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Buckling of restrained steel columns due to fire conditions

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Abstract. An analytical procedure is presented for the determination of the buckling load and the buckling temperature of a straight, slender, geometrically perfect, axially loaded, translationally and rotationally restrained steel column exposed to fire. The exact kinematical equations of the column are considered, but the shear strain is neglected. The linearized stability theory is employed in the buckling analysis. Behaviour of steel at the elevated temperature is assumed in accordance with the European standard EC 3. Theoretical findings are applied in the parametric analysis of restrained columns. It is found that the buckling length factor decreases with temperature and depends both on the material model and stiffnesses of rotational and translational restraints. This is in disagreement with the buckling length for intermediate storeys of braced frames proposed by EC 3, where it is assumed to be temperature independent. The present analysis indicates that this is a reasonable approximation only for rather stiff rotational springs.

Keywords : restrained steel column; inelastic buckling; Reissner beam; high temperatures; critical temperature.

1. Introduction

When considering the effect of fire on steel structures, a special attention has to be paid to the possible stability failure of columns. Namely, these structural elements are both slender and compressed and are therefore prone to buckling, which happens to be particularly true at high temperature. Often fire remains active only in one compartment and does not spread around. If remaining parts of the structure stay relatively cool, thus being stiff compared to the fire-affected part, they represent an additional support (a 'restraint') to the affected part. Such a restraint, when applied to an isolated column, can be adequately modeled with translational and rotational springs at supports. In recent years the restrained steel columns at elevated temperatures have been extensively studied both experimentally (e.g. Ali and O'Connor 2001, Franssen *et al.* 1998, Tan *et al.* 2007, Wang and Davies 2003) and theoretically (e.g. Ali and O'Connor 2001, Franssen and Dotreppe 1992, Huang and Tan 2003, Huang and Tan 2004, Huang and Tan 2007, Valente and Neves 1999). There the second-order theory is used to determine the critical temperature. Such an approach is well appropriate for the determination of critical temperatures for elastic columns, where the axial deformations do not have an essential influence on results. In contrast, the consideration of axial deformations is essential in materially non-linear buckling anslysis (Krauberger *et al.* 2007). Appropriate

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modifications of the methods based on the second-order theory are necessary to account for the effect of axial deformations properly in the buckling load. The present analytical procedure includes the axial deformations and the material non-linearity in a theoretically consistent way.

In majority of analyses researchers use the approximative methods such as the method of finite elements to determine the effects of boundary conditions on behaviour of steel columns in fire. In frame-like structures the boundary conditions of an isolated column can (Eurocode 2003) described by translational and rotational springs whose stiffnesses are determined from the stiffness of the surrounding cool parts of the structure. The properties of the restraints depend on the properties of both the type of connection and the surrounding structure, and on its temperature. Most analyses, however, have assumed the restraints in the form of the temperature-independent springs. The results indicate that the rotational restraints have a favourable effect on the buckling resistance of columns. In contrast, the resistance is decreased in the presence of the axial restraints (e.g. Huang and Tan 2003). Because in design the buckling length of a column should be known as accurately as possible, these papers often present only effects of the restraints on the buckling length. Ali and O'Connor (2001) experimentally determined the buckling length of a column after it has been cooled down. Exact analytical methods are rare, if not impossible in such a non-linear setting. Huang and Tan (2003) proposed the Rankine approach for the determination of the fire resistance of restrained columns. An alternative solution was presented by Gomes *et al*. (2007). Their formulae try to improve those given in Eurocode 3 (2003).

This paper presents an analytical procedure for the determination of the critical temperature of a geometrically ideal steel column exposed to fire, i.e., a column which is straight, geometrically perfect, slender, axially loaded and translationally and rotationally restrained. The stiffnesses of translational and rotational restraints are assumed to be temperature-independent. The column is modelled by a planar geometrically exact beam model of Reissner (1972) with the effect of shear strain being neglected due to the assumed high slenderness of the column. The material model and the temperature-dependent material parameters are taken according to Eurocode 3 (2003). There, the viscous strain is not considered as a separate strain component, but is rather combined with the plastic strain, thus assuming the material to be time independent. A uniform temperature field in the column is assumed, yet somewhat delayed with regard to the temperature of the surrounding gas Eurocode 3 (2003). In the first part of the paper, the thermo-mechanical equations are set up and the fundamental equilibrium state of the column is obtained. Next the perturbation of the fundamental equilibrium state is derived with the help of the linearization. The condition for the existence of the non-trivial solution of the homogeneous linearized equations supply us with the value of the buckling temperature. In the final part of the paper, numerical results for the buckling length factor of a number of restrained steel columns are presented and analysed for the realistic range of values of translational and rotational spring constants. The main findings are gathered in the conclusions.

2. Fire analysis of restrained steel columns

2.1 Preliminaries

We consider a straight, slender steel column of initial, undeformed length L and a constant I-shaped cross-section with rotational restraints ρ^1 and ρ^2 at both ends and with translational restraints μ_H and μ_V on the top. The restraints are assumed to maintain their initial stiffnesses during any deformation or heating of the column. The column is axially loaded with a compression force F while being simultaneously exposed to fire (Fig. 1). The plane of deformation of the column is the plane (x, z) of the Cartesian coordinate system

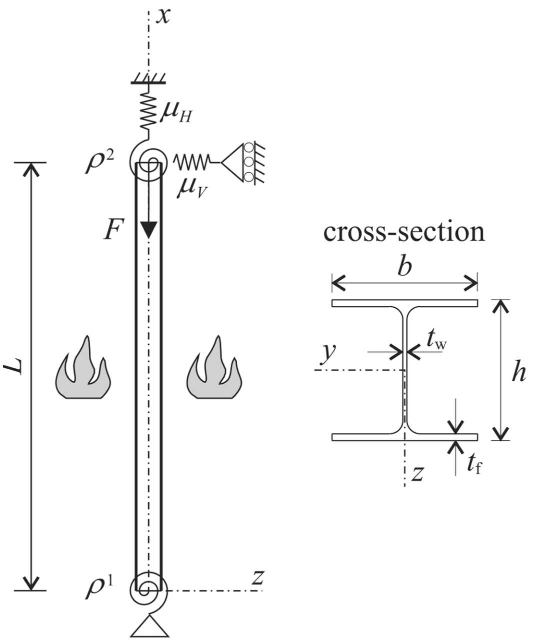

Fig. 1 Mathematical model of the restrained steel column and a typical cross-section

(x, y, z). The reference axis of the column is assumed to coincide with its centroidal axis.

The analysis of steel structures in fire assumes two independent steps. The first step comprises the determination of the temperature field due to the given temperature regime in the fire compartment. In the second step of the analysis, the stress and strain fields due to the combined effect of mechanical and thermal loads are obtained. In what follows, we only briefly describe the first step, while the second step is described in detail.

2.2 The temperature field

In most cases of fire analyses gas temperature in the fire compartment is determined with the help of the so-called parametric temperature-time curves (Magnusson and Thelandersson 1970) or by other simplified numerical models (Lopes *et al.* 2005). The development of gas temperature in the fire compartment depends on many parameters and is therefore both a complex task to do and unreliable. In order to make the engineering design deterministic, the standard temperature-time curves are introduced which uniquely define explicit characteristic relationships between gas temperature in the compartment and time for a number of typical situations, accounting for the amount of fire load and thethermo-dynamical and geometrical properties of the compartment. Once the variation of the gas temperature is known, the temperature distribution within the structure is determined by integrating the differential equation of heat conduction (Eckert and Drake 1972). For a typical I-shaped steel cross-section uniformly heated over its surfaces, it is appropriate to assume an approximately uniform temperature distribution over the cross-section. One such simplified solution, which is also used in this article, is proposed in the EC3 standard (Eurocode 2003). Fig. 2(a) shows different temperature-time curves. For instance, with the use of fire curves EC 1 and SDHI-M, we can describe a short fire with high intensity, typical for office buildings, while the fire in a warehouse with a high amount of combustible material is

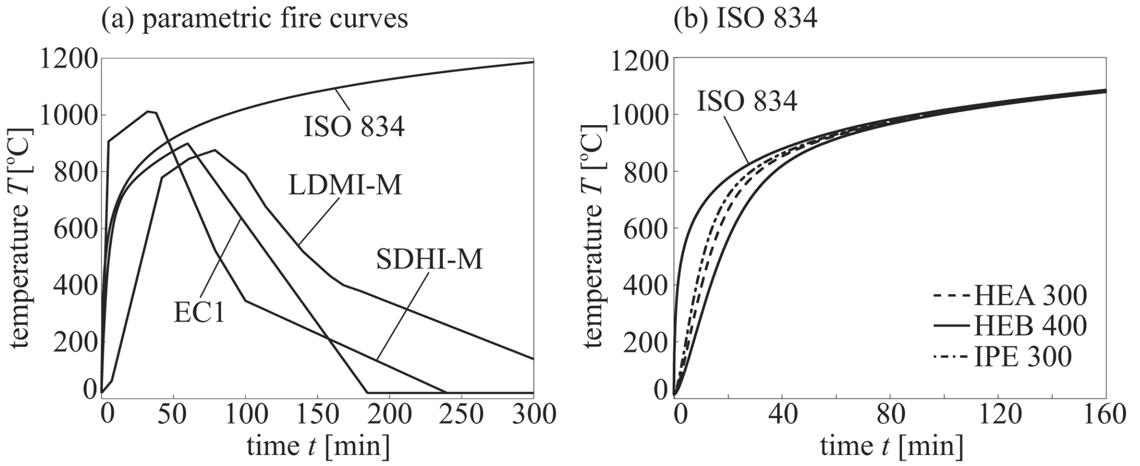

Fig. 2 (a) Parametric fire curves. (b) Temperature-time curves for different steel profiles for fire curve ISO 834

Table 1 Geometrical parameters of cross-sections

	$m\,[\mathrm{m}^{-1}]$	$J\,[\mathrm{cm}^4]$	$A\,[\mathrm{cm}^2]$	h [cm]	b [cm]	t_f [cm]	t_w [cm]
HEA 300	153	18 260	113	29	30	1.40	0.85
HEB 400	97	57 680	198	40	30	2.40	1.35
IPE 300	216	8 360	53.8	30	15	1.07	0.71

well described with the LDMI-M curve. The development of temperature with time in the steel cross-section for a standard fire curve ISO 834 (ISO 834 1975) for three different standard I-shaped cross-sections with commercial labels HEA 300, HEB 400 and IPE 300 is depicted in Fig. 2(b). Geometrical parameters of their sections are presented in Table 1.

2.3 The stress-strain field

After the temperature variation in time has been obtained, we may start the mechanical analysis of the structure. We find the solution in an incremental way. We divide the time of duration of fire into time intervals $[t^{i-1}, t^i]$ ($i = 1,2,3,...$) and determine iteratively the stress and strain state at each time t^i, $i = 1,2,3,...$. The column is modelled by Reissner's geometrically exact beam theory (Reissner 1972), but with the effect of shear deformations being neglected. The corresponding governing equations of such a beam model are (Krauberger *et al.* 2007):

$$f_1 = 1 + u^{i\prime} - (1 + \varepsilon^i)\cos\varphi^i = 0 \tag{1}$$

$$f_2 = w^{i\prime} + (1 + \varepsilon^i)\sin\varphi^i = 0 \tag{2}$$

$$f_3 = \varphi^{i\prime} - \kappa^i = 0 \tag{3}$$

$$f_4 = \mathcal{H}^{i\prime} = 0 \tag{4}$$

$$f_5 = \mathcal{V}^{i\prime} = 0$$

$$f_6 = \mathcal{M}^{i\prime} - (1 + \varepsilon^i)\mathcal{Q}^i = 0, \tag{6}$$

$$f_7 = \mathcal{N}^i = \mathcal{H}^i \cos\varphi^i - \mathcal{V}^i \sin\varphi^i, \tag{7}$$

$$f_8 = \mathcal{Q}^i = \mathcal{H}^i \sin\varphi^i + \mathcal{V}^i \cos\varphi^i, \tag{8}$$

$$f_9 = \mathcal{N}^i = \int_A \sigma^i dA, \tag{9}$$

$$f_{10} = \mathcal{M}^i = \int_A z\sigma^i dA. \tag{10}$$

Here $(\bullet)'$ denotes the derivative with respect to x; u^i and w^i are the components of the displacement vector of the centroidal axis of the column in x and z directions, φ^i is the cross-sectional rotation around y; ε^i is the extensional strain of the centroidal axis, and κ^i is its bending strain (also termed the 'pseudocurvature'). $\mathcal{N}^i$ and $\mathcal{Q}^i$ are the axial and the shear force and $\mathcal{M}^i$ is the bending moment. $\mathcal{H}^i$ and $\mathcal{V}^i$ are the components of the resulting cross-sectional force with respect to spatial axes x and z, respectively. Eqs. (9) and (10) represent the constitutive equations of the cross-section, relating the axial force $\mathcal{N}^i$ and the bending moment $\mathcal{M}^i$ to the normal stress σ^i. These constitutive quantities depend on the material model for steel at elevated temperatures; in this article we use the material model according to EC 3 (2003). Natural and essential boundary conditions corresponding to Eqs. (1)-(10) are (Fig. 1):

bottom, $x = 0$:

$$u^i(0) = 0 \tag{11}$$

$$w^i(0) = 0, \tag{12}$$

$$s_1^1 \mathcal{M}^i(0) - \rho^1 \varphi^i(0) = 0, \tag{13}$$

top, $x = L$:

$$s_H(\mathcal{H}^i(L) + F) + \mu_H u^i(L) = 0, \tag{14}$$

$$s_1^2 \mathcal{V}^i(L) + \mu_V w^i(L) = 0, \tag{15}$$

$$s_2^2 \mathcal{M}^i(L) + \rho^2 \varphi^i(L) = 0. \tag{16}$$

Any combination of boundary conditions can be obtained by choosing an appropriate combination of parameters s_1^1, s_1^2, s_2^2, s_H, μ_H and $\mu_V \in \{0,1\}$.

We assume that the stress-strain state and temperature at time t^{i-1} are given. At time t^i, only temperature is known but not the stress-strain state. The longitudinal normal strain D^i at time t^i in any point of the column is assumed to be the sum

$$D^i = D^{i-1} + \Delta D^i \tag{17}$$

where ΔD^i ($i = 1,2,3,...$) is the increment of the normal strain (here also termed the 'geometrical deformation') in time interval i. Considering the principle of additivity of strains and the material model of steel at elevated temperatures according to EC 3 (Eurocode 2003), we take that the normal strain increment ΔD^i is the sum of the strain increments due to temperature, ΔD^i_{th}, and due to stress, ΔD^i_{σ}. In this material model, viscous strains are assumed to be included in ΔD^i_{σ} and are thus not treated separately:

$$\Delta D^i = \Delta D^i_{th} + \Delta D^i_{\sigma} \tag{18}$$

The temperature strain increment, ΔD^i_{th}, is calculated from the EuroCode 3 formula (2003), where the total temperature strain, D_{th}, is given with a formal expression $D_{\mathrm{th}} = f(T)$; thus, $\Delta D^i_{\mathrm{th}} = f(T^i) - f(T^{i-1})$. The stress-dependent strain increment, ΔD^i_{σ} also termed the 'mechanical strain increment', is assumed to be equal to the sum of elastic and plastic strains, $\Delta D^i_{\sigma} = \Delta D^i_{\mathrm{e}} + \Delta D^i_{\mathrm{p}}$. The assumed stress-strain law for steel at high temperatures is depicted in Fig. 3(a). In this material model, the variation of material parameters of steel with temperature is considered via the reduction factors $k_{p,T}$, $k_{y,T}$ and $k_{E,T}$. Fig. 3(b) shows their variation with temperature. For a full description of the EC3 material model, see Eurocode 3 (2003). The development of the stress and strain state in a steel column during fire is thus fully determined by the system of ten non-linear algebraic and differential Eqs. (1)-(10) and six boundary conditions (11)-(16) for sixteen unknown functions and parameters.

3. Linearized buckling analysis

We wish to find the buckling load of a column with the help of the linear theory of stability (Keller 1970). To this end we first have to derive the solution for the fundamental equilibrium state of the column. If we consider the fact that the column, when subjected to a centric axial force, remains straight and vertical prior to buckling, we easily recognize that its fundamental equilibrium solution is characterized by the condition $\varphi^i = 0$. Inserting $\varphi^i = 0$ in Eqs. (1)-(10) gives:

$$u^i = \varepsilon^i x, \tag{19}$$

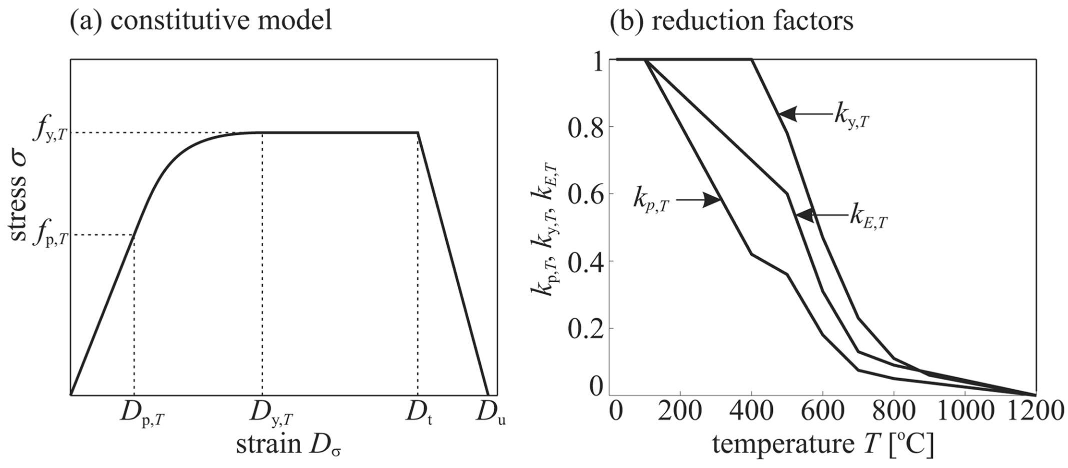

Fig. 3 (a) Stress-strain relationship for steel in tension and compression according to EC 3; (b) temperature dependent reduction factors

$$w^i = 0, \tag{20}$$

$$\varphi^i = 0, \tag{21}$$

$$\mathcal{H}^i = \mathcal{H}^i(0) = \text{const.}, \tag{22}$$

$$\mathcal{V}^i = 0, \tag{23}$$

$$\mathcal{M}^i = 0, \tag{24}$$

$$\mathcal{N}^i = \mathcal{H}^i = \text{const.}, \tag{25}$$

$$\mathcal{Q}^i = \mathcal{V}^i = 0, \tag{26}$$

$$\mathcal{N}^i = \ \sigma^i(\varepsilon^i,\ \kappa^i = 0,\ D^i_{th},\ T^i)A, \tag{27}$$

$$\kappa^i = 0. \tag{28}$$

The solution of the above equations must satisfy boundary condition 14.

$$s_H(\mathcal{H}^i(L) + F) + \mu_H u^i(L) = 0, \tag{29}$$

Table 2 displays two different fundamental solutions corresponding to $s_H = 0$, $F = 0$ and $s_H = 1$, respectively. Due to the non-linearity of the constitutive Eq. (27) the solution requires an iterative algorithm, but otherwise is exact within the numerical precision of the computer. Once the fundamental equilibrium solution is known, we linearize.

Eqs. (1)-(8) around the fundamental solution to obtain

$$\delta f_1 = \delta u^{i\prime}\ -\ \delta\varepsilon^i = 0. \tag{30}$$

$$\delta f_2 = \delta w^{i\prime} + (1 + \varepsilon^i)\delta\varphi^i = 0, \tag{31}$$

$$\delta f_3 = \delta\varphi^{i\prime} - \delta\kappa^i = 0, \tag{32}$$

$$\delta f_4 = \delta\mathcal{H}^{i\prime} = 0, \tag{33}$$

$$\delta f_5 = \delta\mathcal{V}^{i\prime} = 0, \tag{34}$$

$$\delta f_6 = \delta\mathcal{M}^{i\prime} - (1+\varepsilon^i)\delta\mathcal{Q}^i = 0. \tag{35}$$

Table 2 The fundamental solutions

	boundary condition	$\varepsilon^i(x)$	$u^i(x)$	$\mathcal{N}^i(x) = \mathcal{H}^i(x)$
(1)	$u^i(L) = 0$	0	0	$\sigma^i(\varepsilon^i = 0,\ \kappa^i = 0,\ D^i_{\mathrm{th}}, T^i)\mathrm{A} = \text{const.}$
(2)	$\mathcal{H}^i(L) = -F - \mu_H^* u^i(L)$	const. $\neq 0$	$\varepsilon^i x$	$-F - \mu_H^* \varepsilon^i L$

$^*\mu_H \neq \infty$

$$\delta f_7 = \delta\mathcal{N}^i = \delta\mathcal{H}^i \tag{36}$$

$$\delta f_8 = \delta\mathcal{Q}^i = \delta\mathcal{V}^i + \mathcal{N}^i\delta\varphi^i \tag{37}$$

$$\delta f_9 = \delta\mathcal{N}^i = C_{11}^i(\varepsilon^i,\ \kappa^i = 0,\ D_{th}^i,\ T^i)\ \delta\varepsilon^i + C_{12}^i(\varepsilon^i,\ \kappa^i = 0,\ D_{th}^i,\ T^i)\delta\kappa^i \tag{38}$$

$$\delta f_{10} = \delta\mathcal{M}^i = C_{21}^i(\varepsilon^i,\ \kappa^i = 0,\ D_{th}^i,\ T^i)\ \delta\varepsilon^i + C_{22}^i(\varepsilon^i,\ \kappa^i = 0,\ D_{th}^i,\ T^i)\delta\kappa^i \tag{39}$$

In Eqs. (38) and (39), C_{11}^i, $C_{12}^i = C_{21}^i$, C_{22}^i are the components of the current tangent constitutive matrix of the cross-section. Due to the symmetry of the cross-section about the z-axis, and because the temperature field in the column is uniform, these components assume a rather simple form:

$$C_{11}^i(\varepsilon^i, \kappa^i = 0, D_{\mathrm{th}}^i, T^i) = \frac{\partial\sigma^i}{\partial\varepsilon^i}A = E_{\mathrm{t}}^i A = \mathrm{const.}, \tag{40}$$

$$C_{12}^i(\varepsilon^i, \kappa^i = 0, D_{\mathrm{th}}^i, T^i) = C_{21}(\varepsilon^i, \kappa^i = 0, D_{\mathrm{th}}^i, T^i) = 0, \tag{41}$$

$$C_{22}^i(\varepsilon^i, \kappa^i = 0, D_{\mathrm{th}}^i, T^i) = \frac{\partial\sigma^i}{\partial\varepsilon^i}J = E_{\mathrm{t}}^i J = \mathrm{const.} \tag{42}$$

where $E_t^i = \dfrac{\partial\sigma^i}{\partial\varepsilon^i} = \dfrac{\partial\sigma^i}{\partial D^i}$ is the tangent modulus of steel at time t^i and temperature T^i. The corresponding set of boundary conditions is obtained through the linearization of Eqs. (11)-(16) around the fundamental solution:
bottom, $x = 0$:

$$\delta u^i(0) = 0, \tag{43}$$

$$\delta w^i(0) = 0, \tag{44}$$

$$s_1^1\ \delta\mathcal{M}^i(0) - \rho^1\delta\varphi^i(0), \tag{45}$$

top, $x = L$:

$$s_H\delta\mathcal{H}^i(L) + \mu_H\delta u^i(L) = 0, \tag{46}$$

$$s_1^2\ \delta\mathcal{V}^i(L) + \mu_V\delta w^i(L) = 0, \tag{47}$$

$$s_2^2\ \delta\mathcal{M}^i(L) + \rho^2\delta\varphi^i(L) = 0. \tag{48}$$

After a systematic elimination of the unknowns is made, we end up with the system of two linear differential equations with constant coefficients for δu^i and δw^i:

$$\delta u^{i\prime\prime} = 0, \tag{49}$$

$$\delta w^{iIV} + k^{i2}\delta w^{i\prime\prime} = 0, \tag{50}$$

in which the buckling load parameter k^i is introduced as

$$k^{i^2} = \frac{(1+\varepsilon^i)|\mathcal{N}^i|}{E_t^i J} > 0, \tag{51}$$

The general solutions of Eqs. (49) and (50) are

$$\delta u^i(x) = \mathcal{K}_1^i x + \mathcal{K}_2^i, \tag{52}$$

$$\delta w^i(x) = \mathcal{C}_1^i \cos k^i x + \mathcal{C}_2^i \sin k^i x + \mathcal{C}_3^i x + \mathcal{C}_4^i \;, \tag{53}$$

The unknown integration constants, $\mathcal{K}_1^i$, $\mathcal{K}_2^i$, $\mathcal{C}_1^i$, $\mathcal{C}_2^i$, $\mathcal{C}_3^i$ and $\mathcal{C}_4^i$, in Eqs. (52) and (53) are obtained by imposing the boundary conditions (43)-(48) to the solutions (52) and (53). This way we get a system of six homogeneous linear algebraic equations for six unknown integration constants $\mathcal{K}_1^i$, $\mathcal{K}_2^i$, $\mathcal{C}_1^i$, $\mathcal{C}_2^i$, $\mathcal{C}_3^i$, $\mathcal{C}_4^i$. It is well known that the non-trivial solution of a homogeneous system of linear algebraic equations is possible only if the determinant of the system matrix is zero (e.g. Planinc and Saje 1999, Shanley 1947):

$$\det \boldsymbol{L}_T^i = 0. \tag{54}$$

Because Eqs. (52)-(53) are separated, the determinant of the system matrix, $\det\boldsymbol{L}_T^i$, becomes a product of two independent determinants, $\det\boldsymbol{H}_T^i$ and $\det\boldsymbol{K}_T^i$. Matrix $\boldsymbol{H}_T^i$ depends solely on $\mathcal{K}_1^i$, $\mathcal{K}_2^i$, while matrix $\boldsymbol{K}_T^i$ is expressed only with $\mathcal{C}_1^i$, $\mathcal{C}_2^i$, $\mathcal{C}_3^i$, $\mathcal{C}_4^i$. It is easy to show that $\det\boldsymbol{H}_T^i \neq 0$ for any $\mathcal{K}_1^i$ and $\mathcal{K}_2^i$; therefore, condition (54) requires $\det\boldsymbol{K}_T^i = 0$. For a complete derivation, see, e.g., Hozjan *et al.* (2007), Krauberger *et al.* (2007). Here only the final forms of matrix $\boldsymbol{K}_T^i$ and its determinant are given:

$$K_T^i = \begin{bmatrix} 1 & 0 & 0 & 1 \\ \dfrac{C_{22}^i k^{i2} s_1^1}{1+\varepsilon^i} & \dfrac{k^i \rho^1}{1+\varepsilon^i} & \dfrac{\rho^1}{1+\varepsilon^i} & 0 \\ \mu_V \cos[k^i L] & \mu_V \sin[k^i L] & -\dfrac{C_{22} k^{i2} s_2^1}{(1+\varepsilon^i)^2} + L\mu_V & \mu_V \\ \dfrac{k^i (C_{22}^i s_2^2 k^i \cos[k^i L] + \rho^2 \sin[k^i L])}{1+\varepsilon^i} & \dfrac{k^i(-\rho^2 \cos[kL] + C_{22} s_2^2 k \sin[kL])}{1+\varepsilon^i} & -\dfrac{\rho^2}{1+\varepsilon^i} & 0 \end{bmatrix},$$

$$\det \boldsymbol{K}_T^i = \mathcal{A}^i + \mathcal{B}^i \cos(k^i L) + k_{cr} \mathcal{C}^i \sin(k^i L) \tag{55}$$

where

$$\mathcal{A}^i = 2(1+\varepsilon^i)^2 \mu_V \rho^1 \rho^2$$

$$\mathcal{B}^i = -2(1+\varepsilon^i)^2 \mu_V \rho^1 \rho^2 + k^{i4} C_{22}^2 s_1^2 (s_2^2 \rho^1 + s_1^1 \rho^2) - k^{i2} L (1+\varepsilon^i)^2 C_{22}^i \mu_V (s_2^2 \rho^1 + s_1^1 \rho^2),$$

$$\mathcal{C}^i = C_{22}^i s_2^2 (C_{22}^i k^{i2} s_1^1 (-C_{22}^i k^{i2} s_1^2 + L(1+\varepsilon^i)^2 \mu_V) + (1+\varepsilon^i)^2 \mu_V \rho^1) +$$

$$(C_{22}^{i}s_1^{1}(1+\varepsilon^{i})^2\mu_V + C_{22}^{i}k^{i2}s_1^{2}\rho^{1} - L(1+\varepsilon^{i})^2\mu_V\rho^{1})\rho^{2}.$$

Finally, to determine the buckling load, the following set of non-linear algebraic equations for the three unknowns - axial force $\mathcal{N}_{cr}$, axial strain ε_{cr} and temperature T_{cr}- has to be solved:

$$\det \boldsymbol{K}_T^{i} = 0 \tag{56}$$

$$\mathcal{N}_{cr} + F_{cr} + \mu_H \varepsilon_{cr} L = 0 \tag{57}$$

$$\mathcal{N}_{cr} - \sigma_{cr} A = 0 \tag{58}$$

For a particular case of the fully axially restrained and unloaded column ($\mu_H = \infty$, $F_{cr} = 0$), the buckling occurs due to the increase of temperature; thus, $\varepsilon_{cr} = 0$ and $F_{cr} = 0$, and the buckling load follows from Eqs. (56) and (58). This time the only unknowns are $\mathcal{N}_{cr}$ and T_{cr}. Generally, however, Eqs. (56)-(58) are solved with the Newton iterative solution method for three critical values, T_{cr}, $\mathcal{N}_{cr}$ and ε_{cr}. The determination of the related critical time t_{cr} is described in, e.g., Hozjan *et al.* (2007). Once the critical temperature is known, the critical time is extracted from Fig. 2(b) for the chosen cross-section and the fire curve (in this case the ISO 834 fire curve).

4 Restrained columns

We consider steel columns with translational and rotational end restraints (Fig. 4). Using this type of boundary conditions, we can approximately, but with a sufficient accuracy model the stability of columns in unbraced (RCA), partially braced (RCB) and totally braced (RCC) frames (Kishi *et al.* 1998, Liu and Xu 2005, Mahini and Seyyedian 2006, White and Hajjar 1997, Xu and Liu 2002). The

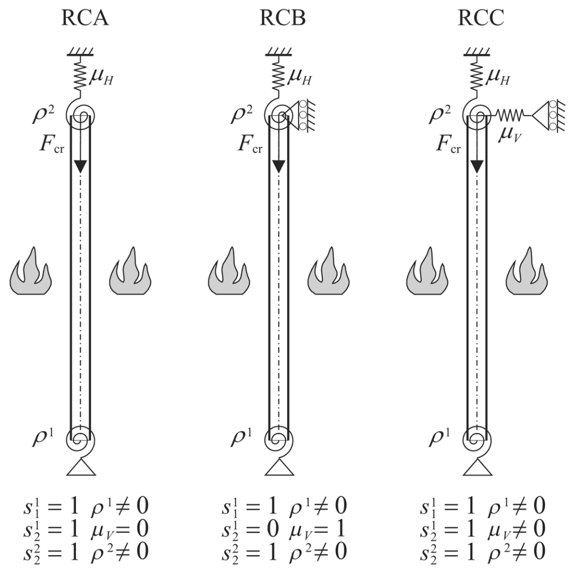

Fig. 4 Steel columns with end restraints

methodology for the calculation of translational and rotational spring stiffnesses of columns as parts of a structure is discussed in literature (Xu *et al.* 2001, Xu *et al.* 2002).

The consideration of specific boundary conditions of the columns RCA in Eq. (55) gives

$$\det\boldsymbol{K}_T = \frac{1}{(1+\varepsilon_{cr})^4}\Big(C_{22,cr}k_{cr}^4(C_{22,cr}k_{cr}(\rho^1+\rho^2)\cos(k_{cr}L) + (-C_{22,cr}^2 k^{i2}_{cr} + \rho^1\rho^2)\sin(k_{cr}L)\Big) = 0\,. \quad (59)$$

Expressions of $\det\boldsymbol{K}_T$ for RCB and RCC columns are similar in form; for the RCB column we obtain

$$\det\boldsymbol{K}_T = \frac{1}{(1+\varepsilon_{cr})^2}\Big((k_{cr}(2\rho^1\rho^2 - (2\rho^1\rho^2 + C_{22,cr}k_{cr}^2L(\rho^1+\rho^2))\cos(k_{cr}L) + k_{cr}^2(C_{22,cr}(C_{22,cr}k_{cr}^2L+\rho^1)$$
$$+ (C_{22,cr} - L\rho^1)\rho^2\big)\sin(k_{cr}^iL)\Big)=0 \quad (60)$$

and for the RCC column we have

$$\det\boldsymbol{K}_T = \mathcal{A} + \mathcal{B}\cos(k_{cr}L) + k_{cr}\,\mathcal{C}\,\sin(k_{cr}L) = 0, \quad (61)$$

where

$$\mathcal{A} = 2(1+\varepsilon_{cr})^2\mu_V\rho^1\rho^2,$$

$$\mathcal{B} = -2(1+\varepsilon_{cr})^2\mu_V\rho^1\rho^2 + k_{cr}^4C_{22,cr}^2(\rho^1+\rho^2) - k_{cr}^2L(1+\varepsilon_{cr})^2C_{22,cr}\mu_V(\rho^1+\rho^2),$$

$$\mathcal{C} = C_{22,cr}(C_{22,cr}k_{cr}^2(-C_{22,cr}k_{cr}^2 + L(1+\varepsilon_{cr})^2\mu_V) + (1+\varepsilon_{cr})^2\mu_V\rho^1) +$$
$$(C_{22}(1+\varepsilon_{cr})^2\mu_V + C_{22,cr}k_{cr}^2\rho^1 - L(1+\varepsilon_{cr})^2\mu_V\rho^1)\rho^2\,.$$

If we consider the linear elastic material along with the inextensibility condition $\varepsilon_{cr} = 0$, Eqs. (59)-(61) can be further simplified:

RCA column:

$$\det\boldsymbol{K}_T = E_{s,T}\,J\,k_{cr}\,(\rho^1+\rho^2)\cos(k_{cr}L) + (\rho^1\rho^2 - E_{s,T}^2\,J\,k_{cr}^2)\,\sin(k_{cr}L) = 0; \quad (62)$$

RCB column:

$$\det\boldsymbol{K}_T = 2\rho^1\rho^2 - \big(2\rho^1\rho^2 + E_{s,T}Jk^2_{cr}L(\rho^1+\rho^2)\big)\cos(k_{cr}L) + k_{cr}\big(E_{s,T}J(E_{s,T}Jk_{cr}^2L+\rho^1) + (E_{s,T}J - L\rho^1)\rho^2\big)\sin(k_{cr}L) = 0 \quad (63)$$

RCC column:

$$\det\boldsymbol{K}_T = \mathcal{A} + \mathcal{B}\cos(k_{cr}L) + k_{cr}\,\mathcal{C}\,\sin(k_{cr}L) = 0; \quad (64)$$

this time, $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ are given as

$$\mathcal{A} = 2\mu_V\,\rho^1\rho^2\,;$$

$$\mathcal{B} = -2\mu_V \rho^1 \rho^2 + k_{cr}^4 (E_{s,T} J)^2 (\rho^1 + \rho^2) - k_{cr}^2 L^2 E_{s,T} J \mu_V (\rho^1 + \rho^2),$$

$$\mathcal{C} = E_{s,T} J \,(E_{s,T} J k_{cr}^2 (-E_{s,T} J k_{cr}^2 + L\mu_V) + \mu_V \rho^1) + (E_{s,T} J \mu_V + E_{s,T} J k_{cr}^2 \rho^1 - \mathrm{L} \mu_V \rho^1)\rho^2$$

As we expected, expressions (62)-(64) agree with those presented in Wang *et al.* (2004). In what follows we analyse effects of various end restraints on the buckling resistance of steel columns in fire. Columns are taken to be made of section HEA 300 and steel S 235. We are particularly interested in the effective buckling length factor K (Cheong-Siat-Moy 1997), defined as

$$L_u = KL. \tag{65}$$

This factor will be determined from Eqs. (56)-(58). For a simply supported column, the condition det$\boldsymbol{K}_T$ = 0 can be further simplified; its solution is $k_{cr}L = \pi$ (Hozjan *et. al* 2007). The critical force $\mathcal{N}_{cr}$ is determined from Eq. (58). The critical temperature is found from Eq. (51). So for $\mu_H \neq \infty$, we can replace the condition det$\boldsymbol{K}_T$ = 0 with

$$(1 + \varepsilon_{cr})\,|\mathcal{N}_{cr}| = \frac{E_t J \pi^2}{L_u^2}. \tag{66}$$

and for $\mu_H = \infty$ with

$$\mathcal{N}_{cr} = A\,|\sigma_{cr}| = \frac{E_t J \pi^2}{L_u^2}. \tag{67}$$

The influence of the material model of steel at high temperatures on the effective buckling length factor is depicted in Fig. 5. The analysis has been made for the RCB column ($\mu_H = 0$, $\mu_V = \infty$). The variation of the effective buckling length factor with respect to spring constants ρ^1 and ρ^2 is shown for $T_{cr} = 300^{\circ}$C and $T_{cr} = 600^{\circ}$C. It is changing from $K = 0.5$ (fixed-fixed column) to $K = 1$ (pinned-pinned column). For both an elastic (Figs. 5a and 5b) and plastic column (Figs. 5c and 5d), the buckling length L_u decreases with temperature. This is, however, an expected result, because the bending stiffness of both elastic and plastic columns decreases with temperature. The variation of the effective buckling length factor with temperature is depicted in Fig. 6 for the column with rotational restraints $\rho^1 = \rho^2 = 1 \cdot 10^5$ kNcm (Fig. 6a) and $\rho^1 = \rho^2 = 5 \cdot 10^6$ kNcm (Fig. 6b). We see that the buckling length is bigger in case of the purely linear elastic material. In case of less stiff rotational springs, the change of the buckling length with temperature is more pronounced for the non-linear material (Fig. 6a), while for more stiff rotational restraints this is not the case (Fig. 6b). For sufficiently stiff rotational springs and for the non-linear material model (Fig. 6b), the effective buckling length factor is insensitive to the temperature change. For this particular column, the material failure may take place prior to buckling, if material is assumed plastic and the temperature lower than 200°C. This is to be contrasted to EC 3, where the effective buckling length factor for intermediate storeys is taken to be 0.5. Such an assumption is reasonable only if top and bottom rotational springs are sufficiently stiff. Thus EC 3 may yield unsafe results with the possible difference being as much as about 35%,if the rotational springs are not sufficiently stiff. As we can observe from Figs. 7(e), 7(f), 8(e), 8(f), 9(e) and 9(f), the buckling length factor for braced frames ($\mu_V = \infty$) varies from 1 to 0.5 for any axial restraint. The change of the

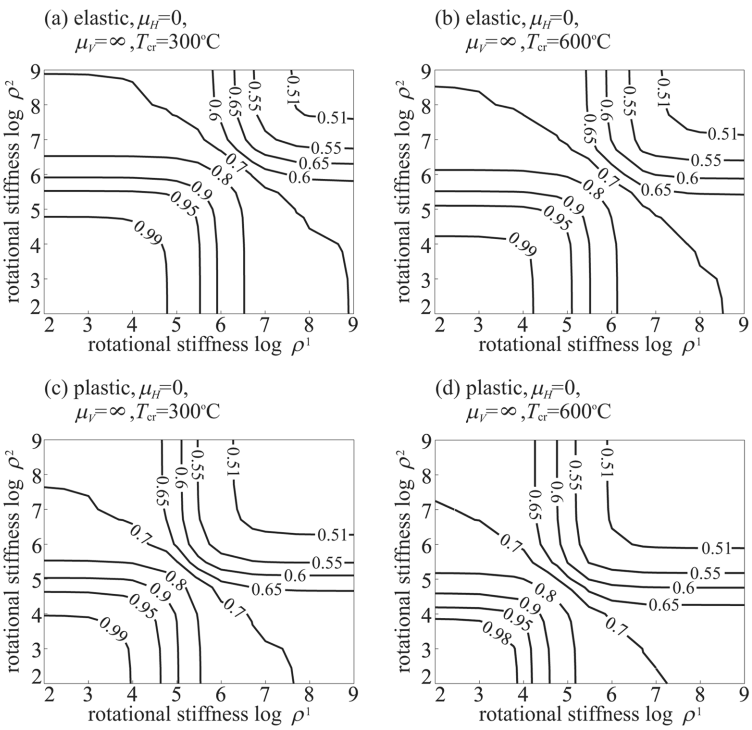

Fig. 5 Restrained steel columns. Variation of effective buckling length factor K with log ρ^1 and log ρ^2 of RCB column for temperatures T_{cr} = 300ºC and T_{cr} = 600ºC. (a) and (b) elastic material model, (c) and (d) plastic material model

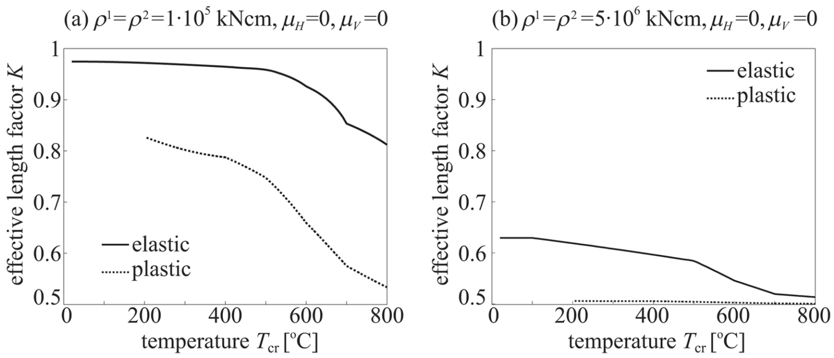

Fig. 6. Variation of buckling length factor K with temperature. (a) $\rho^1 = \rho^2 = 1 \cdot 10^5$ kNcm, (b) $\rho^1 = \rho^2 = 5 \cdot 10^6$ kNcm.

buckling lengths with temperature is smaller, if the rotational springs are more stiff. This indicates that the constant value, 0.5, for the intermediate storeys can be used indeed, if the remaining, unheated part of the structure can offer a sufficient rotational stiffness to the column, which occurs when fire remains localized in a separate compartment with a sufficiently high fire resistance. According to EC 3 (Eurocode 2003), this is achieved when the fire resistance of building components of the compartment are not less

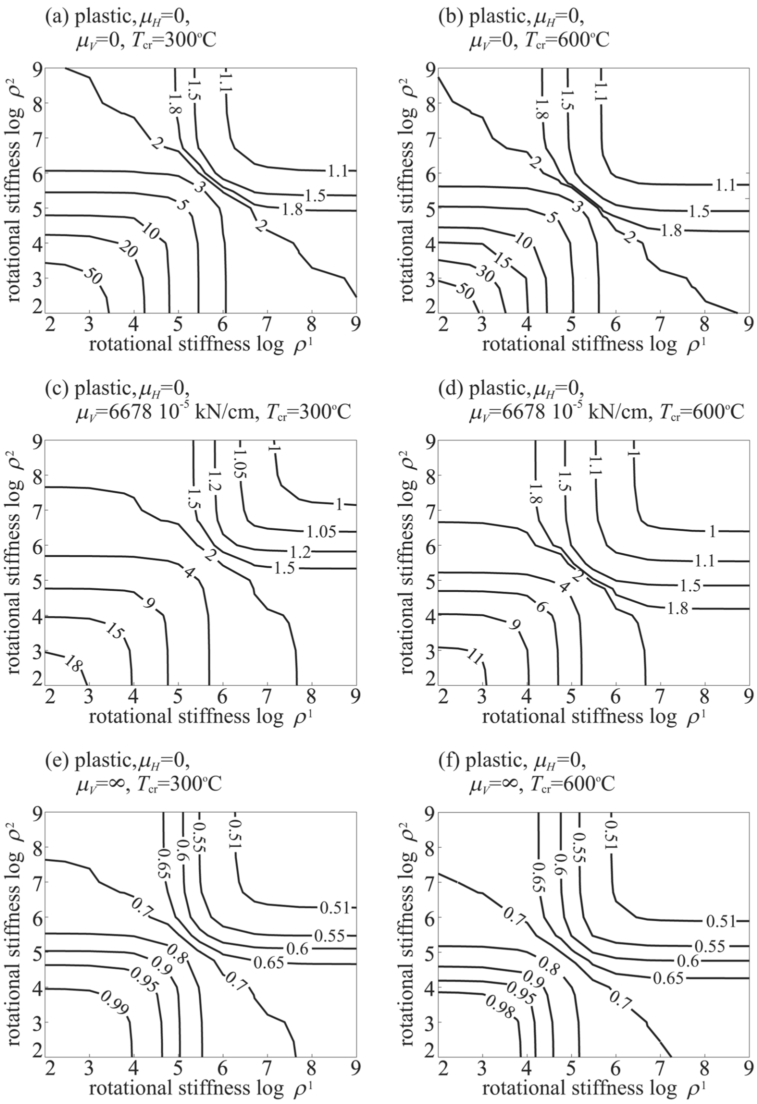

Fig. 7 Restrained steel columns. Variation of effective buckling length factor K with log ρ^1 and log ρ^2 of axially unrestrained column ($\mu_H = 0$) for temperatures $T_{cr} = 300$°C and $T_{cr} = 600$°C. (a) and (b) $\mu_V = 0$, (c) and (d) $\mu_V = 6678 \cdot 10^{-5}$ kN/cm, (e) and (f) $\mu_V = \infty$

fire resistant than the resistance of the column.

Next we analyze the influence the combined effect of the axial μ_H, transverse μ_V and rotational restraints ρ^1 and ρ^2 on the buckling resistance of the column. The analysis is performed for two temperatures, $T_{cr} = 300$°C and $T_{cr} = 600$°C, and two rotational restraints, $\rho^1 = \rho^2 = 1 \cdot 10^6$ kNcm, unless stated otherwise. Results for the axially unrestrained column ($\mu_H = 0$) are depicted in Fig. 7. For the transversely unsupported columns ($\mu_V = 0$), the effective buckling length factor varies from 1 to ∞ (Figs. 7a and 7b). Again, we can observe only a minor decrease of K with temperature. This also holds true for the partly or fully transversely restrained columns (Figs. 7c-7f). Here the change of the effective buckling length factor is about 16% and 3%, respectively. We see that the variation of the buckling length with temperature is most intense for the partly transversely and the smallest for fully transversely restrained columns, where K varies from 0.5 for fixed-fixed column to $K = 1$ for pinned-pinned columns. The buckling length also decreases with the increase of the stiffness of the transversal restraint. The decrease is particularly important for the fully transversely restrained column, resulting in the difference of about 50% for both temperatures, 300ºC and 600ºC. A similar effect of spring constants μ_V, ρ^1 and ρ^2 on the buckling length of columns can be observed for axially restrained columns with $\mu_H = 667.8$ kN/cm (Fig. 8), and for fully axially restrained columns, $\mu_H = \infty$ (Fig. 9). For an axially restrained column with

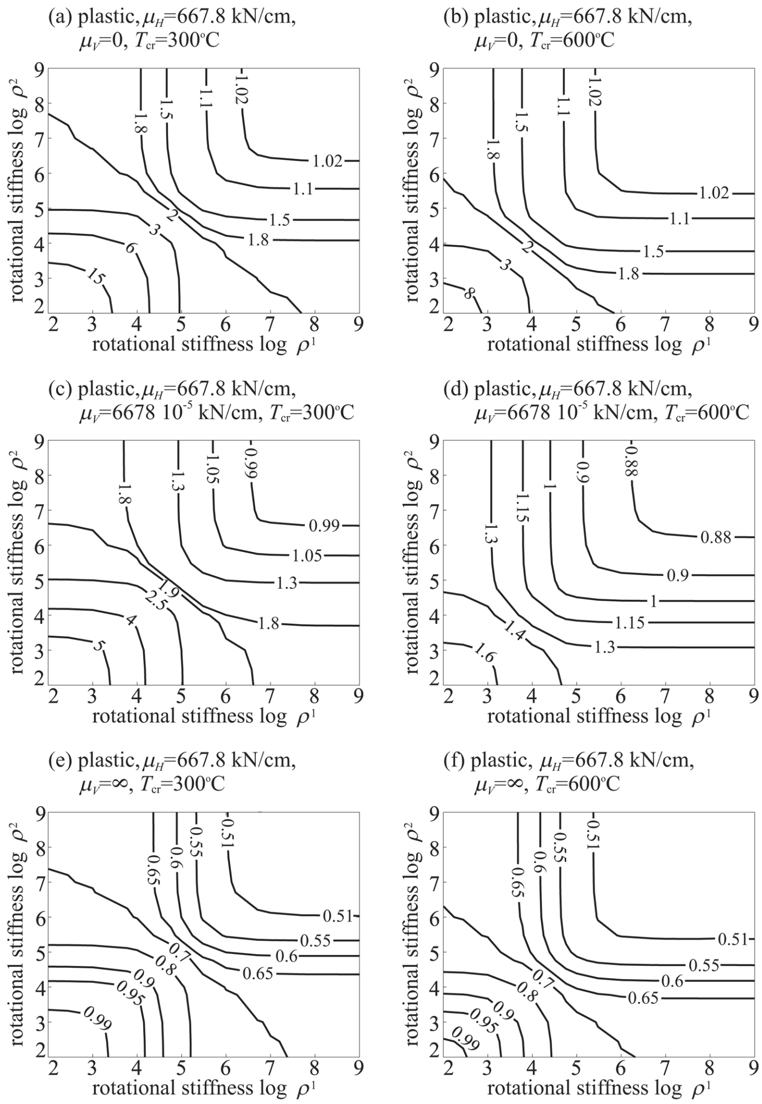

Fig. 8 Restrained steel columns. Variation of effective buckling length factor K with log ρ^1 and log ρ^2 of axially restrained column ($\mu_H = 667.8$ kN/cm) for temperatures $T_{cr} = 300$ºC and $T_{cr} = 600$ºC. (a) and (b) $\mu_V = 0$, (c) and (d) $\mu_V = 6678 \cdot 10^{-5}$ kN/cm, (e) and (f) $\mu_V = \infty$

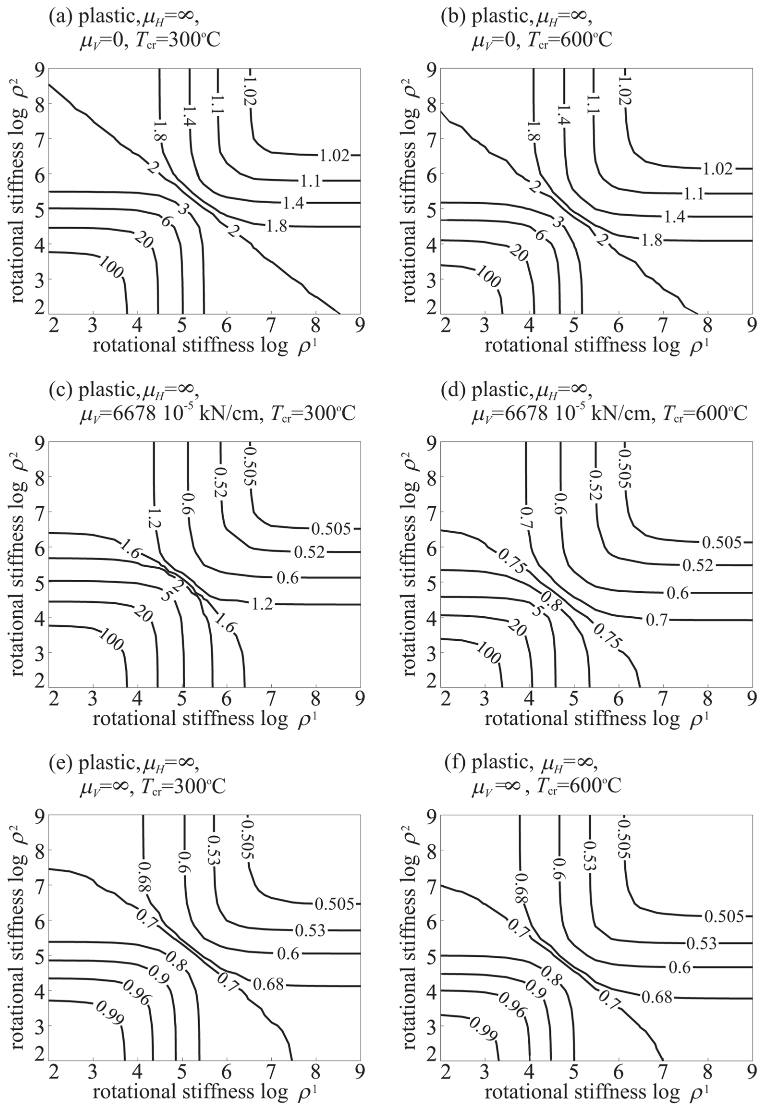

Fig. 9 Restrained steel columns. Variation of effective buckling length factor K with log ρ^1 and log ρ^2 of axially fully restrained column ($\mu_H = \infty$) for temperatures $T_{cr} = 300^\circ$C and $T_{cr} = 600^\circ$C. (a) and (b) $\mu_V = 0$, (c) and (d) $\mu_V = 6678 \cdot 10^{-5}$ kN/cm, (e) and (f) $\mu_V = \infty$

$\mu_H = 667.8$ kN/cm, the change of the buckling length with temperature is roughly 5%, if $\mu_V = 0$, 15% if $\mu_V = 0.6678$ kN/cm and 3% if $\mu_V = \infty$. For a fully axially restrained column, these figures are: 6% if the column is not restrained ($\mu_V = 0$), 3% if $\mu_V = 0.6678$ kN/cm and 3% if $\mu_V = \infty$. As in the case of axially unrestrained columns, the decrease of K due to the transverse restraint is roughly the same, i.e., about 50% for both types of axial restraints and temperatures. Figs. 7(a), 8(a) and 9(a) ($T_{cr} = 300^\circ$C) and Figs. 7(f), 8(f) and 9(f) ($T_{cr} = 600^\circ$C) show the influence of the axial restraint on the variation of the buckling length. If the stiffness of the axial restraint increases, the buckling length of the column decreases. Therefore, for the same buckling resistance in terms of the critical temperature, a more stiff axial restraint and a higher load F_{cr} are required, leading to a higher axial force $\mathcal{N}_{cr}$. Generally, the influence the axial restraint is much more important, if the column is unsupported in the transverse direction, in contrast to the only transversely supported column, where the effect is practically negligible. An overall decrease of K due to the axial restraint is, however, small, and ranges from1% to 12%

5. Conclusions

As the result of the present extensive parametric analyses, the following findings can be stated:

- The effective buckling length factor decreases with temperature. The decrease is also affected by the stiffness of rotational springs. The highest decrease of about 35% has been found with the non-linear material model.
- The (Eurocode 2003) rule on the buckling length, L_u = 0.5L, for the intermediate storeys is found reasonable for rather stiff rotational restraints, and unsafe otherwise. Substantial differences for the buckling length between the present and the EC 3 values have been found (up to 35%).
- The application of the non-linear material model results in smaller buckling lengths compared to those of the linear material model.
- The buckling length also decreases with the increase of the stiffness of lateral restraint, the highest decrease being as much as 50%.

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Notation

μ_H stiffness of horizontal translational restraint at the top of the column

μ_V stiffness of vertical translational restraint at the top of the column

F compression force

x, y, z coordinates

L undeformed length of a column

t^i time at time step i

u^i udisplacement in x direction at time step i

w^i displacement in z direction at time step i

φ^i cross-sectional rotation around y at time step i

ε^i axial strain at time step i

κ^i bending strain at time step i

$\mathcal{N}^i$ axial force at time step i

$\mathcal{Q}^i$ shear force at time step i

$\mathcal{M}^i$ bending moment at time step i

$\mathcal{H}^i, \mathcal{V}^i$ x and z components of the resulting cross-sectional force at time step i

s_1^1, s_1^2, s_2^2 parameters deng restraints $\{0,1\}$

D^i strain at time step i

ΔD^i strain increment at time step i

ΔD_{th}^i temperature strain increment at time step i

ΔD_σ^i stress dependent strain increment at time step i

ΔD_{e}^i elastic strain increment at time step i

ΔD_{P}^i plastic strain increment at time step i

$k_{\text{p},T}$, $k_{\text{y},T}$ and $k_{E,T}$ reduction factor for steel at elevated temperatures according to EC 3

$C_{11}^i, C_{12}^i = C_{21}^i, C_{22}^i$ components of the tangent constitutive matrix of the cross-section

E_t^i tangent modulus of steel at time step i

D_{th}^i temperature strain at time step i

J moment of inertia of cross-section

A area of cross section

T^i temperature at time step i

σ^i stress at time step i

k^i buckling load parameter at time step i

$\mathcal{K}_1^i, \mathcal{K}_2^i, \mathcal{C}_1^i, \mathcal{C}_2^i, \mathcal{C}_3^i$ *and* $\mathcal{C}_4^i$ integration constants at time step i

$\det \boldsymbol{L}_T^i$ determinant of system matrix at time step i

$\det \boldsymbol{H}_T^i$, $\det \boldsymbol{K}_T^i$ determinants of submatrices $\boldsymbol{H}_T^i$, $\boldsymbol{K}_T^i$ at time step i

$\mathcal{N}_{cr}$ critical axial force

F_{cr} critical compression force

ε_{cr} critical deformation

σ_{cr} critical stress

L_u buckling length

K buckling length factor

ρ^1 stiffness of lower rotational restraint

ρ^2 stiffness of upper rotational restraint

T_{cr} critical temperature

DN