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Robust design of liquid column vibration absorber in seismic vibration mitigation considering random system parameter

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Abstract. The optimum design of liquid column dampers in seismic vibration control considering system parameter uncertainty is usually performed by minimizing the unconditional response of a structure without any consideration to the variation of damper performance due to uncertainty. However, the system so designed may be sensitive to the variations of input system parameters due to uncertainty. The present study is concerned with robust design optimization (RDO) of liquid column vibration absorber (LCVA) considering random system parameters characterizing the primary structure and ground motion model. The RDO is obtained by minimizing the weighted sum of the mean value of the root mean square displacement of the primary structure as well as its standard deviation. A numerical study elucidates the importance of the RDO procedure for design of LCVA system by comparing the RDO results with the results obtained by the conventional stochastic structural optimization procedure and the unconditional response based optimization.

Keywords: seismic vibration control; liquid column vibration absorber; random system parameters; robust optimization

1. Introduction

With the use of high-strength materials and advanced construction techniques, high rise buildings are becoming integral part of modern urban infrastructure. These structures are relatively lighter, flexible and lightly damped. The effects of vibrations caused by the environmental hazards such as earthquake, wind etc. are consequently much more in the present day’s structures. The traditional structural design approach has limited capacity of load resistance and energy dissipation. To circumvent these limitations, the uses of practical, effective and cost saving devices for suppression of vibration effects of structures have gain a momentum in the recent past. In fact, it has opened up a new area of research in the last decades. The applications of passive vibration control devices are widely accepted and have frequently been implemented to civil engineering structures in last three decades due to their advantages of low maintenance requirements leading to

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overall economy. Extensive research works have been done in the area of passive vibration control to mitigate the vibration effect of structures with particular emphasis on alleviation of wind and seismic effects. Amongst several alternatives, the Liquid Column Damper (LCD) was widely employed in the past for mitigating both the wind and earthquake induced vibration of structures. The present article deals with the passive vibration control using LCD. To be specific, the optimum performance of a special form of such system termed as liquid column vibration absorber (LCVA) is studied which allows better efficiency, versatility and architectural adaptability as the natural frequency is obtained not only by the length of the liquid column, but also by the area ratio of the horizontal and vertical columns.

From its inception (Sakai *et al*. 1991, Balendra *et al*. 1995), the effectiveness of LCDs to mitigate the effect of wind and seismic induced vibration effect has attracted considerable attention to the vibration control research community (Xu *et al*. 1992, Won *et al*. 1996, Balendra *et al*. 1999, Shum 2009, Lee *et al*. 2011). In fact, the optimum design procedure for LCDs in passive vibration control is proposed in the literature (Gao *et al*. 1997, Yalla and Kareem 2000). The damper parameters are usually obtained by minimizing the mean square responses of structure in the random vibration framework assuming deterministic system parameters i.e., the so called stochastic structural optimization (SSO) approach. A major limitation of such deterministic approach is that the uncertainty information about the decision variables cannot be used in the optimization process. But, the complete information about a dynamical system is rarely available. Therefore, the design of LCD system by SSO procedure may fail to create a control system that provides satisfactory performance. The efficiency of the system may reduce if the parameters are not tuned to the vibrating mode it is designed to suppress due to unavoidable presence of uncertainty and sometimes poor tuning may even amplify the vibration. Hence, for efficient design of LCD system, various uncertainties associated with the input excitation as well as mechanical model of the system should be explicitly taken into account. Thereby, the problem of vibration control of structure considering uncertain system parameters has attracted a great deal of interest in the recent past. Such studies based on minimizing the unconditional expected value of the mean square responses have found substantially different optimum TMD configuration (Jensen *et al*. 1992, Papadimitriou *et al*. 1997, May and Beck 1998, Ferrara and Giacomini 2009, Chakraborty and Roy 2011) and LCD (Taflanidis *et al*. 2007, Debbarma *et al*. 2010a, b). The optimum design of damper system to consider the effect of parameter uncertainty as mentioned above primarily apply the total probability theory to obtain the unconditional system response which is subsequently used as the performance index. However, such design approach does not consider the possible dispersion of the system performance and the damper parameters so designed may be sensitive to the variations of the input system parameters due to uncertainty. But, it is desirable to achieve a damper system which will not only yield optimum performance with regard to reduction of vibration level of structures but also assure less sensitivity with respect to the variations of system parameters due to uncertainty. To achieve this, the dispersion of the performance index from its nominal value is required to be introduced in the optimization process (Huang and Du 2007) which can be achieved by robust design optimization (RDO).

The developments on RDO in different scientific disciplines in the recent past are noteworthy (Zang *et al*. 2004, Park *et al*. 2006, Beyer and Sendhoff 2007). However, there have been a few applications of RDO with respect to reduction of vibration levels of structures (Hwang *et al*. 2001, Son and Savage 2007). The RDO in seismic vibration control were studied in recent past for TMD system (Marano *et al*. 2008, 2010). But, unlike TMDs, the governing equation of LCD motion is nonlinear due to the drag-type forces induced by the orifice. In the design of LCDs, there will be a dependence of the optimum head-loss coefficient on the excitation intensity (Papadimitriou *et al*. 1997, Yalla and Kareem 2000). As a consequence, the tuning may not be optimal for intensities different than the nominal one used in the design and may affect the optimum performance of the system. Thus, the nonlinearity of the system should be considered by including the excitation intensity as an uncertain parameter. Keeping this in view; the robust optimum design of LCVA to mitigate seismic vibration effect considering random system parameters is investigated so that an LCVA configuration is possible to achieve in which the final response reduction capability of the system will be less sensitive. The RDO is obtained by minimizing the weighted sum of the mean value of the performance index and its standard deviation. The maximum root mean square displacement (rmsd) of the primary structure is considered as the performance index. The SSO assuming deterministic system parameters and usually adopted unconditional response based optimization procedure under system parameter uncertainty in the framework of total probability theory are also performed to demonstrate the relevance and importance of the proposed RDO approach. A numerical study illustrate the effectiveness of the proposed RDO of LCVA system by comparing the RDO results with the results obtained by the SSO and unconditional response based optimization procedures.

2. Stochastic response of LCVA- structure system

The basic mechanical model of LCVA is represented by a single degree of freedom (SDOF) system with properties in accordance with the specified mode of vibration intended to control. The structure-damper system is subjected to base motion due to earthquake. If $x(t)$ and $y(t)$ represents the horizontal displacement of the SDOF system relative to the ground and the displacement of the liquid surface, respectively under base acceleration $\ddot{z}_b(t)$ due to earthquake motion, the equation of motion of the liquid column can be approximated as

$$\rho A_h L_{ee} \ddot{y}(t) + \frac{1}{2}\rho A_h \xi \left|\dot{y}(t)\right| \dot{y}(t) + 2\rho g A_h y(t) = -\rho A_h B_h \left\{\ddot{x}(t) + \ddot{z}_b(t)\right\} \tag{1}$$

Where, A_h, A_v, B_h and ρ represent the horizontal and vertical cross sectional area, length of the horizontal portion of the liquid tube and density of the liquid mass, respectively. Following notations are further introduced in the formulation: damper mass $m_l=(\rho A_h B_h+2\rho h A_v)$, mass ratio, $\mu=m_l/m_0$, area ratio, $r=A_v/A_h$, liquid column length, $L_e=(2h+B_h)$, length ratio, $p=B_h/L_e$, frequency of the structure, $\omega_0 = \sqrt{k_0/m_0}$, m_0 is the mass and k_0 is the stiffness of the structure, liquid frequency, $\omega_l=\sqrt{2\mathrm{g}/\mathrm{L_{ee}}}$, $L_{ee} = B_h r + (L_e - B_h) = L_e[1+p(r-1)]$ and tuning ratio: $\gamma=\omega_l/\omega_0$. The damping constant ξ is the coefficient of head loss controlled by the opening ratio of the orifice.

The equation of motion of the liquid column as described by Eq. (1) is non-linear in nature due to the drag-type forces induced by the orifice as indicated by the second term of the left hand side of Eq. (1). Using equivalent linearization techniques, it can be approximated as (Iwan and Yang 1972)

$$\rho A_h L_{ee} \ddot{y}(t) + 2\rho A_h C_p \dot{y}(t) + 2\rho A_h g y(t) = -\rho A_h B_h \left\{\ddot{x}(t) + \ddot{z}_b(t)\right\} \tag{2}$$

In the above, C_p represents the damping co-efficient of the equivalent linear system. It is

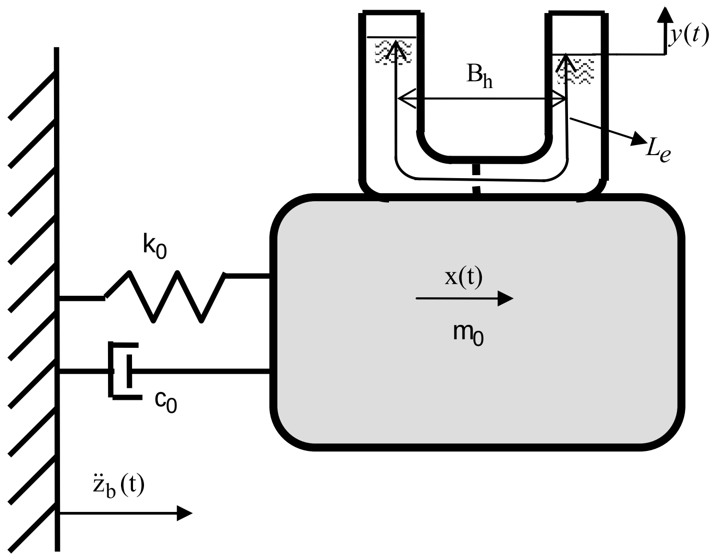

Fig. 1 The liquid column vibration absorbers model

determined by minimizing the mean square value of the damping force, $\in=(1/2)\rho A_h r\xi|\dot{y}|y - 2\rho A_h C_p\dot{y}$. Assuming $\dot{y}$ is a zero mean stationary Gaussian process, the value can be obtained as: $C_p = \sigma_{\dot{y}}\xi r^2/\sqrt{2\pi}$, $\sigma_{\dot{y}}$ is the standard deviation of liquid velocity. It can be noted that C_p depends on $\sigma_{\dot{y}}$ which is not known a priori and required an iterative solution procedure to obtain it. Normalizing Eq. (2) with respect to the liquid mass in the container ($\rho A_h L_{ee}$) yields

$$\ddot{y}(t)+\frac{2C_p}{L_{ee}}\dot{y}(t)+\frac{2g}{L_{ee}}y(t)+p\frac{L_e}{L_{ee}}\ddot{x}(t)=-p\frac{L_e}{L_{ee}}\ddot{z}_b(t) \tag{3}$$

The vibrating SDOF system has the mass of m_0, stiffness of k_0 and structural damping of c_0 (damping ratio of ξ_0) as shown in Fig. 1. The normalized (with respect to m_0) equation of motion of the primary structure attached with LCVA can be written as

$$\{1+\mu\}\ddot{x}(t)+2\xi_0\omega_0\dot{x}(t)+\omega_0^2 x+\ \mu\, pL_e/L_{em}\ \ddot{y}(t)=-\{1+\mu\}\ddot{z}_b(t) \tag{4}$$

Where, $L_{em}=(B_h/r+2h)$. Now, rewriting Eq. (3) and (4) in matrix form yields

$$\mathbf{M}\ddot{\mathbf{Y}}(t)+\mathbf{C}\dot{\mathbf{Y}}(t)+\mathbf{K}\mathbf{Y}(t)=-\mathbf{M}\bar{\mathbf{r}}\ddot{z}_b \tag{5}$$

In which **M**, **C** and **K** represent the mass, damping and stiffness of the combined system defined as

$$\mathbf{M}=\begin{bmatrix}1 & pL_e/L_{ee}\\ \mu pL_e/L_{em} & (1+\mu)\end{bmatrix},\mathbf{K}=\begin{bmatrix}2C_p/L_{ee} & 0\\ 0 & 2\xi_0\omega_0\end{bmatrix}\text{and}\,\mathbf{C}=\begin{bmatrix}2g/L_{ee} & 0\\ 0 & \omega_0^2\end{bmatrix} \tag{6}$$

And $\mathbf{Y}=[y,x]^T$ is the relative displacement vector and $\bar{\mathbf{r}}=[0\ \ 1]^T$. Introducing the state space vector, $\mathbf{Y}_S=(y,x,\dot{y},\dot{x})^T$, Eq. (6) can be written in the state space form as (Lutes and Sarkani 1997)

$$\dot{\mathbf{Y}}_s = \mathbf{A}_s\mathbf{Y}_s + \mathbf{r}\ddot{z}_b(t) \tag{7}$$

Where, $\mathbf{A}_s = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{H}_k & \mathbf{H}_c \end{bmatrix}$ is the structural system matrix having, $\mathbf{H}_k=\mathbf{M}^{-1}\mathbf{K}$, $\mathbf{H}_c=\mathbf{M}^{-1}\mathbf{C}$, $\mathbf{I}$ is 2×2 unit matrix and $\mathbf{0}$ is a null matrix, respectively and $\mathbf{r}=[0, 0, 1, 1]^T$.

The primary structure is excited at base due to seismic acceleration, $\ddot{z}_b(t)$. The well-known Kanai-Tajimi stochastic load model (Tajimi 1960) which can characterize the input frequency content for a wide range of practical situations is adopted in the present study to represent the stochastic earthquake process. The process of excitation at the base can be described as

$$\ddot{x}_f(t) + 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f = -\omega(t) \tag{8}$$

$$\ddot{z}(t) = \ddot{x}_f(t) + \omega(t) = 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f \tag{9}$$

Where, $\omega(t)$ is a stationary Gaussian zero mean white noise representing the excitation at the bed rock, ω_f is the base filter frequency and ξ_f is the filter or ground damping. Introducing the global state space vector, $\mathbf{Z} = [y, x, x_f, \dot{y}, \dot{x}, \dot{x}_f]^T$, Eq. (6) and (9) leads to the following Lyapunov equation (Lutes and Sarkani 1997)

$$\mathbf{AR} + \mathbf{RA}^T + \mathbf{B} = 0 \tag{10}$$

Where, the state space matrix $\mathbf{A}$ involves the properties of structure, damper and load model parameter. All terms in the $\mathbf{B}$ matrix is zero except, $\mathbf{B}(6,6)=2\pi S_0$. The space state covariance matrix $\mathbf{R}$ is obtained as the solution of the above Lyapunov equation. The covariance matrix is represented by the sub-matrices $R_{zz}, R_{z\dot{z}}, R_{\dot{z}z}$ and $R_{\dot{z}\dot{z}}$. The rmsd of the structure can be readily obtained as

$$\sigma_x = \sqrt{R_{ZZ}(2,2)} \tag{11}$$

3. LCVA parameters optimization: conventional SSO approach

The optimization of LCD system of protection requires determination of tuning ratio and coefficient of linear equivalent damping. The conventional optimization problem for system subject to stochastic load is transformed into a standard nonlinear programming problem. The optimum design variables are obtained by minimizing the rmsd of the structure for a known mass ratio μ and deterministic system properties i.e. the so called SSO problem is defined as

$$\text{Find } \bar{b} = (\gamma \;\; \xi)^T \text{ to minimize: } f = \sigma_x \tag{12}$$

The above SSO problem intuitively assumes that the parameters characterizing the structure and earthquake load are completely known. However, the uncertainty in these parameters may lead to an unexpected excursion of responses affecting the desired safety of structure (Zhao *et al.* 1999, Chaudhuri and Chakraborty 2006). The sources of uncertainty in seismic response analysis include both the structural system and the seismic actions. The frequency of the mechanical model representing the stiffness and mass distribution may be afflicted by significant variation during the service life of a structure. It is often difficult to predict the frequency accurately. In modelling of dynamic system, the proper characterization of energy dissipation of a system is very difficult and depends on various interacting complex parameters. One would always expect to consider the presence of uncertainty in the damping properties of a structure. Thus, these two parameters describing the mechanical model of the primary structure are considered to be uncertain. On the contrary, the stochastic spectra are traditionally used to consider the effect of random nature of seismic motion. The load model parameters i.e., S_0, ω_f and ξ_f are normally derived from few analyses on specific accelerograms which were subsequently generalized to a generic class of soils, such as rigid, medium and soft, simply referred to a single studied seismic event. But, in practical applications, the operators usually apply lexical and formal criteria for their identification. It can be reasonably affirmed that proper evaluation of these parameters and the related uncertainty is indeed an essential topic for professional engineers. So, these three parameters describing the stochastic load model are also taken as uncertain. Therefore, the uncertainties considered in the system parameter in the present study are in ω_0, ξ_0, ω_f, ξ_f and S_0 are denoted by a vector **u.**

4. RDO of LCVA parameters under uncertainty

The response of LCVA system of protection depends on the various parameters as mentioned in the previous section as the matrix **A** and **B** are functions of those parameters. Thus, evaluation of stochastic response using Eq. (11) and subsequent solution of the optimization problem i.e., Eq. (12) to obtain LCVA parameters are conditional i.e., the system parameters are deterministic. But, apart from the stochastic nature of earthquake load**,** the uncertainties with regard to these parameters are expected to have considerable influences**.** To estimate the unconditional stochastic response of structures, one needs to perform sensitivity analysis of stochastic dynamic system (Bhattachrayya and Chakraborty 2002, Chaudhuri and Chakraborty 2004, Jensen 2005). To obtain the sensitivities of responses, the first and second order derivatives of basic Lyapunov equation can be obtained by differentiating Eq. (10) with respect to these parameters as

$$\mathbf{A}\mathbf{R}_{,u_i} + \mathbf{R}_{,u_i}\mathbf{A}^{\mathbf{T}} + \mathbf{B}_1 = 0, \text{where}\, \mathbf{B}_1 = \mathbf{A}_{,u_i}\mathbf{R} + \mathbf{R}\mathbf{A}^T_{,u_i} + \frac{\partial}{\partial u_i}(\mathbf{B}) \tag{13a}$$

$$\mathbf{A}\mathbf{R}_{,u_i u_j} + \mathbf{R}_{,u_i u_j}\mathbf{A}^T + \mathbf{B}_2 = 0, \text{where } \mathbf{B}_2 = 2[\mathbf{A}_{,u_i}\mathbf{R}_{,u_i} + \mathbf{R}_{,u_i}\mathbf{A}^T_{,u_i}] + [\mathbf{A}_{,u_i u_j}\mathbf{R} + \mathbf{R}\mathbf{A}^T_{,u_i u_j}] \tag{13b}$$

The sensitivity of the response (the rmsd as considered herein) can be obtained directly by differentiating Eq. (11) with respect to the i-th random variable u_i as following

$$\frac{\partial}{\partial u_i}(\sigma_x)\ i.e.\ \sigma_{x,u_i} = \frac{1}{2}\frac{\mathbf{R}_{,u_i}(2,2)}{\sqrt{\mathbf{R}(2,2)}} \tag{14}$$

In which, $\mathbf{R}_{,u_i}$ (2, 2) is obtained by solving Eq. (13a). The second order sensitivity of the rmsd can be further obtained by differentiating Eq. (14) with respect to the j-th random variable u_j as following

$$\sigma_{x,u_iu_j} = \frac{1}{2\sqrt{\mathbf{R}(2,2)}}\left\{\mathbf{R},_{u_iu_j}(2,2) - \frac{1}{2}\frac{[\mathbf{R},_{u_i}(2,2)\mathbf{R},_{u_j}(2,2)]}{\mathbf{R}(2,2)}\right\} \quad 15)$$

The random responses of structure under stochastic earthquake load depend on the system parameters can be expanded in the Taylor series about the mean value of the random system parameter (with the assumptions that the random variation is small). The *i*-th such parameter u_i can be viewed as the superposition of the deterministic mean component ($\bar{u}_i$) with a zero mean deviatoric component (Δu_i). The Taylor series expansion of the rmsd of the primary structure about the mean values of the random system parameters can be expressed as

$$\sigma_x = \sigma x(\bar{u}_i) + \sum_{i=1}^{nv} \sigma_{x,u_i} \Delta u_i + \frac{1}{2}\sum_{i=1}^{nv}\sum_{j=1}^{nv} \sigma_{x,u_iu_j} \Delta u_i \Delta u_j + ... \quad (16)$$

In the above, *nv* is the total number of random variables involve in the problem, $\sigma_x(\bar{u}_i)$ is the mean part of the rmsd of the structure. The unconditional expected value of the rmsd can be obtained by the quadratic approximation of Eq. (16) assuming uncorrelated random variables

$$\begin{aligned} &\sigma_x = \sigma x(\bar{u}_i) + \sum_{i=1}^{nv} \sigma_{x,u_i} E\{\Delta u_i\} + \frac{1}{2}\sum_{i=1}^{nv}\sum_{j=1}^{nv} \sigma_{x,u_iu_j} E\{\Delta u_i \Delta u_j\} \\ &i.e.\ \ \sigma_x = \sigma x(\bar{u}_i) + \frac{1}{2}\sum_{i=1}^{nv} \sigma_{x,u_iu_i}\sigma_{u_i}^2 \end{aligned} \quad (17)$$

Where, σ_{u_i} is the standard deviation (SD) of the *i*-th random parameter. The linear approximation of Eq. (16) furnishes the variance of the rmsd as following

$$\text{Var.}(\sigma_x) = \sum_{i=1}^{nv}\left[\sigma_{x,u_i}\right]^2 \sigma_{u_i}^2 \quad (18)$$

In the total probability theorem, the LCVA parameters optimization problem as defined by Eq. (12) is redefined by considering the unconditional rmsd of the primary structure obtained by Eq. (14). Thus, the optimization problem considering the effect of random system parameters is defined as following

$$\text{Find } \bar{\text{b}} = (\gamma, \xi)^T \text{ to minimize: } f = \sigma_x = \sigma x(\bar{u}_i) + \frac{1}{2}\sum_{i=1}^{nv} \sigma_{x,u_iu_i}\sigma_{u_i}^2 \quad (19)$$

The minimization of unconditional rmsd of the structure by the total probability theorem to consider the effect of system parameters uncertainty as described above do not consider the possible dispersion of the design performance with respect to the variations of the system parameters due to uncertainty. As already discussed, the objective of an ideal design is to achieve the optimum performance as well as its' less sensitivity with respect to the variations of the system parameters. This necessitates minimizing the performance function as well as its dispersion. The two design criteria often conflicts with each other. In the context of RDO, it is dealt as a multi-objective optimization problem, where the conventional objective function and its standard deviation are the two objective functions that need to be optimized. This leads to a two-criterion optimization problem. The problem can be stated as finding the design vector to minimize $\{f(\bar{\mathbf{u}}), \sigma_f(\mathbf{u})\}$, in which the mean part of the objective function, $f(\bar{\mathbf{u}})=\sigma_x(\bar{\mathbf{u}})$ and its SD can be obtained as

$$\sigma_f(\mathbf{u}) = \sqrt{\text{Var.}(\sigma_x)} = \sqrt{\sum_{i=1}^{nv} \left[\sigma_{x,u_i}\right]^2 \sigma_{u_i}^2} \tag{20}$$

Finally, the dual criteria performance objective function is transformed to an equivalent single objective function as

$$\text{minimize } \phi = \lambda \frac{f(\bar{\mathbf{u}})}{\bar{f}^*} + (1-\lambda)\frac{\sigma_f(\mathbf{u})}{\sigma_f^*} \tag{21}$$

Where, λ is a weighting factor in the bi-objective optimization problem, $\bar{f}^*$ and σ_f^* are the two ideal optimal solutions correspond λ=0.0 (optimum solution is obtained simply by minimizing the standard deviation) and λ=1.0 (optimization without any consideration for robustness). Larger the value of λ implies that the designer puts more significance on the mean value of the performance function over its dispersion. The multi-objective optimization strategy for creation of Pareto front as presented here is the so called Weighted Sum Method (WSM). However, various other approaches are also applied for solving such problems e.g., Compromise Programming method, Physical Programming method and the genetic algorithm (GA). The standard gradient based MATLAB optimization routine is used here to solve the problem. However, for more complex configuration, GA based technique are robust choice for solving the associated optimization problem as the approach is independent from the choice of an initial point and does not require any information regarding the gradient of the objective function. Moreover, GA can be utilized to directly obtain the global convex or non-convex Pareto front for multi-objective optimization problems without converting it to an equivalent single objective function. However, the GA is computationally expensive than the other methods, especially for large scale practical problems. More details may be seen in an excellent state-of-the-art review on multi-objective optimization procedure in Marler and Arora (2004).

5. Numerical study

A SDOF structure with an attached LCVA as shown in Fig. 1 is undertaken to elucidate the proposed RDO of LCVA system for seismic vibration control of structures considering random system parameters. Unless specifically mentioned, the following nominal values are assumed in the present numerical study: T=2sec, ξ_0=1%, μ=3%, p=0.7, r=1.5, ω_f=9π rad/sec, ξ_f=0.6, S_0=2.361×10^{-3} m^2/sec^3, coefficient of variation (cov) of each random parameter =0.1. The rmsd of the unprotected system i.e. without LCVA is found to be 11.08 cm.

The rmsd of the structures with attached LCVA is optimized by the proposed RDO procedure as described by Eq. (21). The optimum rmsd of the structure versus mass ratio is plotted in Fig. 2 for different degree of robustness imposed on the design by different weight factors λ. The rmsd values are normalized with respect to the rmsd of the unprotected structure (11.08 cm) for convenient to study the nature of variation of the performance and the efficiency of the LCVA by the proposed RDO approach with reference to SSO and unconditional response based optimization

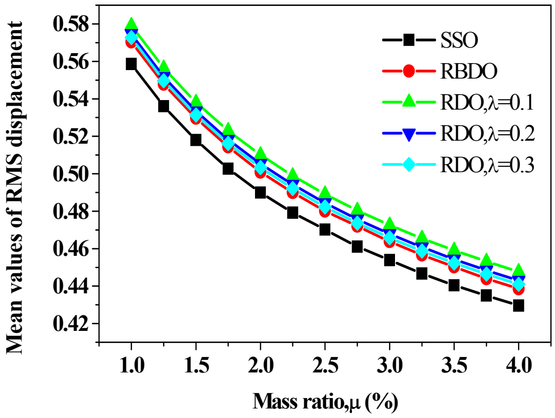

Fig. 2 The variation of the mean value of the rmsd of the primary structure with increasing mass ratio

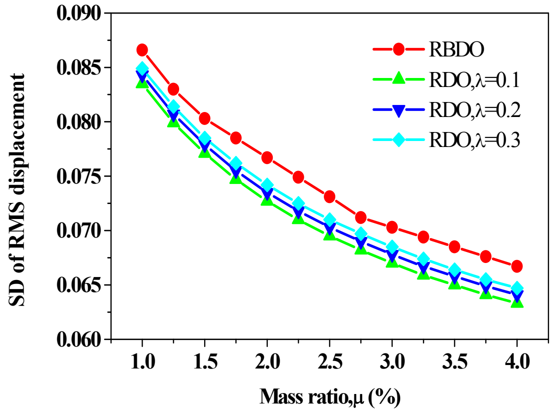

Fig. 3 The variation of the SD of the rmsd of the primary structure with increasing mass ratio

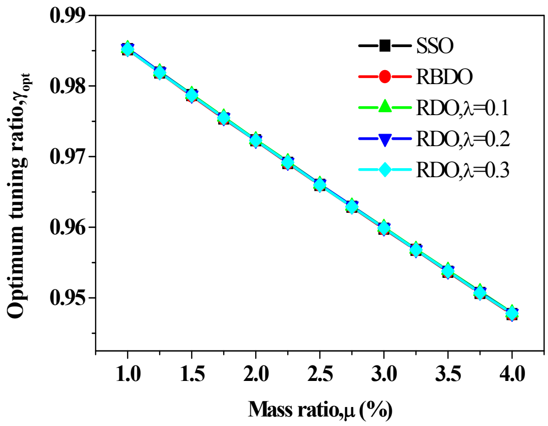

Fig. 4 The optimum tuning ratio with increasing mass ratio

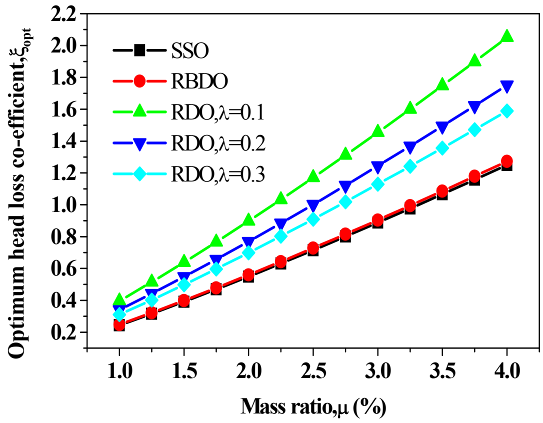

Fig. 5 The optimum head loss coefficient with increasing mass ratio

approach. The associated dispersions i.e., the SD of rmsd of the structure are shown in Fig. 3. The SD of rmsd values are also normalized. The corresponding optimum tuning ratio and head loss coefficient are shown in Figs. 4 and 5, respectively. The results obtained by solving the conventional SSO defined by Eq. (12) and by minimizing the unconditional rmsd to consider the effect of system parameter uncertainty as described by Eq. (16) are also obtained. The results are shown in the same plot for ease in comparison with the present RDO results. The conventional SSO results are denoted by deterministic and the unconditional rmsd based results are denoted as RBDO in all such plots. It can be readily observed from the plots that though the efficiency of vibration reduction is not completely eliminated with respect to that of SSO, the efficiency of the LCVA reduced in case of unconditional rmsd based optimization approach. There is a further marginal reduction in the efficiency of the LCVA performance for various RDO cases as obtained for different settings of λ. However, the dispersion of the designs performance i.e., the SD of the rmsd is less in RDO approach than the SD of rmsd obtained by usually adopted unconditional

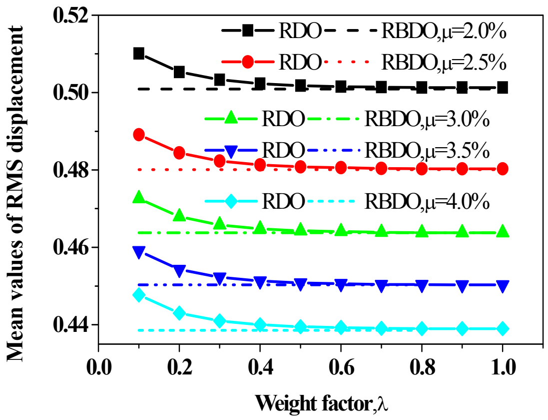

Fig. 6 The variation of the mean value of the rmsd of the primary structure with varying weight factor, λ for different mass ratio

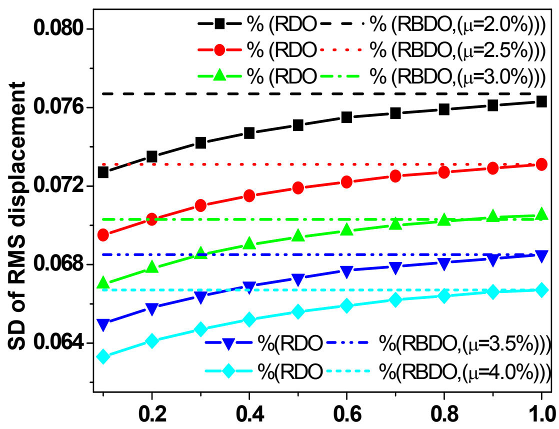

Fig. 7 The variation of the SD of the rmsd of the primary structure with varying weight factor, λ for different mass ratio

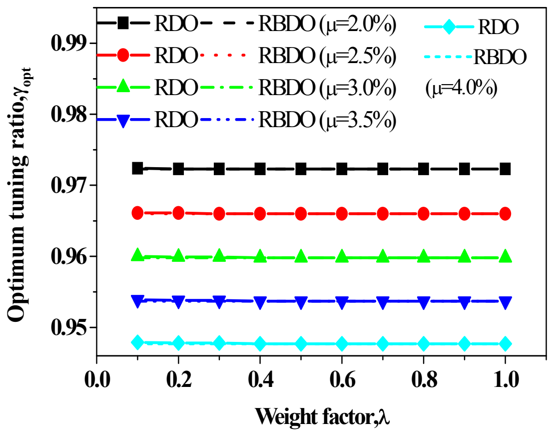

Fig. 8 The optimum tuning ratio with varying weight factor, λ for different mass ratio

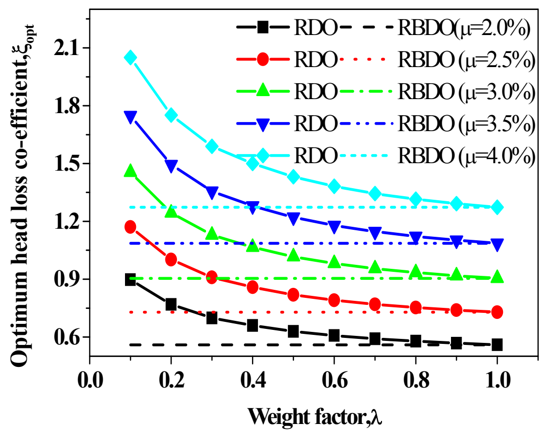

Fig. 9 The optimum head loss coefficient with varying weight factor, λ for different mass ratio

rmsd based optimization procedure. Fig. 4 shows that the tuning ratio is not sensitive for different robust design for a fixed mass ratio. However, the optimum head loss coefficients as shown in Fig. 5 changes notably to achieve various RDO solutions for different setting of weight factor λ. The RDO solutions show the tendency of greater damping values requirements in comparison to those required by the conventional SSO or unconditional rmsd based optimization approach. The tendencies are more marked when λ decreases which is obvious as lower λ values correspond to more importance to the performance variation to achieve more robustness in the design.

The normalized mean value of the rmsd of the structure versus the weight factor λ is plotted in Fig. 6 for different mass ratio. The associated normalized SD of rmsd is shown in Fig. 7. It can be readily observed from the plots that the rate of increase in the rmsd is much less than the rate of decrease in its SD for all values of λ. This clearly indicates that by a marginal sacrifice of the

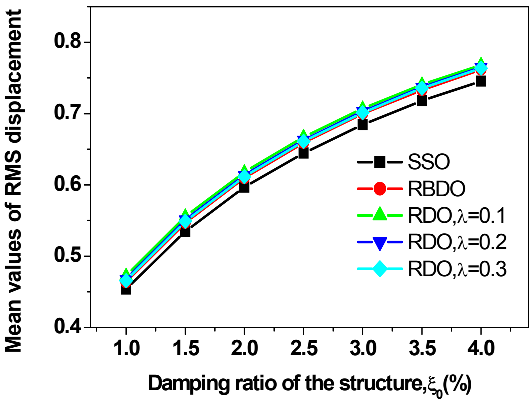

Fig. 10 The variation of the mean value of the rmsd of the primary structure with increasing damping ratio

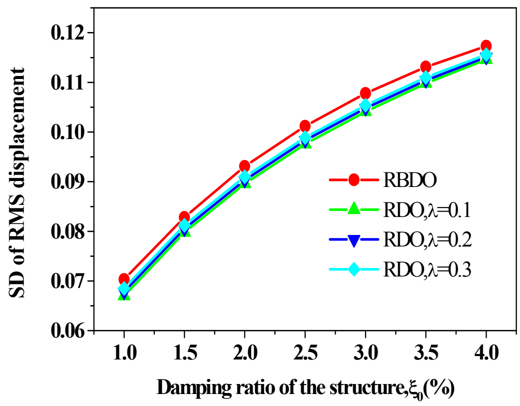

Fig. 11 The variation of the SD of the rmsd of the primary structure with increasing damping ratio

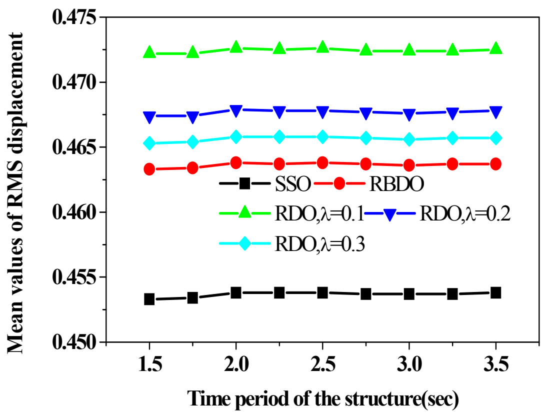

Fig. 12 The variation of the mean value of the rmsd of the primary structure with varying time period

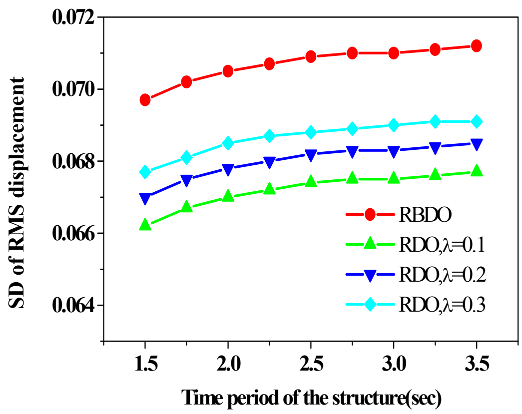

Fig. 13 The variation of the SD of the rmsd of the primary structure with varying time period

performance of LCVA, a reasonable improved robustness in its performance is possible to achieve. The variations of the optimum damper parameters with varying λ values are shown in Figs. 8 and 9. As earlier, it is observed that the optimum tuning ratios do not changes but the head loss coefficients get adjusted to yield improvement to the robustness of the performance of the system.

The sensitivities of robust solution of LCVA with respect to various parameters involved are further studied. The normalized mean value of the rmsd of the structure and its' associated SD with different values of weight factor λ for increasing damping ratios of the structure are studied in Figs. 10 and 11, respectively. The normalized mean and SD of the rmsd with varying time period of the structure are plotted in Figs. 12 and 13, respectively for different values of λ. The reduction of efficiency of the LCVA system by the unconditional rmsd based optimization and the RDO with respect to the efficiency achieved by the conventional SSO approach as shown in Figs. 10 and 12 is obvious as it does not consider the effect of system parameter uncertainty. However, the

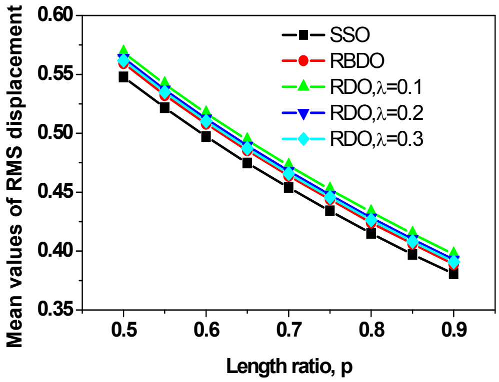

Fig. 14 The variation of the mean value of the rmsd of the primary structure with varying length ratio

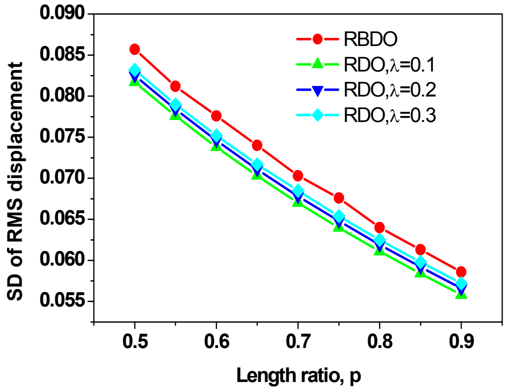

Fig. 15 The variation of the SD of the rmsd of the primary structure with varying length ratio

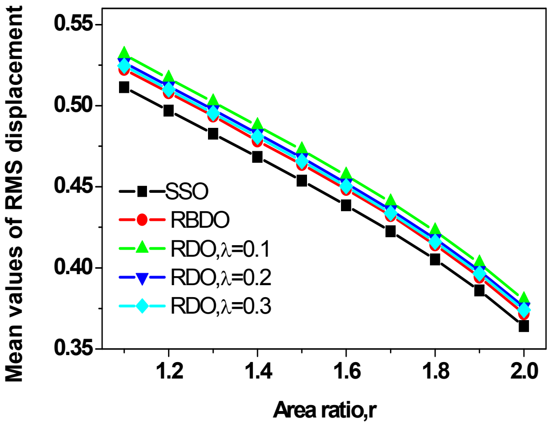

Fig. 16 The variation of the mean value of the rmsd of the primary structure with varying area ratio

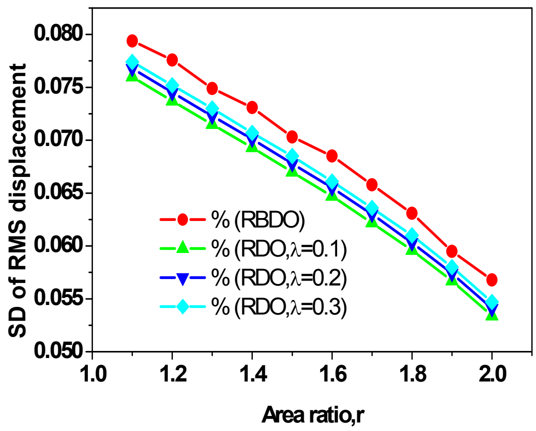

Fig. 17 The variation of the SD of the rmsd of the primary structure with varying area ratio

reduction in the SD of rmsd indicating more robustness in the design by the proposed RDO approach is clearly observed over a wide range of damping ratio and time period of the structure.

Similar results are developed for varying length ratio and area ratio of the LCVA. The results are shown in Figs. 14 and 15 for varying length ratio and in Figs. 16 and 17 for varying area ratio. The results show that the trends of RDO results remain same over wide range of length ratio and area ratio.

The trade-off scenario between the performance objective and its robustness, the typical characteristics of any multi-objective optimization procedure is studied further in term of Pareto front, in Fig. 18 for different mass ratio. The optimum rmsd of the structure and its associated SD as obtained by the unconditional rmsd based optimization procedure is also shown in the figure (same symbol, but inside not filled). It is clear from the figure that more robustness is achieved at the cost of sacrificing the optimum performance. It can be observed that though the unconditional

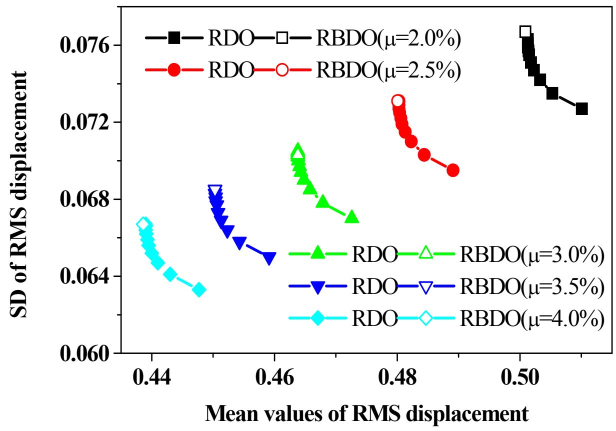

Fig. 18 The Pareto fronts of the LCVA system optimization problem

rmsd based optimization yields better performance in terms of reduction of vibration level; but the SD of the rmsd is also quite high. It may be further noted that the SD obtained by the usually adopted unconditional rmsd based optimization to allow system parameter uncertainty is a fixed value and the designers have no control over it. Whereas, in case of proposed RDO procedure for LCVA system of protection, the designer has the necessary flexibility to reduce the SD of the rmsd of the structure reasonably by marginally sacrificing the efficiency.

6. Conclusions

The effectiveness of applying RDO procedure compare to the conventional unconditional rmsd based optimization procedure for design of LCVA system to mitigate the seismic vibration of structures under system parameter uncertainty is presented. It is generally observed that the efficiency of the LCVA system of protection is marginally less by the proposed RDO approach compare to that of the unconditional rmsd based optimization. However, the dispersion of the design i.e., the SD of rmsd is reasonably reduced compare to the SD of rmsd as obtained by the unconditional rmsd based optimization approach. It is important to note that it is possible to achieve a desired level of performance efficiency and associated dispersion by RDO procedure under uncertain parameters through suitable choice of parameters λ. Thus, the proposed approach can provide for more realistic and cost-effective trade-offs between the control performance and its robustness with due importance to the unavoidable presence of system parameter uncertainty. Moreover, in many real life problems, the mini-max criteria may provide a range of variations indicating the implications of the presence of uncertainty. These estimates, though unsuitable for unconditional rmsd based optimization, can be integrated into a RDO process. However, these need further studies. It is to be noted here that the proposed RDO approach estimates the robustness measure following linear perturbation based approximation of functions around the mean values of the uncertain system parameter. Thus, the approach is applicable so long the level of uncertainty is small and for comparatively larger levels of uncertainty, other alternative approaches to the linear perturbation analysis e.g., stochastic simulation should be applied. The present study is based on stationary stochastic earthquake load model. For more realistic nonstationary earthquake model will require to deal with time dependent response statistics evaluation and performance function in the optimization procedure. This of course needs further study.

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