Steel and Composite Structures, Vol. 7, No. 5 (2007) 405-419
DOI: http://dx.doi.org/10.12989/scs.2007.7.5.405

Behaviour of axially loaded RC columns strengthened by steel angles and strips

J. M. Adam

ICITECH, Departamento de Ingenieria de la Construccion y Proyectos de Ingenieria Civil, Universidad Politecnica de Valencia, Camino de Vera s/n, 46071 Valencia, Spain

S. Ivorra†

Departamento de Ingenieria de la Construccion, OO.PP. e Infraestructura Urbana, Universidad de Alicante, Apartado de Correos 99, 03080 Alicante, Spain

E. Giménez, J. J. Moragues and P. Miguel

ICITECH, Departamento de Ingenieria de la Construccion y Proyectos de Ingenieria Civil, Universidad Politecnica de Valencia, Camino de Vera s/n, 46071 Valencia, Spain

C. Miragall

Departamento de Mecanica de los Medios Continuos y Teoria de Estructuras, Universidad Politecnica de Valencia, Camino de Vera s/n, 46071 Valencia, Spain

P. A. Calderón

ICITECH, Departamento de Ingenieria de la Construccion y Proyectos de Ingenieria Civil, Universidad Politecnica de Valencia, Camino de Vera s/n, 46071 Valencia, Spain
(Received August 8, 2007, Accepted October 22, 2007)

Abstract. This paper presents the development of some numerical models based on the results of laboratory tests performed on axially loaded RC columns strengthened with steel angles and strips. These numerical models consider the nonlinearity of the building materials and the effects of the contact interfaces between different materials. The results of the finite element models accurately describe the general behaviour of the strengthened columns. This study allows engineers to assess the relative importance of the mechanisms acting on the strengthened RC columns. Constructive recommendations are also provided in this paper.

Keywords: RC columns; strengthening; steel angles; strips; finite element modelling; nonlinear analysis.

---

†Corresponding author, E-mail: sivorra@ua.es

1. Introduction

Structural retrofitting is becoming more and more frequent to maintain, repair or improve the load-bearing capacity of a structure. Columns are the elements that support building structures; thus failures in column behaviour may lead to collapse of the whole structure. Column strengthening is therefore a very important aspect in a building structure.

There are different strengthening techniques for reinforced concrete (RC) columns; the more widely used being concrete jacketing, composite-based strengthening systems (FRP) and the use of steel jacketing.

Steel jacketing is one of the most common strengthening techniques (Tamai *et al*. 2000, Wu *et al*. 2006). In Europe, one of the most common strengthening techniques is based on the use of steel angles and strips (Cirtek, 2001a, Ramírez 1996, Adam *et al*. 2005, Giménez *et al*. 2006, Adam *et al*. 2006). In this kind of strengthening technique it is essential to guarantee the correct load transmission from the beam to the strengthened column. There are two ways of designing the column-beam connection. One of them consists of welding steel capitals to the ends of the strengthening angles attached to the lower part of the beam, so that the beam load is transferred to the strengthened column through the strengthening element. The other way of designing the column-beam connection is without these steel capitals.

The usual method for designing this kind of strengthening consists of using steel capitals, although some literature on the topic does not recommend their use in the following situations (Regalado 1999):

1. When beam resistance is insufficient to absorb the loads transmitted by the steel capitals
2. When columns are individually strengthened with no continuity between floors
3. In seismic areas.

In these cases, the column ends are banded using steel strips of adequate dimensions and spacing.

Despite the research works on strengthened RC columns using steel angles and strips (Cirtek 2001a, Cirtek 2001b, Ramírez 1996, Ramírez and Bárcena 1975, Ramírez *et al*. 1977), the effects of the mechanisms that act on the strengthened column are still unknown.

With the purpose of characterising the behaviour of axially loaded RC columns strengthened by steel angles and strips, full-size specimens were tested at the laboratory of the Institute of Concrete Science and Technology (ICITECH) of the Technical University of Valencia (Giménez *et al*. 2004, Calderón *et*

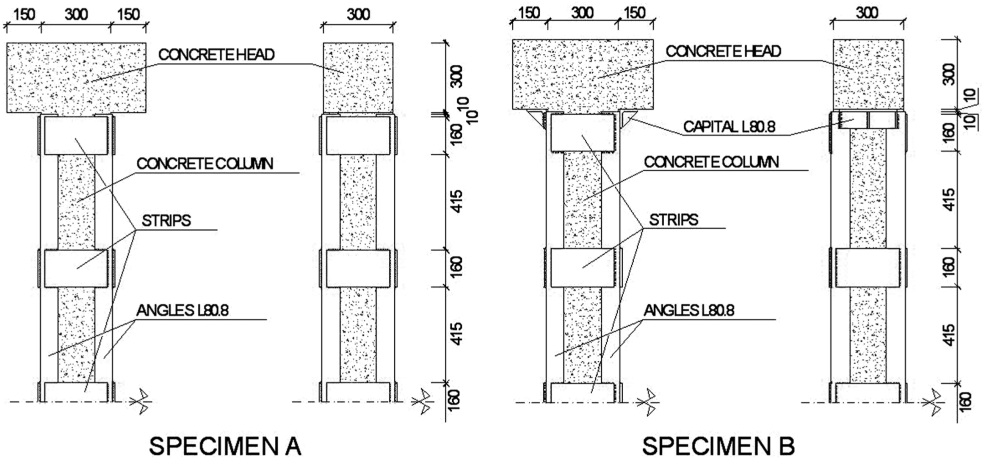

Fig. 1 Geometry of specimens A and B (dimensions in mm)

al. 2006). This paper presents also the numerical analysis of the specimens used to determine the influence of the following factors on the strengthened structural element:

1. Presence or absence of steel capitals at the column ends for direct load transmission.
2. Effect of confinement by the steel strips.
3. Effect of the interface material between the steel of the strengthening element and the concrete mixture of the column on shear stress transmission.

2. Experimental study

2.1 Specimen layout

Full-size 2500 mm long and 300 × 300 mm cross section columns were tested. Reinforced concrete heads, of dimensions 300 × 300 × 600 mm, were placed at the top and bottom of the column, simulating beam-column connection, as shown in Fig. 1.

The column heads and the column were built using two different types of concrete. The concrete mixture used in the column heads had a compressive strength of 90 MPa, and the concrete used in the column had variable compressive strength ranging between 10.6 and 15.5 MPa, depending on the sample specimen. The reason for using high strength concrete (HSC) for the column heads was to avoid either crushing or cracking by stress concentration in the areas near the load-bearing points. The reinforcement consisted of four 12 mm diameter longitudinal rods with 6 mm diameter cross ties every 0.20 mm. It should be emphasized that the reinforcement used was the minimum permitted under Spanish regulations (Ministerio de Fomento 1998) for RC columns and is very close to most international codes (CEB-FIB 1991, Eurocode No. 2 1991). The columns were strengthened by angles L80.8 and rectangular strips 270 × 160 × 8 mm, of steel type Fe430 (Eurocode No. 3 1993). The strips were welded to the angles.

Three types of specimens, called Test, A and B, were used in the experiments, and two specimens of each type were tested at the laboratory. The specimen Test consisted of a non-strengthened column, and specimens A and B differed in the type of joint between the steel strengthening and the concrete heads.

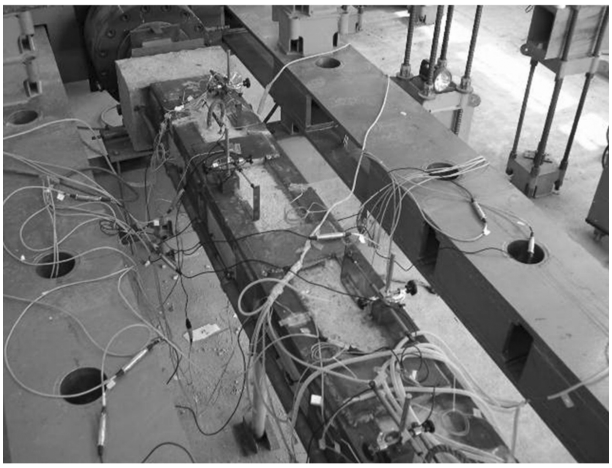

Fig. 2 View of the testing position of the specimen

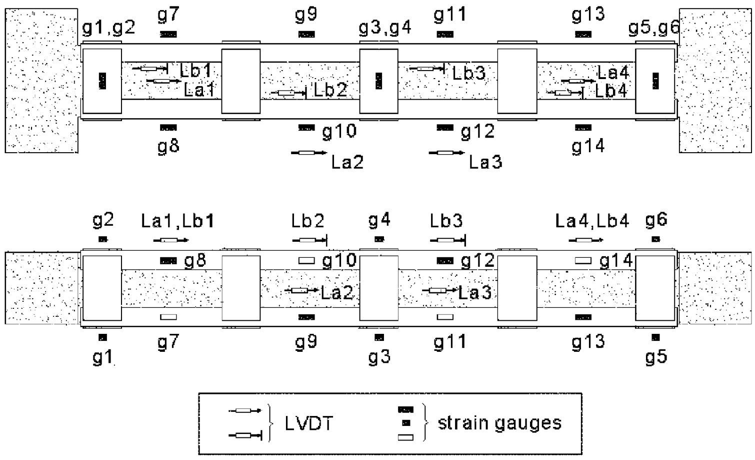

Fig. 3 Instrumentation used in the tests

In specimen A there is no connection between the strengthening elements and the heads, the strengthening elements ending with strips welded to the angles. In specimen B, the connection between the strengthening elements and the heads consisted of capitals formed by steel L80.8 angles, with no steel strips placed in the area of the capitals. Fig. 1 shows a detailed illustration of the ends of the strengthening elements. The strengthening steel elements were bonded to the column with cement mortar.

2.2 Instrumentation and testing procedure

Each set of specimens was placed horizontally on a steel frame and loaded with a hydraulic jack capable of applying a maximum force of 5000 kN. The specimens were tested when the concrete had reached an age over 60 days. Load was applied in a displacement control mode with a constant rate of 0.5 mm/min.

Each specimen was fitted with a minimum of 14 strain gauges mounted on the steel strengthening

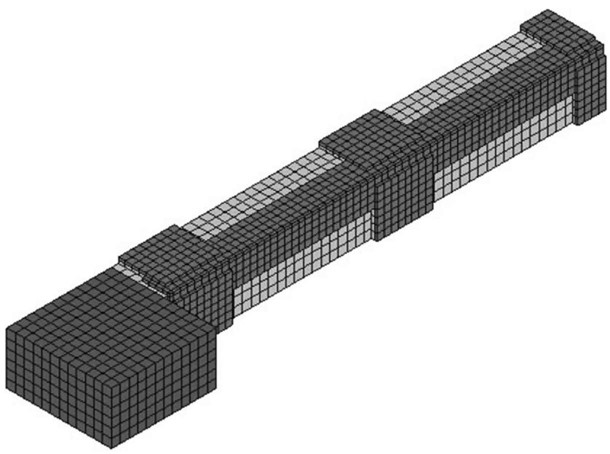

Fig. 4 Finite element model

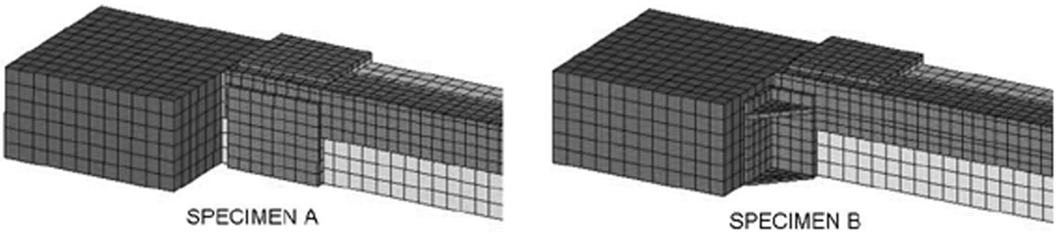

Fig. 5 Detail of the ends of specimens A and B

elements and with 8 linear variable differential transducers (LVDT) to measure the lateral and axial strains in steel strengthening and in concrete. Four LVDT were used for measuring axial strain in concrete deformations and other four LVDT for measuring the slip displacement between the column concrete and the strengthening steel angles. Fig. 2 shows an illustration of the steel frame with the specimen in the testing position, and Fig. 3 shows the instrumentation used in the tests.

3. Numerical study

3.1 Description of the elements used, boundary conditions and loads applied

A numerical study was carried out through the finite element method (FEM) to analyse the specimens tested at the laboratory. The ANSYS (2005) general-purpose FE program was used in this study. Only the top 1/8 of the specimen was modelled due to the symmetry of the loads and geometry, as shown in Fig. 4. Fig. 5 shows a detail of the ends of specimens A and B 8-noded brick SOLID45 elements were used to model the concrete of the specimen ends, with a 25 × 25 × 25 mm finite element (FE) mesh. The strengthening steel and the cement mortar of the interface bond were modelled using 20-node solid hexahedral SOLID95 elements, with a 20 × 20 × 8 mm FE mesh. Both SOLID45 and SOLID95 allow consider of nonlinearity caused by material behaviour.

The column concrete was modelled with 8-node solid hexahedral SOLID65 elements, with a 25 × 25 × 25 mm FE mesh. SOLID65 consider concrete cracking and crushing, as well as other nonlinearities. For reinforcement simulation, either LINK8 elements or smeared procedures inside the elements can be used. According to recent research works (Barbosa and Ribeiro 1998, Erduran and Yakut 2004) both techniques yield similar results. In our analysis, the smeared procedure was adopted due to its numerical simplicity.

The contact between the strengthening steel and the cement mortar was simulated by contact elements TARGET170 and CONTACT174. These elements allow the development of a Coulomb friction law, enabling the separation of the elements in the interface under tensile stress. On the other hand, the contact between the cement mortar of the interface bond and the concrete column was considered as a fixed contact. This supposition is based on the results obtained at the laboratory, in which mortar and concrete remained bonded.

Loading was applied in a displacement control mode to simulate the axial loads at the ends of the specimens, recording the loading level for each offset increment. Due to the nonlinear nature of the models used, the iterative Newton-Raphson model was used in this study.

3.2 Properties of the materials and constitutive models

3.2.1 Concrete elements

Concrete used at the specimen heads was considered to be a linear elastic material with Poisson ratio $\upsilon = 0.2$ and with the elastic modulus recommended in ACHE (2004), defined in Eq. (1).

$$E_{ci} = 11000(f_c)^{0.3} \tag{1}$$

Column concrete was considered to be a nonlinear material based on Drucker and Prager (1952) yield criterion. Unlike other yield criterion such as Von Mises, the yield criterion adopted in this study considers the influence of triaxial compressive stresses. The Poisson ratio used for this concrete was $\upsilon = 0.2$, whereas for the elastic modulus the coefficient recommended in CEB-FIB (1991) are used:

$$E_{ci} = E_{co}(f_c/f_{cmo})^{1/3} \tag{2}$$

For the Drucker Prager yield criterion is necessary to know the values of c, $\O$ and $\O_r$. There are different expressions to determine the values of c and $\O$; in this study are used Eqs. (3) and (4)

$$f'_{co} = \frac{2c\ \cos\phi}{1-\sin\phi} \tag{3}$$

$$k_1 = \frac{1+\sin\phi}{1-\sin\phi} \tag{4}$$

Factor k_1 is used to calculate compressive strength for concrete under lateral confinement pressure. There are other expressions to estimate the values of c and $\O$, like those reported in Rochette and Labossière (1996) or Adam *et al*. (2007), which do not consider the influence of factor k_1. As mentioned above, in this study are used expressions (3) and (4) because these expressions showed a better approximation to the real behaviour of the specimens tested at the laboratory.

Most of the confinement models propose a compressive strength for confined concrete as:

$$\frac{f'_{cc}}{f'_{co}} = 1 + k_1\frac{f_1}{f'_{co}} \tag{5}$$

Eq. (5) was first proposed by Richart *et al*. (1928) with $k_1 = 4.1$, whereas Miyauchi *et al*. (1999) proposed $k_1 = 2.98$ for FRP strengthened RC columns. In our FE model is used $k_1 = 3$, similar to the value employed by Mirmirian *et al*. (2000) and Johansson and Akesson (2001). Once the values of k_1 and f'_{co} for each of the sample specimens were known, are obtained the values of c and $\O$ presented in Table 1. In all models is adopted $\O_r$=0.

The steel of the reinforcement had an elastic modulus $E_s = 210000$ MPa, a Poisson ratio $\upsilon = 0.3$ and an elastic limit $f_y = 400$ MPa, using Von Mises yield criterion.

3.2.2 Cement mortar

The cement mortar used to bond the concrete material and the steel strengthening elements was considered to have a linear elastic behaviour, with an elastic modulus $E_m = 25000$ MPa and a Poisson ratio $\upsilon = 0.2$. When is analyzed the behaviour of cement mortar during the laboratory tests, is observed that it never failed, presenting negligible deformation. Therefore, it is reasonable to consider that it is a linear elastic material.

Table 1 Concrete material properties used in the FE analysis

Specimen	f_c (MPa)	Drucker Prager parameters			Elastic parameters	
		c (MPa)	ϕ (º)	ϕ_r (º)	E_{ci} (GPa)	υ
A	15.5	4.47	30	0	24.88	0.2
B	10.6	3.06	30	0	21.92	0.2

3.2.3 Strengthening steel

A rate-independent plasticity material model has been used for the strengthening steel, with no hardening rule and elastic-perfectly plastic behaviour. The material response is bilinear, using the Von Mises yield criterion. The elastic modulus adopted was $E_s = 210000$ MPa, Poisson ratio $\upsilon = 0.3$, and elastic limit $f_y = 275$ MPa.

3.3 Interaction between steel and cement mortar

To simulate the contact between the strengthening steel and the cement mortar, Coulomb's frictional law was used. This model allows simulating the transmission of shear stresses between the connecting elements. According to the Coulomb frictional law, the maximum shear stress to be transmitted through the bonding interface is:

$$\tau = a + \mu p \tag{6}$$

The friction coefficient between concrete and strengthening steel ranges between 0.2 and 0.6 (Baltay and Gjelsvik 1990). In our study is developed an iterative procedure for model calibration, obtaining good results with a value $\mu = 0.20$, similar to Ellobody and Young (2006), Ellobody *et al*. (2006), Hu *et al*. (2005), Gupta *et al*. (2007) and Lu *et al*. (2006). Adhesion was considered zero as it only occurs at the first loading levels, that is, when the relative displacements are small with a maximum value close to 0.1 MPa.

4. Experimental results and calibration of the numerical model

For the development of the numerical model, the parameters that a priori present the highest degree of uncertainty are k_1 from which $Ø$ is obtained, and μ, which may range between 0.2 and 0.6. These two parameters, k_1 and μ, are the independent variables in the fitting of the numerical model to the experimental results.

The following experimental relations have been used for model calibration:

- Relation between load applied by the hydraulic jack and column shortening
- Relation between load applied by the hydraulic jack and deformation or stress of the angle's core fibre in the nearest section to the column heads (deformation measured by strain gauges g7, g8, g13 and g14). In this way, load transmission from the RC column to steel angles can be analyzed.
- Relation between load applied by the hydraulic jack and deformation or stress of the steel strips close to the column heads (deformation measured by strain gauges g1, g2, g5 and g6). In this way, confinement provided by the strips can be analyzed in detail.

Figs. 6, 7 and 8 show the results of the experimental tests and the results of the numerical model for different values of k_1 and μ. As can be observed, the best fitting is obtained for values $k_1 = 3$ and

$\mu = 0.20$. As we can see in Figs. 6, 7 and 8, the numerical model accurately characterises the overall behaviour of the columns, the load transmission between the RC column and the steel strengthening, and the effect of strips confinement.

With respect to load transmission from the RC column to the steel angles, it can be observed in Fig. 7 that the numerical model does not fit well at the first loading levels. In the case of specimen B, the differences found between the numerical model and the experimental results are due to the fact that in the sample specimens, the contact between the capital and the concrete head is not perfect as a consequence of the contraction experienced by the capital after welding. This gives lower load values in the steel angles than expected if contact between capital and concrete heads was perfect.

5. Results analysis and discussion

In addition to the aspects afore mentioned, the numerical model accurately reproduces the patterns of column behaviour as observed in the laboratory. Both in the experimental study and in the numerical model failure in specimens A and B occurs at the column head concrete placed between the two steel strips closest to the column heads. As predicted in the numerical model,

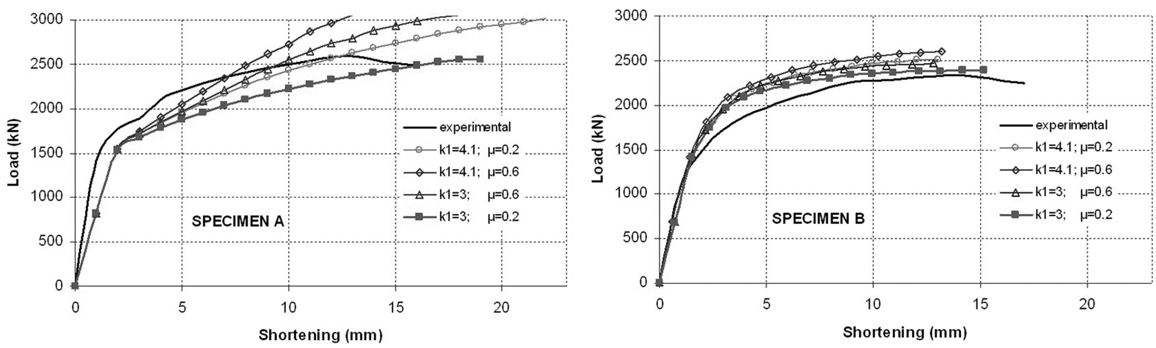

Fig. 6 Relation between hydraulic jack load and column shortening. Comparison of the experimental results and the numerical model for different values of k_1 and μ

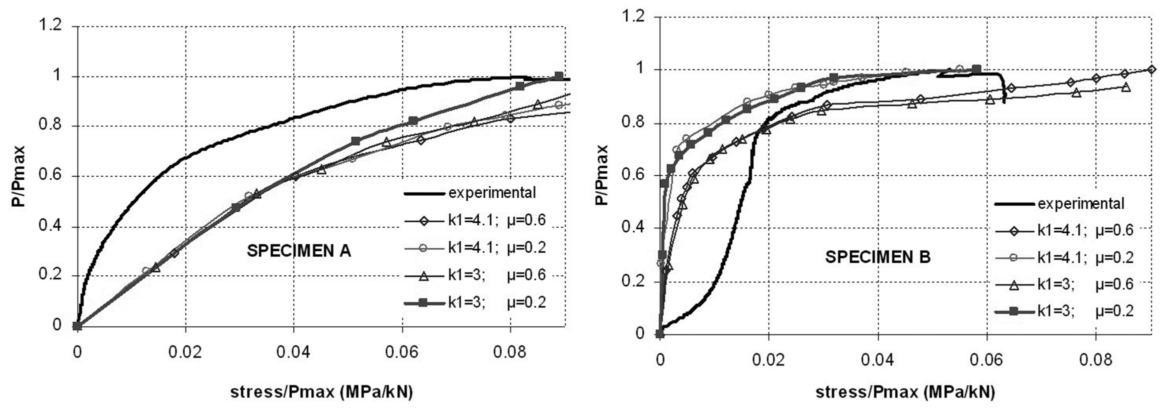

Fig. 7 Relation between hydraulic jack load and average stress value at strain gauges g7, g8, g13 and g14 (values relative to P_{max}). Comparison of the experimental results and the numerical model for different values of k_1 and μ

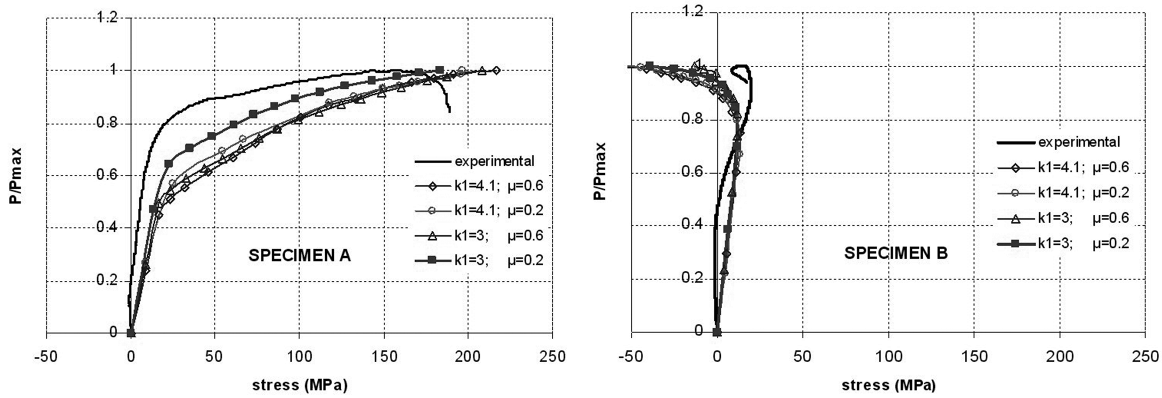

Fig. 8 Relation between hydraulic jack load and average stress value at strain gages g1, g2, g5 and g6 (values relative to $P_{\max}$). Comparison of the experimental results and the numerical model for different values of k_1 and μ

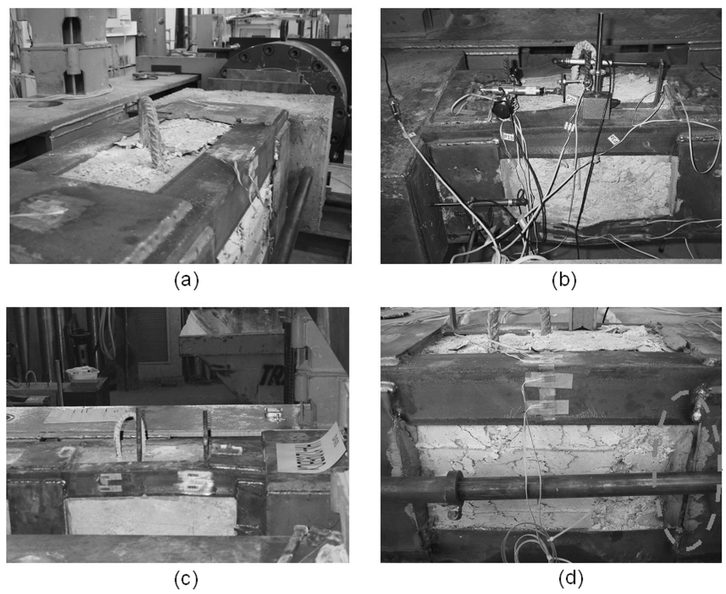

Fig. 9 Failure of the sample specimens. (a) Generalized failure of the concrete at the ends of Specimen A; (b) Angle plasticity at the ends of specimen A; (c) Angle plasticity at the ends of specimen B; (d) Relative displacement mortar-steel at the ends of specimen A

most of the sample specimens also presented plasticity of the steel angles due to the combination of the axial stresses and the cross-sectional stresses generated by the column concrete when expanding laterally by the Poisson effect, causing flexion in the direction of the main axes of the strengthening angles.

Both the numerical and experimental models show a considerable relative displacement between the strengthening steel and the cement mortar, this effect being more noticeable at the column heads.

Fig. 9 illustrates the failure of the sample specimens, including the aspects mentioned above. Figs. 10, 11 and 12 present a summary of the results obtained with the FE model for failure in the strengthening

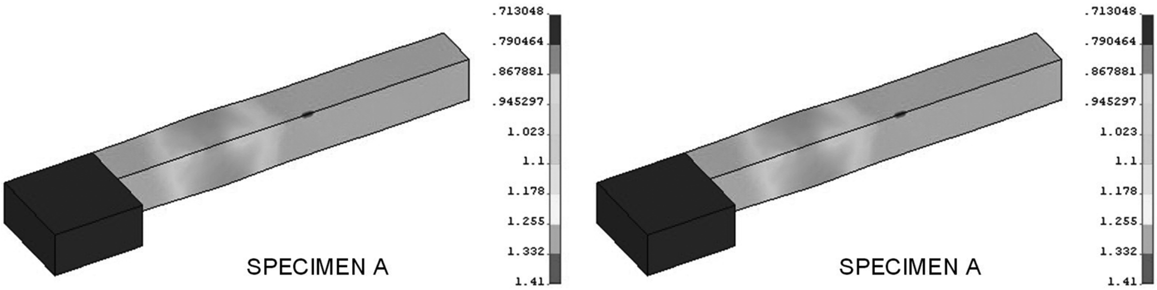

Fig. 10 Concrete plasticity according to the stress state ratio

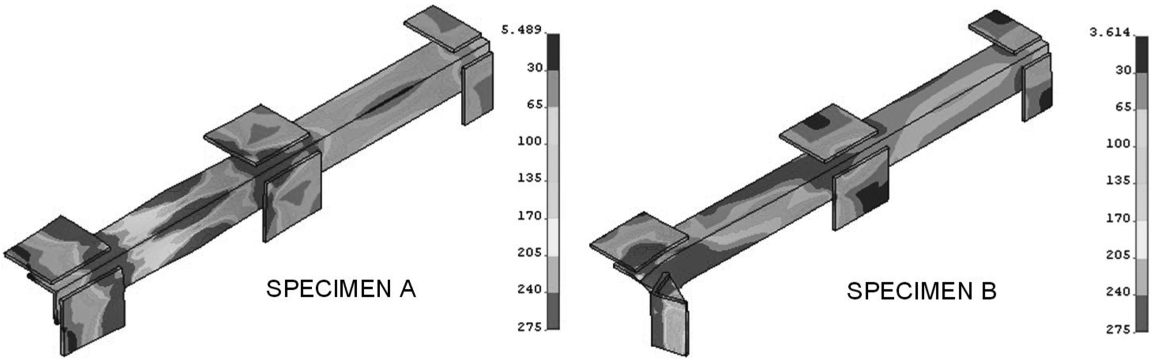

Fig. 11 Stress in steel strengthening according to Von Mises criterion (MPa)

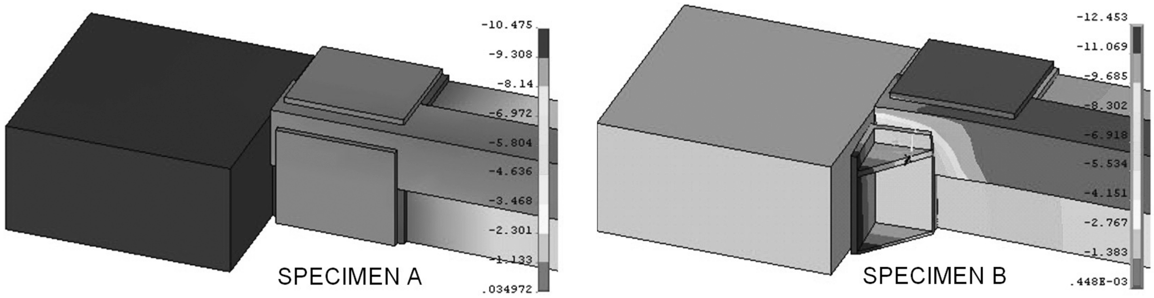

Fig. 12 View of the relative displacement between cement mortar and steel strengthening at the ends of the specimen (mm)

element, presenting a similar behaviour to the patterns observed in the sample specimens.

Following are the most relevant details of column behaviour.

5.1 Confinement effect

The steel strips caused significant confinement in the concrete at the ends of specimen A, increasing the compressive strength of the concrete in this part of the column and as a consequence, failure in the part of the column with lower confinement. This effect can be seen in Fig. 10.

As mentioned above, the effect of strips confinement can also be studied by the tensile stresses of the metal angles resulting from the cross-sectional dilation of the concrete material. These tensile

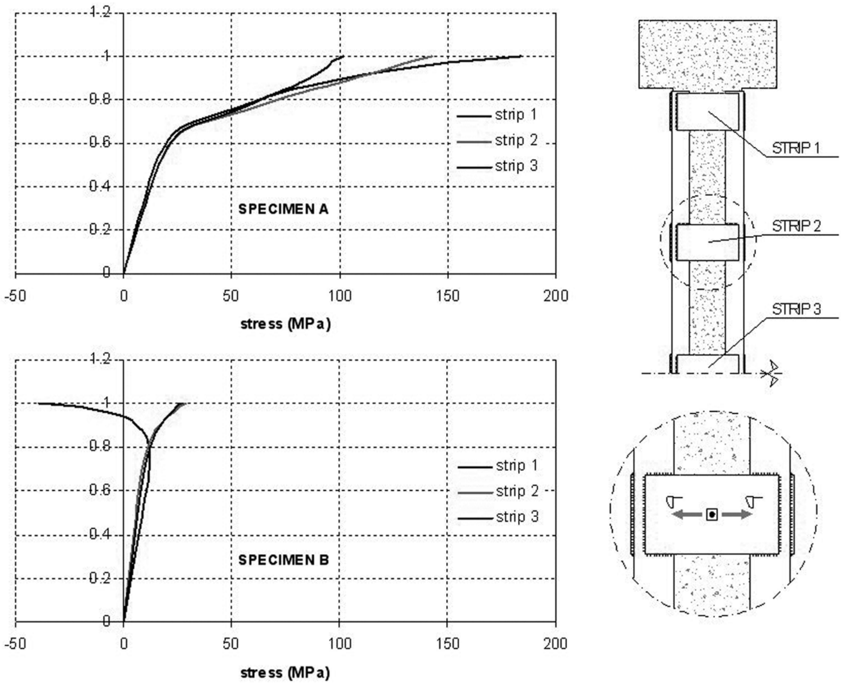

Fig. 13. Strip stresses for specimens A and B depending on the loading level induced by the hydraulic jack (see the numeration of the strips, as well as point for stress estimation). Results obtained with the numerical models

stresses are higher at the end strips, particularly during the ultimate loading levels, and decrease in the central strips (see Fig. 13(a)). It can be observed in Fig. 13(b) that for Specimen B strip tensile stress is almost negligible; as a consequence the effect of strip confinement can also be neglected, especially when compared to specimen A.

5.2 Effect of the cement mortar at the interface

One of the mechanisms affecting the behaviour of the strengthened columns is load transmission by shear stresses at the interface bond between the cement mortar material and the strengthening steel. Fig. 6 shows how an increase in the friction coefficient between the strengthening steel and the mortar of the interface bond causes an increase in the ultimate loading level of the strengthened structure. It has been observed that if the μ value varies from 0.2 to 0.6, this gives an increase in strength in Specimen A of 20 %, whereas in Specimen B this increase is only of 6 %.

Fig. 14 represents the load absorbed by the concrete material and the strengthening steel for each load value applied to the specimen ends. The X axis shows the load fraction transferred to the concrete material and the strengthening steel, whereas the Y axis shows the length of the analyzed cross section with respect to the end heads. An increase in μ leads to better load distribution between concrete and strengthening steel, in particular when the strengthened column works at loads higher than $2/3P_{max}$. This improvement in load transmission is better observed in Specimen A, whereas changes in the value of μ scarcely affect Specimen B.

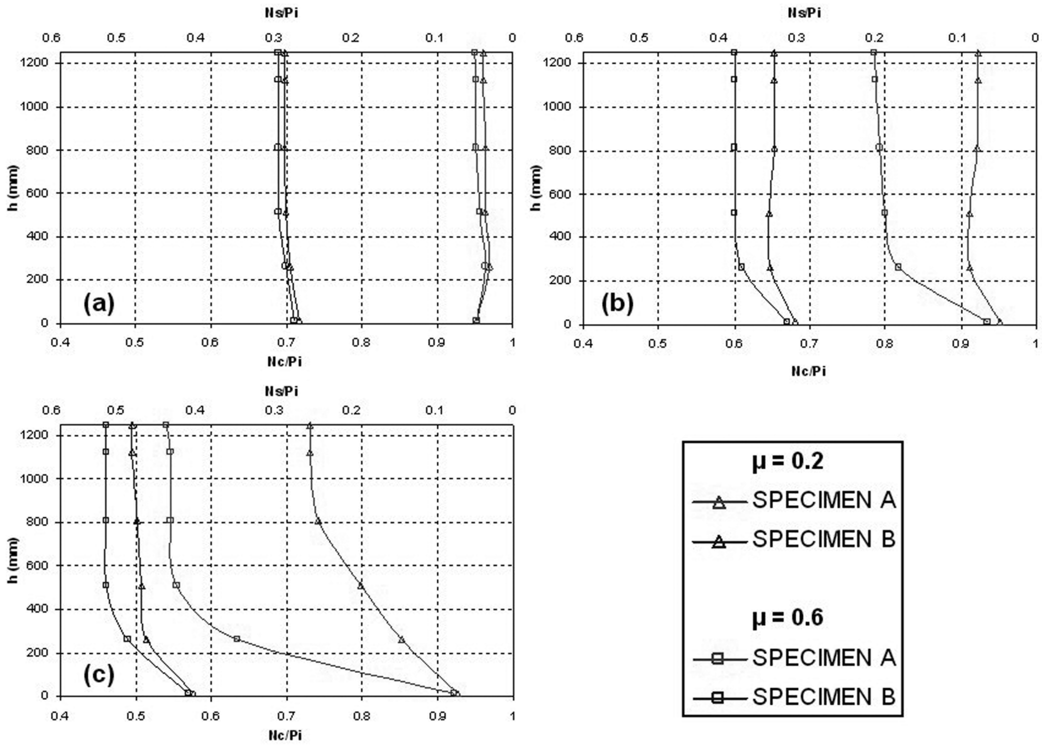

Fig. 14 Load distribution between the steel strengthening and the column concrete material for different loading values ((a) $P_i = 1/3\ P_{\max}$; (b) $P_i = 2/3\ P_{\max}$; (c) $P_i = P_{\max}$) and for different μ values. Results of the numerical models

5.3 Effect of the capitals

The presence of capitals allows the direct transmission of some fraction of the load to the strengthening elements. Fig. 14 shows an improvement in Specimen B as compared to Specimen A in terms of load transmission to steel metallic angles, in particular in the zone closest to the strip ends. Although the concrete material of Specimen B is less stressed, failure occurs at loading values similar to those found in Specimen A. This is due to the fact that the concrete material at the ends of Specimen B is not sufficiently confined.

Analyzing Fig. 11 with more detail, it can be observed that plasticity in the strengthening steel occurs differently in the two types of sample specimens. In Specimen B, both in the laboratory tests and in the numerical model, steel angles plasticity occurs in the zone near the capital as a result of the direct transmission of the load by the concrete head. This behaviour differs from that observed in Specimen A, in which plasticity occurs in the area between the two first steel strips.

6. Practical recommendations for the design

Based on the numerical and experimental analysis it is possible to determine a number of practical recommendations for the design of RC columns strengthening systems using steel angles and strips. The analysis is divided into two sections depending on whether the strengthening technique uses capitals at the column ends or not.

6.1 Without capitals

It has been observed in Specimen A that two effects strongly affect the behaviour of strengthened columns without capitals at the column ends; these effects are strip confinement pressure and the influence of the interface bonding material.

Since failure in the columns always occurs at the column ends, and since the steel strips exert confinement pressure on concrete, it is advisable to reduce strip spacing at the ends of the column. In this way, the confinement effect will increase, improving column performance at the column ends as it is in this part where load transmission between the RC column and the strengthening steel structure takes place.

The numerical model showed that an increase in the friction coefficient between the strengthening steel and the cement mortar causes a significant increase in the failure load of the strengthened column. It is thus recommended during the constructive phase, to be particularly careful when setting the mortar interface, avoiding the formation of air bubbles or the use of expanding mortars.

6.2 With capitals

In the tests with Specimen B, the confinement effect decreased by removing the steel strip below the capital, which isolated the effect of load transmission through the capital. In these specimens the confinement pressure of the strengthening structure on the column concrete material is negligible. To optimize the use of steel in the strengthening structure it is advisable to place strips below the capital, as this will result in a confinement effect similar to that observed in Specimen A. In this way, a small increase in cost will lead to considerable improvement in column strength.

Since the resistance of the strengthened element is practically independent of the fictional characteristics of the mortar used in the interface bond between the strengthening steel and the column concrete material, the bonding mortar does not require any special properties, as opposed to the specimens without capitals.

In Fig. 14 it can be observed that most loads are directly transmitted through the capitals; thus a more detailed study of capital stiffness is required, analyzing the most suitable capital dimensions and welding stiffeners to join their wings.

7. Conclusions

An experimental and numerical study has been developed to study the behaviour of axially loaded RC columns, strengthened with steel angles and strips. This study intendes to characterise the relative importance of the different parameters acting on the strengthened RC column. The numerical model accurately represents the behaviour of the strengthened RC column with and without capitals until it fails. The model allowed us to analyze the influence of the main mechanisms affecting the behaviour of the strengthening structure: confinement pressure induced by the strengthening, load transmission to the steel angles by friction, and load transmission through the steel capitals.

It has been observed that the steel strips exert high confinement pressure on concrete at the column ends, that the cement mortar plays an important role in load transmission between the concrete material and the strengthening steel in columns with no capitals at the ends, and that the use of capitals greatly improves load transmission between concrete material and strengthening steel, in particular at the ends of the strengthened column. Some practical recommendations for strengthening design in RC columns are included in the paper.

Acknowledgements

The authors wish to express their gratitude for the financial support received from the Spanish Ministry of Science and Technology under the investigation project MAT 2003-08075, co-financed with FEDER funds.

Referentes

ACHE (2004), Recomendaciones para el proyecto de estructuras de hormigón de alta resistencia. Comisión 1, Grupo de trabajo 1/2. Asociación Científico-técnica del Hormigón Estructural [in Spanish].

Adam, J. M., Ivorra, S., Giménez, E. and Calderón, P. A. (2005), "Estudio numérico sobre el comportamiento estructural de soportes de hormigón armado reforzados mediante angulares metálicos y presillas, sometidos a compresión simple", fib Simposio "El Hormigón Estructural y el Transcurso del Tiempo", La Plata [in Spanish].

Adam, J. M., Calderón, P. A., Giménez, E., Hidalgo, C. and Ivorra, S. (2006), "A study of the behavior of the cement mortar interface in reinforced concrete columns strengthened by means of steel angles and strips", *Structural Faults and Repair*, Engineering Technics Press, Edinburgh.

Adam, J. M., Pallarés, F. J., Calderón, P. A. and Payá, I. J. (2007), "A study of the conditions of use of a new safety system for the building industry", *Eng. Struct.* **29**, 1690-1697.

ANSYS theory reference 9.0, 2004. ANSYS Inc.

Baltay, P. and Gjelsvik, A. (1990), "Coefficient of friction for steel on concrete at high normal stress", *J. Mater. Civ. Eng.,* **2**(1), 46-49.

Barbosa, A. F. and Ribeiro, G. O. (1998), "Analysis of reinforced concrete structures using ANSYS nonlinear concrete model", *Computational Mechanics: New Trends and Applications*, Barcelona.

Calderón, P. A., Giménez, E., Adam, J. M. and Ivorra, S. (2006), "Full scale testing of RC columns strengthened with steel angles and battens", *Structural Faults and Repair*, Engineering Technics Press, Edinburgh.

CEB-FIB Model Code 90, 1991. Laussane.

Cirtek, L. (2001a), "RC columns strengthened with bandage – experimental programme and design recommendations", *Constr. Build. Mater.* **15**, 341-349.

Cirtek, L. (2001b), "Mathematical model of RC banded column behaviour", *Constr. Build. Mater.,* **15**, 351-359.

Drucker, D. C. and Prager, W. (1952), "Soil mechanics and plastic analysis or limit design", *Quaterly of Applied Mathematics* **10**, 157-165.

Ellobody, E. and Young, B. (2006), "Design and behaviour of concrete-filled cold-formed stainless steel tube columns", *Eng. Struct.,* **28**, 716-728.

Ellobody, E., Young, B. and Lam, D. (2006), "Behaviour of normal and high strength concrete-filled compact steel tube circular stub columns", *J. Constr. Steel Res.,* **62**, 706-715.

ENV 1992-1-1 (Eurocode No. 2), 1991. Design of concrete structures. Part 1: General rules and rules for buildings.

ENV 1993-1-1 (Eurocode No. 3), 1993. Design of steel structures. Part 1: General rules and rules for Buildings.

Erduran, E. and Yakut, A. (2004), "Drift based damage functions for reinforced concrete columns", *Comput. Struct.* **82**, 121-130.

Giménez, E., Calderón, P. and Serna, P. (2004), "Contribution to the study of the strengthening of reinforced concrete column using steel angles and steel battens", *5th International Phd Symposium in Civil Engineering*, Delft.

Giménez, E., Adam, J. M., Calderón, P. A. and Ivorra, S. (2006), "Numerical and experimental study of the strengthening of reinforced concrete columns using steel angles and strips", *Proceedings of The Tenth East Asia_pacific Conference on Structural Engineering & Construction (EASEC-10)*, Bangkok.

Gupta, P. K., Sarda, S. M. and Kumar, M. S. (2007), "Experimental and computational study of concrete filled steel tubular columns under axial loads", *J. Constr. Steel Res.,* **63**, 182-193.

Hu, H. T., Huang, C. S. and Chen, Z. L. (2005), "Finite element analysis of CFT columns subjected to an axial compressive force and bending moment in combination", *J. Constr. Steel Res.,* **61**, 1692-1712.

Johansson, M. and Akesson, M. (2001), "Finite element study of concrete-filled steel tubes using a new confinement-sensitive concrete compression model", *Nordic Concrete Research* **27**(2), 43-62.
Lu, F. W., Li, S. P., Li, D. W. and Sun, G. (2006), "Flexural behaviour of concrete filled non-uni-thickness walled rectangular steel tube", *J. Constr. Steel Res.*, doi:10.1016/j.jcsr2006.09.006.
Ministerio de Fomento, 1998. Instrucción de hormigón estructural. EHE [in Spanish].
Mirmirian, A., Zagers, K. and Yuan, W. (2000), "Nonlinear finite element modelling of concrete confined by fiber composites", *Finite Elem. Anal. Des.* **35**(1), 79-96.
Miyauchi, K., Inoue, S., Kuroda, T. and Kobayashi, A. (1999), "Strengthening effects of concrete columns with carbon fiber sheet", *Transactions of the Japan Concrete Institute* **21**, 143-150.
Ramírez, J. L. and Bárcena, J. M. (1975), Eficacia resistente de pilares de hormigón armado de baja calidad reforzados por dos procedimientos diferentes. Informes de la Construcción **272**, 89-98 [in Spanish].
Ramírez, J. L., Bárcena, J. M. and Feijóo, J. M. (1977), "Comparación resistente de cuatro métodos de refuerzo de pilares de hormigón armado", *Informes de la construcción* **290**, 57-68 [in Spanish].
Ramírez, J. L. (1996), "Ten concrete column repair methods", *Constr. Build. Mater.*, **10**(3), 195-202.
Regalado, F. (1999), Los pilares. Criterios para su proyecto cálculo y reparación. Alicante: CYPE Ingenieros [in Spanish].
Richard, F. E., Brantzaeg, A. and Brown, R. L. (1928), "A study of de failure of concrete under combined compressive stresses", *Bulletin 185*. Engineering Experiment Station, University of Illinois, Urbana, IL.
Rochette, P. and Labossière, P. A. (1996), "A plasticity approach for concrete columns confined with composite materials", *Proceedings Advanced Composite Materials in Bridges and Structures*, CSCE.
Tamai, S., Sato, T. and Okamoto, M. (2000), "Hysteresis model of steel jacketed RC columns for railway viaducts", *Proceedings of the 16th Congress of IABSE*, Lucerne.
Wu, Y. F., Liu, T. and Oehlers, D. J. (2006), "Fundamental principles that govern retrofitting of reinforced concrete columns by steel and FRP jacketing", *Adv. Struct. Eng.*, **9**(4), 507-533.

Notation

a	: adhesion according to Coulomb's frictional law
c	: cohesion according to Drucker Prager yield criterion
E_{ci}	: Tangencial elasticity modulus of concrete
E_{co}	: 2.15e10^4 MPa according to CEB-FIB Model Code 90
E_m	: Elasticity modulus of cement mortar
E_s	: Elasticity modulus of steel
f_c, f'_{co}	: ultimate uniaxial compressive strength
f'_{cc}	: ultimate triaxial compressive strength
f_{cmo}	: 10 MPa according to CEB-FIB Model Code 90
f_l	: lateral confining pressure
f_y	: elastic limit of steel
h	: distance from column end
k_1	: confinement effectiveness coefficient
Nc	: Load absorbed by column concrete
Ns	: Load absorbed by strengthening steel
P, P_i	: Load applied by the hydraulic jack at the specimen end
$P_{\max}$	: maximum load absorbed by the strengthened column
p	: normal stress to the contact element
μ	: friction coefficient at the steel-mortar interface
τ	: shear stress at the steel-mortar interface
υ	: Poisson ratio
$\emptyset$	: internal friction angle according to Drucker Prager yield criterion
$\emptyset_r$	: dilatancy angle according to Drucker Prager yield criterion

DN