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# The comparative analysis of optimal designed web expanded beams via improved harmony search method

Ferhat Erdal*

*Department of Civil Engineering, Akdeniz University, 07058, Antalya, Turkey*

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**Abstract.** This study aims at comparing the optimum design of two common types open web expanded beams: with hexagonal openings, also called castellated beams and beams with circular openings referred to as cellular beams. The minimum weights of both beams are taken as the objective functions while the design constraints are respectively implemented from The Steel Construction Institute Publication Numbers 5 and 100. The design methods adopted in these publications are consistent with BS5950 parts. The formulation of the design problem considering the limitations of the above mentioned turns out to be a discrete programming problem. Improved harmony search algorithm is suggested to compare the optimum design of mentioned web-expanded beams to analysis the performance of both beams. The design algorithms based on the technique select the optimum Universal Beam sections, dimensional properties of hexagonal and circular holes and total number of openings along the beam as design variables.

**Keywords:** structural optimization; web-expanded beams; castellated beams; cellular beams; harmony search algorithm

## 1. Introduction

Web-expanded beams provide economical solution and pleasing appearance for large clear-span structures. Decrease in story height reduces interior volume and exterior surface of building and these results in cost saving. Furthermore, in comparison with solid web and web opening beams (Redwood and Cho 1993), web-expanded beams can easily increase the shear capacities, vertical bending stiffness and capacities of structure. Open web-expanded beams can be fabricated where architectural or structural solutions dictate standard steel sections inappropriate. This is achieved by cutting the web of a hot rolled beam in a certain pattern and then welding two halves together to form a deeper section. As a result of these cutting and welding back processes, beams will have a deeper section and greater resistance to deflection than a comparable original solid section. Open web expanded beams are of two general types: castellated and cellular beams. Castellated beams are initially split along their length by a profiled single flame cut as shown in Fig. 1(a). Two halves of the beam are then separated and welded back together (Dougherty 1993). The fabrication process of cellular beams is slightly different from castellated beams (Lawson 1988). These beams are manufactured by twice cutting an original rolled beam web in a half

*Corresponding author, Assistant Professor, E-mail: eferhat@akdeniz.edu.tr


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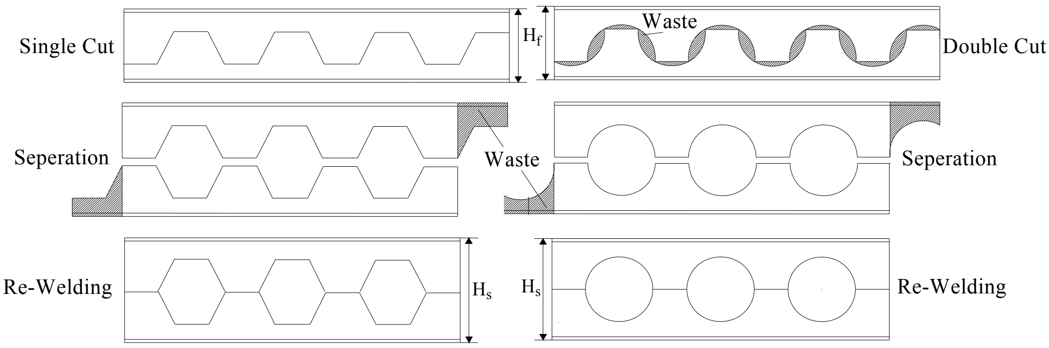


(a) Basic process of castellated beam (b) Basic process of cellular beam

Fig. 1 Basic fabrication processes of web-expanded beams

circular pattern along its centerline, then separating two tee parts and re-welding these two halves as shown in Fig. 1(b). Increasing the stiffness of original beam with no weight increase in the steel beam has been the purpose of the selection of these beams from designers. This study is concerned with the application of harmony search algorithm for the mentioned web-expanded beams. Harmony search method originated by Geem and Kim (2001) is based on the musical performance process that takes place when a musician searches for a better state of harmony. Jazz improvisation seeks musically pleasing harmony similar to the optimum design process which seeks to find the optimum solution. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is determined by the set of values assigned to each decision variable. The minimum weight design of castellated and cellular beams requires the selection of beams from standard steel UB section list such that both beams satisfy the strength and serviceability constraints.

## 2. The design of cellular beams

The design of a cellular beam has need for the selection of a rolled beam from which the cellular beam is to be produced, the selection of circular hole diameter and the selection of spacing between the centers of these circular holes or total number of holes in the beam as shown in Fig. 2.

In consequence the number of the rolled beam sections in the standard steel sections tables, the circular opening diameter and the total number of holes are taken as design variables in the optimum design problem considered. For that purpose a design pool is prepared which consists of list of standard rolled beam sections, a list of various diameter sizes and a list of integer number starting from 2 to 40 for the total number of holes in a cellular beam. The optimum design problem formulated considering the design constraints explained in The Steel Construction Institute Publication titled "Design of Composite and Non-composite Cellular Beams" (Ward 1990) which are consistent with BS5950 (2000); Part 1 and 3 yields the following mathematical model. Find a integer design vector $\{I\}=\{I_1, I_2, I_3\}^T$ where $I_1$ is the sequence number for rolled beam section in the standard steel sections list, $I_2$ is the sequence number for the hole diameter in the discrete set which contains various diameter values and $I_3$ is the total number of holes for the cellular beam. Once $I_1$ is selected, then the rolled steel beam designation becomes known and all cross sectional

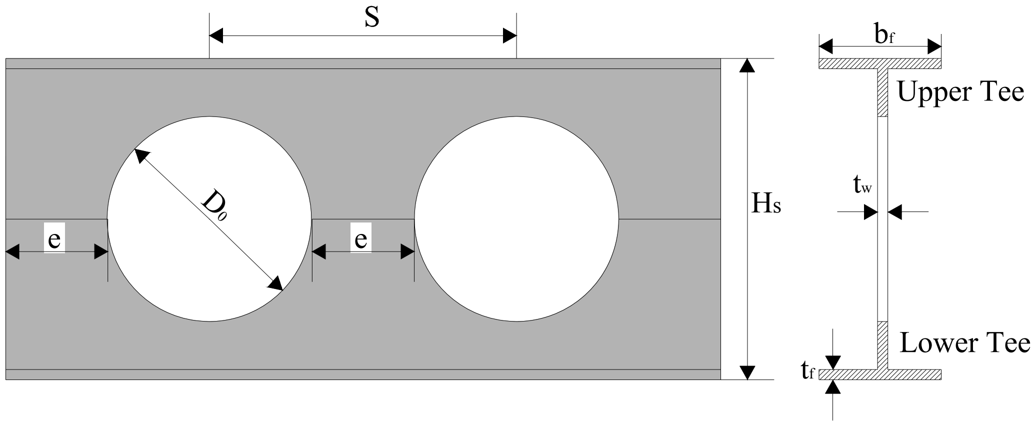


Fig. 2 Design variables for a cellular beam

properties of the beam becomes available for design. The corresponding values to $I_2$ and $I_3$ in the design sets makes the hole diameter and the total number of holes available for the cellular beam. Hence the design problem turns out to be

Minimize the weight of the cellular beam

$$W_{cel} = \rho_s A L - \rho_s \left( \pi \left( \frac{D_0}{2} \right)^2 N_H \right) \tag{1}$$

Subject to

$$g_1 = 1.08 \times D_0 - S \leq 0 \tag{2}$$

$$g_2 = S - 1.6 \times D_0 \leq 0 \tag{3}$$

$$g_3 = 1.25 \times D_0 - H_S \leq 0 \tag{4}$$

$$g_4 = H_S - 1.75 \times D_0 \leq 0 \tag{5}$$

$$g_5 = M_U - M_P \leq 0 \tag{6}$$

$$g_6 = V_{\max\sup} - P_v \leq 0 \tag{7}$$

$$g_7 = V_{O\max} - P_{vy} \leq 0 \tag{8}$$

$$g_8 = V_{H\max} - P_{vh} \leq 0 \tag{9}$$

$$g_9 = M_{A-A\max} - M_{w\max} \leq 0 \tag{10}$$

$$g_{10} = V_{Tee} - 0.5 \times P_{vy} \leq 0 \tag{11}$$

$$g_{11} = \frac{P_0}{P_u} - \frac{M}{M_p} - 1 \leq 0 \tag{12}$$

$$g_{12} = y_{\max} - L/360 \le 0 \tag{13}$$

where $W_{cel}$ is the weight of the cellular beam, $D_0$ is hole diameter, $\rho_s$ is density of steel, $A$ is total area of profile, $N_H$ is number of holes, $H_S$ is overall depth of cellular beam, $L$ is span of cellular beam and $S$ is distance between centers of holes. $M_U$ is maximum moment under loading, $M_P$ is plastic moment capacity $V_{\max\,\sup}$ is maximum shear at support $V_{O\max}$ is maximum shear at opening, $V_{H\max}$ is maximum horizontal shear, $M_{A\text{-}A\max}$ is maximum moment at $A$-$A$ section shown in Fig. 3, $M_{w\max}$ is maximum allowable web post moment, $V_{Tee}$ is vertical shear on tee, $P_0$, $M$ are forces on the section and $y_{\max}$ is maximum deflection at the beam. Although the diameter of holes and spacing between their centers are left to designer to select, the geometric limitations given in constraints (2)-(4) are required to be observed. Eq. (6) represents overall beam flexural capacity limitation. Under applied load combinations the cellular beam should have sufficient flexural capacity to be able to resist the external loading.

Eqs. (7)-(9) represents shear capacity checks. There are three shear checks in the design of cellular beams. The first one is shear check at the support. Eq. (7) makes sure that shear at the support does not exceed the shear capacity of the section. It is also necessary to check two more shear failure modes additionally. The first shear failure mode check Eq. (8) is the vertical shear capacity check of the beam. The sum of the shear capacities of the upper and lower tees gives the vertical shear capacity of the beam. The factored shear force in the beam should not exceed allowable vertical shear. The other Eq. (9) is the horizontal shear check. The horizontal shear is developed in the web post due the change in axial forces in the tee as shown in Fig. 3. The horizontal shear force in the web post of beam should not exceed allowable horizontal shear. The details of the computations of shear force and bending moment at a section of cellular beam is given in Erdal (2011).

The flexural capacity of the upper and lower tees under bending is also critical in steel cellular beams. The transfer of shear forces across a single opening causes secondary bending stresses. Eqs. (10-12) are required for the flexural and buckling strength of web post. The details of the computation of the maximum moment at section A-A shown in Fig. 3. $M_{A\text{-}A\max}$ and the maximum allowable web-post moment $M_{w\max}$ are also given in Erdal (2011). The last Eq. (13) is the serviceability requirement that the cellular beam has to satisfy. The design steps of cellular beams are summarized very briefly in the paper due to space limitations, yet the detailed implementation specifics of them can be found in Erdal *et al*. (2011).

## 3. Design of castellated beams

Since the 1950's the high strength to weight ratio of castellated beams has been a desirable item to structural engineers in their efforts to design even lighter and more cost efficient steel structures. The design process of castellated beams is different from cellular beams as they do not have same geometrical properties. The strength of a castellated beam shall be determined based on the interaction of flexure and shear at the hexagonal opening. Design constraints include the displacement limitations, overall beam flexural capacity, beam shear capacity, overall beam buckling strength, web post flexure and buckling, vierendeel bending of upper and lower tees, local buckling of compression flange and practical restrictions between hexagonal hole dimensions and the spacing between openings. The design procedure given here is taken from "The Steel Construction Institute Publication No: 005 titled "Design of Castellated Beams". The design methods are consistent with BS5950 part 1 and 3, and BS449.

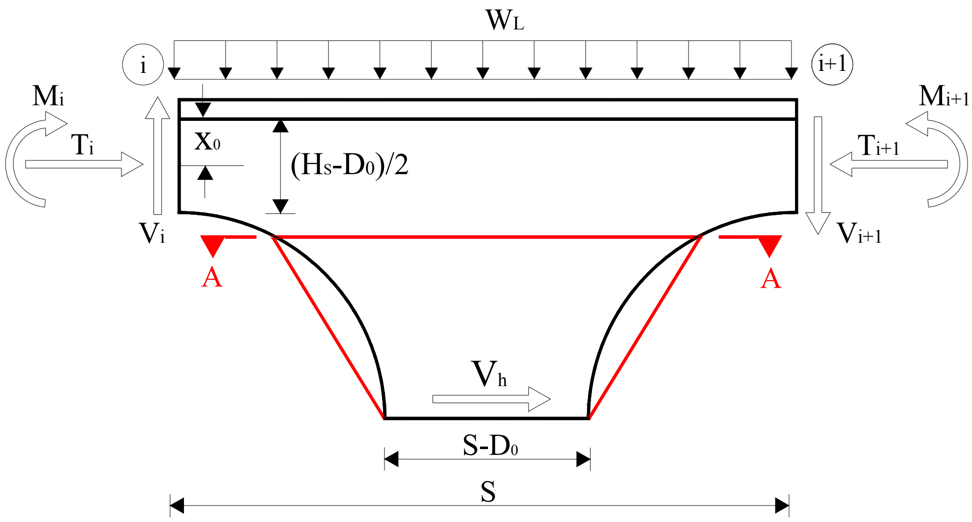


Fig. 3 Horizontal shears in web post of a cellular beam

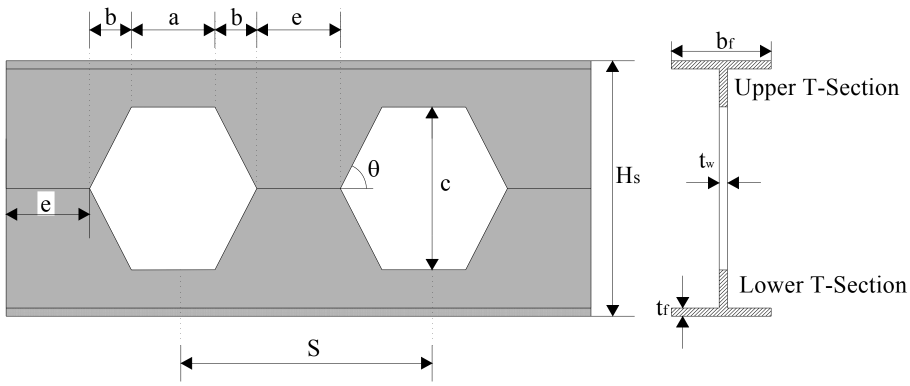


Fig. 4 Geometry and notation for castellated beam

The standart profile geometry and notations used for castellated beams are shown in Fig. 4. The dimensions of the beam are described as following Eqs. (14)-(17).

$$a = 0.5 \times (S - 2 \times c \times \cot \theta) \tag{14}$$

$$b = c \times \cot \theta \tag{15}$$

$$S = 2 \times (a + b) \tag{16}$$

$$H_S = h_f + c \tag{17}$$

Where, $S$ is spacing between centers of the holes, $h_f$ is the depth of original section, $H_S$ is the final depth of the castellated beam, $a$, $b$ and $c$ are the dimensions of the hexagonal holes. Design properties and dimensions of the castellated beam are considered as design constraints.

### 3.1 Optimum design problem of castellated beam

The optimum design of a castellated beam requires the selection of the design variables called the sequence number of a universal beam sections in the standard steel sections tables, the hexagonal hole depth, angle between the edges and the total number of hexagonal holes. For this purpose a design pool is prepared which consists of list of standard UB beam sections starting from 254×102×28 to 914×419×388, a list of various hexagonal depth sizes, a list of angle values and a list of integer numbers starting from 2 to 40 for the total number of holes in a cellular beam. Find an integer design vector $\{I\}=\{I_1, I_2, I_3, I_4\}^T$ where $I_1$ is the sequence number for UB beam section in the standard steel sections list, $I_2$ is the sequence number for the hexagonal depth size in the discrete set which contains various depth values, $I_3$ is the angle between the edges and $I_4$ is the total number of holes for the castellated beam. Hence the design problem turns out to be minimizing the weight of the cellular beam

$$W_{cas} = \rho_s A L - \rho_s \left(N_H \times c(a+b)\right) \tag{18}$$

Where $W_{cas}$ denotes the weight of the castellated beam, $\rho_s$ is the density of steel. $A$ represents the total cross-sectional area of the universal beam section selected for the castellated beam, $L$ is the span of the castellated beam, $c$ is the depth of hexagonal holes and $N_H$ is the total number of holes in the castellated beam. The castellated beam is also subjected to number of behavioral restrictions as given in Eqs. (19)-(22). Depending on the values of hole diameters, spacing between the hole centers and the final depth of the beam determined; following geometrical constraints must be satisfied;

### 3.2 Maximum stress capacity

In the elastic design method the maximum stress in the beam can be expressed as following equations. Under applied load combinations maximum stress ($\sigma_{\max}$) in a castellated beam should not exceed an allowable stress capacity ($\sigma_{allow}$).

$$K_1 = 1/(A_{tee} \times h_t) \tag{19}$$

$$K_2 = a/(4 \times Z_{tee}) \tag{20}$$

$$\sigma_{\max} = (K_1 \times M + K_2 \times V) \tag{21}$$

$$\sigma_{\max} \le \sigma_{allow} \tag{22}$$

Where, $A_{tee}$ is area of tee, $h_t$ is distance between centroids of top and bottom tees, $Z_{tee}$ is section modulus of tee, $K_1$ and $K_2$ are behavioral coefficients about of beam. Stresses owing to bending and shear are shown in Fig. 5.

### 3.3 Beam shear capacity

It is necessary to check three shear failure modes in castellated beams. The first one is the web-post shear capacity check of the beam. The factored shear force in the web-post should not exceed $P_{vy}$

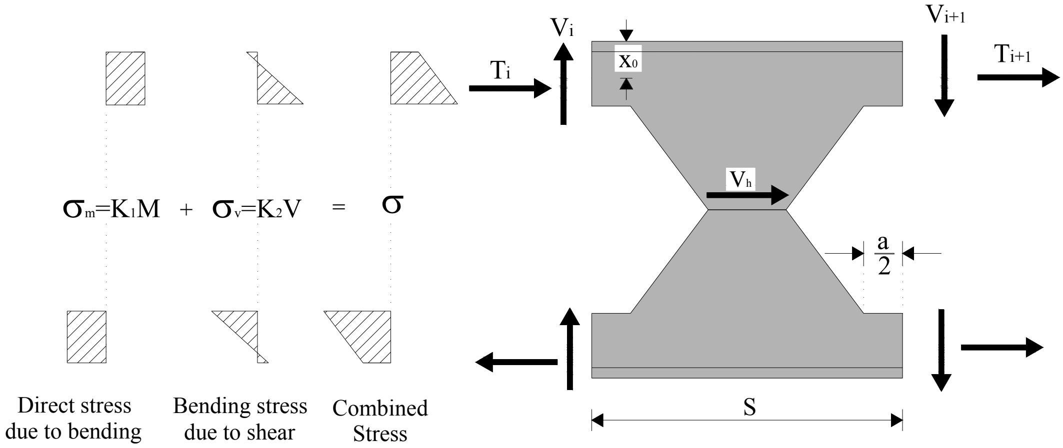


Fig. 5 Stress in tees of the castellated beam

$$P_{vy} = 0.6 \times p_y \times (0.9 \times Minimum \;\; Area \;\; of \;\; web \;\; post) \tag{23}$$

The other is the horizontal shear check. The horizontal shear is developed in the web post due the change in axial forces in the tee as also shown in Fig. 3. The horizontal shear capacity in the web post of beam should not exceed $P_{vh}$ (Eq. (24))

$$P_{vh} = 0.6 \times p_y \times (0.9 \times \; Area \;\; of \;\; webs \;\; upper \;\; and \;\; lower \;\; tees) \tag{24}$$

Considering the vertical equilibrium and the rate of the variation of bending moment, the following equations can be written.

$$V_{i+1} = V_i \tag{25}$$

$$M_i = T_i \times (H_S - 2x_0) \tag{26}$$

$$V_{i+1} = \frac{dM}{dx} = \frac{M_{i+1} - M_i}{S} = (T_{i+1} - T_i) \times \frac{(H_S - 2x_0)}{S} \tag{27}$$

For horizontal equilibrium

$$V_h = T_{i+1} - T_i = V_{i+1} \frac{S}{H_S - 2x_0} \tag{28}$$

Where $V$ is shear force, $T$ is axial force and $M$ is bending moment at the cross section of the cellular beam, $S$ is distance between hexagonal hole centers and $x_0$ is the distance between the axial force to flange. These are all shown in Fig. 3.

### *3.4 Web buckling capacity of beam*

In this study the compression flange of the castellated beam is assumed to be sufficiently restrained through the floor system it is attached to. Hence the overall buckling strength of the castellated beam is omitted. Experimental tests on castellated beams have shown that the web post flexural and buckling capacity is checked using the following equations according to BS5950 method (Eqs. (29)-(31)).

$$\lambda_r = \frac{(H_s - 2 \times t_f) \times \sqrt{3}}{t_w} \tag{29}$$

$$P_w = H_s \times t_w \times P_c \tag{30}$$

$$V_{\max} \leq P_w \tag{31}$$

In these Eqs. (29)-(31); $\lambda_r$ is slenderness ratio of web and $H_s$ is overall depth of castellated beam. $P_c$ value is obtained from Table 27(c) in BS 5950 according to $\lambda_r$ and $P_y$ values.

### *3.5 Vierendeel bending of upper and lower tees*

The flexural capacity of the upper and lower tees under Vierendeel bending is critical. The transfer of shear forces across a single opening causes secondary bending stresses. The Vierendeel bending stresses around the opening may be calculated using interaction curves. For a symmetrical section, the shear force is resisted by the upper and lower web sections in proportion to their depth squared. Therefore, the shear force is divided equally between upper and lower web sections. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as following Eqs. (32)-(34).

$$P_u = \frac{\sigma_{allow}}{K_2} \tag{32}$$

$$M_u = \frac{\sigma_{allow}}{K_1} \tag{33}$$

$$\frac{P_o}{P_u} + \frac{M}{M_p} \leq 1.0 \tag{34}$$

Where $P_o$ and $M$ are the force and moment on the section due to external loading respectively. $P_u$ is the maximum allowable shear force and $M_p$ is the maximum allowable bending moment in the castellated steel beam.

### *3.6 Deflection of castellated beam*

The limiting values for deflection of a beam under applied load combinations are given in BS5950, Part 1. According to these limitations the maximum deflection of a castellated beam should not exceed span/360. The deflection of castellated beam is computed using the virtual work method which is explained in detail in Knowles (1980). Fig. 5 shows points of inflection at sections $i$ and $i$+1.

Shear force under applied load combination is distributed equally tees, the axial and horizontal forces in the upper and lower tee are given by

$$N_i = \frac{M_i}{h} \quad \text{and} \quad T_i = \frac{S(V_i + V_{i+1})}{2h} \tag{35}$$

Where; $h$ is distance between the centre of upper and lower tees and $S$ is distance between centrals of holes. The deflection at each point is found by applying a unit load at that point. Internal forces under a unit load are given by $\overline{V_i}/2$, $\overline{N_i}$, $\overline{T_i}$.

Deflection due to bending moment in tee

$$y_{mt} = \frac{a^3}{24EI_T}(V_i\overline{V_i}) \tag{36}$$

Deflection due to bending moment in web post of beam

$$y_{wp} = \frac{3\times c^3}{Eb^3 t_w}\left[\log_e\left(\frac{a+2b}{a}\right)+\left(\frac{2a)}{a+2b}\right)-\frac{1}{2}\left(\frac{(a/2)^2}{((a/2)+b)^2}\right)-\frac{3}{2}\right]T_i\overline{T_i} \tag{37}$$

Deflection due to axial force in tee

$$y_{at} = \frac{2S}{EA_T}(N_i\overline{N_i}) \tag{38}$$

Deflection due to shear in tee

$$y_t = \frac{(a/2)}{GA_{TWEB}}(V_i\overline{V_i}) \tag{39}$$

Deflection due to shear in web post

$$y_w = \frac{c}{Gbt_w} X \log_e\left(\frac{a+2b}{a}\right) T_i\overline{T_i} \tag{40}$$

Where $E$ is elasticity modulus of steel, $I_T$ is total moment of inertia of beam, $G$ is shear modulus and $X$ is the web post form factor. The total deflection of a single opening under applied load (Eq. (41)) is obtained by summing the deflections computed in Eqs. (36)-(40). On the other hand, the deflection of the castellated beam is calculated by multiplying the deflection of each opening by the total number of openings in the beam as given from Toprac and Cooke (1980).

$$y_T = y_{mt} + y_{wp} + y_{at} + y_t + y_w \tag{41}$$

## 4. Metaheuristic search techniques in optimization

The solution methods available among the mathematical programming techniques to obtain optimum results to discrete programming problems are not very efficient for practical use. Fortunately, the emergence of innovative stochastic search techniques that are based upon the mimicking of paradigms found in nature has changed this situation altogether. The basic idea behind search techniques is to simulate the natural phenomena, such as survival of the fittest in genetic algorithms (Goldberg 1989), flock migration in swarm intelligence (Kennedy *et al*. 2001), shortest path to food source in ant colony optimization (Dorigo and Stützle 2004 ), the cooling process of molten metals through annealing into a numerical algorithm (Kirkpatrick *et al*. 1983), looking for a prey in hunting search algorithm (Oftadeh *et al*. 2010), accelerations of charge particles in charged system search (Kaveh and Talatahari 2010) and best harmony of instruments in harmony search technique (Geem and Lee 2004, 2005) that is automated by nature to achieve the task of optimization of its own. The design algorithms developed using these techniques are very effective for global search owing to their capability of the finding optimum solutions in the search space at an affordable time. An improved version of harmony search algorithm (IHS) is proposed in this paper as an efficient algorithm for solving web-expanded beams optimization problems. The robustness of the algorithm lies in its capability to implement the aforementioned HS parameters dynamically and update them during the search for the most efficient optimization process.

### *4.1 Improved harmony search method*

Harmony search (HS) algorithm is one of the recent editions to such stochastic search techniques founded on musically pleasing simulation to solve combinatorial optimization problems. This approach utilizes the experience of a musician for searching pleasing harmony similar to the optimum design process which seeks to find optimum solution. The pitch of each instrument determines the aesthetic quality; in just the same way as the objective function value is determined by the set of values assigned to each decision variable. Although HS method has been successfully applied to different practical optimization problems since its origination, the applications of the method in structural optimization are still immature and require a substantial amount of further research. Up until this time only a limited number of publications in the literature are carried out where the application of the technique in different problem areas encountered in the field. Amongst these restricted studies that look at the effectiveness of the HS method, Lee and Geem (2004) used the technique for minimum weight design of planar and space truss structures. In 2009, Saka *et al*. (2009) and Değertekin (2009) focused to examine the optimum design of steel frames formulated according to BS5950 and LRFD-AISC design codes with HS, respectively. Later, the success of the method in optimum W-sections for the transverse and longitudinal beams of grillage systems was investigated in Erdal *et al*. (2009, 2010, 2013). Mainly small scale applications that consist of a small number of design variables were used in these aforementioned studies and all of them were concluded that HS algorithm was a very rapid and effective method for optimum design of such systems. Conversely, Hasançebi *et al*. (2009, 2010) evinced a comprehensive performance evaluation of the technique in the optimum design of real size trusses and frames where the design problem was formulated according to ASD-AISC (1989) in evinced a completely opposite outlook. In comparison to those of other metaheuristic techniques, the performance of HS algorithm was qualified substandard with its slow convergence rate and unreliable search efficiency. An improvement of the technique was recommended for its application to new structural optimization problems, which in fact led to the motivation of the present study.

In the classical HS method the parameters harmony memory considering rate ($\eta$) and pitch adjusting rate ($\rho$) are selected prior to the application of the method and they are kept constant until the end of the iterations. The numerical applications have shown that the selection of values for $\eta$ and $\rho$ is problem dependent and the initial values selected affect the performance of the algorithm. Consequently, in order to determine the appropriate values of the harmony search parameters it is necessary to solve the optimization problem several times with different values and select the solution with minimum weight. It is apparent that such application devaluates the efficiency of the algorithm. In order to overcome this discrepancy, numbers of improvements are suggested in the literature. First, Mahdavi *et al*. (2007) have proposed an improved harmony search algorithm that uses variable $\rho$ and $bw$ in improvisation step where $bw$ is an arbitrary distance bandwidth. Then, Omran and Mahdavi (2008) have used the concepts from swarm intelligence to enhance the performance of HS method. Later, Taherinejad (2009) has proposed a new function which could help the algorithm to explore vast search space while focusing well on local and global optimums. And then, Hasançebi *et al*. (2011) suggested adaptive harmony search method where $\eta$ and $\rho$ are adjusted by the algorithm itself automatically using probabilistic sampling of control parameters. Hence the algorithm tunes these parameters to advantageous values online during search. Eventually, Carbas and Saka (2012, 2013) have used the improved version of algorithm for latticed steel domes and some engineering problems, respectively. In the present study, different strategies are proposed for $\eta$ and $\rho$ to compare the minimum weight design of steel castellated beams and cellular beams. $\rho$ is updated using the concept suggested by Coelho and Bernert (2009). Before initiating the design process, a set of steel beam sections selected from an available UB profile list are collected in a design pool. Each steel section is assigned a sequence number that varies between 1 to total number of sections ($N_{sec}$) in the list. During optimization process selection of sections for design variables is carried out using these numbers. The basic components of the improved harmony search algorithm can now be outlined as follows.

#### *4.1.1 Initialization of a parameter set*

First a harmony search related optimization parameter set is specified. This parameter set consists of four entities known as a harmony memory size ($\mu$), a harmony memory considering rate ($\eta$), a pitch adjusting rate ($\rho$) and a maximum search number ($N_S$). Out of these four parameters, $\eta$ and $\rho$ are dynamic parameters that vary from one solution vector to another, and are set to initial values of $\eta^{(0)}$ and $\rho^{(0)}$ for all the solution vectors in the initial harmony memory matrix. It is worthwhile to mention that in the standard harmony search algorithm these parameters are treated as static quantities, and hence they are assigned to suitable values chosen within their recommended ranges of $\eta \in [0.70, 0.95]$ and $\rho \in [0.20, 0.50]$.

#### *4.1.2 Initialization of harmony memory matrix*

Harmony memory matrix **H** is generated randomly initialized next. This matrix represents a design population for the solution of a problem under consideration, and incorporates a specified number of solutions referred to as harmony size ($\mu$). Each solution vector ($\mathbf{I}^i$) consists of $N_d$ design variables integer number between 1 to $N_s$ (number of values) selected randomly each of which corresponds sequence number of design variables in the design pool, and is represented in a separate row of the matrix; consequently the size of **H** is ($\mu \times N_d$). $I_i^j$ is the sequence number of the $i^{th}$ design variable in the $j^{th}$ randomly selected feasible solution.

$$\mathbf{H} = \begin{bmatrix} I_1^1 & I_2^1 & \dots & I_{N_d}^1 \\ I_1^2 & I_2^2 & \dots & I_{N_d}^2 \\ \dots & \dots & \dots & \dots \\ I_1^\mu & I_2^\mu & \dots & I_{N_d}^\mu \end{bmatrix} \begin{matrix} \phi(\mathbf{I}^1) \\ \phi(\mathbf{I}^2) \\ \dots \\ \phi(\mathbf{I}^\mu) \end{matrix} \tag{42}$$

#### 4.1.3 Evaluation of harmony memory matrix

($\mu$) solutions shown in Eq. (42) are then analyzed, and their objective function values are calculated. The solutions evaluated are sorted in the matrix in the increasing order of objective function values, that is $\phi(\mathbf{I}^1)\leq\phi(\mathbf{I}^2)\leq\ldots\leq\phi(\mathbf{I}^{\mu})$.

#### 4.1.4 Improvising a new harmony

Upon sampling of a new set of values for parameters, the new solution vector $\mathbf{I}' = [I_1', I_2',.., I_{nv}']$ is generated. In the harmony memory consideration, each design variable is selected at random from either harmony memory matrix or the entire discrete set. The probability that a design variable is selected from the harmony memory is controlled by a parameter called harmony memory considering rate ($\eta$). To execute this probability, a random number $r_i$ is generated between 0 and 1 for each variable $I_i$. If $r_i$ is smaller than or equal to $\eta$, the variable is chosen from harmony memory in which case it is assigned any value from the $i$-th column of the $\mathbf{H}$, representing the value set of variable in $\mu$ solutions of the matrix (Eq. (43)). If $r_i>\eta$, a random value is assigned to the variable from the entire discrete set.

$$I_i' = \begin{cases} I_i' \in \{I_i^1, I_i^2,..., I_i^{\mu}\} & if\ r_i \leq \eta \\ I_i' \in \{1,.., N_S\} & if\ r_i > \mu \end{cases} \tag{43}$$

If a design variable attains its value from harmony memory, it is checked whether this value should be pitch-adjusted or not. Pith adjustment simply means sampling the variable's one of the neighboring values, obtained by adding or subtracting one from its current value. Similar to $\eta$ parameter, it is operated with a probability known as pitch adjustment rate ($\rho$), Eq. (43). If not activated by $\rho$, the value of the variable does not change.

$$I_i'' = \begin{cases} I_i' \pm 1 & if\ r_i \leq \rho \\ I_i' & if\ r_i > \rho \end{cases} \tag{44}$$

##### 4.1.4.1 Updating parameters

$$\rho_{(I)} = \rho_{(MIN)} + (\rho_{(MAX)} - \rho_{(MIN)}) \times Deg_{(I)} \tag{45}$$

where, $\rho_{(\mathrm{I})}$ is the pitch adjusting rate for generation $I$, $\rho_{(\mathrm{MIN})}$ is the minimum adjusting rate, $\rho_{(\mathrm{MAX})}$ is the maximum adjusting rate, and $i$ is the generation number. The $Deg_{(I)}$ is updated according to the following expression

$$Deg_{(I)} = \frac{(HCOST_{MAX(I)} - HCOST_{MEAN})}{(HCOST_{MAX(I)} - HCOST_{MIN(I)})} \tag{46}$$

where, $HCOST_{\mathrm{MAX}(I)}$ and $HCOST_{\mathrm{MIN}(I)}$ are the maximum and minimum function objective values in generation $I$, respectively; $HCOST_{\mathrm{MEAN}}$ is the mean of objective function value of the harmony memory matrix. The improvisation of $\eta$ is carried out using the following expression

$$\eta_{(I)} = \eta_{(MAX)} - (\eta_{(MAX)} - \eta_{(MIN)}) \times Deg_{(I)} \tag{47}$$

where, $\eta_{(\mathrm{I})}$ is the harmony memory considering rate for generation $I$, $\eta_{(\mathrm{MAX})}$ is the maximum considering rate, $\eta_{(MIN)}$ is the minimum considering rate, and $I$ is the generation number.

#### *4.1.5 Adaptive constraint handling*

Once the new harmony vector is obtained using the above-mentioned rules, it is then checked whether it violates problem constraints. If the new harmony vector is severely infeasible, it is discarded. If it is slightly infeasible, it is included in the harmony memory matrix. In this way the violated harmony vector which may be infeasible slightly in one or more constraints is used as a base in the pitch adjustment operation to provide a new harmony vector that may be feasible. This is carried out by using larger error values initially for the acceptability of the new design vectors and then this value is adjusted during the design cycles according to the expression given below

$$Er(i) = Er_{MAX} - \frac{(Er_{MAX} - Er_{MIN})}{\sqrt{N_S}} \times \sqrt{i} \tag{48}$$

where, $Er(i)$ is the error value in iteration $i$, $Er_{MAX}$ and $Er_{MIN}$ are the maximum and the minimum errors defined in the algorithm respectively, $N_s$ is the maximum iteration number until which tolerance minimization procedure continues. Eq. (48) provides larger error values in the beginning of the design cycles and quite small error values towards the final design cycles. Hence when the maximum design cycles are reached the acceptable design vectors remain in the harmony memory matrix and the ones which do not satisfy one or more design constraints smaller than the error tolerance would be pushed out during the design iterations.

#### *4.1.6 Update of Harmony matrix*

After generating the new harmony vector, its objective function value is calculated. If this value is better than that of the worst harmony vector in the harmony memory, it is then included in the matrix while the worst one is discarded out of the matrix. The updated harmony memory matrix is then sorted in ascending order of the objective function value.

#### *4.1.7 Termination*

Steps 3 and 4 are repeated until a pre-assigned maximum number of cycles $N_{cyc}$ is reached. The number is selected large enough such that within this number no further improvement is observed in the objective function.

## 5. Design examples

In the first part of numerical examples, a benchmark problem (welded beam design) chosen from the literature is studied to verify the effectiveness of the proposed solution algorithms employed for the HS techniques, as well as to check the implementation with those of others in the literature. The optimum solutions located by the algorithms and the number of structural analyses required to obtain these solutions are reported in Table 1.

In the second part, two structural design examples selected to minimize and compare the weights of optimally designed steel castellated and cellular beams. Design examples are also used to compare the performance of improved harmony search (IHS) optimization software over the standard one (HS) as well as to demonstrate its improvisation ability under different sets of initial values chosen for the control parameters. The solution algorithms of IHS ve HS algorithms for

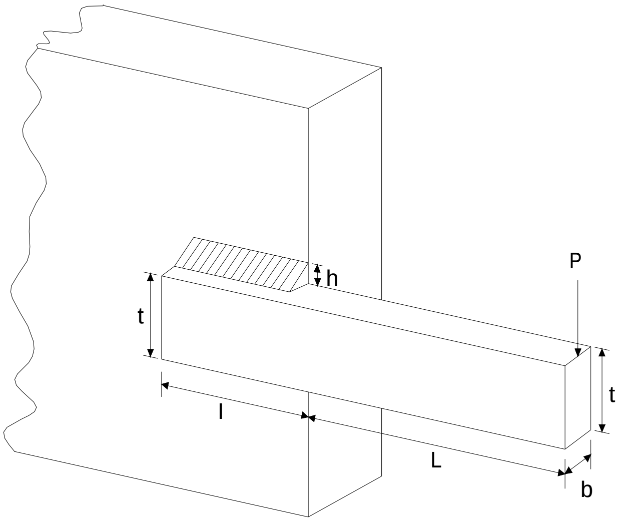


Fig. 6 Welded beam design

these web expanded beams are computerized in four optimization software that are both compiled in *FORTRAN* source code. It is common practice to use universal beam (UB) sections. Among the steel section list of these UB sections starting from 254×102×28 to 914×419×388 are chosen to constitute the discrete set consisting of 64 steel sections from which the design algorithm selects the sectional designations for the beam members. In both design examples, the following material properties of the steel are used: modulus of elasticity ($E$)=205 kN/mm$^2$ and Grade 50 steel is adopted for the steel which has the design strength ($P_y$)=355 MPa.

### 5.1 Benchmark problem

A rectangular beam shown in Fig. 6 has been frequently used in the literature for testing and comparing various optimization techniques. The optimization problem involves four design variables: the thickness of the weld $h=x_1$, the length of the welded joint $l=x_2$, the width of the beam $t=x_3$ and the thickness of the beam $b=x_4$. The values of $x_1$ and $x_2$ are coded with integer multiples of 0.0065 .#Although the formulation of the objective function (Eq. (49)) is the same, this benchmark problem comes with two different forms in the literature according to the behavioral constraints, side constraints and the ranges of mentioned design variables.#In the present study, this rectangular beam is designed as a cantilever beam to carry a certain load with minimum overall cost of fabrication. There are eleven constrains (second form), which involve shear stress ($\tau$), bending stress in the beam ($\sigma$), buckling load on the bar ($P_c$), deflection of the beam ($\delta$) and side constraints (Mahdavi *et al*. 2007). The welded beam problem is stated as following Eq. (49).

Minimize

$$f(x) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2) \tag{49}$$

In the first form, $f(x)$ is subjected to the following behavioral and side constraints.

Shear stress $\rightarrow$ $g_1(x) = \tau(x) - \tau_{max} \le 0$ (50)

Bending stress $\rightarrow$ $g_2(x) = \sigma(x) - \sigma_{max} \le 0$ (51)

End deflection $\rightarrow$ $g_3(x) = \delta(x) - \delta_{max} \le 0$ (52)

Buckling load $\rightarrow$ $g_4(x) = P - P_c(x) \le 0$ (53)

Side constraints $\rightarrow$ $g_5(x) = x_1 - x_4 \le 0$ (54)

Side constraints $\rightarrow$ $g_6(x) = 0.125 - x_1 \le 0$ (55)

Side constraints $\rightarrow$ $g_7(x) = 0.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5 \le 0$ (56)

In the second form, $f(x)$ is subjected to the following behavioral and side constraints.

Shear stress $\rightarrow$ $g_1^{'}(x) = \tau_{max} - \tau(x) \ge 0$ (57)

Bending stress $\rightarrow$ $g_2^{'}(x) = \sigma_{max} - \sigma(x) \ge 0$ (58)

End deflection $\rightarrow$ $g_3^{'}(x) = 0.25 - \delta(x) \ge 0$ (59)

Buckling load $\rightarrow$ $g_4^{'}(x) = P_c(x) - P \ge 0$ (60)

Side constraint $\rightarrow$ $g_5^{'}(x) = x_4 - x_1 \ge 0$ (61)

Where

$$\tau(x) = \sqrt{(\tau^{'})^2 + 2\,\tau^{'}\tau^{''}\frac{x_2}{2R} + (\tau^{''})^2} \quad (62)$$

$$\tau^{'} = \frac{P}{\sqrt{2}x_1 x_2} \quad and \quad \tau^{''} = \frac{MR}{J} \quad (63)$$

$$M = P\left(L + \frac{x_2}{2}\right) \quad and \quad R = \sqrt{\frac{x_2^{\,2}}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \quad (64)$$

$$J = 2\left\{\frac{x_1 x_2}{\sqrt{2}}\left[\frac{x_2^{\,2}}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \quad and \quad \delta(x) = \frac{4PL^3}{E x_3^{\,3} x_4} \quad (65)$$

$$\sigma(x) = \frac{6PL}{x_4 x_3^{\,2}} \quad and \quad P_c(x) = \frac{4.013\sqrt{\frac{(E G x_3^2 x_4^6)}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \quad (66)$$

$$P = 6000\ lb, \quad L = 14\ in., \quad E = 30\times10^6\ psi, \quad G = 12\times10^6\ psi$$
$$\tau_{max} = 13{,}600\ psi, \quad \sigma_{max} = 30{,}000\ psi, \quad \delta_{max} = 0.25\ in. \quad (67)$$

Table 1 Optimum solutions of welded beam design

| Design Variables | IHS | SQP | FA | $GA_s$ | EA | PSO |
|---|---|---|---|---|---|---|
| $x_1(h)$ | **0.203907** | 0.20572 | 0.2015 | 0.20880 | 0.1829 | N.A. |
| $x_2(l)$ | **3.499898** | 3.47060 | 3.5620 | 3.42050 | 4.0483 | N.A. |
| $x_3(t)$ | **9.063898** | 9.03682 | 9.0414 | 8.99750 | 9.3666 | N.A. |
| $x_4(b)$ | **0.205594** | 0.20572 | 0.2057 | 0.21000 | 0.2059 | N.A. |
| $f(x)$ | **1.729661** | 1.7248 | 1.73121 | 1.74830 | 1.82455 | 1.92199 |

In the first form, the ranges for the design variables are given as follows

$$\begin{aligned} 0.1 \le x_1 \le 2.0, \quad 0.1 \le x_2 \le 10 \\ 0.1 \le x_3 \le 10, \quad 0.1 \le x_4 \le 2.0 \end{aligned} \tag{68}$$

In the second form, the ranges for the design variables are given as follows

$$\begin{aligned} 0.125 \le x_1 \le 5.0, \quad 0.1 \le x_2 \\ x_3 \le 10, \quad 0.1 \le x_4 \le 5.0 \end{aligned} \tag{69}$$

To apply IHS algorithm to the both form of the welded beam, the four design variables $x_1$, $x_2$, $x_3$ and $x_4$ were assumed to be discrete variables, and their possible values for each form have shown above. The IHS algorithm does not require the initialization of search parameters. The values of control parameters for harmony memory considering rate ($\eta$) and pitch-adjusting rate ($\rho$) are dynamically adjusted by the proposed algorithm during optimization cycles by the use of Eqs. (43)-(45). The values of $\eta_{MAX}$ and $\rho_{MAX}$ are taken as 0.99 and the 0.01 is assigned to $\eta_{MIN}$ and $\rho_{MIN}$. The maximum number of searches is taken as 30000 in the design case.

In the literature, a plenty of different solutions of the welded beam benchmark design problem ranging between 1.73121 and 1.92199 are reported with different numerical techniques according to the different design constraints and design variables. Some of them are as follows: 1.92199 by Parsopoulos and Vrahatis with a unified particle swarm optimization (PSO), 1.82455 by Coello with evolutionary multi objective optimization technique (EA), 1.74830 by Coello with self-adaptive penalty approach for genetic algorithms ($GA_s$), 1.73121 by Gandomi *et al.* with firefly algorithm (FA) and Fesanghary *et al.* with hybridizing harmony search algorithm with sequential quadratic programming (SQP). All the mentioned results are compared against those obtained from IHS algorithm. The optimum solutions and comparison of results for the welded beam design problem are also tabulated with more detail in Table 1. This table also demonstrates that the proposed algorithm is performed very well locating an optimum value for the objective function with 1.729661. Consequently, the IHS technique is recommended for its application to optimization of the two different web expanded beam problems.

### *5.2 5-m span intermediate steel beam*

A simply supported beam shown in Fig. 7 is selected as first structural design example in order to compare the minimum weight of optimally designed steel castellated and cellular beams. The beam has a span of 5 m and is subjected to 5 kN/m dead load including its own weight. Two concentrated live loads with 40 kN weight also act at the beam as shown in the same figure. The

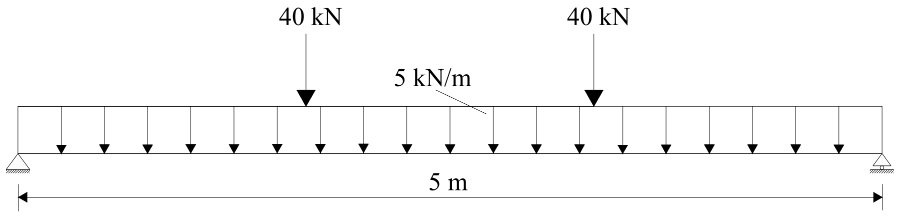


Fig. 7 Loading of 5-m simply supported beam

maximum displacement of the beam under these loads is restricted to 14 mm while other design constraints are implemented from BS5950 as explained in Section 1 and 3.

Considering the stochastic nature of HS technique, castellated and cellular beams with 5m span are separately designed with both improved and standard algorithms. The parameterization of the technique is conducted in line with the recommendations of the former studies (Lee and Geem, Saka, Değertekin, Erdal *et al.*), and thus the following parameter value set is used in solving the problem: a harmony memory size of $\mu$=50, a maximum search number of $N_s$=5000 are kept the same for both improved and standard HS algorithms. A harmony memory considering rate of $\eta$=0.90, and a pitch adjusting rate of $\rho$=0.30. It is important to note that these values of control parameters for $\eta$ and $\rho$ remain unchanged in the standard HS algorithm. Contrary to standard HS method, the values of $\eta_{MAX}$ and $\rho_{MAX}$ parameters in the IHS algorithm are taken as 0.99 and the 0.01 is assigned to $\eta_{MIN}$ and $\rho_{MIN}$. These values are dynamically updated by the proposed algorithm during the optimization process.

The optimum results obtained by improved and standard versions of technique as well as the sectional designations and geometric dimensions for both beams are given in Table 2. It is apparent from the same table that improved HS has produced the lightest beam for steel cellular beams that has the minimum weight of 133.71 kg. The controlling interaction ratios of cellular beam are 0.99 for vierendeel bending, 0.78 for web-post buckling and 0.62 for horizontal shear. The next lightest design is obtained by classical version of HS for again steel cellular beams which is 144.86 kg; 8.34% heavier than the one found by IHS. The third lightest design is attained by IHS algorithm for castellated beam which is 151.59 kg; 13.38% heavier than the overall lightest cellular beam. The controlling interaction ratios of castellated beam are 0.99 for vierendeel bending, 0.49 for web-post buckling and 0.46 for horizontal shear. Finally, classical HS algorithm has accomplished the heaviest design with castellated beam which is 159.82 kg; 19.53% heavier for same 5 m span.

These results demonstrate that steel cellular beam produces less weight than castellated beam in all circumstances and notwithstanding the performances of search techniques for 5-m span. It is

Table 2 Optimum solutions of 5-m simply supported beam

| | **CASTELLATED BEAM** | | | | | **CELLULAR BEAM** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Section Design (UB) | Depth of Hole | Number of Holes | Max. Strength Ratio | Min. Weight (kg) | Section Design (UB) | Diameter of Hole | Number of Holes | Max. Strength Ratio | Min. Weight (kg) |
| **IHS** | 254×146×31 | 218 | 14 | 0.99 | **151.59** | 254×102×28 | 239 | 15 | 0.99 | **133.71** |
| **HS** | 305×102×33 | 202 | 15 | 0.97 | **159.82** | 254×146×31 | 296 | 13 | 0.93 | **144.86** |

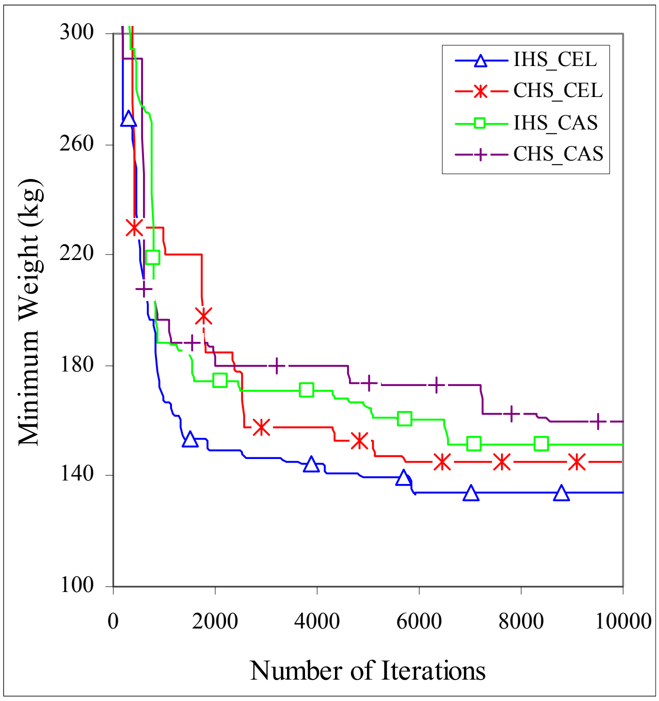


Fig. 8 Design history graph of 5-m simply supported beam

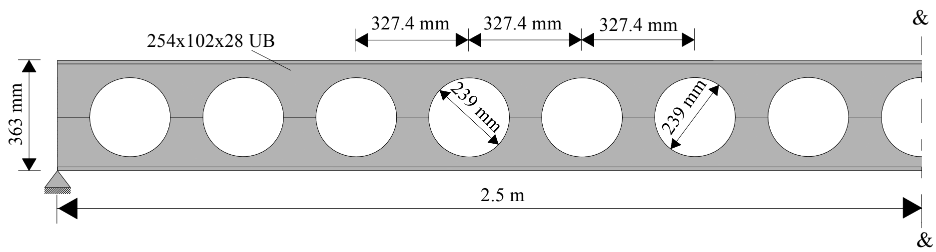


Fig. 9 Optimum profile section of the 5-m cellular beam

also shown that the proposed algorithm improves the performance of HS technique and it renders unnecessary the initial selection of the harmony search parameters. Consequently, the improved version of HS technique is recommended for its application to optimization of 12-m span intermediate steel beam problem presented in the next design example. The design history curves for improved and standard versions of the technique for castellated and cellular beams are shown in Fig. 8. This figure reveals the fact that IHS method has the faster convergence rate than classical HS algorithm for both beams.

Within 5,000 analyses the proposed technique approaches a design in the vicinity of the optimum results. The maximum values of vierendeel bending moment ratio are 0.99 and 0.93 for cellular beams and 0.99 and 0.97 for castellated beams which are almost upper bound for both beams. This clearly reveals the fact that, in both beams, vierendeel bending moment constraints are dominant in the design problem. The IHS design algorithm presented selects 254×102×28 UB

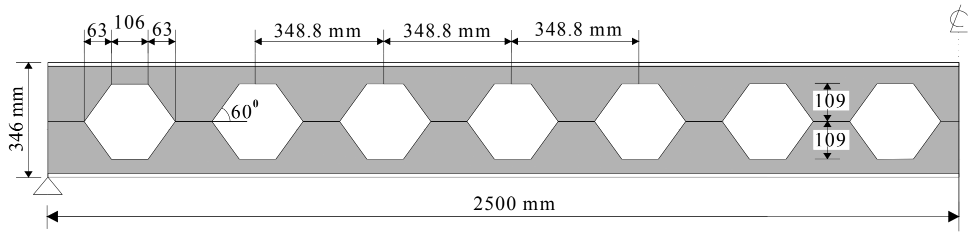


Fig. 10 Optimum profile section of the 5-m castellated beam

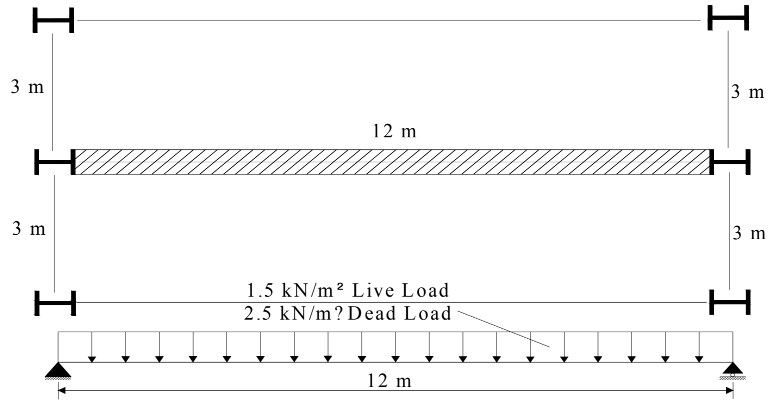


Fig. 11 Loading of 12-m intermediate steel beam

Table 3 Optimum solutions of 12-m span intermediate steel beam

| | Improved Harmony Search Algorithm | | | | | |
|---|---|---|---|---|---|---|
| | Section Design (UB) | Diameter/Depth of Hole | Number of Holes | Value of Angle | Max.Strength Ratio | Minimum Weight (kg) |
| Cellular Beam | 356×127×39 | 366 | 25 | - | 0.99 | **436.7** |
| Castellated Beam (Varying Angle) | 356×127×39 | 359 | 24 | 57 | 0.98 | **457.2** |
| Castellated Beam (Fixed Angle) | 356×171×45 | 341 | 26 | 60 | 0.94 | **528.4** |

section for the cellular root beam and 254×146×31 UB section for the castellated root beam shown in Table 2. The optimum cellular beam should be produced such that it should have 15 circular holes each having 239 mm diameter. The optimum shape of the cellular beam obtained from HS method is demonstrated in Fig. 9. On the other hand, the optimum castellated beam should be produced such that it should have 14 hexagonal holes each having 218 mm depth. The optimum shape of the castellated beam obtained from HS method is demonstrated in Fig. 10.

### *5.3 12-m span intermediate steel beam*

A typical 12-m span intermediate steel beam shown in Fig. 11 is considered as a second structural design example in order to compare the minimum weight of optimally designed steel

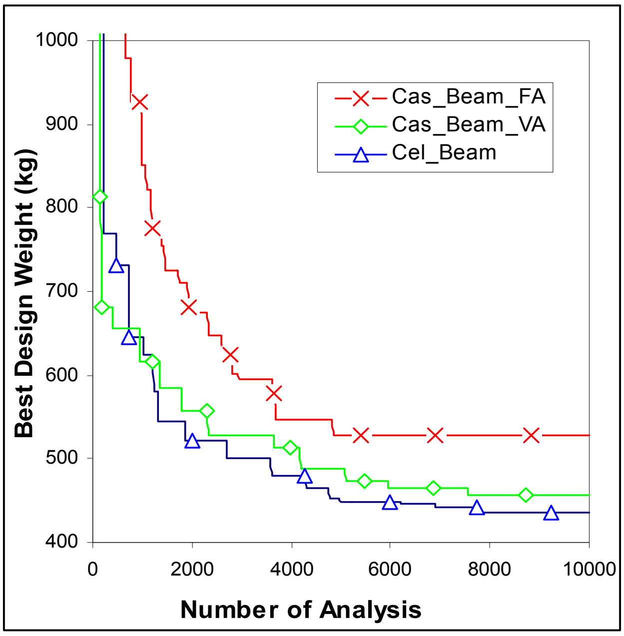


Fig. 12 Design history graph of 12-m intermediate steel beam

castellated and cellular beams. The beam is subjected to the uniform dead load of 2.5 kN/m$^2$ including concrete slab, steel deck, reinforcement and steel beam and a live load of 1.5 kN/m$^2$ in addition its own dead weight. The upper flange of the beam is laterally supported by the floor system that it supports. Beam spacing is 3 m. The maximum displacement of the beam under the live-load is restricted to be less than $L/360$, where $L$ is the length of the beam.

This design example is separately solved as castellated and cellular beams using improved harmony search algorithm. The size of the maximum number of generations is kept the same for both beams. The values of $\eta_{\mathrm{MAX}}$ and $\rho_{\mathrm{MAX}}$ parameters in the IHS algorithm are taken as 0.99 and the 0.01 is assigned to $\eta_{\mathrm{MIN}}$ and $\rho_{\mathrm{MIN}}$. These values are dynamically updated by the proposed algorithm during the optimization process as a feature of the proposed technique. It is apparent from the Table 3 that IHS algorithm produces a least weight for a cellular beam which is equal to 436.7 kg. IHS design algorithm presented selects 356×127×39 UB section for cellular beam. The optimum cellular beam shown in Fig. 12 should be produced such that it should have 25 circular holes each having 366 mm diameter. The controlling interaction ratios of steel cellular beam are 0.99 for bending moment, 0.78 for web-post buckling and 0.62 for horizontal shear.

Changing the angle of hexagonal hole in the optimum design of castellated beams has a considerable effect on the minimum weight and it is more appropriate to consider parameters as an additional design variable if a better design is looked for. Besides the sequence number of a universal beam sections, the depth of hexagonal hole and the total number of hexagonal holes, angle between the edges which are varied from 50° to 70° is added as fourth design variable to demonstrate this effect and design of castellated beam. The optimum castellated beam shown in Fig. 13 is obtained by considering four design variables. It is apparent from Table 3 that the optimum design has the minimum weight of 457.2 kg which selects 356×127×39 UB section for the root beam, total of 24 holes in the beam each having 359 mm depth and 57° angle of each

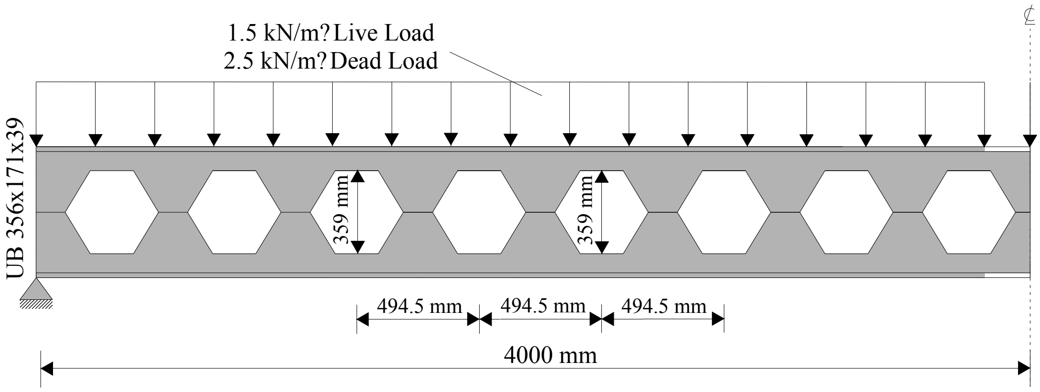


Fig. 14 One-third of optimum cellular beam with 12-m span

hexagonal hole. The controlling interaction ratios of steel cellular beam are 0.98 for bending moment, 0.81 for web-post buckling and 0.65 for horizontal shear. When the optimum design problem is carried out considering only fixed angle ($\theta$=60°), the minimum weight of the castellated beam turns out to be 528.4 kg; 15.57% heavier than the castellated beam with varying angle. IHS algorithm selects 356×127×39 UB section and the optimum castellated beam should be produced such that it should have 26 hexagonal holes each having 341 mm depth. The controlling interaction ratios of steel cellular beam are 0.94 for bending moment, 0.80 for web-post buckling and 0.53 for horizontal shear. These results clearly reveal the fact that, in both beams, bending moment constraints are dominant in this particular problem. The design history curve for cellular beam, castellated beam with fixed angle and castellated beam with varying angle is shown in Fig. 12. It is apparent from the same figure that IHS method performs the nearly same convergence rate and produces same steel sections for steel cellular beam and castellated beam with varying angle. Inasmuch as cellular beams are fabricated by cutting the beam using a double half circular pass and wasting amount of material in this double cutting process, IHS method finds again the better solution for steel cellular beam in this design problem. The optimum shapes of the cellular beam and castellated beam with varying angle obtained from IHS method is demonstrated in Fig. 13 and Fig. 14, respectively.

## 6. Effect of random number sequences in IHS algorithm

Since stochastic methods are based on eventual random decisions in operators, it is required to carry out a series of independent runs for castellated and cellular beam design examples. Random number sequences always produce same number for different runs of the programs provided that the same seed value is used in each run. If the subroutine SEED is not called before the first call to subroutine RANDOM in FORTRAN, RANDOM always begins a seed value of one. However the use of different seed values in each run generates different random numbers. Since the IHS method also employs random number sequences in making decisions, the final result attained naturally is dependent upon the random numbers generated within each search.

To investigate the effect of random number sequences generated during the design procedure to the final result obtained by IHS technique, two design examples for castellated and cellular beam

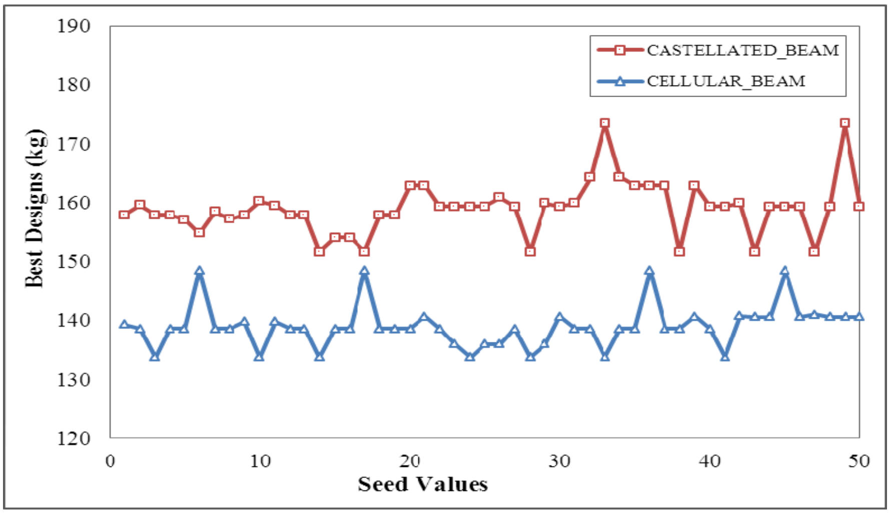


Fig. 15 Effect of Seed Values for 5-m span beam with 500 iterations

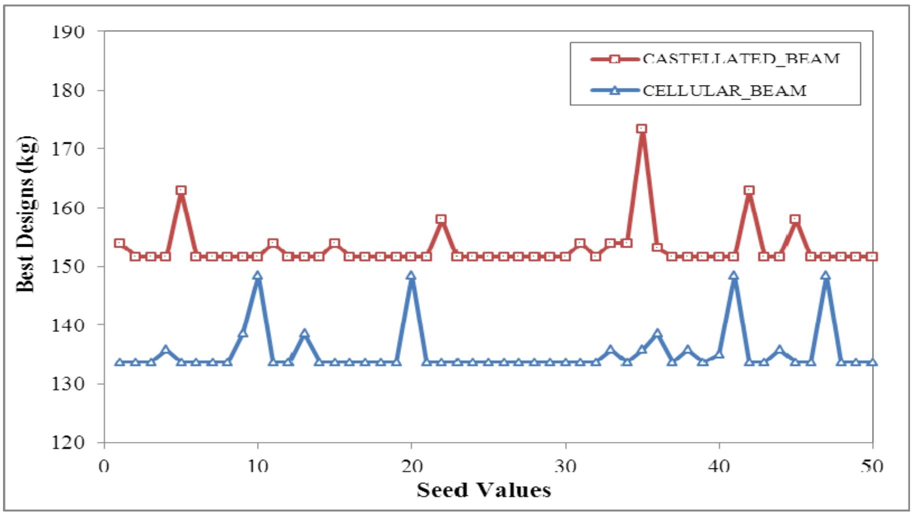


Fig. 16 Effect of Seed Values for 5-m span beam with 5000 iterations

are re-designed several times by using different seed value in each run. Firstly, 5-meter intermediate steel beam is optimized 50 times by running the program with different seed values. In the first run seed value of 1 is given in the beginning of the FORTRAN program, in the second run the seed value of 2 is assumed and in the 50$^{th}$ run the seed value of 50 is adopted. These runs are collected in two groups in order to investigate the effect of the initially selected maximum number of iterations in the IHS technique and variation of the seed value within that group of runs. In the first group of runs the maximum number of iterations is taken as 500 and seed values are changed from 1 to 50 in the each separate runs. In the second group of runs this number is taken as 5000. The minimum weights obtained in each run for the 5-meter intermediate steel beam are shown in Fig. 15 and Fig. 16 depending on the maximum number of iterations adopted in both group of run. It is apparent from the comparison of these two figures that the use of different seed

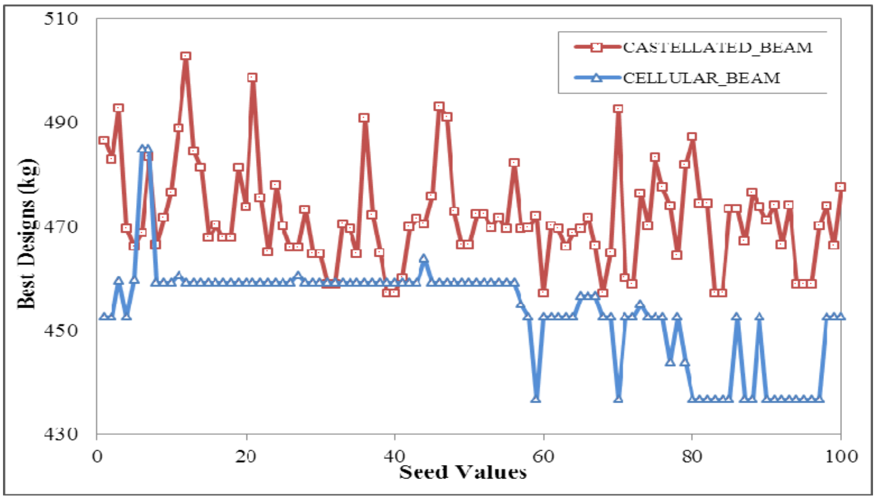


Fig. 17 Effect of Seed Values for 12-m span beam with 1000 iterations

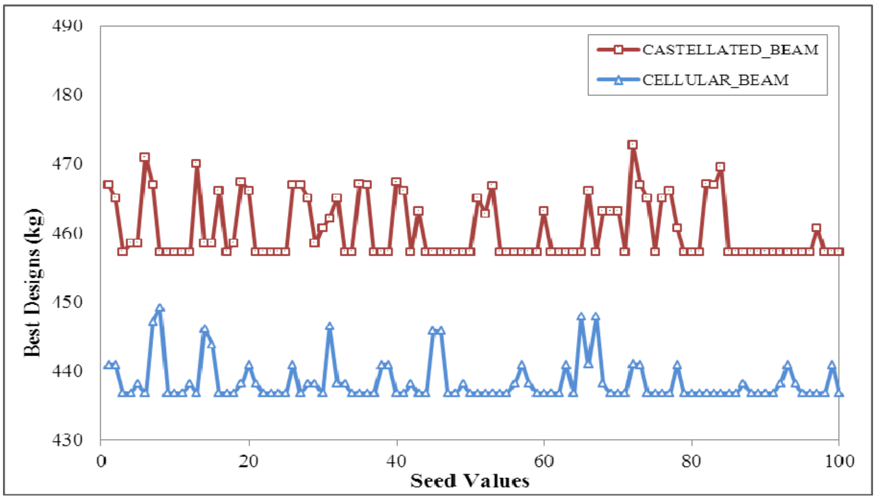


Fig. 18 Effect of Seed Values for 12-m span beam with 10000 iterations

values affects the minimum weight obtained in each run though some of the runs produce the same minimum weight. However this effect becomes less if the maximum number of iterations in each run is selected as a large number.

The same procedure is applied to the last design example of 12-meter span intermediate steel beam system. Firstly, this beam is considered and 100 runs are carried out each of which having a different seed value. In the first group of runs the maximum number of iterations is taken as 1000 and seed values are changed from 1 to 100 in the each separate runs. In the second group of runs this number is taken as 10000. The variation of the minimum weights with the seed values are given in Fig. 17 and Fig. 18, respectively. Once more it is apparent that the seed value adopted in each run has an effect on the final result obtained. It is clear from these figures that the use of different seed values strongly affects the minimum weight obtained. These figures also exhibit step variations between the minimum weight values attained in each run and the seed value selected in that particular run though the same final results are obtained with some of the different seed values. The situation becomes better when the maximum number of iterations in each run is increased to 10000. Consequently it can be concluded that the random number selection affects the final result obtained in IHS method. However use of large number of iterations in each harmony search run improves the performance of the harmony search method.

## 7. Conclusions

The present research is the first study to cover a comparison of the optimally designed castellated and cellular beams, as well as a comparison of the performance of the adaptive and classical versions of harmony search algorithm during the optimization process of mentioned web expanded beams. An improved version of harmony search algorithm is also developed in this paper as a robust method for effectively dealing with a rectangular welded beam problem. Unlike the classical algorithm where the update parameters, harmony memory consideration rate and pitch adjusting rate, of the technique are assigned to constant values throughout the search, the proposed algorithm benefits from updating these control parameters to advantageous values online during the iteration process. The summary of the results obtained by the application of seven search techniques for the welded beam design problem is tabulated in Table 1. This table also demonstrates that the proposed IHS algorithm is performed very well locating an optimum value for the objective function with 1.729664. Noticing the fact that the differences between the minimum values of objective function attained by IHS and CHS in the optimum designs of welded beam is 37.64%. It can be also concluded that IHS approach is the most effective algorithm amongst the seven techniques. Consequently, the IHS technique is recommended for its application to optimization of the two different web expanded beam problems. Then, the efficiency of the improved harmony search algorithm in structural optimization is numerically examined using two examples on size optimum design of castellated and cellular beams. The design history graph generated for the 5-meter simply supported beam problem using improved and classic harmony search algorithms clearly evince a significant performance improvement achieved with the former. Moreover, a comparison of optimally designed steel cellular and castellated beams attained with these techniques in Table 2 confirms that cellular beam produces less weight than castellated beam. Fig. 8 reveals the fact that IHS method has the faster convergence rate than classical HS algorithm for both beam types. In the last design example, Table 3 tabulated for the 12-meter intermediate steel beam demonstrates that castellated beam with varying angle produces 15.57% lighter weight than the castellated beam with fixed angle. More exactly, changing the angle of hexagonal hole in the optimum design of steel castellated beams has a considerable effect on the optimum design and it is more appropriate to consider this parameter as an additional design variable if a better design is looked for. It is apparent from the same table that IHS method finds same sections for steel cellular beam and castellated beam with varying angle but the proposed method finds the less weight for steel cellular beam due to the two cutting process and waste parts between the half circles. The results obtained by the application of improved harmony search algorithms demonstrate that steel cellular beams produce a more cost-effective solution than castellated beams as a result of their flexible geometry and they have several different diameters of circular hole are possible without change in the fabrication process and therefore at no extra cost for the same beam section. Similarly, the effect of random number generation to the final result in the case of IHS algorithm is also investigated by running the optimum design program with different seed values. The minimum weights obtained in each run with different seed value for the both beams.

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## Appendix: Classification of web-expanded beams

The computation of the nominal moment strength $M_p$ of a laterally supported beam necessitates first the classification of the open web-expanded beam. The beam can be plastic, compact, non-compact or slender. In compact sections, local buckling of the compression flange and the web does not occur before the plastic hinge develops in the cross section. On the other hand in compact sections, the local buckling of compression flange or web may occur after the first yield is reacted at the outer fiber of the flanges. Classification I-shaped sections are carried out according to Table 4 that is given in BS5950.

Table 4 Limiting width to thickness ratios

| Type of Element | Plastic | Compact | Semi-compact |
|---|---|---|---|
| Outstand Element of Compression Flange | $\frac{b_f}{2t_f} \le 8.5 \in$ | $\frac{b_f}{2t_f} \le 9.5 \in$ | $\frac{b_f}{2t_f} \le 15 \in$ |
| For web, with neutral axis at mid-depth | $\frac{H_S - 2t_f}{t_w} \le 79 \in$ | $\frac{H_S - 2t_f}{t_w} \le 98 \in$ | $\frac{H_S - 2t_f}{t_w} \le 120 \in$ |

The moment capacity is calculated as $M_p = P_y \times S$ for plastic or compact sections and as $M_p = P_y \times Z$ for semi-compact sections where $\varepsilon = (275/P_y)^{1/2}$ is constant, $\lambda_f = b_f/(2t_f)$ for I-shaped member flanges and the thickness in which $b_f$ and $t_f$ are the width and the thickness of the flange in which $S$ is the plastic modulus and $Z$ is the elastic modulus of section about relevant axis. $P_y$ is the design strength of steel. $\lambda_w = h/t_w$ for beam web, in which $h = H_s - 2t_f$ plus allowance for undersize inside fillet at compression flange for rolled I-shaped sections. $H_s$ is the overall depth of the section and $t_w$ is the web thickness. $h/t_w$ values are readily available in UB-section properties table.