Structural Engineering and Mechanics, *Vol. 53, No. 4 (2015) 833-845*
DOI: http://dx.doi.org/10.12989/sem.2015.53.4.833

A teaching learning based optimization for truss structures with frequency constraints

Tayfun Dede and Vedat Toğan*

Department of Civil Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey

(Received May 26, 2014, Revised December 4, 2014, Accepted January 13, 2015)

Abstract. Natural frequencies of the structural systems should be far away from the excitation frequency in order to avoid or reduce the destructive effects of dynamic loads on structures. To accomplish this goal, a structural optimization on size and shape has been performed considering frequency constraints. Such an optimization problem has highly nonlinear property. Thus, the quality of the solution is not independent of the optimization technique to be applied. This study presents the performance evaluation of the recently proposed meta-heuristic algorithm called Teaching Learning Based Optimization (TLBO) as an optimization engine in the weight optimization of the truss structures under frequency constraints. Some examples regarding the optimization of trusses on shape and size with frequency constraints are solved. Also, the results obtained are tabulated for comparison. The results demonstrated that the performance of the TLBO is satisfactory. Additionally, TLBO is better than other methods in some cases.

Keywords: teaching learning based optimization; frequency constraints; shape and size optimization; trusses

1. Introduction

Especially after making progress in the computer technology and the numerical approaches to be used to solve the problems, the optimal design of structures has advanced rapidly as an active research area for many years. Mathematical programming techniques employed to solve the structural optimization problems exhibited the limited practicability. Therefore, the usage of optimization methods, which are gradient-free and inspire from the natural or the physical phenomena, attract increasingly attention. They have been successfully applied in the optimal design of different type of optimization problems of truss structures under distinct constraints for five or six decades (Rajeev and Krishnamoorthy 1992, Camp *et al.* 2005, Toğan and Daloğlu 2006, 2008, Hasançebi 2008, Lamberti 2008, Li *et al.* 2009, Kaveh and Talatahari 2010, Gandomi *et al.* 2012, Dede 2013). In addition, it is found well application and comparison studies in other engineering fields related the optimization techniques evolutionary-based (Yıldız 2012a, 2013a, b)

A designer can manage the structural design by accomplishing the size and shape optimization of a truss under frequency constraints. So, the natural frequencies of the structural system can be taken far away from the excitation frequency in order to avoid or reduce from the destructive

*Corresponding author, Associate Professor, E-mail: togan@ktu.edu.tr

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

effects of dynamic loads. In addition, a design can be obtained as lightweight as possible (Gandhi 1993, Gomes 2011, Lingyun *et al.* 2005, Zuo *et al.* 2011, Kaveh and Zolghadr 2011). This optimization problem requires the collocation of the nodal coordinates; cross-sectional areas as design variables and the solution of the eigenvalue problem. However, aforementioned requirements cause difficulties when the shape and size optimization of trusses under frequency constraints is performed, i.e., the changes in the values of shape and size variables are of widely different order of magnitude and their representations have fundamentally different physical meaning (Wang *et al.* 2004). Moreover, the frequency constraint being highly non-linear, non-convex and implicit with respect to the design variables makes the gradient calculation difficult or even impossible (Lingyun *et al.* 2005). Also, when optimizing for mass, vibration modes can switch and this causes convergence difficulties (Gandhi 1993, Gomes 2011). Hence, because of the difficulties mentioned above it seems to be crucial to utilize a meta-heuristic optimization technique rather than the gradient-based methods for obtaining the solution of such problems.

Pantelides and Tzan (1997) proposed a modified simulating annealing method. They used the proposed method to solve the optimal design problems of structures subjected to dynamic loading with displacement and stress constraints. Lingyun *et al.* (2005) formed a hybrid algorithm by simplex search and genetic algorithms following a nature based scheme of Niche. In their work, three truss structures were analyzed using the hybrid algorithm called Niche Genetic Hybrid Algorithm and the results were compared with literature results. Torkzadeh *et al.* (2008) and Gholizadeh *et al.* (2008) optimized the weight of structures subjected to multiple natural frequency constraints. For this purpose, they used a Genetic Algorithm (GA) and a Neural Network (NN) together and replaced the structural analysis requiring much computational time in the optimization process by a properly trained neural network with radial basis function (RBF) and a wavelet radial basis function (WRBF) neural network. Zuo *et al.* (2011) presented GA-based structural optimization procedure for the optimization of the truss structures with frequency constraints. They also proposed an adaptive eigenvalue reanalysis method derived on the Kirsch's combined approximation method in order to reduce the computational time. A particle swarm optimization (PSO) algorithm and enhanced charged system search (CSS) algorithm are also employed by Gomes (2011) and by Kaveh and Zolghadr (2011), respectively, as an optimization engine in a truss mass optimization problem on size and shape with frequency constraints.

One of the recently developed meta-heuristic algorithms is Teaching Learning Based Optimization (TLBO), which mimics teaching-learning process in a class between the teacher and the students (learners). To implement the TLBO two key steps known as "Teaching Phase" and "Learning Phase" must be performed, respectively. The "Teaching Phase" produces a random ordered state of points called learners within the search space. Then a point is considered as the teacher, who is highly learned person and shares his or her knowledge with the learners. However, the learning process is represented by interaction between each learner in the "Learning Phase". After a number of sequential Teaching-Learning cycles, the distribution of the randomness within the search space becomes smaller and smaller about to point considering as teacher, which means that knowledge level of the whole class is close to teacher's level and the algorithm converges to a solution. Rao *et al.* (2011) presented and applied first the TLBO for solving the mechanical design optimization problems taken from the literature. Then TLBO algorithm is employed different kind of optimization problems (Hosseinpour *et al.* 2011, Satapathy and Naik 2011, Toğan 2012, 2013, Dede and Ayvaz 2013, Niknam *et al.* 2012, Nayak *et al.* 2012, Yıldız 2012b) to demonstrate the applicability and efficiency of the method.

This study presents the performance evaluation of the Teaching Learning Based Optimization

(TLBO) as an optimization engine in the weight optimization of the truss structures under frequency constraints. Some examples taken from the literature regarding the optimization of trusses on shape and size with frequency constraints are solved and the results are tabulated for comparison.

2. Formulation of the optimization problem

As mentioned before, it is possible to perform the shape and size optimization of a structure with the natural frequency constraints so as to be far away from the excitation frequency. The structural topology is kept the same as prescribing in advance and it is preserved throughout the solution process. The nodal coordinates and element cross-sectional areas of structures, however, are changed within a restricted region continuously. Hence, the optimization problem can be described mathematically as follows

$$\begin{aligned} &\text{find} \quad \mathbf{X} = \{x_1, x_2, ..., x_{ndv-1}, x_{ndv}\} \\ &\text{minimize} \quad f(\mathbf{X}) = \sum_{e=1}^{ne} L_e \rho_e A_e \end{aligned} \tag{1}$$

subject to

$$\begin{aligned} &\omega_j \geq \omega_j^* \ \text{ for some natural frequencies } j \\ &\omega_k \leq \omega_k^* \ \text{ for some natural frequencies } k \\ &x_{\min,i} \leq x_i \leq x_{\max,i} \end{aligned} \tag{2}$$

where $\mathbf{X}$ is a vector for design variables consisting of nodal coordinates and sectional areas; *ndv* represents total number of design variables; $f(\mathbf{X})$ is the objective function, which is taken as the weight of the structure; L_e, ρ_e ,and A_e are the length, material density and cross-sectional area of the *e*th element while *ne* demonstrates the total number of elements in the structure; ω_j is the *j*th natural frequency and ω_j^* is its lower bound; ω_k is the *k*th natural frequency and ω_k^* is the corresponding specified upper limit; x_i is the *i*th design variable that is either nodal coordinate or sectional area, while $x_{min,i}$ and $x_{max,i}$ are its lower bound and upper bound, respectively. In engineering applications, to keep the structure symmetric and/or limit the numbers of variables a number of shape or sizing variables are linked. So, *ndv* demonstrates the number of the independent design variables in Eq. (1).

The constraints treated as $g_j(\mathrm{X})=1-\omega_j/\omega_j^*\leq 0$ and $g_k(\mathrm{X})=\omega_k/\omega_k^*-1\leq 0$ in this paper are handled by using the concept of penalty functions so the objective function is modified as

$$f(\mathbf{X}) = \begin{cases} f(\mathbf{X}) & \text{if } \mathbf{X} \in \mathrm{FS} \\ f(\mathbf{X})(1+PF) & \text{otherwise} \end{cases} \tag{3}$$

where *PF* is the penalty function, *FS* denotes the feasible search space. The form of the penalty function employed in this paper is indicated as

$$PF = K \sum_{i=1}^{nc} (\max(0, g_i(\mathbf{X}))) \tag{4}$$

in which K is adjusting coefficient taken as 10 for truss structures (Rajeev and Krishnamoorthy 1992) and nc is the number of frequency constraints. Eq. (4) causes to obtain greater objective function value for the solution that violates the constraints.

In this paper an evolutionary algorithm developed in recent years is employed as an optimization method. The main concepts of the algorithm are briefly explained in next section.

3. Teaching Learning Based Optimization (TLBO)

TLBO mimics teaching-learning ability of teacher and learners in a class, which is known as the teaching learning process. The method assumes the teacher and the learners as being the two vital components of the algorithm and divides the teaching learning process into two basic modes of the learning: a) Teacher Phase and b) Learner Phase. During the first phase, teacher tries to impart her/his knowledge directly to the learners. However, during the second phase, it is aimed that a learner may increase interacting with other learners. With analogously other population based methods, in the TLBO process, a group of learners is assumed as population. Different subjects, which the learners are taught by teacher, are taken as different design variables. The 'fitness' value of the optimization problem is represented throughout the learners' result and the best solution within the entire population is adopted as the teacher. "Teacher phase" and "Learning phase" being the two main parts of TLBO as mentioned before are explained below in detail.

3.1 Teaching phase

The first part of the TLBO is to teach learners through the teacher. For this purpose, an initial population is randomly generated with pre-defined size ($np=pop\_size$) for that population. An individual ($\mathbf{X}_i$) within the population represents a single possible solution to a particular optimization problem. $\mathbf{X}_i$ is a real-valued vector with ndv elements, where ndv is the dimension of the problem, which represents the number of the subjects to be taught within the TLBO context. The best individual ($\mathbf{X}_{best}$) is assigned as a teacher ($\mathbf{X}_{teacher}$) who is responsible for teaching role. Then, it is tried by means of Eq.(5) to enhance other individuals ($\mathbf{X}_i$,i=1,…,np) in the population by moving their positions towards the position of the $\mathbf{X}_{teacher}$ by taking into account the current mean value, which represents the qualities of all students from the current generation, of the individuals ($\mathbf{X}_{mean}$).

$$\mathbf{X}_{new} = \mathbf{X}_i + r\,(\mathbf{X}_{teacher} - T_F\,\mathbf{X}_{mean}) \tag{5}$$

Eq. (5) demonstrates the amount of the influences of the learner depending on the difference between the teacher's knowledge and the qualities of all students. In here, $\mathbf{X}_{new}$ and $\mathbf{X}_i$ are the modified and existing solution of i, r is a random number varying between 0 and 1, T_F is a teaching factor, which is decided randomly with equal probability as T_F = round[1 + rand (0,1) (2-1)]. $\mathbf{X}_i$ is only replaced if his/her new solution $\mathbf{X}_{new}$ is better than $\mathbf{X}_i$.

3.2 Learning phase

Unlike teacher phase which aims to convey the knowledge from the teacher to the learner, the main idea behind of Learning phase is to give opportunity to the learners to increase their knowledge through interaction between themselves. A learner might be in interaction randomly

Set the algorithm parameters*: number of design variables ndv, population size np, maximum number of cycles* (C_{max})*, lower and upper bound for design variables* $\boldsymbol{x}_{min}$ *and* $\boldsymbol{x}_{max}$

Create initial random population (class) and evaluate the solution (student or learner)
For each learner i in the class
    Calculate the $f_i(\boldsymbol{X})$
End

Iterates with the TLBO to find a student with design variables that leads to a minimum objective function
Loop until criteria of maximum iteration ($C < C_{max}$) *is met*
    Set the best objective function of the current class as the teacher $\boldsymbol{X}_{teacher} = \min(f(\boldsymbol{X}))$
    Calculate the mean of each group of learners $\boldsymbol{X}_{mean}$
    For each design variable i in the learners
        $x_T = 0$
        For each learner j in the class
            $x_T = x_T + x_{j,i}$
        End
        $\bar{x}_i = x_T / np$
    End
    $\boldsymbol{X}_{mean} = [\ \bar{x}_1\ \ \bar{x}_2\ \ \ldots\ \ \bar{x}_{ndv}\ ]$
    Update the learner and her/him objective function
    For each learner i in the class
        $r = rand\ [0,1]$
        $T_F = round\ (1 + rand\ [0,1]\ (2-1))$
        $\boldsymbol{X}_{new} = \boldsymbol{X}_i + r\ (\boldsymbol{X}_{teacher} - T_F\ \boldsymbol{X}_{mean})$
        If $f(\boldsymbol{X}_{new}) < f(\boldsymbol{X}_i)$ *then* $\boldsymbol{X}_i = \boldsymbol{X}_{new}$
    End
    Interact the learners with each other and update their objective function
    For each learner i in the class
        $r = rand\ [0,1]$
        Select $\boldsymbol{X}_j$ *randomly for* $\boldsymbol{X}_i$ *,* $i \neq j$
        If $f(\boldsymbol{X}_j) < f(\boldsymbol{X}_i)$ *then* $\boldsymbol{X}_{new} = \boldsymbol{X}_i + r\ (\boldsymbol{X}_j - \boldsymbol{X}_i)$ *else* $\boldsymbol{X}_{new} = \boldsymbol{X}_i + r\ (\boldsymbol{X}_i - \boldsymbol{X}_j)$
        If $f(\boldsymbol{X}_{new}) < f(\boldsymbol{X}_i)$ *then* $\boldsymbol{X}_i = \boldsymbol{X}_{new}$
    End
End

Fig. 1 Pseudo-code for the simple teaching-learning based optimization

with other learners present in the class. This interaction can be through formal communication, group discussion, presentation, etc. TLBO process allows a learner $\mathbf{X}_i$ to interact randomly with another learner $\mathbf{X}_j$, where i is unequal to j, in order to learn new things from her or him. However, as is well known, $\mathbf{X}_i$ can improve knowledge and move own level to the $\mathbf{X}_j$ (Eq. (6)) if X_j has more knowledge than $\mathbf{X}_i$. Otherwise, $\mathbf{X}_i$ modifies itself by means of Eq. (7).

$$\mathbf{X}_{new} = \mathbf{X}_i + r(\mathbf{X}_j - \mathbf{X}_i) \tag{6}$$

$$\mathbf{X}_{new} = \mathbf{X}_i + r(\mathbf{X}_i - \mathbf{X}_j) \tag{7}$$

In the case that the newly produced learner $\mathbf{X}_{new}$ generates better results by following Eq. (6) or Eq. (7), he/she is maintained. Otherwise, preserve $\mathbf{X}_i$. Both phases are repeated until reaching the maximum number of generations. A pseudo-code for the implemented teaching learning based optimization algorithm is illustrated in the Fig. 1.

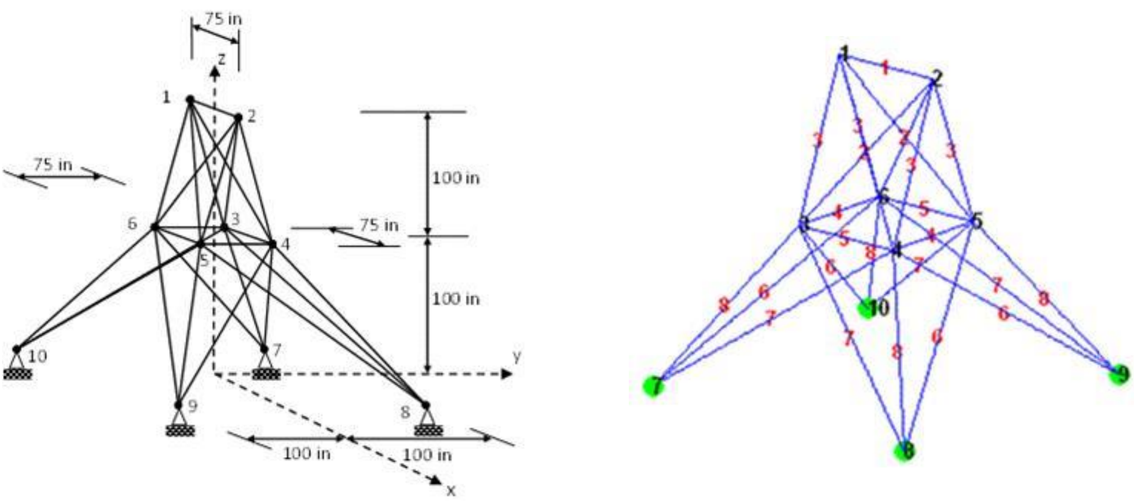
4. Numerical examples

Three typical truss optimization examples are considered to demonstrate the feasibility and validity of TLBO for solving shape and size optimization of trusses with multiple frequency constraints. The TLBO is implemented by MATLAB while the finite element analysis software based on the matrix-displacement concept and also coded in MATLAB is integrated to perform modal analysis to obtain the corresponding natural frequencies of structure. For every example, the optimization computation process is repeated 10 times. At every turn, the population used in the solution process is generated independently and randomly. Moreover, in the figures depicted for the examples, the number on the lines show the group number of elements and the others illustrate the nodal points.

4.1 25 bar space truss

Configuration of 25 bar space truss structure is given in Fig. 2. This structure is optimized by Tong and Liu (2001) by taking into account the stress, displacement and dynamic constraint for size optimization. The displacements at nodes 1 and 2 in the directions of x and y are limited to ±0.35 in., stress is constrained to ±40 ksi and the allowable minimum fundamental frequency of the structure is limited to 55.0 Hz.

Available discrete design variables are changes from 0.1 in^2 to 3.5 in^2 with the 0.1 in^2 increments. The modulus of elasticity and density of the material are assumed to be 6.89×10^{10}



(a) Geometrical scheme (b) member grouping scheme

Fig. 2 25 bar space truss structure

Table 1 Loading case for 25-bar truss

Nodes	F_x (lbf)	F_y (lbf)	F_z (lbf)
1	1000	−10000	−10000
2	0	−10000	−10000
3	500	0	0
6	600	0	0

Table 2 Optimal design comparison for the 25-bar space truss

Design Variables (area in^2)	Optimal cross-sectional area (in^2)	
	Tong and Liu (2001)	Present Study (TLBO)
A_1	0.1	0.1
A_2	0.5	0.2
A_3	3.4	2.3
A_4	0.1	0.1
A_5	0.1	1.0
A_6	0.8	0.6
A_7	1.5	0.3
A_8	3.4	2.3
Weight (kg)	237.52	217.7549
Frequency > 55Hz	57.02 Hz	59.6406 Hz

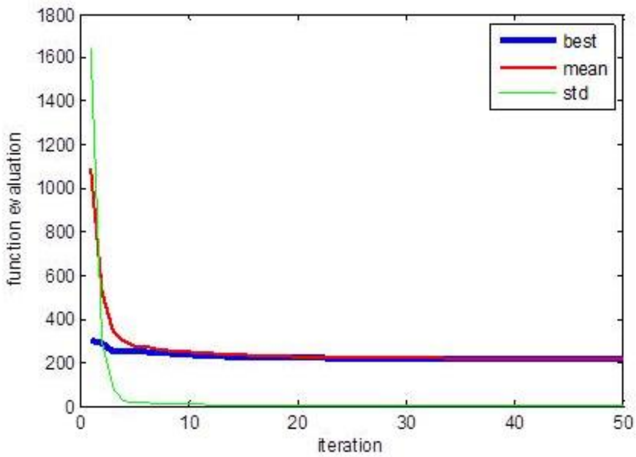


Fig. 3 Best solution, mean solution and standard deviaiton of 25 bar space truss structure

N/m^2 and 2770 kg/m^3, respectively. The loading of this structure is given in the Table 1. The geometry properties of this structure are the same as given in Tong and Liu (2001).

At the end of the optimization process, the weight of this structure is obtained as 217.7579 kg without any violations and the fundamental frequency is 59.6406 Hz. The comparison of the results with the other results presented in the technical literature is given in Table 2.

Fig. 3 shows the function evaluations of this structure as best solution (weight of structure), mean solution and the standard deviation. As seen from this figure, after almost five iterations, the mean solution is parallel to the best solution and the standard deviation is close the zero.

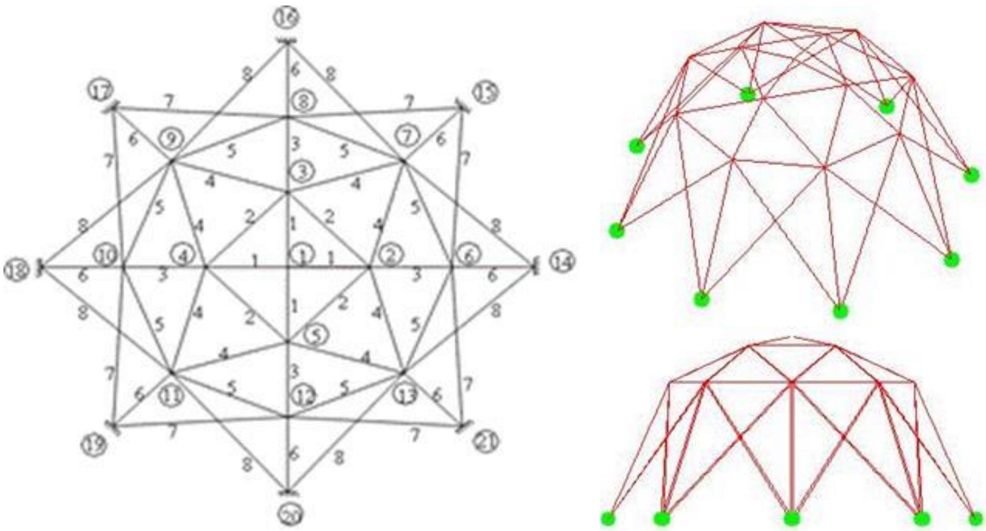


Fig. 4 52 bar dome structure

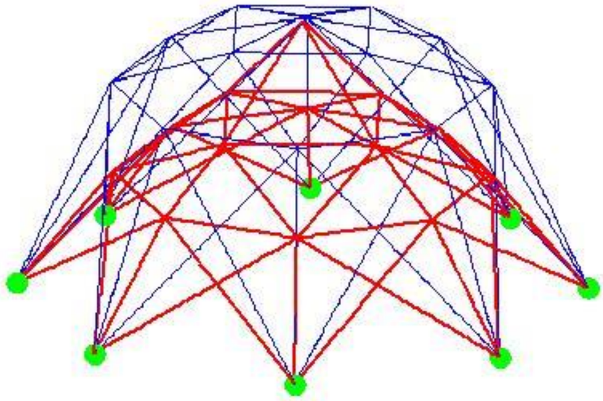


Fig. 5 Optimized shape (thick lines) with intial shape (thin lines) of 52 dome structure

4.2 52 bar dome structure

52 bar dome structure was previously optimized by Kaveh and Zolghadr (2012), Kaveh and Mahdavi (2013), Lingyun *et al.* (2005). Configuration of 52 bar dome structure is given in Fig. 4. In Fig. 4, coordinates of the nodes 1, 2, 6, and 14 are [0 0 6 m], [2 m 0 5.7 m], [4 m 0 4.5 m] and [6 m 0 0] in *x*, *y*, and *z* directions, respectively.

Non-structural masses of m=50 kg are added to the free nodes. Continues design variables are changes from 0.0001 m^2 to 0.001 m^2. The modulus of elasticity is 2.1×10^{11} N/m^2 and the material density is 7800 kg/m^3. The symmetry of the structure is protected in the shape optimization process. In shape optimization, ±2 m is allowed for each movable node. For frequency constraints, first frequency is assumed little or equal to 15.916 Hz and second frequency is assumed greater or equal to 28.649 Hz. Thus, the total constraints consist of two natural frequency, five shape variables, and the eight size variables. Element of the structure is arranged in eight groups.

At the end of the optimization process, the weight of this structure is obtained as 193.2721 kg without any violations and the first and second fundamental frequency are 11.5337 and 28.6482 Hz, respectively. Fig. 5 demonstrates the optimized shape of structure (thick lines) with the initial shape of structure (thin lines).

The comparison of the results with the other results obtained in the distinct studies is given in Table 3. Fig. 6 shows the histories of the function evaluation of the best solution (weight of structure), the mean solution and the standard deviation. As seen from the figure, the mean solution is parallel to the best solution and the standard deviation is close the zero after almost 30 iterations.

4.3 120 bar dome structure

Configuration of 120 bar dome structure is given in Fig. 7. This structure is studied by Kaveh and Zolghadr (2012) and Kaveh and Mahdavi (2013) for size optimization only under dynamic constraints. Coordinates of the nodes 1, 2, 14, and 38 are [0 0 7 m], [6.94 m 0 5.85 m], [12.04 m 0 3 m] and [15.89 m 0 0] in *x*, *y*, and *z* directions, respectivley.

Non-structural masses of *m*=3000 kg, 500 kg and 100kg are added to the free nodes, 1, 2-13, and 14-37, respectively. Continues design variables are changes from 0.0001 m^2 to 0.01293 m^2. The modulus of elasticity is 2.1×10^{11} N/m^2 and the material density is 7971.810 kg/m^3. For frequency constraints, first frequency is assumed greater or equal to 9 Hz and second frequency is assumed little or equal to 11 Hz.

Table 3 Optimal design comparison for the 52-bar dome structure

Optimal Results		Initial	Kaveh and Zolghadr (2011)	Kaveh and Mahdavi (2013)	Lingyun *et al.* (2005)	Present Study (TLBO)
Coordinates (m)	Z_1	6.000	5.331	5.4785	5.8851	5.8939
	X_2	2.000	2.134	2.4517	1.7623	2.2647
	Z_2	5.700	3.719	3.7027	4.4091	3.7086
	X_6	4.000	3.935	4.1190	3.4406	3.9608
	Z_6	4.500	2.500	2.5000	3.1874	2.5000
cross-sectional area (cm^2)	A_1	2.0	1.0000	1.0000	1.0000	1.0000
	A_2	2.0	1.3056	1.3620	2.1417	1.1164
	A_3	2.0	1.4230	1.2585	1.4858	1.1932
	A_4	2.0	1.3851	1.3809	1.4018	1.5255
	A_5	2.0	1.4226	1.3551	1.9110	1.3985
	A_6	2.0	1.0000	1.0000	1.0109	1.0000
	A_7	2.0	1.5562	1.3485	1.4693	1.5662
	A_8	2.0	1.4485	1.5730	2.1411	1.4002
Frequency (Hz)	1	22.69	12.987	15.9154	12.81	11.5337
	2	25.17	28.648	28.8070	28.65	28.6482
	3	25.17	28.679	28.8070	28.65	28.6494
	4	31.52	28.713	28.8070	29.54	28.6494
	5	33.80	30.262	30.0307	30.24	28.7237
Weight (kg)		338.69	197.309	193.3183	236.046	193.2721

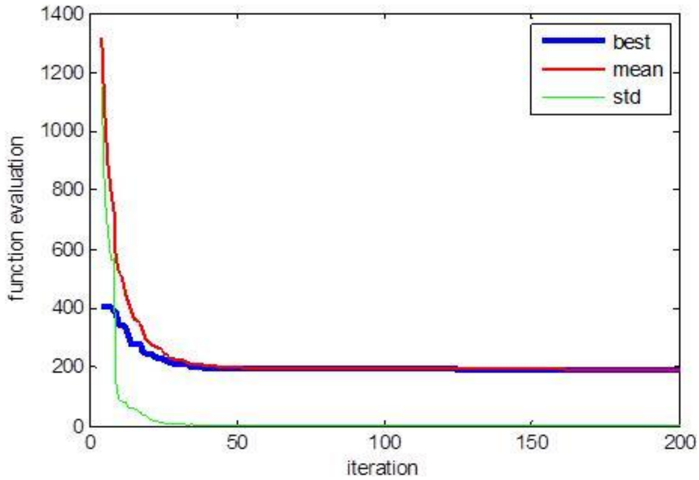


Fig. 6 Best solution, mean solution and standard deviaiton of 52 bar dome structure

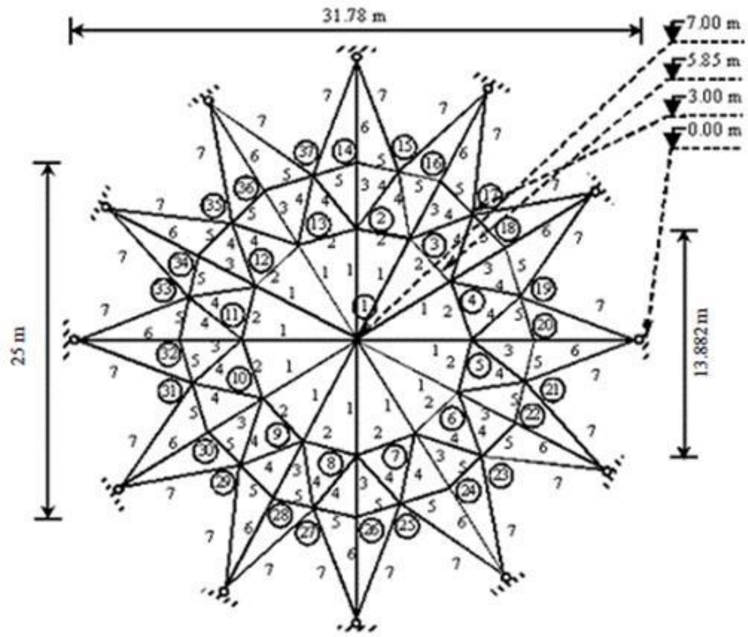


Fig. 7 120 bar dome structure

At the end of the optimization process, the weight of this structure is obtained as 6602.3421 kg without any violations and the first and second fundamental frequency are 9.000 and 9.001 Hz, respectively. The comparison of the results with the other results produced by different researchers is given in Table 4. Fig. 8 presents the function evaluations of this structure as best solution (weight of structure), mean solution and the standard deviation. As seen from the figure, after almost 40 iterations, the mean solution becomes parallel to the best solution and the standard deviation gets close the zero.

Table 4 Optimal design comparison for the 120-bar dome structure

Optimal Results		Kaveh and Zolghadr (2011)	Kaveh and Mahdavi (2013)	Present Study (TLBO)
cross-sectional area (cm^2)	A_1	17.478	20.0325	22.4967
	A_2	49.076	38.2935	37.3414
	A_3	12.365	11.7403	8.9873
	A_4	21.979	21.9118	12.7422
	A_5	11.190	10.200	8.7352
	A_6	12.590	10.9328	7.8429
	A_7	13.585	14.6337	7.5146
Frequency (Hz)	1	9.000	9.000	9.000
	2	11.007	11.000	9.001
	3	11.018	11.002	9.001
	4	11.026	11.0210	9.4961
	5	11.048	11.0863	9.5517
Weight (kg)		9046.34	8769.50	6602.3421

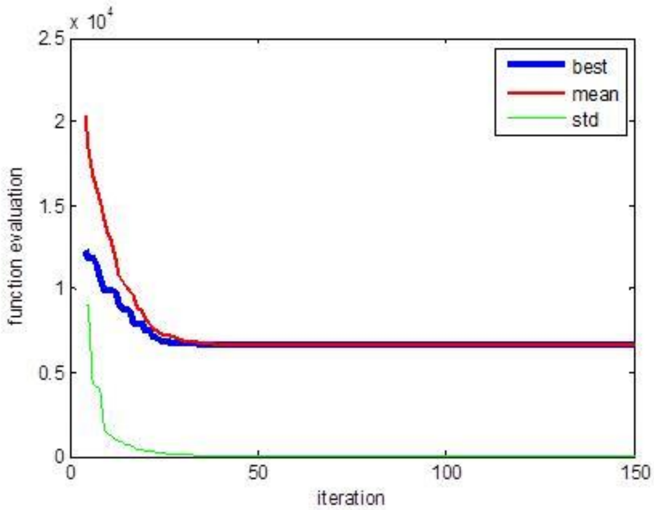


Fig. 8 Best solution, mean solution and standard deviaiton of 120 bar dome structure

5. Conclusions

Considering the natural frequencies as additional constraints in a structural optimization process makes the problem even more complex than classical optimization problems under displacement and stress constraints only. For this reason, to solve an optimization problem with the natural frequency constraints the optimization algorithm used in the solution process should be robust. The TLBO has shown better performance when it is compared with the other optimization algorithms studied in the previous researches. Therefore, a teaching-learning-based optimization algorithm is presented in this study for size and shape optimization of space truss and dome structure including the natural frequency constraints. The results obtained in the study are compared with the ones obtained in other studies by other meta-heuristic methods. And finally, it can be concluded that TLBO can be preferred as an effective optimization tool for optimum design of structures with frequency constraints.

References

Camp, C.V., Bichon, B.J. and Stovall, S.P. (2005), "Design of steel frames using ant colony optimization", *J. Struct. Eng.*, ASCE, **131**(3), 369-379.

Dede, T. (2013), "Optimum design of grillage structures to LRFD-AISC with teaching-learning based optimization", *Struct. Multidis. Opt*., **48**(5), 955-964..

Dede, T. and Ayvaz, Y. (2013), "Structural optimization with teaching-learning based optimization algorithm", *Struct. Eng. Mech.*, **47**(4), 495-511.

Gandomi, A.H., Talatahari, S., Yang, X.S. and Deb, S. (2012), "Design optimization of truss structures using cuckoo search algorithm", *Struct. Des. Tall Spec. Build.*, **22**(17), 1330-1349.

Gholizadeh, S., Salajegheh, E. and Torkzadeh, P. (2008), "Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network", *J. Sound. Vib.*, **312**, 316-331.

Gomes, H.M (2011), "Truss optimization with dynamic constraints using a particle swarm algorithm", *Expert. Syst. Appl.*, **38**, 957-968.

Grandhi, R.V. (1993), "Structural optimization with frequency constraints - A review", *AIAA J.*, **31**, 2296-2303.

Hasançebi, O. (2008), "Adaptive evolution strategies in structural optimization: enhancing their computational performance with applications to large-scale structures", *Comput. Struct.*, **87**, 119-132.

Hosseinpour, H., Niknam, T. and Taheri, S.I. (2011), "A modified TLBO algorithm for placement of AVRs considering DGs", *Proceedings of 26th International Power System Conference*, Tehran, Iran, October-November.

Kaveh, A. and Mahdavi, V.R. (2013), "Optimal design of structures with multiple natural frequency constraints using a hybridized BB-BC/Quasi-Newton algorithm", *Asian J. Civil Eng., Build. Hous.*, **57**, 27-38.

Kaveh, A. and Talatahari, S. (2010), "Optimum design of skeletal structures using imperialist competitive algorithm", *Comput. Struct.*, **88**, 220-1229.

Kaveh, A. and Zolghadr, A. (2011), "Shape and size optimization of truss structures with frequency constraints using enhanced charged system search algorithm", *Asian J. Civil Eng., Build. Hous.*, **12**, 487-509.

Lamberti, L. (2008), "An efficient simulated annealing algorithm for design optimization of truss structures", *Comput. Struct.*, **86**, 1936-1953.

Li, L.J., Huang, Z.B. and Liu, F. (2009), "A heuristic particle swarm optimization method for truss structures with discrete variables", *Comput. Struct.*, **87**, 435-443.

Lingyun, W., Mei, Z., Guangming, W. and Guang, M. (2005), "Truss optimization on shape and sizing with frequency constraints based on genetic algorithm", *J. Comput. Mech.*, **25**, 361-368.

Nayak, M.R., Nayak, C.K. and Rout, P.K. (2012), "Application of multi-objective teaching learning based optimization algorithm to optimal power flow problem", *Procedia Technol.*, **6**, 255-264.

Niknam, T., Azizipanah-Abarghooee, R. and Aghaei, J. (2012), "A new modified teaching-learning algorithm for reserve constrained dynamic economic dispatch", *Power Syst., IEEE Trans.*, **99**, 1-15.

Pantelides, C.P. and Tzan, S.R. (1997), "Optimal design of dynamically constrained structures", *Comput. Struct.*, **62**, 141-149.

Rajeev, S. and Krishnamoorthy, C.S. (1992), "Discrete optimization of structures using genetic algorithms", *J. Struct. Eng.*, ASCE, **118**, 1233-1250.

Rao, R.V., Savsani, V.J. and Vakharia, D.P. (2011), "Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems", *Comput. Aid. Des.*, **43**, 303-315.

Satapathy, S.C. and Naik, A. (2011), "Data clustering based on teaching-learning-based optimization. Swarm, Evolutionary, and Memetic", *Comput. Lec. Note. Comput. Sci.*, Springer-Verlag, Berlin.
Toğan, V. (2012), "Design of planar steel frames using teaching-learning based optimization", *Eng. Struct.*, **34**, 225-232.
Toğan, V. and Daloğlu, A. (2006), "Optimization of 3d trusses with adaptive approach in genetic algorithms", *Eng. Struct.*, **28**, 1019-1027.
Toğan, V. and Daloğlu, A. (2008), "An improved genetic algorithm with initial population strategy and self-adaptive member grouping", *Comput. Struct.*, **86**, 1204-1218.
Toğan, V. (2013), "Design of pin jointed structures using teaching-learning based optimization", *Struct. Eng. Mech.*, **47**(2), 209-225.
Tong, W.H. and Liu, G.R. (2001), "An optimization procedure for truss structures with discrete design variables and dynamic constraints", *Comput. Struct.*, **79**, 155-162.
Torkzadeh, P., Salajegheh, J. and Salajegheh, E. (2008), "Efficient methods for structural optimization with frequency constraints using higher order approximations", *Int. J. Struct. Stab. Dyn.*, **8**, 439-450.
Wang, D., Zhang, W.H. and Jiang, J.S. (2004), "Truss optimization on shape and sizing with frequency constraints", *AIAA J.*, **42(**3), 622-630.
Yıldız, A.R. (2012a), "A comparative study of population-based optimization algorithms for turning operations", *Inform. Sci.*, **210**, 81-88.
Yıldız, A.R. (2012b), "Optimization of multi-pass turning operations using hybrid teaching learning-based approach", *Int. J. Adv. Manu. Tech.*, **66**(9-12), 1319-1326.
Yıldız, A.R. (2013a), "Comparison of evolutionary-based optimization algorithms for structural design optimization", *Eng. App. Artif. Int.*, **26**(1), 327-333.
Yıldız, A.R. (2013b), "A new hybrid bee colony optimization approach for robust optimal design and manufacturing", *Appl. Soft. Comput.*, **13**(5), 2906-2912.
Zuo, W.W., Xu, T., Zhang, H. and Xu, T. (2011), "Fast structural optimization with frequency constraints by genetic algorithm using adaptive eigenvalue reanalysis methods", *Struct. Multidis. Optim.*, **43**, 799-810.

CC