode shape of cracked beam for ξ_1/L=0.20-ξ_2/L=0.80 and r/a=0.4

(ξ_1/L=0.4–ξ_2/L=0.6) and crack ratio r/a=0.4. The change in the mode shapes are clearly apparent at the crack sections.

4.3. Implications for the inverse problem

The inverse problem in this case is the identification of discrepancies between measured structural properties and some *a priori* model (Doebling *et al.* 1996, Mottershead and Friswell 1993, Mottershead *et al.* 1996, and Dimarogonas 1996). The direct problem, analysed here, gives some

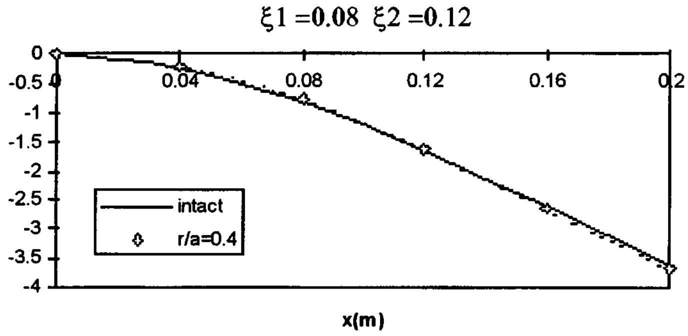

Fig. 9 First mode shape of cracked beam for ξ_1/L=0.40-ξ_2/L=0.60 and r/a=0.4

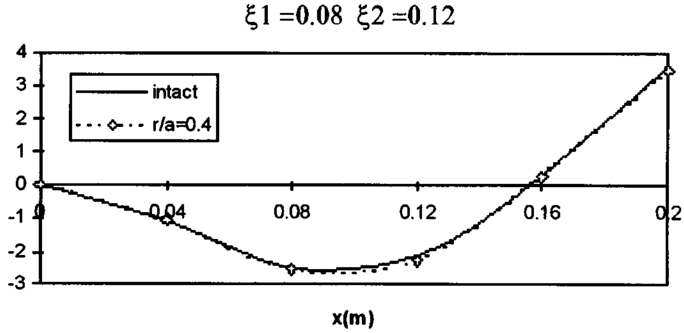

Fig. 10 Second mode shape of cracked beam for ξ_1/L=0.40-ξ_2/L=0.60 and r/a=0.4

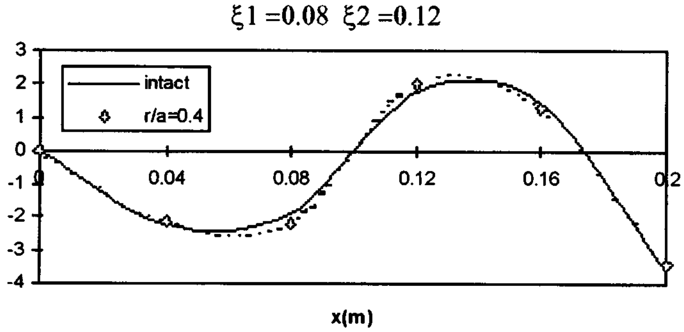

Fig. 11 Third mode shape of cracked beam for ξ_1/L=0.40-ξ_2/L=0.60 and r/a=0.4

insights into the ability of the inverse methods to identify multiple cracks. In addition to the work of Doebling *et al.* (1996), the reader is referred to the work by Ostachowicz and Krawczuk (1991), Mares *et al.* (1999), Liang *et al.* (1992) and Hu and Liang (1993).

As has been remarked, both experimental and theoretical studies have been largely limited to the detection and quantification of a single structural defect. What has become apparent to the authors in the current study is that, perhaps paradoxically, the detection of multiple defects in structures may be more difficult than the identification of a single fault, particularly when the defects are close together. For example, where the single defect is identifiable by a local change in curvature, two close defects may interact to make the curvature anomaly of each less apparent.

5. Conclusions

In this paper the analysis of the vibration properties of structures containing multiple cracks has been presented. The method integrates the fracture mechanics and the joint interface mechanics to couple otherwise linear substructures (using a component mode formulation). The method has been benchmarked against known cases from experimental and theoretical studies.

Whilst the results generally confirm existing theory, it is apparent that it may be more difficult to identify multiple cracks close together than the same cracks occurring in isolation.

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