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# Application of shakedown analysis technique to earthquake-resistant design of ductile moment-resisting steel structures

Han-Seon Lee †

*Department of Architecture, Yeungnam University, Kyongsan 712-749, Korea*

Vitelmo V. Bertero ‡

*Department of Civil Engineering, University of California, Berkeley, CA 94720, U.S.A.*

**Abstract.** The motivations of the application of shakedown analysis to the earthquake-resistant design of ductile moment-resisting steel structures are presented. The problems which must be solved with this application are also addressed.

The illustrative results from a series of static and time history nonlinear analyses of one-bay three-story steel frame and the related discussions have shown that the incremental collapse may be the critical design criterion in case of earthquake loading.

Based on the findings, it was concluded that the inelastic excursion mechanism for alternating load pattern, such as in earthquake, should be the sidesway mechanism of the whole structure for the efficient mobilization of the structural energy dissipating capacity and that the shakedown analysis technique can be used as a tool to ensure this mechanism.

**Key words:** shakedown analysis; earthquake-resistant design; incremental collapse; alternating plasticity; instantaneous plastic collapse; plastic deformation; plastic hinge; energy dissipation.

## 1. Introduction

The loading on a structure may vary considerably during its lifetime. In particular, a building structure located in a seismically active region will undergo severe structural responses generated by earthquake ground excitation. The magnitudes of these loads at any particular instant can not be foreseen, although their distributions or maximum values may be estimated, so that the sequence of loading is unpredictable. This type of loading is termed variable repeated loading or generalized loading(Neal 1977, Horne 1979, Hodge 1959, Maier 1937).

Many researchers (Bertero and Popov 1965, 1973, Popov and Pinkney 1969, Yamada 1969, Bertero et al. 1976, Zohrei 1982, Lashkari 1983, and MaCabe and Hall 1987) in earthquake engineering have paid attention to the problems which arise in the inelastic response of

† Full-time Lecturer
‡ Professor Emeritus

structure to variable repeated loading generated by earthquake ground excitations. The failures by incremental collapse and low cycle fatigue have been thought of as some of the most probable failure types of structure under the maximum credible earthquake ground shakings. Nevertheless, it is surprising to note that very little effort has been devoted to solve these problems through the use of shakedown analysis technique itself.

However, recently, Guralnick et al. (1984,1986) have shown, by using linear programming and energy approach for shakedown analysis, that much of hysteretic energy dissipation is concentrated in a relatively few dominant plastic hinges and that strengthening structural components at those locations can significantly alter the incremental collapse behavior of the entire structure. But the assumed load history was not realistic from the standpoint of earthquake engineering.

Therefore, the motivations of and difficulties with the application of shakedown analysis to earthquake-resistant design will be discussed. Then, a simple one-bay three-story example structure is taken for the illustration of usefulness of shakedown analysis technique in earthquake-resistant design.

## 2. Motivations of application of shakedown analysis to seismic design

*(1) Load pattern in earthquake:* The earthquake ground excitations are of a variable repeated nature. Thus, the real pattern of inertial forces developed during the response of a structure to earthquake ground motions is usually variable repeated in nature rather than monotonically and proportionally increasing in one direction, as assumed in the conventional design practice.
*(2) Problem of plastic design:* Simple plastic design, particularly minimum-weight plastic design, does not directly take into consideration the requirements of stiffness and strength for serviceability. Thus, the drift and deflection limit should be checked later and the resulting revised design may not be necessarily the minimum-weight design.
*(3) Problem of elastic design:* The basic concept of elastic design that the stress does not exceed the proportional limit, is fictitious because the real stresses in structures can be quite different from the stresses calculated by elastic analysis, due to such factors as residual stresses originated from lack of fit, differential foundation settlement and so on. For this reason, while elastic analysis is needed for the serviceability checks on deflection at service level, the calculation of elastic stresses at serviceability load would seem to be devoid of real significance(Horne and Morris 1982).
*(4) Characteristics of shakedown load analysis:* Shakedown analysis deals with variable repeated loading like the one generated by earthquake ground excitation. From analysis of the shakedown theorem and its application, it can be found that shakedown analysis involves plastic analysis and thus the presence of initial residual moments due to lack of fit, fabrication process or movements of foundations does not affect the shakedown load factor, which is also independent of load history. Furthermore, shakedown analysis also involves elastic analysis because the calculation of shakedown load factor needs the elastic moments assuming an initial stress-free condition while the calculation of the instantaneous plastic collapse load factor does not. Hence the investigation of the concept of shakedown analysis reveals that this analysis technique has the possibility to be used as a tool for meeting the requirements of earthquake-resistant design for serviceability and safety against collapse simultaneously.

## 3. Problems to be solved in applying shakedown analysis to seismic design

*(1) Difficulty in estimating shakedown deformations:* The shakedown theorem just specifies the conditions which, if satisfied, ensure that plastic flow will eventually cease. But by using this theorem alone, it is impossible to determine or to place upper-bounds on the deformations that may develop in a frame subjected to variable repeated loading with its load factor smaller than or equal to shakedown load factor(Neal 1977).

*(2) Probabilities of failures by shakedown phenomena:* If a frame is subjected to variable repeated loading and if its load factor exceeds the shakedown load factor, but is less than the instantaneous collapse load factor, then incremental collapse or low cycle fatigue by alternating plasticity, which are called hereafter as shakedown phenomena, may occur.

This raises the questions as to whether the indefinite continuation of plastic flow, by either alternating plasticity or incremental collapse, is a more relevant ultimate limit state than is the plastic collapse, for this type of loading. However, Horne(1954) has shown that this is unlikely to be the case for the floor and wind loads. The reasoning is as follows.

First, in case of the incremental collapse, the principal point is that plastic collapse requires only a single application of the appropriate load combination with load factor $\lambda$=plastic collapse load factor $\lambda_{p.\ c.}$ whereas incremental collapse requires a number of load applications with $\lambda \geq$incremental collapse load factor $\lambda_{inc.\ c.}$.

Let

$$P_r(\lambda \geq \lambda_{p.\ c.}) = Probability\ of\ \lambda \geq \lambda_{p.\ c.} = p \tag{1-a}$$
$$P_r(\lambda \geq \lambda_{inc.\ c.}) = Probability\ of\ \lambda \geq \lambda_{inc.\ c.} = q \tag{1-b}$$

Then, unacceptably large deformations would only develop after, say, $n$ applications of load at a load factor greater than $\lambda_{inc.\ c.}$. Thus

$$P_r(n\ applications\ of\ \lambda \geq \lambda_{inc.\ c.}) = q^n \tag{2}$$

Even though $p < q$, generally $p \gg q^n$ where $n$ is of the order 10. Therefore, the probability of incremental collapse has been thought of as very low when compared with that of plastic collapse.

Secondly, alternating plasticity, if it occurs, does not cause the growth of large deflections. The only risk involved is that of fracture due to low cycle fatigue. Various investigators have shown that the life of mild steel beams subjected to large reversals of strain, several times the yield stain, is of the order of $10^2$ to $10^4$ cycles. The implication is that alternating plasticity is most unlikely to be a relevant ultimate state for the floor and wind loading.

In the case of variable repeated loading generated by earthquake ground shakings, considerable research has been conducted, mainly to study the failure by low cycle fatigue at beam-column or column-foundation connections through experiment and computer simulations (Bertero and Popov 1965, 1975, Popov and Pinkney 1969, Yamada 1969, Zohrei 1982, Lashkari 1983, and, MaCabe and Hall 1987). But, generally the scope of research was confined to the level of structural components or to the behavior of simplified Single-Degree-Of-Freedom fictitious model. Furthermore, there has been almost no probabilistic and analytic study, not to mention experimental research, on the failure by incremental collapse during or after the structural response to severe earthquake ground shaking though this type of failure was believed to be more critical than failure by low cycle fatigue(Bertero, Herrera, and Mahin 1976).

*(3) Analysis versus design for shakedown load:* Minimum-weight plastic design for single or multiple loadings does not ensure that the design will shakedown under any sequence of loads within the limits. Design for shakedown is essentially a nonlinear process though analysis for shakedown load can be performed by a linear programming technique(Livesley 1975). Therefore, direct design methods, and particularly minimum-weight procedures for shakedown load, have been thought of as generally impracticable(Horne 1979). Only remarkable progress in nonlinear programming made recently, appears to make practical direct design methods.

## 4. Shakedown phenomena for earthquake load

Since there is little information on the prediction of plastic deformations under the state of shakedown, alternating plasticity and incremental collapse, it is considered important to discover the characteristics of inelastic behaviors under these phenomena by directly conducting nonlinear static and time history analyses. Hence a simple example structure is taken and the behavior of this structure under the pattern of fixed vertical load and alternating lateral load is first investigated using static event to-event nonlinear analysis with respect to shakedown phenomena. Next, time history nonlinear analyses with earthquake ground accelerograms of different characteristics are conducted to recheck the shakedown phenomena found in static analyses.

Then the incremental collapse, heretofore thought to be improbable in the case of wind and floor loads, is proved to be the critical problem in earthquake-resistant design.

### *4.1. Design of one-bay three-story structure*

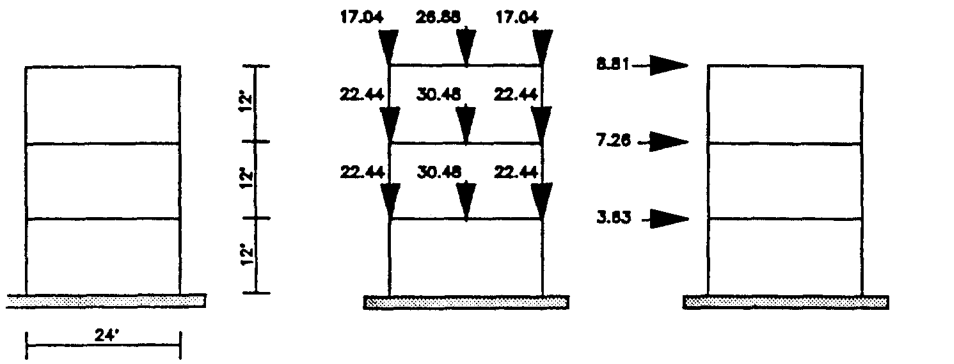
The geometry of example structure and assumed gravity loads are shown in Fig.1(a) and Table 1 respectively. From the assumed gravity load and by using 1982 UBC earthquake regulation, the lateral load distribution equivalent to the earthquake load at service level is shown is Fig 1.(c) while the gravity loads are assumed to be concentrated at the ends and midspan of girders as shown in Fig 1.(b).

Table 1 Load assumptions for 1-bay 3-story structure

| | floor | roof | ext. wall |
|---|---|---|---|
| dead load* | 90 | 75 | 30** |
| live load* | 37 | 37 | |

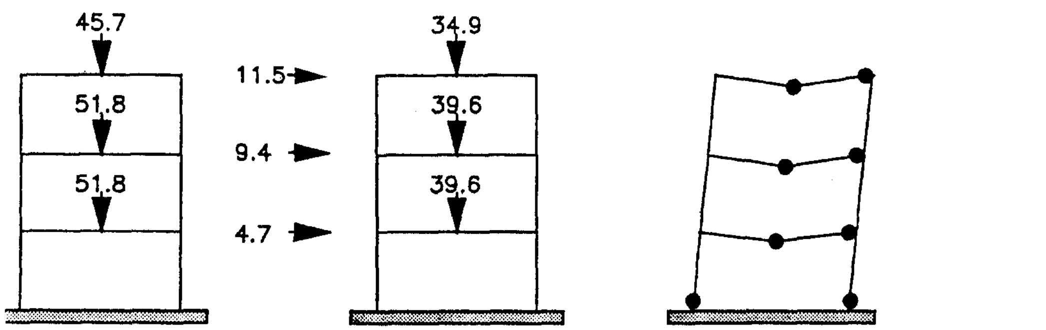
* unit=psf, ** for vertical surface

For simplicity of design, the structure is assumed to have two design variables, girder plastic moment $M_P^G$ and column plastic moment $M_P^C$ under the load conditions given by the Part Two in AISC Specifications, which are shown in Fig 2(a) and (b). The minimum-weight procedure based on plastic design has resulted in the selection of compact sections, $W16\times36(M_P=162\ kip\text{-}ft)$ for girder and $W14\times26(M_P=121\ kip\text{-}ft)$ for column. The critical failure mechanism for this design is shown in Fig.2(c).




(a) Geometry (bent spacing:20ft) (b) Gravity load(unit:kip) (c) Earthquake load(unit:kip)

Fig. 1 Geometry of structure and load conditions



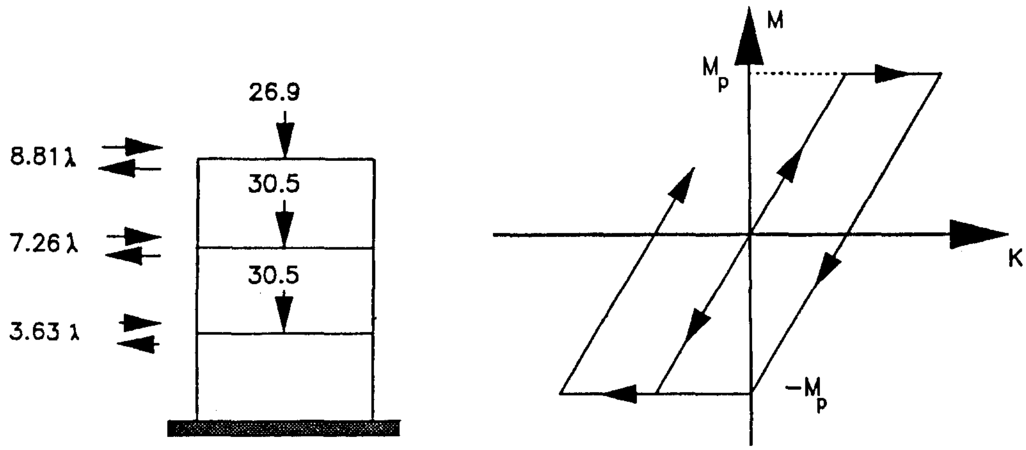

(a) $1.7(D+L)$ (unit: kip) (b) $1.3(D+L+EQ)$ (unit: kip) (c) Mechanism for $1.324(D+L+EQ)$

Fig. 2 Load cases and controlling mechanism for minimum-weigth plastic design

### *4.2. Shakedown analysis of one-bay three-story structure for earthquake load*

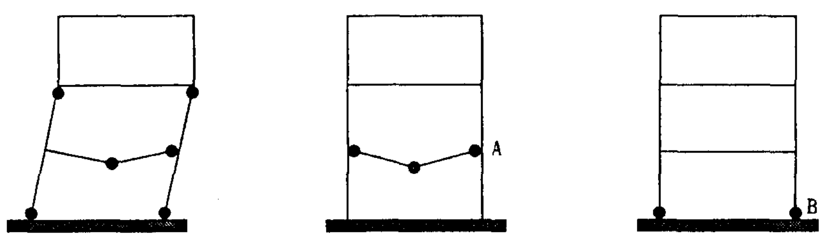
During the short duration of an earthquake, it seems that the load condition assumed as $1.3(D+L+EQ)$ does not represent the real excitation. Even though it has not been proved acceptable by the probabilistic approach, the earthquake load condition is now assumed in such a way that the vertical gravity load remains fixed at the level of service load $1.0(D+L)$, but that the lateral load can be variable repeated from $-\lambda(EQ)$ to $+\lambda(EQ)$.

Then, the shakedown analysis is imposed under the load condition of $1.0(D+L)\pm\lambda(EQ)$. The load pattern and the assumed moment-curvature relation are shown in Fig.3. Also the failure mechanism and corresponding load factors determined by the shakedown analysis are shown in Fig.4. In this case, the shakedown load factor $\lambda_{s.d.}$ is the smaller of incremental collapse load factor, $\lambda_{inc.c.} = 1.654$, and alternating plasticity load factor, $\lambda_{a.p.} = 1.550$, i.e., $\lambda_{s.d.} = 1.550$.




(a) Load pattern (unit: kip) (b) Moment-curvature relationship

Fig. 3 Load pattern and assumed moment-curvature relationship for shakedown analysis




($\lambda_{p.c.}=1.785$) ($\lambda_{inc.c.}=1.654$) ($\lambda_{a.p.}=1.550$)

(a) Plastic collapse (b) Incremental collapse (c) Alternating plasticity

Fig. 4 Failure mechanism and load factor obtained through shakedown analysis

### *4.3. Results from static event-to-event nonlinear analyses*

Variable repeated loading or generalized loading in shakedown theorem comprises any load history as long as the assumed load pattern is retained. But for the convenience of illustration, the history of variable repeated loading is simplified in such a way that the lateral earthquake load is applied back and forth repeatedly with the same intensity while the gravity load remains constant. The program INSA(Powell 1985) was used to conduct nonlinear event-to-event analyses.

Three load factors ($\lambda_1=1.69$, $\lambda_2(=\lambda_{inc.\ c.})=1.654$ and $\lambda_3=1.62$) are used to perform these nonlinear analyses. The progresses of deflection at the midspan of the second floor, as the number of load cycles increases, are shown in Fig.5 corresponding to each load factor.

Also the histories of plastic rotation at the right end of second-floor girder and at the right base support of structure are shown in Fig.6(a) and (b) respectively. It should be noted that the load factor $\lambda=1.69$ is only 2 % larger than load factor of incremental collapse $\lambda_{inc.\ c.}=1.654$ whereas load factor $\lambda=1.62$ is 2% smaller than $\lambda_{inc.\ c.}=1.654$. However, it is clear in

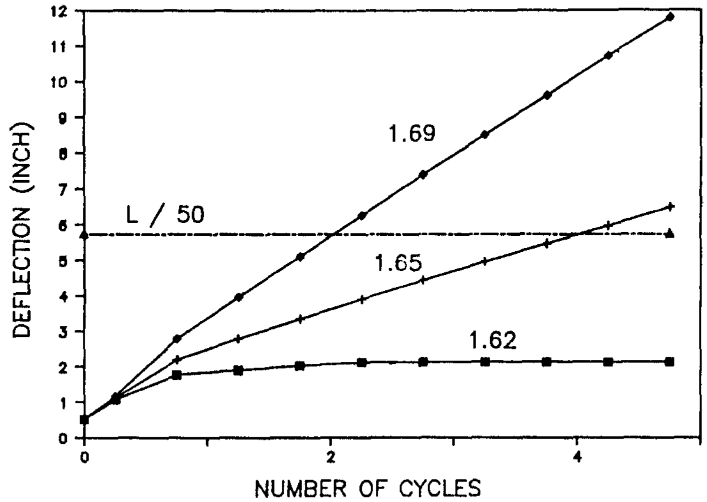
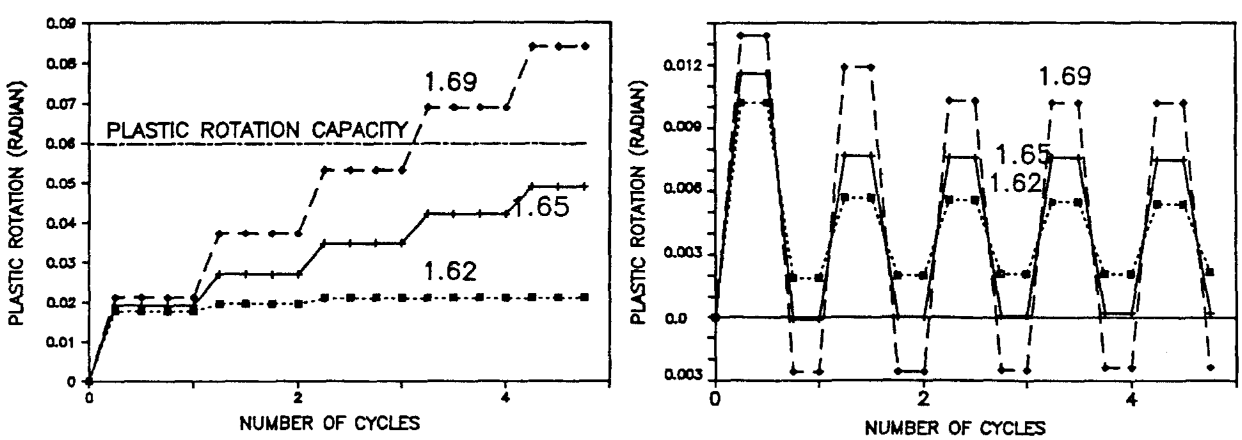



Fig. 5 Vertical deflection instability at midspan of second-floor girder by phenomenon of incremental collapse

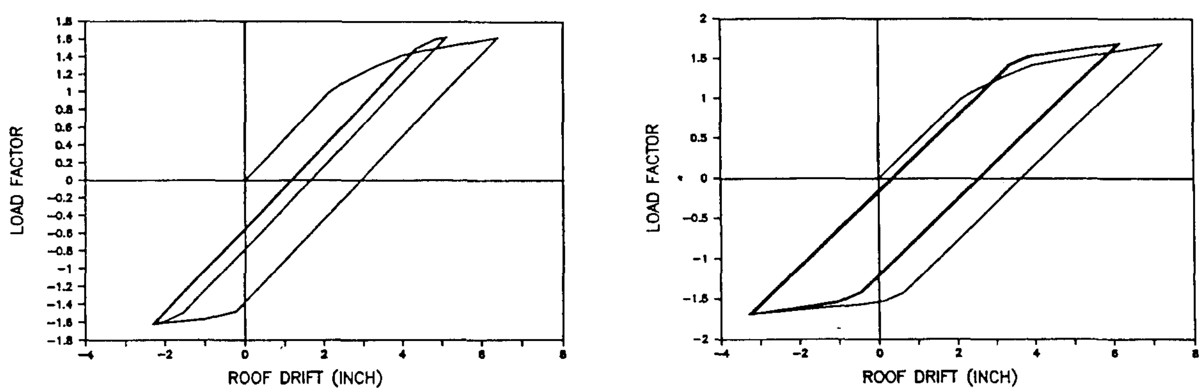
Fig.5 that for load factor $\lambda = 1.69$, the second-floor girder undergoes deflection instability at midspan but that for load factor $\lambda = 1.62$ permanent deflection approaches asymptotically a certain bound and does not exceed this value no matter how many load cycles are applied. It can be found in Fig.6(a) that the accumulated plastic rotation per cycle at plastic hinge formed at the right end of second-floor girder is so large that only 4 load cycles can lead to the exhaustion of the plastic rotation capacity, 0.060 radian, whereas the plastic behavior corresponding to $\lambda = 1.62$ shows no increase of plastic rotation after reaching a certain acceptable value. However, in Fig.6(b), all three load factors cause alternating plasticity at right base of structure because they are all larger than the load factor of alternating plasticity, $\lambda_{a.\ p.} = 1.55$, whose critical region is this plastic hinge.




(a) Plastic rotations at plastic hinge A in Fig. 4(b) (b) Plastic rotations at plastic hinge B in Fig. 4(c)

Fig. 6 Incremental collapse and alternating plasticity

The graphs of roof drift versus applied earthquake lateral load are shown in Fig.7. Though the hysteretic loops do not exactly represent the total dissipated energy of the structure, general performance of the structure in association with energy dissipation can be observed. That is, when load factor $\lambda$ approaches the load factor of plastic collapse, the amount of energy dissipation per cycle greatly increases. But the load factor in the range of $\lambda_{inc.\ c}(=1.654) \leq \lambda \leq \lambda_{p.\ c}$ $(=1.785)$, such as $\lambda=1.69$, induces the rapid exhaustion of plastic deformation capacity by incremental collapse, as shown in Fig.6(a). Therefore, the structural behavior for this range of load factor has both desirable (increase in energy dissipation) and undesirable (too large accumulation of plastic rotation) aspects simultaneously.

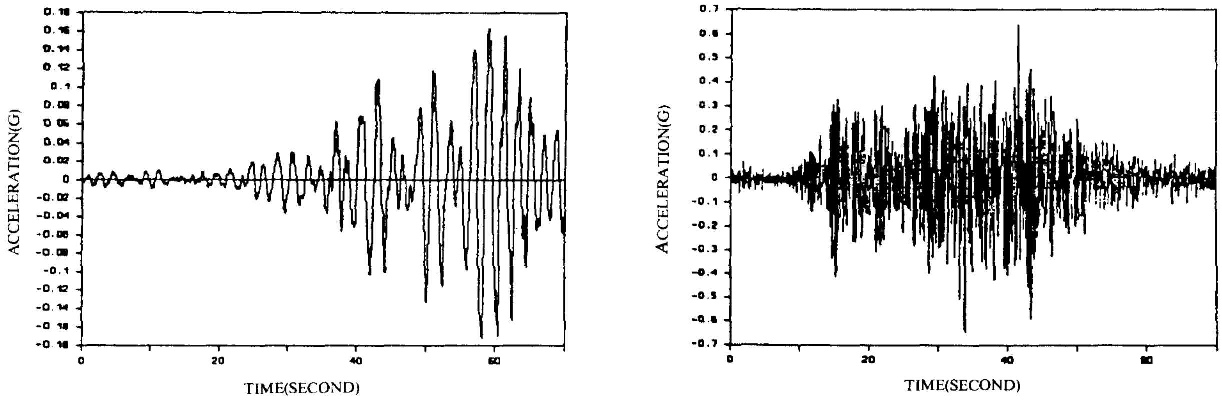



(a) $\lambda=1.62$ (b) $\lambda=1.69$

Fig. 7 Hysteretic behavior for different load factors

### 4.4 Results from time history nonlinear analyses

So far, the lateral load distribution has been assumed to have a constant shape from linear elastic behavior to the plastic collapse. This clearly does not represent the real earthquake excitation. Therefore, it is necessary to show that the same phenomena of incremental collapse and alternating plasticity occur at the same locations during realistic earthquake ground motions. Thus the same structure is tested with two recorded earthquake ground accelerograms. One is the one recorded as SCT EW component, Mexico City in 1985, and the other is the one recorded as N10E component, LLolleo Chile in 1985(Bertero 1986). These two accelerograms have quite different characteristics as can be observed in Fig.8. The damping ratio is assumed to be 2 % and the mechanical behavior of the member (critical regions) is assumed to be linear-elastic plastic with a deformation hardening of 1 %. Dynamic model and characteristics of the one-bay three-story structure are shown in Fig.9. The program DRAIN-2D (Kanaan and Powell 1973) was used for the analyses.




(a) 1985 Mexico City earthquake (SCT, EW component, PGA=0.172g)

(b) 1985 Chile LLolleo earthquake (N10E component, PGA=0.668g)

Fig. 8 Earthquake ground accelerograms used for time history analyses

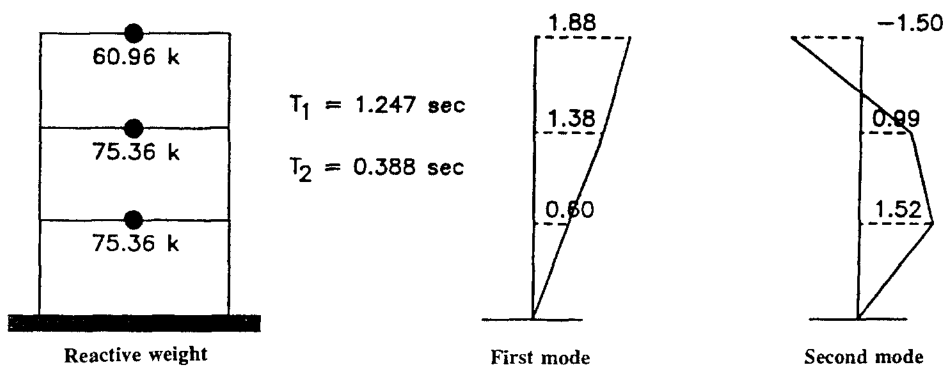



Fig. 9 Dynamic characteristics of example structure

*(1) 1985 Mexico City Earthquake (SCT, EW):* The time histories of plastic hinge rotation at the left end, midspan and right end of second-floor girder are shown in Fig.10(a). It can be noted in this figure that the accumulations of plastic rotation by the phenomenon of incremental collapse have occurred twice up to $t=55$ seconds. One is by the plastic rotations of the midspan and right end of girder around $t=49$ seconds and the other by those of the midspan and left end of girder around $t=51$ seconds. However, around $t=60$ seconds, the phenomena of incremental collapse were followed by that of alternating plasticity. Here the combination of phenomena of incremental collapse and alternating plasticity implies that even at the same location in the global structure the mode of inelastic failure can be different depending upon the type of applied or acting extreme excitations. The maximum plastic rotation at the right end of the girder is shown to be over 0.05 radian, which has almost reached the available capacity of 0.06 radian.

The history of vertical deflection at the midspan of the second-floor is shown in Fig.10(b). The increase of vertical deflection at the instant of the accumulation of plastic rotations by the phenomena of incremental collapse is so large that the final permanent deflection appears to

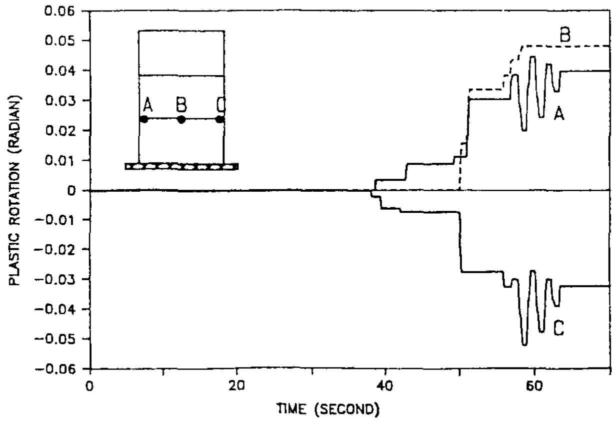



Fig. 10(a) Time history of plastic rotations at girder of second floor for Mexican earthquake(PGA=0.172g)

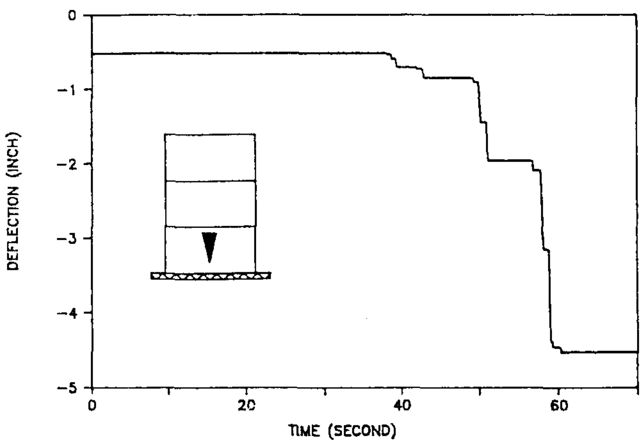



Fig. 10(b) Time history of vertical deflection at midspan of second-floor girder for Mexican earthquake (PGA=0.172g)

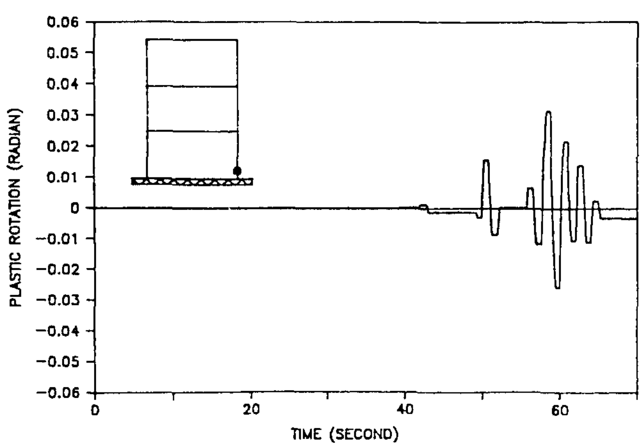



Fig. 10(c) Time history of plastic rotation at right base of example structure for Mexican earthquake (PGA=0.172g) be over 4.7 inches which corresponds to 1.6 % of span length. It is also interesting to note in Fig.10(c) that the manner of plastic deformations at the support of structure is quite different from that at second-floor girder. One shows the alternation of plastic deformation whereas the other reveals the accumulation of plastic deformation. Therefore, if the lateral force distribution similar to the actual one can be assumed, the shakedown analysis appears to give satisfactory prediction on the critical locations of structure and their probable failure modes in inelastic behaviors.

*(2) 1985 Chile LLolleo Earthquake(N10E):* In Fig.11(a), six times of the accumulation of plastic hinge rotation by the phenomenon of incremental collapse can be noticed. Also, the vertical deflection instability at the midspan of the second-floor girder is shown in Fig.11(b). The time history of plastic rotation at the right support of structure in Fig.11(c) clearly represents the phenomenon of alternating plasticity. Basically, the same observations as in the case of the Mexico City (1985,SCT) record can be made for this earthquake ground motion.

## 5. Justification of usefulness of shakedown analysis technique in seismic design

### *5.1. Alternating plasticity in earthquake-resistant design*

The alternating plasticity has been usually treated as one of the possible limit states even though the probability of failure by this in building structure is shown to be very low for the nonseismic load as wind and floor load(Neal 1977, Horne 1979, Hodge 1959). But from the viewpoint of earthquake-resistant design, which allows ductile inelastic behavior against severe earthquake, the alternating plasticity is actually the main source of energy dissipation without any accumulation of plastic deformations or without the exhaustion of given plastic deformation capacities.

Therefore, it is desirable to design a structure with as many plastic hinges of alternating-plasticity type as possible. But it is necessary to check the possibility of the failure by low cycle fatigue under maximum credible earthquake ground motions. The details at the beam-column and column-foundation connections should be carefully designed to prevent this type of failure, which could otherwise cause a sudden and catastrophic collapse of the whole structure. However the number of cycles of large inelastic strain reversals necessary to attain fracture of the structural material is usually so great that it is doubtful that failure by low cycle fatigue can be developed by the number of severe long pulses that could exist in even the longest conceivable strong motion of an actual earthquake(Bertero, Herrera and Mahin 1976).

### *5.2. Incremental collapse as limit state in seismic design*

In an earthquake, the imminent or incipient plastic collapse or instantaneous collapse does not necessarily imply that the structure has failed, because (i) the inertial force due to earthquake is not a sustained load but varies with time, and (ii) the plastic collapse by mechanism flow is not only acceptable but highly desirable as long as the deflection is not excessive and the energy dissipation does not exceed the energy dissipation capacity of the structure. With the cyclic load pattern such as earthquake excitations just a few number of load cycles having enough intensity can cause the explosive accumulation of plastic deformation at the critical region of structure as aforementioned. Thus, the plastic deformation capacity will be rapidly exhausted and the plastic hinge will break down or eventually unacceptably large permanent de-

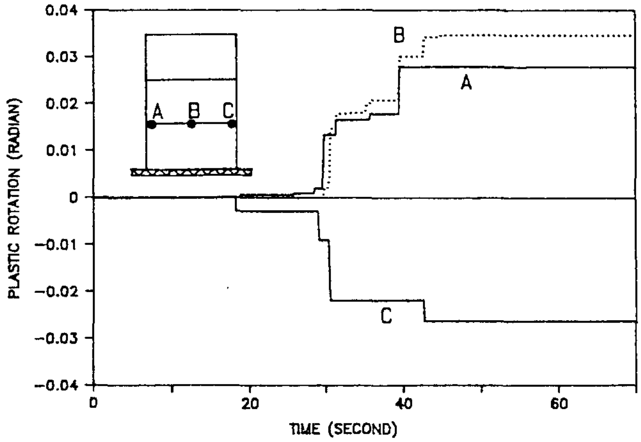



Fig. 11(a) Time history of plastic rotations at girder of second floor for Chile LLolleo earthquake (PGA=0. 54g)

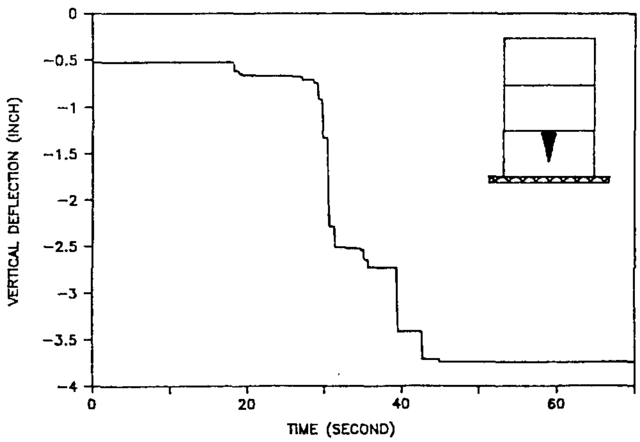



Fig. 11(b) Time history of vertical deflection at midspan of second-floor girder for Chile LLolleo earthquake (PGA=0.54g)

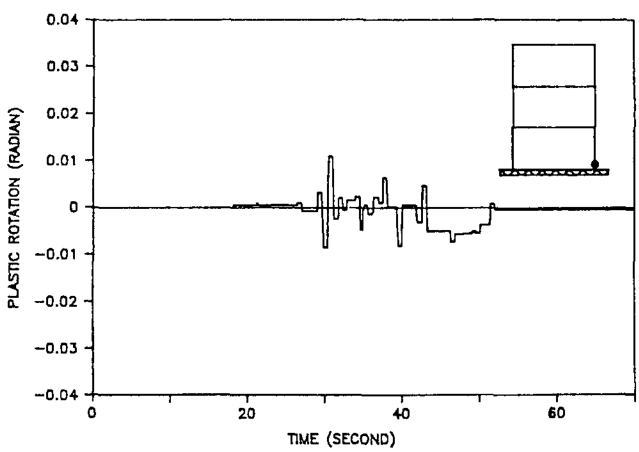



Fig. 11(c) Time history of plastic rotation at right base for Chile LLolleo earthquake (PGA=0.54g) flection will result. At this point, this type of failure, i.e. incremental collapse, should be prevented and is more clearly defined as failure than the incipient instantaneous collapse in the case of earthquake-resistant design.

However, if the permanent deflection at incipient instantaneous collapse is unacceptably large, then it has to be considered a failure of structure. The principal point against accepting incremental collapse as critical limit state for the floor and wind loading is that the probability of incremental collapse is very low when compared with that of instantaneous collapse. However, it is necessary to check if this is true in case of earthquake loading.

Let $P_{r\ cond}$ (A/B) be the conditional probability of event A's occurrence in the case that B has already occurred. Once a set of inertial forces due to earthquake ground shaking acted in one direction, then it would be highly probable that the other set of inertial forces in the reversed direction with the same or higher level of intensity would follow. Then, the probability of incremental collapse which requires $n$ times of load cycles can be expressed as follows.

$$P_r(n \text{ cycles of } \lambda \geq \lambda_{inc.c.}) = P_r(\lambda \geq \lambda_{inc.c.}) \times P_{r\ cond}(\lambda_2 \geq \lambda_{inc.c.} / \lambda_1 \geq \lambda_{inc.c.})^{2n-1} \tag{3}$$

where $\lambda_1$ means load factor applied in one direction and $\lambda_2$ load factor applied in the reversed direction consecutively.

Now let eq.(3) be applied to the case of one-bay three-story example structure. The probabilities of $\lambda \geq 1.785 (= \lambda_{p.\ c.})$ and $\lambda \geq 1.69 (\geq \lambda_{inc.\ c.})$ can be calculated by assuming the normal distribution of probability density function. By assuming $\nu T = 10^3$ in Fig.12 (Clough and Penzien 1975), reliability index $\beta_{p.\ c.}$ corresponding to $\lambda_{p.\ c.} = 1.785$ is obtained as 3.90. Then, $\beta_{inc.\ c.}$ is calculated by proportion as 3.69. Thus from numeric tables of normal distribution (Ang and Tang 1975),

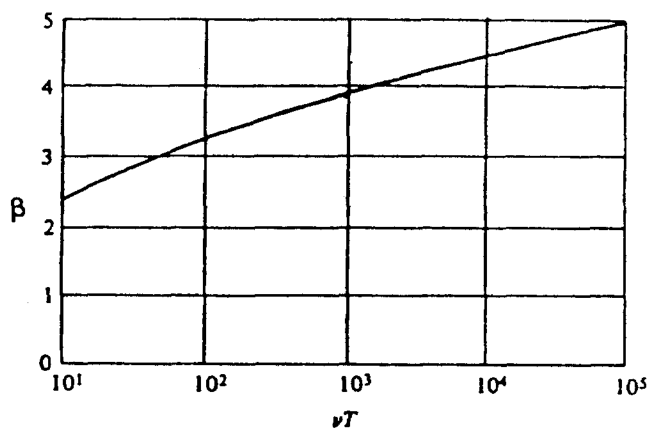



Fig. 12 Normalized mean extreme value($\beta$) versus $\nu T$(Clough and Penzien 1975)

$$P_r(\beta = 3.90) = 0.0000481, \qquad P_r(\beta = 3.69) = 0.000112. \tag{4}$$

Therfore, using eq.(3), the probabilities, $P_r(n \text{ cycle of } \lambda \geq \lambda_{inc.\ c.})$, with respect to varied number of load cycles and conditional probabilities are given in Table 2. Noting that the required number of load cycles to exhaust available plastic rotation capacity is shown to be 4 in

Fig.6(a), the probability of incremental collapse in Table 2 ranges from $4.97 \times 10^{-5}$ to $7.83 \times 10^{-5}$ with conditional probability varying from 0.85 to 0.95. When these probabilities are compared with that of incipient instantaneous collapse, $4.81 \times 10^{-5}$, it can be found that the probability of incremental collapse is larger than or at least competitive with that of instantaneous collapse.

Table 2 Probability of ($n$ cycles of $\lambda \geq \lambda_{inc.\ c}$)

| $Pr_{cond}$ \ no. of cycles | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| 0.95 | | 7.828 | 7.065 | 6.376 | 5.750 | 5.193 | 4.687 |
| 0.90 | 8.172 | 6.619 | 5.362 | 4.343 | | | |
| 0.85 | 6.884 | 4.974 | 3.594 | | | | |

* unit=0.00001

### 5.3. Usefulness of shakedown analysis technique in seismic design

Shakedown analysis based on the shakedown theorem provides the shakedown load under which a structure eventually behaves completely elastically after unknown finite plastic deformation. But engineers, who are more concerned with the energy dissipation capacity of structure through inelastic deformations under severe earthquake, are generally not so much interested in the eventual elastic behavior and thus in the shakedown load itself.

Furthermore, as stated before, alternating plasticity is one of the main sources of energy dissipation without causing the rapid exhaustion of plastic deformation capacity at critical regions and hence should be allowed for the minimization of the weight of structure because the more energy is dissipated the less strength will be required and thus the less material used. Also, as pointed out earlier, the incremental collapse should be prevented because of the explosiveness in the accumulation of plastic deformation leading to the rapid exhaustion of available plastic deformation capacity and excessive permanent deflection.

Therefore, it is desirable to design a structure to have as many plastic hinges of alternating plasticity as possible. From this point of view, the most efficient collapse mode for alternating load pattern such as earthquake loading is the sidesway mechanism of the whole structure. However, in order to ensure this mechanism, it is necessary to use shakedown analysis technique for the prediction of incremental collapse load factor. Then this incremental collapse can be prevented by introducing the constraint of $\lambda_{inc.\ c}$(Incremental collapse) $\geq \lambda_{p.\ c}$(Sidesway mechanism of the whole structure).

But, there remains one important problem to overcome. That is, the design against the incremental collapse load needs nonlinear programming while the analysis requires only linear programming because the elastic moments used in shakedown analysis are proportional to the displacements and moments of inertia(i.e. design variables in case of moment-resisting steel frame) which have nonlinear relations with respect to each other in the elastic equilibrium equations.

## 6. Conclusions

The advantage of ductile moment-resisting steel frame in seismic design lies in the fact that a significant portion of input energy can be dissipated through the plastic deformations. One type of energy dissipation is by the accumulation of plastic deformation while the other is by the alternation of plastic deformation. The former can lead to the exhaustion of plastic deformation capacity of a critical region in structures with only a few load cycles(i.e. incremental collapse) whereas the latter (i.e. alternating plasticity) may cause failure by low cycle fatigue only with a sufficient number of load cycles. Since the number of load cycles and the maximum load intensity expected during maximum credible earthquake ground excitation may exceed those required to cause the incremental collapse, the incremental collapse should be prevented and considerd as a limit state of comprehensive earthquake-resistant design. On the contrary, the number of yield reversals and their maximum yield strain expected under the longest conceivable strong motion of an actual earthquake are generally considered not enough to cause the failure by low cycle fatigue in case of ductile moment-resisting steel frame.

Therefore, it is desirable to design a structure to have as many plastic hinges of alternating plasticity as possible. From this point of view, the most efficient collapse mode for alternating load pattern, such as in earthquake, is the sidesway mechanism of the whole structure since plastic hinges develop all over the structure, therefore the mechanism mobilizes the whole capacity of structural members except columns to resist the maximum earthquake excitations, and dissipate the input energy by alternating plasticity without the excessive accumulation of plastic deformation due to incremental collapse. Here, the shakedown analysis technique can be used to predict the load factor $\lambda_{inc.\ c.}$ and mechanism of incremental collapse. Thus, the introduction of design constraint of $\lambda_{inc.\ c.} \geq \lambda_{p.\ c.}$(Sidesway machanism of the whole structure) will prevent the incremental collapse while ensuring the mechanism of $\lambda_{p.\ c.}$.

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