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Optimum design of steel framed structures including determination of the best position of columns

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Abstract. In the present study, an efficient method for the optimum design of three-dimensional (3D) steel framed structures is proposed. In this method, in addition to choosing the best position of columns based on architectural requirements, the optimum cross-sectional dimensions of elements are determined. The preliminary design variables are considered as the number of columns in structural plan, which are determined by a direct optimization method suitable for discrete variables, without requiring the evaluation of derivatives. After forming the geometry of structure, the main variables of the cross-sectional dimensions are evaluated, which satisfy the design constraints and also achieve the least-weight of the structure. To reduce the number of finite element analyses and the overall computational time, a new third order approximate function is introduced which employs only the diagonal elements of the higher order derivatives matrices. This function produces a high quality approximation and also, a robust optimization process. The main feature of the proposed technique is that the higher order derivatives are established by the first order exact derivatives. Several examples are solved and efficiency of the new approximation method and also, the proposed method for the best position of columns in 3D steel framed structures is discussed.

Keywords : optimum design; approximation concepts; higher order approximations; steel structures.

1. Introduction

Three-dimensional steel framed structures are one of the most common types of structures. In such structures, choosing the position of columns is the first stage of structural design. This can affect the overall weight or cost, but because of need for many numbers of trial and error, no engineers pay particular attention to such choice. In fact, the position of columns in structural plan is the primary design consideration, before starting the analysis and structural design. If we use a modular system with equal distances between columns, the number of columns in x- and y-directions of structural plan (N_x, N_y) is considered as the preliminary design variables. Therefore, determination of the best position of columns is an exercise in optimum geometry of the structure. These preliminary variables are discrete and then, optimization methods with discrete variables should be employed.

There are several methods for the optimum design of structures with discrete variables such as the duality method, penalty functions, branch and bound method, simulated annealing, genetic algorithms, and so on. Each of the methods has some limitations and difficulties (Arora *et al.* 1994). In the first three methods, first the continuous solution should be obtained and then the discrete solution is achieved.

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Also, the derivatives of functions under considerations with respect to the design variables are necessary. From the above methods, the simulated annealing and genetic algorithms can be used for the optimum shape design of structures. Another method which may also be used for both continuous and discrete optimization is the Fibonacci method (Edwin and Stanislaw 2001). This is a direct search method without requiring the evaluation of derivatives.

On the other hand, after choosing the number of columns, the structural geometry is formed and the optimum cross-sectional dimensions of the elements are obtained. In addition to satisfying the design constraints, such as side constraints, stresses and displacements, slenderness and buckling constraints, achieving the least-weight of structure is also an objective which can be mathematically expressed as

$$\begin{aligned} &\text{Minimize} && W(X) \\ &\text{Subject to:} && g_j(X) \leq 0 \qquad j = 1,2,\ldots,m \\ & && x_i^l \leq x_i \leq x_i^u \qquad j = 1,2,\ldots,n \end{aligned} \tag{1}$$

where W is weight of the structure and X represents the design variables vector. The notation m is the number of constraints $g_j(X)$. Also, n defines the number of design variables and x_i, x_j^l, x_i^u are the variables and their lower and upper bounds, respectively.

The constraints are, in general, implicit forms of the design variables and cannot be expressed directly in terms of the design variables. Thus, a numerical optimization technique should be employed to solve this optimization problem. On the other hand, objective functions, constraints and their derivatives should be evaluated by numerical methods for achieving the optimum solution. These functions should be evaluated many times and each evaluation needs a structural analysis. Therefore, hundreds of structural analyses are required, thereby making the process very inefficient and lengthy. In order to solve this problem, approximation methods need to be used for the structural analysis with only a few finite element (FE) analyses. In fact, the key for increasing the process efficiency is introducing a high quality approximation. A great number of studies have been carried out to enhance the quality of approximations and some of the higher order approximations are referred to in this study.

A two-point approximation method was employed by implementing three terms of Taylor series for frequency constraints (Salajegheh 2000). The improved two-point approximation method was also outlined for design optimization (Wang *et al.* 1995). Later the convex approximation was proposed, in which the order of approximation for each constraint was automatically adjusted (Chung and Chiou 2001). This method resulted in a better convergence of the optimization process. Also, several modifications of the convex approximations were described and employed for the optimum design of composite structures (Bruyneel and Fleury 2002). Another two-point approximation method was proposed using linear and reciprocal variables and the first order derivatives (Xu *et al.* 2000). A quadratic approximation was outlined, which all the elements of Hessian matrix were estimated from the existing data (Salajegheh and Rahmani 1998). In fact, an approximate Hessian matrix was established using the first order available derivatives, which decreased the number of design iterations.

A three-point approximation was later introduced using the existing data of the three previous design points (Salajegheh 1997, 2000). The second and third order approximation methods were proposed with the diagonal elements of the Hessian and third order derivatives matrices, which increased the quality of the approximation for trusses (E. Salajegheh and J. Salajegheh 2002).

A multi-point approximation method was proposed using the Hermite interpolation and was based on the function and derivatives data obtained at the previous design points (Wang *et al.* 1996). Recently, the approximation concepts have also been used for dynamic loads (Salajegheh *et al.* 2005, and Salajegheh and Heidari 2004, 2005-a, 2005-b, and Salajegheh *et al.* 2008).

In the present study, In order to increase the quality of approximate functions, a new third order approximation (TA) method is proposed and its accuracy is compared with the quadratic approximation (QA) method presented by E. Salajegheh and J. Salajegheh (2002) and the exact method (with no approximation). As mentioned, in the exact method no approximation is employed and when we need to analyze the structure during optimization, the finite element analysis is used. Also, in order to choose the position of columns in structural plan, the one-dimensional Fibonacci method is extended and employed for two discrete variables (N_x, N_y). In this method, the new approximate function is also employed at various stages of the optimum design. Numerical results indicate that this is a quick and suitable method for the above purpose.

2. One-dimensional direct search method

One of the quickest methods for one-dimensional unconstrained optimization (Eq. 2) is the direct search method, without requiring the evaluation of derivatives which is named the Fibonacci method (Edwin and Stanislaw 2001). This method can minimize the functions with continuous and discrete variables.

$$\begin{aligned} &\text{Minimize} && \phi(x) \\ &\text{Subject to:} && a \le x \le b \end{aligned} \tag{2}$$

where ϕ is the objective function and a, b are the lower and upper bounds for variable x, respectively.

The Fibonacci sequence is a numerical sequence and the terms of sequence are obtained from Eq. (3).

$$F_l = \frac{\sqrt{5}}{5 \times 2^{l+1}}[(\sqrt{5}+1)^{l+1} + (-1)^l(\sqrt{5}-1)^{l+1}] \qquad l = 0, 1, 2... \tag{3}$$

where l represents the number of iterations and F_l is the terms of sequence.

The only requirement for the function to be minimized is that it should be a unimodal function (Fig. 1). Two inner points in the interval $[a,b]$ are obtained after conformity of a, b on terms of the Fibonacci sequence. The objective function is evaluated in these points and then, the solutions are compared together. Eqs. (4) and (5) demonstrate the methodology for determining the inner points in the interval $[a, b]$ in each iteration.

$$\lambda_k = \alpha_k + \frac{F_{l-k-1}}{F_{l-k+1}}(b_k - a_k) \qquad k = 1, 2, ..., l-1 \tag{4}$$

$$\mu_k = \alpha_k + \frac{F_{l-k}}{F_{l-k+1}}(b_k - a_k) \qquad k = 1, 2, ..., l-1 \tag{5}$$

If $\phi(\lambda_k) \le \phi(\mu_k)$, then $a_{k+1} = a_k$ and $b_{k+1} = \mu_k$ otherwise, $a_{k+1} = \lambda_k$ and $b_{k+1} = b_k$ in the next iteration and this process is continued until achieving the optimum point x^*.

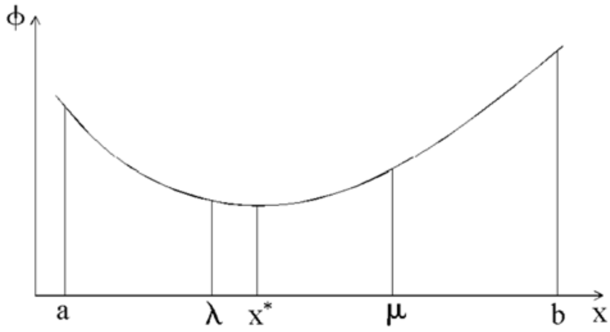

Fig. 1 Unimodal function

3. Extension of the direct search method for two-variable functions

In this study, the one-dimensional Fibonacci method is extended and is employed to minimize two-variable functions. This method does not need the function derivatives to be determined and it can be used when the design variables are discrete. Fig. 2 shows the plot of a two-variable convex function in the design space. It can be stated as

$$\begin{aligned} &\text{Minimize} && \phi(x_1, x_2) \\ &\text{Subject to:} && a_1 \le x_1 \le b_1 \\ & && a_2 \le x_2 \le b_2 \end{aligned} \tag{6}$$

where ϕ is the objective function and a_1, b_1, a_2, b_2 are the lower and upper bounds for variables x_1, x_2, respectively.

Now, let $\varphi(x_1)$ be defined in the following manner

$$\begin{gathered} \varphi(x_1) = \text{Min } \phi(x_1, x_2) \\ \text{Subject to: } a_1 \le x_1 \le b_1 \end{gathered} \tag{7}$$

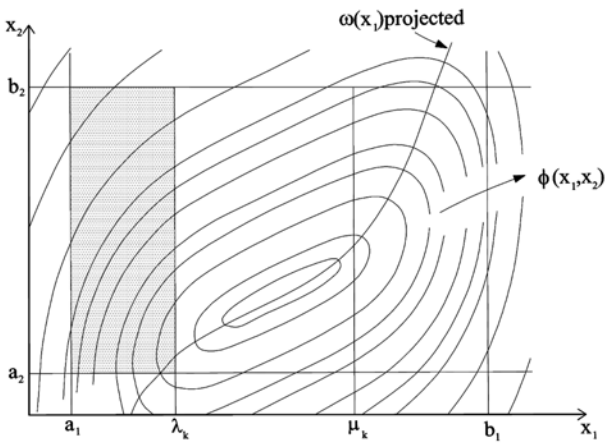

Fig. 2 Plot of a two-variable function in the design space and reducing the interval of uncertainty

When x_2 is kept at a fixed value, $\phi(x_1, x_2)$ becomes a unimodal function in terms of x_1 and then, $\varphi(x_1)$ is the minimum of the unimodal function of x_1 for selected value of x_2. This can be determined by the one-dimensional search method which is described in Sec. 2. Thus, the overall process is as follows:

(a) By keeping x_2 at a fixed value, two inner points λ_k and μ_k in the interval $[a_1, b_1]$ are chosen according to the one-dimensional search method of Sec. 2.

$$a_1 \le \lambda_k \le \mu_k \le b_1 \tag{8}$$

(b) $\varphi(\lambda_k)$ and $\varphi(\mu_k)$ are evaluated and compared according to Sec. 2.

(c) The above procedure can be repeated for each new interval, until

$$\text{Min } \varphi(x_1) = \text{Min } \phi(x_1, x_2) \tag{9}$$

is located.

(d) By keeping x_1 at a fixed value, the above three steps can be repeated for the variable x_2.

(e) This procedure is repeated until conformity of variables in two consecutive stages.

The required solution of the problem is obtained as a result of a sequence of one-dimensional searches. It is to be noted that, since the second variable is kept at a fixed value in the first one-dimensional search, its lower bound can be selected and in subsequent searches, the optimum values of the variables which are obtained from the previous searches are kept at a fixed value.

In the present study, $\phi(x_1, x_2)$ is considered as the weight function of structure in terms of two discrete variables N_x, N_y. In fact, these preliminary design variables which indicate the geometry of structure are chosen by the extended Fibonacci method. After determining them, the remainder of optimization process is achieved until the optimum cross-sectional dimensions of elements are evaluated.

4. The new third order approximation method

Given a function $F(X)$, the third order approximation (TA) is obtained by considering only the diagonal elements of the higher order derivatives which is expressed as

$$F_T(X) = F(X^1) + \sum_{i=1}^{n} F_{,i}(X^1)(x_i - x_{1i}) + \frac{1}{2}\sum_{i=1}^{n} F_{,ii}(X^1)(x_i - x_{1i})^2 + \frac{1}{6}\varepsilon_T \sum_{i=1}^{n}(x_i - x_{1i})^3 \tag{10}$$

where n is the number of variables and X represents variables vector. X^1 is variables vector at the present design point and x_i and x_{1i} are the components of X and X^1, respectively. The notation $F_{,i}(X^1)$ and $F_{,ii}(X^1)$ represent the first and second order derivatives at the present design point, respectively. The notation ε_T represents the diagonal elements of the third order derivatives matrix which are considered equal. The subscript T denotes the third order approximation.

Since the computational cost for evaluating the exact derivatives is increased with increasing the degrees of freedom, the second order approximate derivatives at the present design point $F_{,ii}(X^1)$ are estimated using the first order derivatives. To determine $F_{,ii}(X^1)$, the first order exact and approximate derivatives at the previous design point X^0 are matched, then we have

$$F_{,ii}(X^1) = \frac{F_{,i}(X^0) - F_{,i}(X^1) - \frac{1}{2}\varepsilon_T(x_{0i} - x_{1i})^2}{x_{oi} - x_{1i}} \tag{11}$$

By matching the exact and approximate values of the function at the previous design point and employing Eq. (12), ε_T is obtained in terms of the first order derivatives:

$$\varepsilon_T = \frac{F(X^0) - F(X^1) - \frac{1}{2}\sum_{i=1}^{n}(F_{,i}(X^0) + F_{,i}(X^1))(x_{oi} - x_{1i})}{-\frac{1}{12}\sum_{i=1}^{n}(x_{oi} - x_{1i})^3} \tag{12}$$

It is recognized that for some functions such as nodal displacements, using the reciprocal approximation would result in a more accurate estimation of the function. Thus, the third order approximation with the use of reciprocal variables

$$y_i = \frac{1}{x_i} \tag{13}$$

is arranged as follows by substituting into Eq. (10):

$$F_{TR}(X) = F(X^1) + \sum_{i=1}^{n} F_{,i}(X^1)(x_i - x_{1i})\left(\frac{x_{1i}}{x_i}\right)\left(2 - \frac{x_{1i}}{x_i}\right) + \frac{1}{2}\sum_{i=1}^{n} F_{,ii}(X^1)(x_i - x_{1i})^2\left(\frac{x_{1i}}{x_i}\right)^2 + \frac{1}{6}\varepsilon_{TR}\sum_{i=1}^{n}\left(\frac{1}{x_i} - \frac{1}{x_{1i}}\right)^3 \tag{14}$$

where subscript *TR* denotes the third order approximation with the reciprocal variables. By matching the first order exact and approximate derivatives and also the exact and approximate values of function at the previous design point, $F_{,ii}(X^1)$ and ε_{TR} are obtained in terms of the first order derivatives:

$$F_{,ii}(X^1) = \frac{F_{,i}(X^0) - F_{,i}(X^1)x_{1i}\left(\frac{3x_{1i}}{x_{oi}^2} - \frac{2x_{1i}^2}{x_{oi}^3}\right) - \frac{1}{2}\varepsilon_{TR}\left(\frac{1}{x_{oi}} - \frac{1}{x_{1i}}\right)^2\left(\frac{-1}{x_{oi}^2}\right)}{(x_{oi} - x_{1i})\left(\frac{x_{1i}}{x_{oi}}\right)^3} \tag{15}$$

$$\varepsilon_{TR} = \frac{F(X^0) - F(X^1) - \frac{1}{2}\sum_{i=1}^{n} F_{,i}(X^0)(x_{oi} - x_{1i})\left(\frac{x_{oi}}{x_{1i}}\right) - \frac{1}{2}\sum_{i=1}^{n} F_{,i}(X^1)(x_{oi} - x_{1i})\left(\frac{x_{1i}}{x_{0i}}\right)}{\frac{-1}{12}\sum_{i=1}^{n}\left(\frac{1}{x_{oi}} - \frac{1}{x_{1i}}\right)^3} \tag{16}$$

Therefore, in the new approximation method, firstly ε_T, ε_{TR} are obtained from Eqs. (12), (16) and then, the second order derivatives are estimated from Eqs. (11), (15). The value of function at the present design point is determined by substituting these derivatives into Eqs. (10), (14). It is to be noted that for estimating the higher order derivatives, it is only necessary to obtain the first order derivatives at the two previous design points (two-point approximation).

In this study, intermediate variables are used to obtain an approximate function. Intermediate variables for these structures are taken as cross-sectional areas and moments of inertia which are determined from the cross-sectional dimensions. To determine the approximate forces of the elements, a linear approximation method is used in the first iteration and Eqs. (10) to (12) for the others. Also, to determine approximate displacements, a reciprocal linear approximation method is used in the first iteration and Eqs. (14) to (16) for the others.

5. Numerical results

Computer programs have been developed based on the preceding discussion and theories.

Firstly in order to study the accuracy and quality of the new TA approximation method, two steel framed structures are solved and their results are compared with the QA method proposed by E. Salajegheh and J. Salajegheh (2002) and also the exact method (with no approximation). In optimization process, the DOT (Design Optimization Tool) optimizer presented by Vanderplaats *et al.* (1991) with the modified feasible direction algorithm and the intermediate variables are used.

Secondly two examples are presented for the optimum design of 3D steel framed structures including determination of the best position of columns and hence efficiency of the proposed method is also studied. In this case, in addition to determining the number of columns in two directions, the optimum cross-sectional dimensions of elements are obtained using the TA approximation method. For this purpose, the following stages are performed:

(a) The number of columns in two directions of structural plan (preliminary variables) are chosen based on the extended direct search method.

(b) After determining the position of columns and forming the geometry of structure, the optimum cross-sectional dimensions of elements (main variables) are obtained using only a few FE analyses and the new TA approximation method.

(c) By choosing the new number of columns in two directions, the stages (a) and (b) are repeated until conformity of N_x, N_y in two consecutive stages.

The design constraints are considered as the side constraints, the minimum and maximum cross-sectional dimensions for performance considerations, the drift requirements which are controlled by a quotient of the storey height (generally, h/200 to h/400) and the AISC requirements (2002) for stresses in elements such as tension, compression, bending, shearing, torsion stresses and their combination. Also, the AISC limitations for slenderness and prevention of local buckling of flange and web in box and I-sections (Fig. 3) are accounted for.

It should be noted that in the examples presented, the computational time is CPU time using a Pentium IV 1.80 GHz.

5.1. Example 1. Optimum design of a one-storey structure with a specified geometry

A steel framed structure is shown in Fig. 4. All beams and columns are rigidly connected and this

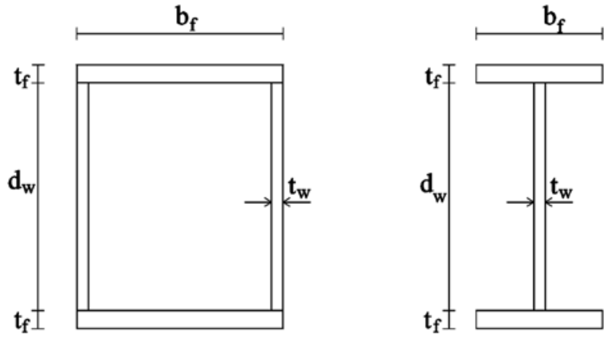

Fig. 3 Box and I-shaped cross-sectional dimensions

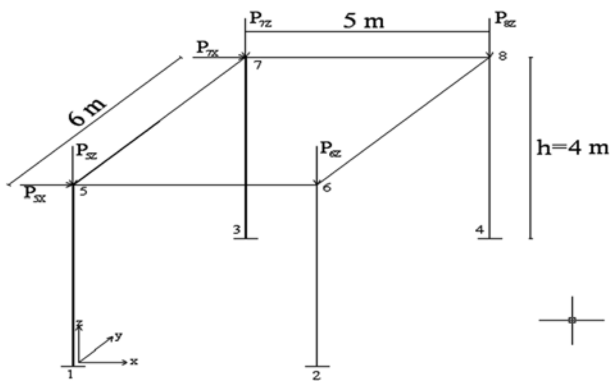

Fig. 4 One-storey framed structure

structure is subjected to the lateral concentrated loads $P_{5x} = P_{7x} = 6{,}000$ kgf on joints 5 and 7 and the gravity concentrated loads $P_{5z} = P_{6z} = P_{7z} = P_{8z} = -30{,}000$ kgf on joints 5 to 8. Moreover, it assumed that a uniformly distributed load 2,000 kgf / m is imposed in negative z-direction on the linking members 5-6 and 7-8. For this structure, one type of box-shaped column and two types of I-section beams (one in x-direction and one in y-direction) are used. The material properties are given as weight density $\rho = 7{,}850$ kgf / m^3, yield stress $f_y = 2{,}400$ kgf/cm^2, Young's modulus $E = 2 \times 10^6$ kgf/cm^2. The initial cross-sectional dimensions are shown in Table 1.

In addition to the mentioned constraints, the minimum size of cross-sectional dimensions and lateral drift limitations are assumed to be 5 mm and h/400, respectively. However, a total of 58 constraints are considered in this example.

Comparison of the results in terms of the number of FE analyses, optimum weight, computational time, maximum constraints and the number of active constraints at the last iteration of optimization using the different methods are presented in Table 2. Since the numerical optimization technique is used, it is considered a tolerance ($-0.05 \le g_j\, X \le 0.03$) for active constraints. It can be seen from Table 2 that although the quality of the results obtained by the QA and the TA methods is high, the accuracy of the TA method is higher. Since the maximum constraints approach 0.03, it also indicates that the quality of the TA method is higher than the QA method. In this example, the computational time for the QA method is slightly less comparing with the TA method. This means that because this is a small structure,

Table 1 Initial cross-sectional dimensions for one-storey structure (cm)

Type	Section	d_w	t_w	b_f	t_f
1	Box	40	1	40	1
2,3	I	40	1	30	1

Table 2 Comparison of the results for one-storey structure

Method	Number of analyses	Optimum weight (kgf)	Time (s)	Maximum constraint	Number of active constraints
Exact	312	1576.2	18.43	0.029945	32
QA	6	1668.7	4.57	0.028899	23
TA	5	1628.0	4.83	0.029929	27

the time required for the analysis is not significant and cannot be considered as a major factor.

The weight histories of the structure at various iterations of the optimum design are presented in Table 3. Iteration number 0 indicates the initial design point. This table demonstrates that the optimum weight at the various iterations by the TA method is less than the QA method. However, the weight of structure has been decreased by about 56% compared to the initial weight using the approximation methods.

One should note that a continuous approximation is carried out for the cross-sectional dimensions. As in practice the plates are chosen from a set of available commercial sections, thus the obtained dimensions may not be practical. A discrete approximation would be outlined in more practical values. A similar approach outlined by Salajegheh (1996) which can be used for converting the continuous variables to discrete values. The approximation concepts presented in this study can also reduce the computational cost of the discrete optimization.

5.2. Example 2. Optimum design of a five-storey structure with a specified geometry

The steel framed structure shown in Fig. 5 has 150 joints and 325 elements. It is subjected to lateral and gravity loadings. The gravity loadings in negative z-direction are assumed as the uniformly distributed loads 2,500 kgf/m and 5,000 kgf/m on all the side and middle beams in x-direction, respectively. The lateral loads are presented in Table 4. In this table, F_y denotes the lateral loads on joints in y-direction.

Five types of elements are used for box-shaped columns (one type for each storey) as well as ten

Table 3 Weight histories for one-storey structure (kgf)

Iteration no.	QA	TA
0	3736.5	3736.5
1	1847.0	1847.0
2	1750.8	1682.1
3	1706.9	1648.2
4	1689.5	1631.8
5	1670.3	1628.0[a]
6	1668.7[a]	

[a]Optimum weight

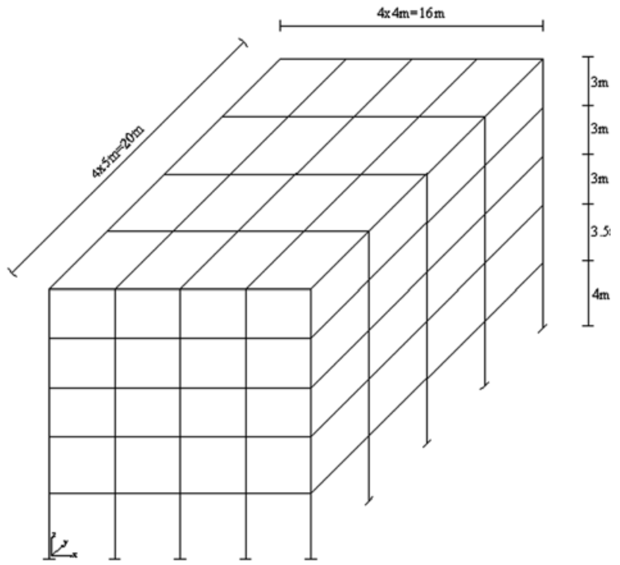

Fig. 5 Five-storey framed structure

Table 4 Lateral loads for five-storey structure (kgf)

Joints no.	F_y
26~30	2500
51~55	5000
76~80	6500
101~105	8000
126~130	9500

types of I-sections for beams (one in x-direction and one in y-direction for each storey). The other properties of elements as well as constraints are similar to those in Example 1. Thus, a total of 2285 constraints are considered. The initial cross-sectional dimensions are shown in Table 5.

The results of the optimum design by the QA, TA and exact methods are presented in Tables 6 and 7. Comparison of the optimum weights, number of FE analyses and active constraints at the last iteration given in these tables, indicates that the quality of the TA method is better than the QA method. Also, as the maximum constraints approach 0.03, the accuracy of the TA method is better. The computational time using the TA method is less comparing with the QA method. It is due the fact that the overall

Table 5 Initial cross-sectional dimensions for five-storey structure (cm)

Type	Section	d_w	t_w	b_f	t_f
1,2	Box	50	1.2	30	1
3,4,5	Box	40	1	30	1
6,8,10,12,14	I	30	1	15	1
7,9,11	I	45	1	15	1
13,15	I	40	1	15	1

Table 6 Comparison of the results for five-storey structure

Method	Number of analyses	Optimum weight (kgf)	Time (s)	Maximum constraint	Number of active constraints
Exact	194	76489.8	10937	0.0291315	359
QA	9	78742.6	1382	0.0257969	281
TA	7	78565.0	1109	0.0281267	306

Table 7 Weight histories for five-storey structure (kgf)

Iteration no.	QA	TA
0	98713.7	98713.7
1	82730.6	82730.6
2	80777.4	80465.2
3	80306.3	79441.8
4	79732.1	79019.6
5	79452.1	78789.0
6	79287.3	78596.6
7	79097.0	78565.0[a]
8	78991.2	
9	78742.6[a]	
10	78742.6	

[a]Optimum weight

computational time in large-scale problems with several hundred degrees of freedom would decrease when using the higher order methods. This seems to be reasonable as the time taken by the analysis is greater than the required time for the optimization process. The overall computational time to achieve an optimum solution depends on the number of design variables, the number of constraints and the size of the problem in terms of degrees of freedom. Usually, all the constraints are not considered in the optimum design and some of the critical or near critical constraints are retained. Thus, only the number of variables and the number of degrees of freedom have a great influence on the computational time.

5.3. Example 3. Optimum design of a one-storey structure with unspecified geometry

The one-storey steel framed structure with dimensions 30 m and 40 m in x- and y- directions, respectively and a height of 4 m is chosen. The total gravity and lateral loads are 1,200,000 kgf and 120,000 kgf, respectively. These loadings are applied on the top. Since the structural geometry varies at various stages of optimization, the gravity loads are assumed as the uniformly distributed loads along all the beams in accordance with their respective covering surface. Also, the lateral loads are equally distributed as the concentrated loads on the joints of the top in each stage. Thus, four independent load conditions, including the gravity loads in negative z-direction plus the lateral loads in both the positive and negative x- and y-directions are considered. For this structure, one type of column box-section and two types of beam I-sections (one in x-direction and one in y-direction) are used. The material properties as well as constraints are similar to those in Example 1. The initial cross-sectional dimensions are shown in Table 8.

With due attention to the architectural requirements, the minimum and maximum distance between

Table 8 Initial cross-sectional dimensions for one-storey structure (cm)

Type	Section	d_w	t_w	b_f	t_f
1	Box	35	1	35	1
2,3	I	35	0.8	20	1

columns in two directions are 2 m and 6 m, respectively. Thus, the minimum and the maximum number of columns are 6 and 16 in x-direction and 8 and 21 in y-direction, respectively.

The optimum design of structure including the best position of columns is performed using the extended direct search and also the new TA method. Histories at selected intervals, the inner points of intervals and the number of selected columns in each one-dimensional search are shown in Table 9. In this table, S is the number of one-dimensional searches and N_x, N_y are the number of columns in x- and y-directions, respectively. The notation K and D represent the number of iteration in each search and the domain of intervals, respectively. λ and μ are the inner points of the intervals. $\phi(\lambda)$ and $\phi(\mu)$ are the optimum weight (kgf) at the selected points. Also, N_x^* and N_y^* denote the optimum number of columns in x- and y-directions, respectively.

Table 9 indicates that if we use the proposed method and suppose $N_y = N_{y\,\min} = 8$ at the first one-dimensional search for variable N_x, the solution of the problem requires five searches. However, a total of 25 iterations of the optimum design procedure is necessary. The structure with $N_x^* = 9$, $N_y^* = 12$ and

Table 9 History at selected intervals and the number of columns in each one-dimensional search for one-storey structure

S	N_x	N_y	K	D	a	b	λ	μ	$\phi(\lambda)$	$\phi(\mu)$	N_x^*	N_y^*
1	Var.	8	1	13	5	18	10	13	36438.5	36907.5		
			2	8	5	13	8	10	37746.9	36438.5		
			3	5	8	13	10	11	36438.5	36572.3		
			4	3	8	11	9	10	36837.5	36438.5	10	-
2	10	Var.	1	13	8	21	13	16	33728.1	34671.8		
			2	8	8	16	11	13	33526.0	33728.1		
			3	5	8	13	10	11	33803.5	33526.0		
			4	3	10	13	11	12	33526.0	33636.5	-	11
3	Var.	11	1	13	5	18	10	13	33526.0	34544.2		
			2	8	5	13	8	10	33626.8	33526.0		
			3	5	8	13	10	11	33526.0	33840.2		
			4	3	8	11	9	10	33378.8	33526.0	9	-
4	9	Var.	1	13	8	21	13	16	33352.5	34308.1		
			2	8	8	16	11	13	33378.8	33352.5		
			3	5	11	16	13	14	33352.5	33778.2		
			4	3	11	14	12	13	33146.9	33352.5	-	12
5	Var.	12	1	13	5	18	10	13	33636.5	34756.9		
			2	8	5	13	8	10	33534.3	33636.5		
			3	5	5	10	7	8	34923.3	33534.3		
			4	3	7	10	8	9	33534.3	33146.9	9	-

"Var." = Variable

the least-weight of 33,146.9 kgf is the optimum solution of this problem. In this case, the distances between columns in x- and y-directions are 3.75 m and 3.636 m, respectively and the structure has 216 joints and 303 elements. The total computational time is 12955(about 3.5 hours) and the optimum cross-sectional dimensions for this structure with the best position of columns are shown in Table 10.

If we wish to determine the best position of columns by the trial and error method, with due attention to the minimum and maximum number of columns in two directions, the optimum design procedure should be performed 154 times and the results should be compared. In order to demonstrate the process of this method, the optimum design is separately performed for the structure with various numbers of columns in x- and y-directions and their results as contour lines are shown in Fig. 6. In this figure, the horizontal and vertical axes are N_x and N_y, respectively.

Although, the minimum and maximum of N_x are 6 and 16 by conformity of these numbers in terms of the Fibonacci sequence, the interval [5,18] is obtained. According to Sec. 2, the objective function should be a unimodal function in this interval. This is used in the interval [5,18] for N_x and the interval [8, 21] for N_y in Fig. 6. This figure demonstrates that the objective function is a unimodal function in each one-dimensional direction and then, we can use the Fibonacci method in this problem. Also, with due attention to the considered intervals, a structure with $N_x^* = 9$, $N_y^* = 12$ and the least-weight of 33,146.9 kgf is the optimum solution of this problem.

Although, $N_y = N_{y\min} = 8$ is used for the first search, Fig. 6 shows that by moving in the each one-

Table 10 Optimum cross-sectional dimensions for one-storey structure with the best position of columns (cm)

Type	d_w	t_w	b_f	t_f
1	21.393	0.524	22.622	0.541
2	26.182	0.599	15.049	0.500
3	24.688	0.507	15.012	0.539

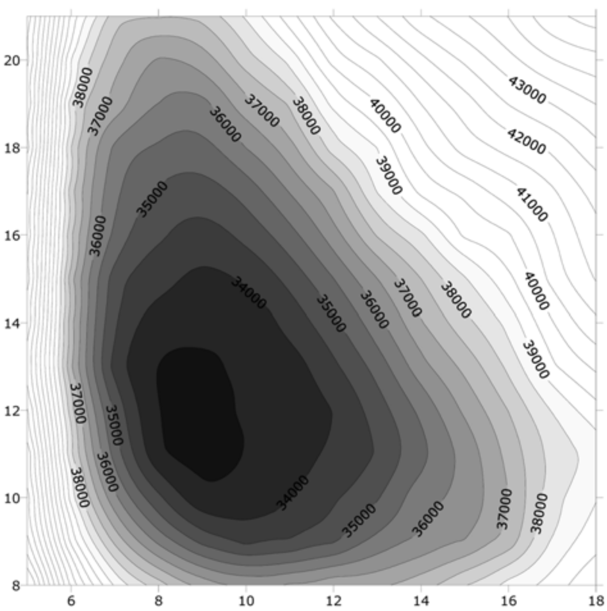

Fig. 6 Plot of objective function (weight of structure) in design space for one-storey structure

dimensional direction, we obtain the solution of the problem with a maximum of five searches and performing a maximum of 25 iteration of the optimum design procedure. This indicates that the efficiency of the proposed method is very high. Although, the ratio between the numbers of optimization processes for the two cases is about 16%, this ratio is much less for the computational times. For structures with largeand, the number of variables and also the computational time are also increased.

5.4. Example 4. Optimum design of an eight-storey structure with unspecified geometry

Consider an eight-storey steel framed structure with dimensions 25 m and 20 m in x- and y-directions, respectively. For this structure, eight types of box-shaped columns (one type for each storey) and sixteen types of I-section beams (one in x-direction and one in y-direction for each storey) are used. The specifications for the structure are shown in Table 11. In this table, h is the height of each storey. The notations F_x and F_y are the total independent lateral loads in x- and y-directions and F_z is the total gravity loads in negative z-direction at each floor level. In the each iteration, the total gravity and lateral loads are distributed on the beams and joints, similarly to Example 3. The notations NC, NB_x and NB_y are the type numbers of columns and beams in x- and y-directions for various floors, respectively. The material properties and imposed constraints are similar to Example 1. The initial cross-sectional dimensions are shown in Table 12. The minimum and maximum distances between columns in two directions are assumed to be 2 m and 7 m. Thus, the minimum and maximum number of columns is 5 and 13 in x-direction and also 4 and 11 in y-direction, respectively.

The optimum design of structure is performed using the proposed method and the results are shown in Table 13. This table shows histories at selected intervals, the inner points of intervals and the number of selected columns in each one-dimensional search. Parameters are similar to those in Example 3. This table indicates that if we assume that $N_y = N_{y\,\min} = 4$ in the first one-dimensional search for variable N_x,

Table 11 Specifications for eight-storey structure

Storey	h (m)	F_x, F_y (kgf)	F_z (kgf)	NC	NB_x	NB_y
1	3.5	10000	-350000	1	9	17
2	3.5	20000	-350000	2	10	18
3	3.5	30000	-350000	3	11	19
4	3.5	40000	-350000	4	12	20
5	3.5	50000	-350000	5	13	21
6	3.5	60000	-350000	6	14	22
7	3.5	70000	-350000	7	15	23
8	3.5	80000	-350000	8	16	24

Table 12 Initial cross-sectional dimensions for eight-storey structure (cm)

Type	Section	d_w	t_w	b_f	t_f
1~5	Box	50	1.5	50	1.5
6~8	Box	50	1.2	50	1.2
9~13	I	50	1.5	25	1.5
14~16	I	50	1.2	20	1.2
17~21	I	50	1.5	25	1.5
22~24	I	50	1.2	20	1.2

Table 13 Histories at selected intervals and the number of columns in each one-dimensional search for eight-storey structure

S	N_x	N_y	K	D	A	b	λ	μ	$\phi(\lambda)$	$\phi(\mu)$	N_x^*	N_y^*
1	Var.	4	1	8	5	13	8	10	306412	320333		
			2	5	5	10	7	8	298419	306412		
			3	3	5	8	6	7	301289	298419	7	-
2	7	Var.	1	8	4	12	7	9	285468	292863		
			2	5	4	9	6	7	283405	285468		
			3	3	4	7	5	6	292121	283405	-	6
3	Var.	6	1	8	5	13	8	10	293742	310869		
			2	5	5	10	7	8	283405	293742		
			3	3	5	8	6	7	285550	283405	7	-

"Var." = Variable

the solution of the problem is obtained using three searches. However, a total of 12 times the optimum design procedure is performed in this problem and a structure with, $N_x^* = 7$, $N_y^* = 6$ and the least-weight of 283,405 kgf is the optimum solution of this problem. In this case, the distances between columns in x- and y-directions are 4.167 m and 4 m, respectively and the structure has 378 joints and 904 elements. In this problem, the total computational time is 47129s(about 13 hours).

If we wish to solve this problem by the trial and error method, the optimum design should be performed 72 times and their results are shown in Fig. 7. This figure demonstrates that the objective function is a unimodal function in both directions and thus we can use the Fibonacci method in this problem. Also, a structure with $N_x^* = 7$, $N_y^* = 6$ and the least-weight of 283405 kgf is the solution of

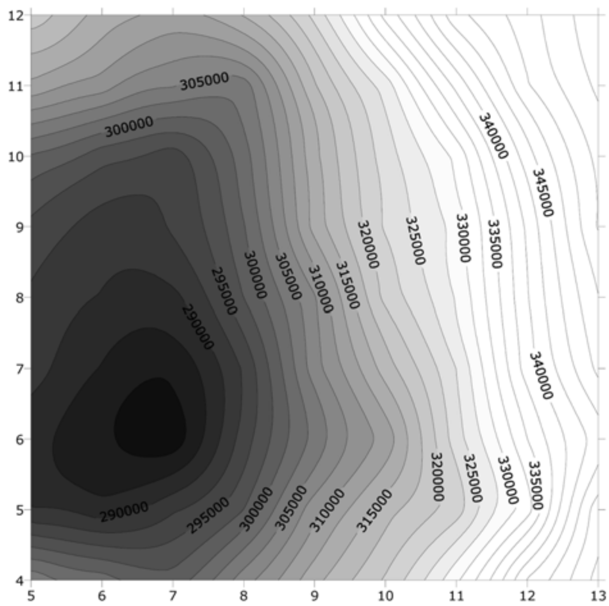

Fig. 7 Plot of objective function in design space for eight-storey structure

this problem. It is observed that by moving in each one-dimensional direction, we obtain the solution of the problem with a maximum of five searches and performing the optimum design procedure for a maximum 20 times. In this problem, the ratio between numbers of optimization processes for the two cases is about 16% which indicates that efficiency of the proposed method is very good.

6. Conclusions

A new third order approximate function has been proposed and its efficiency has been shown during structural optimization. The exact evaluation of functions is computationally expensive, thus the generation of some appropriate approximation allows an efficient and also a robust optimization process. The main feature of this efficient proposed method is that the higher order derivatives are determined from the available first order derivatives. Moreover, only the diagonal elements of the higher order derivatives are estimated and employed in approximate functions. The numerical results indicated that the quality and accuracy of the TA method is higher comparing with the QA method.

Also, efficiency of the extended direct search method has demonstrated in the context of determining the best position of columns in modular steel framed structures. This method is one of the most efficient methods for determining the minimum value of functions with two discrete variables. If such method is not used, then the optimum geometry of the structure could be obtained by many trials and errors, which is computationally expensive. As the dimensions of structure and domain between minimum and maximum the number of columns in structural plan are increased, the efficiency of the method also increases. Firstly, geometry of the structure is formed based on choosing the preliminary variables, i.e., the number of columns in x- and y-directions of structural plan and then, the main variables of cross-sectional dimensions are determined using the optimization procedure outlined above. The present numerical results indicate that the weight of steel framed structures is a unimodal function in terms of the number of columns in two orthogonal directions. Also, the proposed method for the optimum design of these structures including determination of the best position of columns is very efficient and can be used in practical situations.

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