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Application of frequency domain analysis for generation of seismic floor response spectra

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Abstract. This paper presents a case study with a multi-degree-of-freedom (MDOF) system where the Floor Response Spectra (FRS) have been derived from a large ensemble of ground motion accelerograms. The FRS are evaluated by the frequency response function which is calculated numerically. The advantage of this scheme over a repetitive time-history analysis of the entire structure for each accelerogram of the set has been highlighted. The present procedure permits generation of FRS with a specified probability of exceedence.

Key words: ground motion; frequency response function; floor response spectra; exceedence probability.

1. Introduction

Earthquakes constitute a major design consideration for the systems, structures and equipment of a Nuclear Power Plant (NPP). Usually, the design of the structures on ground are based on the ground response spectra. In an NPP application, the normalised ground response spectrum is conservatively taken as the mean-plus-sigma value of the normalised spectral ordinate of a large ensemble of accelerograms recorded on similar sites and covering an adequate band of earthquake parameters (e.g., magnitude and source distance) in the range of interest (USAEC 1973, Seed *et al.* 1976, Ghosh *et al.* 1986 and Ghosh and Sharma 1987). The seismic qualification of an equipment is generally based on the response spectrum of the floor on which it is mounted. The floor response spectrum (FRS) for an internal floor is usually derived from the floor acceleration time-history computed by a dynamic analysis of the structure subjected to a specified ground motion and the latter is a ground response spectrum compatible accelerogram (SCA). The time-history of a single event is not adequate for this purpose since the response spectrum of a single event is not sufficiently broad-banded. Alternative methods of generating the FRS, including the method of direct generation of FRS from the ground response spectrum has been proposed by several authors (for example, Singh 1975). Such methods require quite elaborate calculations based on the dynamic characteristics of the structure i.e., natural frequency, mode shapes and modal participation factors and are based on some relationship between a response spectrum (RS) and its power spectral density function (PSD).

Since the SCA cannot be defined uniquely, and a single SCA may not always produce a FRS with a desired degree of conservatism, use of more than one SCA (say, three) has been suggested by some authors. Smoothening and peak broadening of the FRS have been suggested in some

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regulatory document and Standard (USNRC 1982, ASCE 1980). The FRS should be conservative enough to account for the uncertainties in the ground motion and in the structural parameters. The existing Guides and Standards do not explicitly demand a certain non-exceedence probability for the FRS.

Deriving the FRS from the same set of strong ground motion accelerograms as used to obtain the design basis ground motion response spectrum, would provide for the variations and uncertainty in the ground motion in a consistent manner. The modifications due to uncertainties in the structural parameters should be superimposed on this FRS. Such an approach would permit developing the FRS with a desired level of conservatism.

While a repetitive time-history analysis of the entire structure for each accelerogram of the set will be uneconomical, similar results with fewer computations can be obtained by the Fourier transform approach (Humar and Xia 1993, Hall and Beck 1993). In the present method first, a time-history analysis of the structure is carried out for one ground motion accelerogram by solving the equations of motion. This yields the response of the structure as a function of time. The transfer functions of various responses (e.g., absolute acceleration) at various nodes of the structure are evaluated numerically from the ratio of the Fourier transform of the response to that of the base excitation. The response at any node to other ground motion accelerograms are then obtained through the transfer function for that node. Thus, this approach does not require explicit knowledge of the modal parameters of the structure (i.e., natural frequencies, mode shapes and mode participation factors) nor does it depend on any assumed relationship between PSD and RS. However, the transfer function and the dynamic characteristics have been obtained for a single degree of freedom system and presented in the paper. The calculations in this approach are relatively simple.

This paper presents a study to generate floor response spectra based on this approach. The accuracy and the efficiency of computation using Fourier Transform have been highlighted.

2. Theory

2.1. Frequency response function

Let the response of a linear M-degree-of-freedom system be $\{x_i(t)\}$ (i=1, M) to an input $x_g(t)$ where t is the time. Zero initial conditions are assumed.

The finite Fourier transform $X_i(f)$ of $x_i(t)$ is defined as (Champeney 1973)

$$X_i(f)=F(x_i(t))=\int_0^T x_i(t)\exp(-2\pi jt)dt;\ j=\sqrt{-1} \tag{1}$$

f is the frequency and T is the duration of the signal. The inverse transform is given by

$$x_i(t)=F^{-1}(X_i(f))=\int_0^{f\max} X_i(f)\exp(2\pi jft)df \tag{2}$$

$f\,$max is the highest frequency in the finite transform.

When $x_i(t)$ is discretised at N points at an interval of Δt and there are as many points in the frequency domain,

$$\Delta f = 1/(N\,\Delta t) \tag{3}$$

Eq. (1) can be rewritten in the discrete form as

$$X_i(k\,\Delta f) = \Delta t \sum_n x_i\,(n\,\Delta t)\,\exp(-2\,\pi j\,k\,\Delta f\,n\,\Delta t);\ k = 0, 1, 2... \tag{4}$$

The discrete inverse transform can be similarly defined.
The frequency response function, $H_i(f)$ (also known as the transfer function) for the ith degree of freedom of the system can be defined as

$$H_i(f) = X_i(f)/X_g(f) \tag{5}$$

Thus knowing the frequency response function of the system from the response to a given excitation, the Fourier transform of the response to any other excitation can be obtained. The time-history of the response can then be obtained by inverting its Fourier transform.

The Fourier transform $X_i(f)$ of the function $x_i(t)$ and the frequency response function $H_i(f)$ of the system are complex-valued functions i.e.,

$$H_i(f) = a_i(f) + b_i(f) \tag{6}$$

where a_i and b_i are the real and imaginary parts of $H_i(f)$ respectively.

2.2. Application to a MDOF mechanical vibration system

The equation of motion of a MDOF mechanical vibration system can be written as

$$[M]\,[\ddot{x}] + [C]\,[\dot{x}] + [K]\,[x] = -[M]\,[\ddot{x}_g] \tag{7}$$

where $[x]$ is the relative displacement vector $(x_1, x_2, ..., x_M)^T$ for a M degree of freedom system. $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively.

The transfer function H_i of absolute acceleration at the ith node is evaluated by considering say, the kth ground motion time-history $[\ddot{x}_{g,k}]$ and the corresponding floor time-history $[\ddot{x}_{i,k}]$ obtained by solving Eq. (7). Then,

$$H_i = \frac{F(\ddot{x}_{i,k} + \ddot{x}_{g,k})}{F(\ddot{x}_{g,k})} \tag{8}$$

The transfer function is a characteristic of the system and does not depend on the input excitation.

The floor acceleration time-history, $\ddot{x}_{i,j}$ at node i for another ground motion time-history $\ddot{x}_{g,j}$ is obtained by the inverse transform of the product of the Fourier transform of the new ground acceleration time-history and the transfer function at node i, H_i i.e.,

$$\ddot{x}_{i,j} = F^{-1}\,[F(\ddot{x}_{g,j})H_i] \tag{9}$$

Some explicit analytical results for a single degree of freedom system (SDOF) are given below.

Considering the Fourier transform of Eq. (7) for a SDOF,

$$X(\omega) = \frac{-F(\ddot{x}_g)}{\omega_n^2 + 2j\zeta\omega_n\omega - \omega^2} \tag{10}$$

where ω_n is the natural frequency of the system and its damping ratio is ζ and $\omega = 2\pi f$. The Fourier

Transform of the relative acceleration $\ddot{x}$ is

$$Y_r = F(\ddot{x}) = -\omega^2 X \tag{11}$$

and the Fourier transform of the absolute acceleration is

$$Y_a = F(\ddot{x} + \ddot{x}_g) \tag{12}$$

The frequency response function $H(\omega)$ of absolute acceleration is given by

$$H(\omega) = H_R(\omega) + j\, H_I(\omega) \tag{13}$$

The real part,

$$H_R(\omega) = 1 - \frac{\omega^2(\omega^2 - \omega_n^2)}{(\omega^2 - \omega_n^2)^2 + 4(\zeta\omega\omega_n)^2} \tag{14}$$

and the imaginary part,

$$H_I(\omega) = -\frac{2\zeta\omega^3\omega_n}{(\omega^2 - \omega_n^2)^2 + 4(\zeta\omega\omega_n)^2} \tag{15}$$

For any system, the FRS of the floor acceleration time-history is obtained for all the ground motion time-histories ($j = 1, 2, 3, \ldots\ldots\ldots, K$) and these are statistically combined.

3. Numerical analysis

To establish the accuracy of the numerically computed frequency response function, the $H(f)$ for a SDOF system was evaluated by considering the numerical solution of Eq. (7) to the N-S component of the Imperial Valley earthquake of 18-05-1940 recorded at El Centro (World Data Center 1977). The difference between the analytically and numerically computed $H(f)$ was found

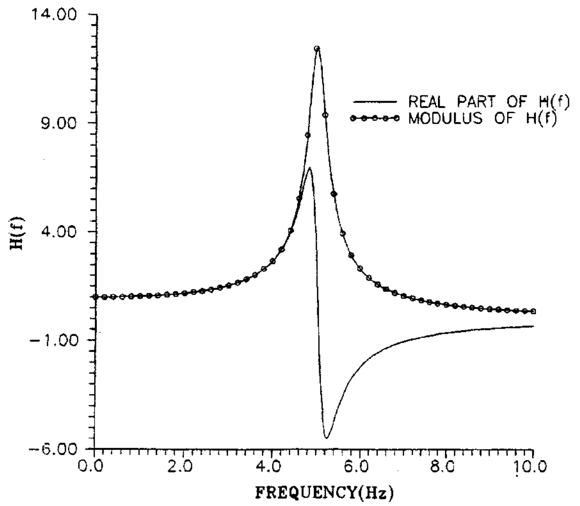

Fig. 1 Transfer function $H(F)$ for absolute acceleration of a SDOF system

to be less than three percent on an average. The real part of $H(f)$ i.e., $H_R(f)$ changes sign, from positive to negative and the modulus of this function attains a maximum at $f=f_n$ where f_n is the natural frequency of the system. The results are shown in Fig. 1 for m=1.7564×10^3 kg, k=1.7555 ×10^6 N/m and ζ=0.04.

Table 1 Data base for the present study

Sl. No.	Earthquake	Station Name & No.	Mag.	Source Dist (Km)	Component	Pk. acc. (g)
1.	Kern county	Taft, CAL	7.6	56.0	N21E	.156
	21/07/52	1095			S69E	.179
2.	San Francisco	Golden Gate	5.25	11.0	N10E	.083
	22/03/57	1077			S80E	.105
3.	Helena Mnt.	Federal Bldg.	6.0	8.0	S00W	.146
	31/10/35	Helena,323			S90W	.145
4.	Wheeler Ridge	Taft, CAL	6.0	51.0	N21E	.064
	12/01/1954	1095			S69E	.067
5.	Parkfield	Temblor, CAL	5.6	7.0	N65W	.269
	27/06/66	1097			S25W	.347
6.	Parkfield	San Luis	5.6	63.6	N36W	.018
	27/06/66	1083			S54W	.013
7.	Borrego Mnt.	SCE Plant	6.5	122.0	N33E	.041
	08/04/68	280			N57W	.046
8.	San Fernando	Pacoima Dam	6.6	3.2	S16E	1.250
	09/02/71	279			S74W	1.240
9.	San Fernando	Pacoima Dam	2.4	3.2	S16E	.021
	09/02/71	279			S74W	.027
10.	San Fernando	Pacoima Dam	3.1	3.2	S16E	.052
	09/02/71	279			S74W	.046
11.	San Fernando	Pacoima Dam	4.0	3.2	S16E	.115
	09/02/71	279			S74W	.112
12.	San Fernando	Pacoima Dam	3.0	3.2	S16E	.032
	09/02/71	279			S74W	.048
13.	San Fernando	Pacoima Dam	2.5	3.2	S16E	.031
	09/02/71	279			S74W	.024
14.	San Fernando	Pacoima Dam	2.4	3.2	S16E	.028
	09/02/71	279			S74W	.019
15.	San Fernando	Castiac Old	6.6	22.8	N21E	.390
	09/02/71	110			N69W	.320
16.	San Fernando	LA Water &	6.6	24.1	N50W	.200
	09/02/71	Power, 137			S40W	.140
17.	San Fernando	LA2011 Zonal	6.6	25.5	S62E	.080
	09/02/71	190			S28W	.070
18.	San Fernando	Pmp Pt.	6.6	35.5	N00E	.150
	09/02/71	Pearblossom 269			N90W	.100

The strong motion records of earthquakes should be selected such that they have similar sources, propagation paths and recording site properties as the controlling earthquakes governing the seismic design of structures at a particular site. The controlling earthquakes are determined from a consideration of the seismotectonics of the region and the seismic potential of the sources. It must be ensured that the recorded motions represent free-field conditions and are free of or corrected for any soil-structure interaction effects. Important source properties include magnitude and, if possible, fault type and tectonic environment. Propagation path properties include distance, depth and attenuation. Relevant site properties include shear wave velocity profile and other factors that might affect amplitude of waves at different frequencies. A sufficiently large number of site-specific time-histories should be used so that a sufficiently broad band spectrum can be developed encompassing various uncertainties (USNRC 1997).

An ensemble of thirty six strong motion time-histories recorded on rock sites (Table 1) (World Data Center 1977) was considered for generating the FRS as well as the ground motion response spectrum. Further studies have been carried out on a seven degree of freedom system the parameters for which are given in Table 2 which also gives the natural frequencies of the system under consideration.

First, a time-history analysis of the system is carried out for ground motion corresponding to the N-S component of the Imperial valley earthquake of 18-05-1940 recorded at EL Centro (see Eq. 7). The frequency response functions of absolute acceleration of the various nodes of the system are then evaluated from the responses obtained from this analysis by making use of Eq. (8).

The real part and the modulus of the frequency response function at node 7 are shown in Fig. 2. From this figure and Table 2, it is seen that the real part of $H(f)$ changes sign at the natural frequencies and modulus of $H(f)$ goes through local maxima at these points. Since the natural frequencies are the characteristics of the whole system, the general patterns of $H(f)$ at all nodes are expected to be similar. The moduli of $H(f)$ for other nodes were also seen to be having the local maxima at the same frequencies. As an example, the modulii of $H(f)$ at nodes 4 and 7 are presented in Fig. 3. The response to other ground motions are then obtained through Eq. (9) and the corresponding FRS at various nodes are generated for each ground motion record. To establish the accuracy of the FRS obtained through the frequency domain analysis, the FRS for node 7 were evaluated by (i) direct computation (solution of Eq. 7 by a time-history analysis) and (ii) by the frequency response function approach (Eq. 9) for another excitation corresponding to the N21E

Table 2a Parameters of the multi-degree of freedom system m_i, k_i and c_i are the same for all degrees of freedom (i.e., $i = 1$ to 7) ζ_j is the modal damping ratio for the jth mode ($j = 1$ to 7)

m_i (kg)	k_i (N/m)	ζ_j
1.7564×10^3	1.7555×10^6	0.04

Table 2b Undamped natural frequency of the system

Mode No.	1	2	3	4	5	6	7
Undamped Natural Frequency (Hz)	1.052	3.103	5.029	6.732	8.149	9.199	9.852

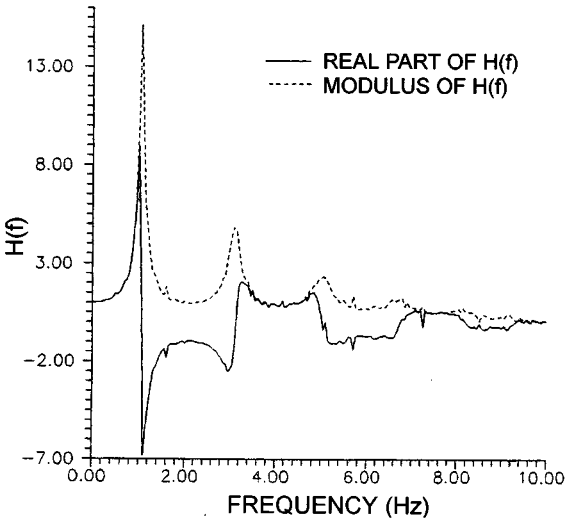

Fig. 2 Real part and modulus of $H(f)$ of absolute acceleration at node 7

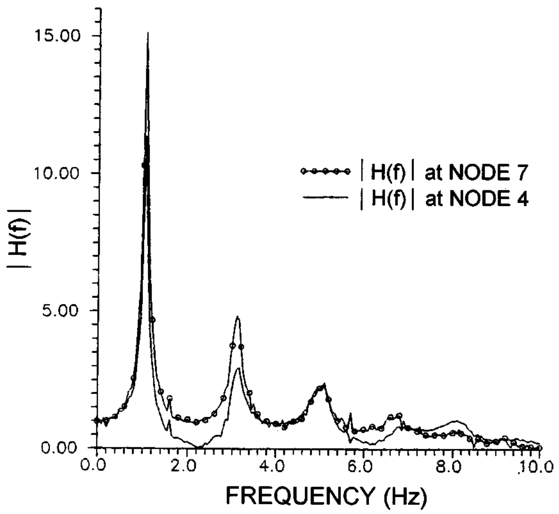

Fig. 3 Comparison of modulus of $H(f)$ at nodes 4 and 7

component of the Taft recording of the Kern County earthquake on 21.07.1952 (item 1, Table 1). The comparison of these FRS are shown in Fig. 4. The two results are found to be practically identical. The foregoing studies thus establish the accuracy of the adopted procedure.

Next, the FRS for a chosen node is evaluated for the floor time-histories obtained from Eq. (9) for all the ground motion accelerograms referred to in Table 1. From a statistical analysis of the various FRS generated, the mean, the standard deviation and the envelope value of FRS of each node and at each frequency are generated. The mean, mean-plus-sigma and the envelope FRS at node 7 for this ensemble of 36 FRS, for a damping value of 5% of critical, are shown in Fig. 5. The individual

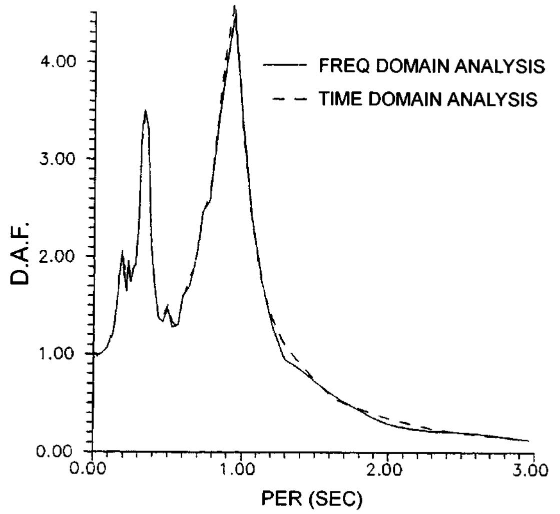

Fig. 4 5% Damping normalised absolute acceleration response spectrum at the 7th DOF

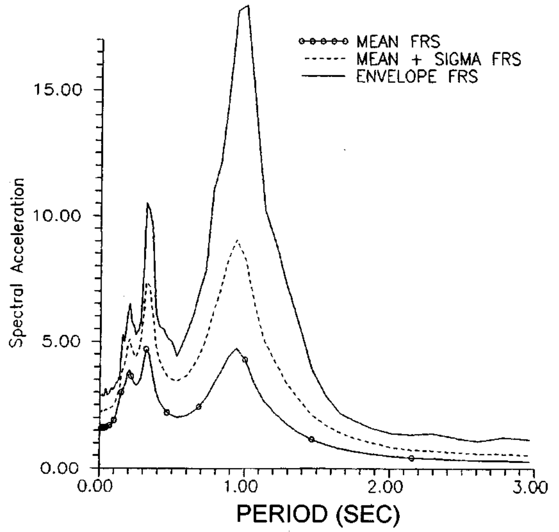

Fig. 5 5% Damping floor response spectra at node 7 with 36 normalised ground accelerograms

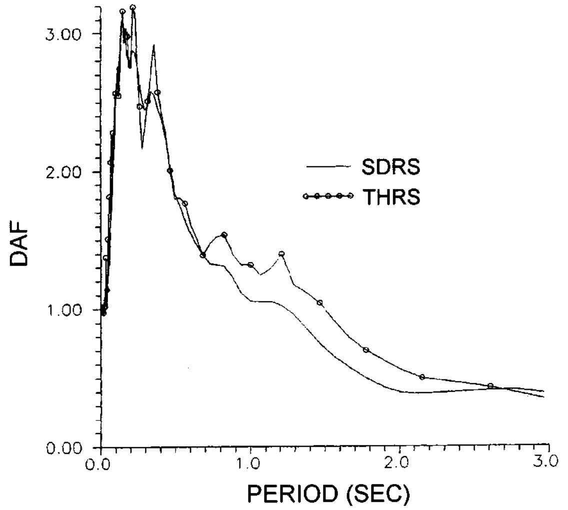

Fig. 6 Ground motion response spectrum; comparison of SDRS and THRS for 5% damping

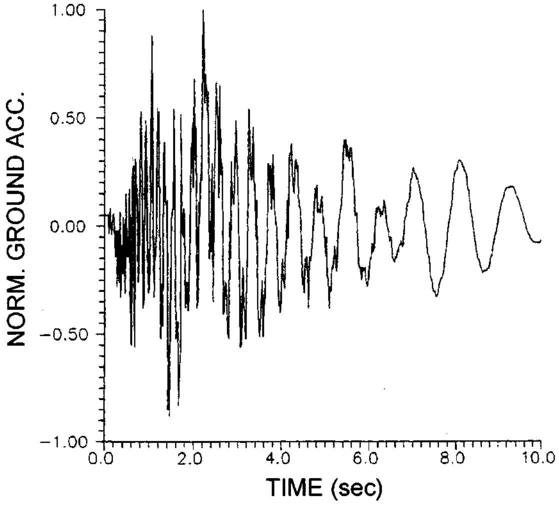

Fig. 7 Ground response spectrum compatible accelerogram

FRS are evaluated for ground accelorgrams normalised with respect to the corresponding peak acceleration. Since the mean and the standard deviation values of the FRS at any node is known, the FRS for any specified value of probability of exceedence can be generated by this procedure.

A ground motion response spectrum (SDRS) corresponding to the mean-plus-sigma value of the ensemble of the response spectra of the 36 ground motion time-histories and an accelerogram (SCA) compatible with the 5% damping ground response spectrum were also generated. These results are presented in Figs. 6 and 7 respectively. The response of the system shown was evaluated for the excitation by the SCA and the FRS was computed. Fig. 8 shows a comparison between the mean-plus-sigma FRS and the FRS evaluated from this SCA for node 7. These FRS have been evaluated for 5% damping. For reference, the time history generated response spectrum (THRS) for

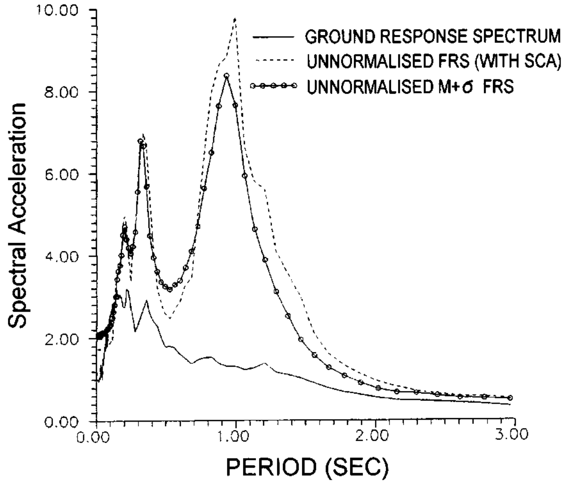

Fig. 8 Comparison of 5% damping floor response spectra at node 7

Table 3 Comparison of the number of computations required in the two algorithms

	Time-history method	Transfer function method
No. of additions/subtractions	$K[(N-1)(9+16M+4M^2)+41M/2+11M^2/2-4]$	$11KMN/2$
No. of multiplications/divisions	$K[(N-1)(10+20M+4M^2)+21M/2+13M^2/2+3]$	$KM[19N+2(N+1)\log_2 N-4]$
No.of funtion calls	$2KM$	$10KM(2N-1)$

where N=No. of discrete data points in a time-history and also the number of points in the Fourier transform of the time-history, K=No. of time-histories; M=No. of degrees of freedom of the MDOF system. The time-history method is based on Wilson-theta method and the system matrices are full square matrices.

the SCA is also shown in Fig. 8.

The number of additions, multiplications and function calls to be performed for obtaining the acceleration time-history of each floor has been estimated for analysis by transfer function method and the time-history method. A comparison of the number of various operations involved in the two algorithms is presented in Table 3. The comparison is for an MDOF system. It may be noted that in a finite element calculation the number of degrees of freedom involved will be higher than the number of floors where the FRS will have to be evaluated.

For the numerical study, the number of data points in the time-history/ Fourier transform, N has been assumed to be 1024. K and M, i.e., the number of time-histories and the number of degrees of freedom respectively, have been varied over a wide range of values. It has been found that number of multiplications in the present method is less than those in the time-history method for $M > 5$ irrespective of the number of time-histories. If it is assumed that the operation of evaluation of the sine, cosine and square root is equivalent to 3 multiplications, then the present method is comparatively advantageous for $M > 20$ irrespective of the number of time-histories considered.

4. Conclusions

The proposed method to evaluate FRS through a Fourier transform approach produces accurate results and involves fewer computations than those required for a number of time-history analysis if the number of degrees of freedom exceeds a certain value. For the case studied the FRS has been presented for various statistical levels. Thus the response spectra at the ground and the floor levels can be determined on a consistent basis. For the case studied the mean, mean-plus-sigma and the envelope values of the FRS have been presented. As indicated earlier, the FRS for any specified probability of exceedence can be generated.

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