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Bicriteria optimal design of open cross sections of cold-formed thin-walled beams

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Abstract. This paper presents a analysis of the problem of optimal design of the beams with two I-type cross section shapes. These types of beams are simply supported and subject to pure bending. The strength and stability conditions were formulated and analytically solved in the form of mathematical equations. Both global and selected types of local stability forms were taken into account. The optimization problem was defined as bicriteria. The cross section area of the beam is the first objective function, while the deflection of the beam is the second. The geometric parameters of cross section were selected as the design variables. The set of constraints includes global and local stability conditions, the strength condition, and technological and constructional requirements in the form of geometric relations. The optimization problem was formulated and solved with the help of the Pareto concept of optimality. During the numerical calculations a set of optimal compromise solutions was generated. The numerical procedures include discrete and continuous sets of the design variables. Results of numerical analysis are presented in the form of tables, cross section outlines and diagrams. Results are discussed at the end of the work. These results may be useful for designers in optimal designing of thin-walled beams, increasing information required in the decision-making procedure.

Keywords: thin-walled beams; cold-formed beams; I-section beams; multicriteria optimization.

1. Introduction

The lack of strength of thin-walled cold-formed beams has been well discussed for many years and is now widely described in the literature. The general approach to the stability of compressed beams or buckling arising during their bending has also been established for a long time. However, these problems still remain a subject of contemporary studies, particularly with regard to local buckling of these members. Extensive discussion of strength and stability problems of thin-walled beams may be found in different monographs, which include references to many other pertinent articles (for example: Bažant and Cedolin 1991, Trahair 1993, Magnucki and Ostwald 2005).

Features of the thin-walled structures are characterized by a set of advantages and disadvantages. The thin-walled beams are distinguished by good strength properties, relatively low weight and the ability to carry high loads. The beneficial relation between the weight and carrying loads is the main advantage of these structures. The beams are manufactured with the use of cold rolling technology and, therefore,

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they may rather easily meet the requirements imposed by users in many branches of mechanical industry and civil engineering. The greatest benefit of these structures is their efficiency, as a large increment of strength may be caused by appropriate choice of cross section shape, with minimal or no weight increase. The benefits that optimally designed this-walled beams offer are generally classified as weight-saving ones, with respect to the load-bearing function and constructional demands. The thin-walled beams are a great area for innovation research and in practice this has proved to be very significant.

Obviously thin-walled beam structures have also some disadvantages. The most important is the susceptibility of these structures to global and local buckling. Calculations of critical loads require applying complex methods of finding satisfactory solution for the stability state of the thin-walled beams. These calculations may be done analytically or numerically (Bažant and Cedolin 1991, Mohri 2003, Trahair 1993).

It is clear that the advantages of thin-walled beams can be better exploited and their faults can be minimized if the basic geometric parameters are calculated with the help of structural optimization. Most of the work in the area of optimum design is subjected to so-called single criterion scalar optimization. In scalar optimization the most commonly used optimality criterion is the weight of a structure (Liu 2004, Tian 2004). Such a criterion may be connected with economic features because material, manufacturing and application costs to a certain degree depend on weight.

The cost of the structure is a universal criterion, however, the difficulties in determining actual and forecasted costs make the problem more complex and difficult resulting in considerable simplification to be required. It is worthwhile to note that accurate determination of the cost depends on the user (decision maker) imposing his point of view to the designers.

In engineering applications of thin-walled structures, several noncomparable criteria must usually be considered in the optimal design of structures. Such a problem leads to multicriteria optimization, where the structure is described with use of several, often conflicting criteria. It is natural that such optimal design is nearer to the technical reality and it much better describes real behavior conditions of structures.

Thin-walled cold-formed beams with open cross section profiles are widely used in many branches of mechanical industry and civil engineering. Recently, an increasing interest for improving these profiles regarding shapes and manufacturing processes can be noticed. This is confirmed by a number of papers published in worldwide journals and at international conferences. In these publications, particularly conference publications, special attention is paid to the general studies of thin-walled cold-formed beams. Although the problems of strength and stability of the thin-walled structures are widely presented, discussions of the specific problems of optimal shaping of cold-formed open cross section beams are not extensive. The monograph by Magnucki and Ostwald (2005) contains a list of 154 references connected with these structures. Most of these works were published 2000-2005 (about 70%). Only 40 (26%) were thematically connected with the problem of optimal design of cold-formed beams with open cross sections.

The beams, as elements of many different mechanical and civil engineering structures, may be subject to longitudinal forces, transverse loads and different combinations of these loads. The modern thin-walled beams of open cross sections are usually made of higher-strength steel. Therefore their walls may be thinner and whole beam may be relatively light. As a result, characteristics of such constructions are limited mainly by general and local stability conditions. From the practical point of view, the geometric constraints which limit the basic dimensions of cross section are very important as well.

A review of studies on general and local stability of cold-formed beams is presented by Davies (2000), Dubina (2004) and Hancock (2003). Some works on optimal design of cold-formed beams are presented by Magnucka-Blandzi (2001, 2004), Magnucki (2000, 2002, 2005) and Stasiewicz (2004). The recapitulation of these works is presented by Magnucki and Ostwald in the form of a monograph (Magnucki and Ostwald 2005). In this work the problem of multicriteria optimization of thin-walled cold-formed beams is formulated and presented.

The bicriteria optimization can better connect the cost and practical features of a structure. In engineering practice, the weight of a structure as the first criterion in bicriteria optimization design may be connected with the economic requirements. This criterion may be expressed for example by minimal area of the beam cross section. The second factor determining practical value of a structure may be represented by one of the strength conditions. The role of the most natural factor improving functional quality of a structure may be fulfilled by the condition of ensuring its appropriate rigidity, expressed, for example by minimization of deflection of the beam middle-point. The second criterion may be expressed in different form, which depends on priorities of a design.

In general multicriteria optimization of thin-walled cold-formed beams is a relatively new research approach supporting the process of designing thin-walled structures. For this reason, the presented multicriteria optimization models with two conflicting criteria (the bicriteria problem) should be considered as innovative.

Problem of multicriteria optimization of thin-walled columns and beams were presented by Manevich and Raksha (2001) and Raksha (2003). Procedures of multicriteria optimization to the optimal design of thin-walled cold-formed beams were adapted by Kasperska, Ostwald and Rodak (2004, 2005) and Kasperska, Magnucki and Ostwald (2005). Presented paper is continuation of this research.

The two types of antisymetrical I-sections of cold-formed thin-walled beams are formulated and numerically solved in the presented paper on the bicriteria optimization problem. The first cross section with double flanges is similar to an I-type section beam, and the second with single bent flanges is generalized of the first cross section. For both sections the weight of the beam, expressed by cross section area, is taken as the first optimization criterion and deflection of the beam is taken as the second.

2. Mathematical model of thin-walled beams with open cross sections

The optimization problem is formulated for a beam loaded with two equal moments M [kN·m] applied to the beam ends (pure bending, see Fig. 1).

For purposes of the optimization calculations, the beam cross sections presented in Fig. 2 are taken into consideration. The first model is double flange I-section, the second one is lipped I-section. Except for parameters H and t, the dimensions of all cross sections parameters are referred to the centerline.

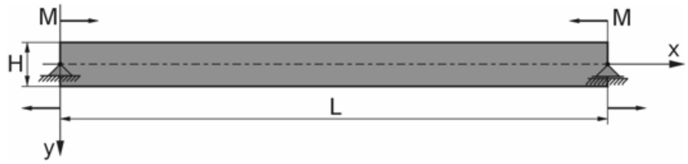


Fig. 1 Model of the thin-walled beam

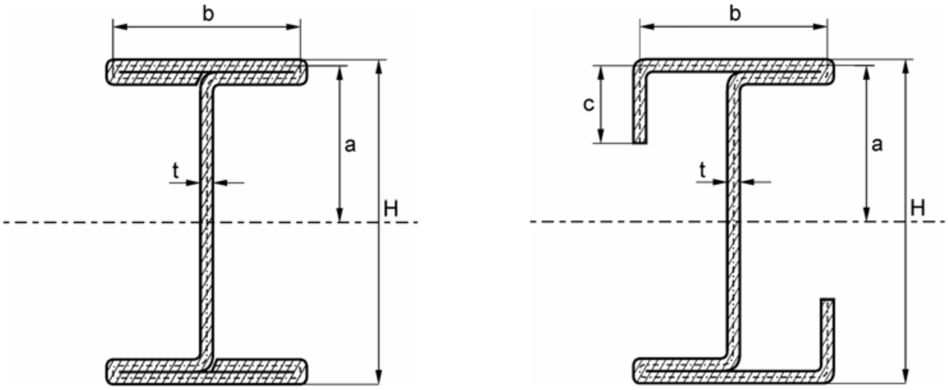


Fig. 2 Antisymmetrical cross sections of the cold-formed beams (Models no 1 and 2)

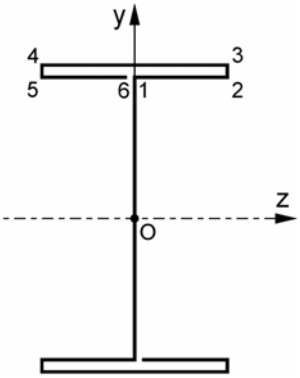


Fig. 3 Scheme of the double flange I-section (Model 1)

For the double flange I-section (Fig. 3) the centroid of the cross section and the shear centre (point O) are located in the centre of the coordinate system yz (yz – the principal axes). The total area A and the geometric stiffness for Saint-Venant torsion J_t of the cross section are in the form:

$$A = 2t(a+2b),\ J_t = \frac{2}{3}t^3(a+2b) \tag{1}$$

Moments of inertia of the cross section area with respect to the y and z axes (J_y and J_z, respectively) and the sectorial (warping) moment of inertia J_ω are as follows:

$$J_y = \frac{1}{3}tb^3 \qquad J_z = 2t\left\{a^2 b + (a-t)^2\left[b + \frac{1}{3}(a-t)\right]\right\},$$

$$J_\omega = 2t\left\{(a-t)\omega_1^2 + \frac{1}{6}b\left[\omega_1^2 + \omega_1\omega_2 + \omega_2^2 + 2(\omega_3^2 + \omega_3\omega_4 + \omega_4^2) + \omega_5^2 + \omega_5\omega_6 + \omega_6^2\right]\right\} \tag{2}$$

The sectorial (warping) functions in selected points of the profile (see Fig. 3) are of the following forms:

$$\omega_1 = -bt - \tilde{\omega}_0, \qquad \omega_2 = -\frac{1}{2}b(a+t) - \tilde{\omega}_0, \qquad \omega_3 = -\frac{1}{2}ba - \tilde{\omega}_0,$$

$$\omega_4 = \frac{1}{2}ba - \tilde{\omega}_0, \qquad \omega_5 = \frac{1}{2}b(a+t) - \tilde{\omega}_0, \qquad \omega_6 = bt - \tilde{\omega}_0,$$

where $\tilde{\omega}_0 = -\frac{tb(a-t)}{a+2b-t}$.

For the lipped I-section (Fig. 4) the total area A and the geometric stiffness for Saint-Venant torsion J_t of the cross section are in the form

$$A = 2t\left(a + \frac{3}{2}b + c\right), \quad J_t = \frac{2}{3}t^3\left(a + \frac{3}{2}b + c\right) \tag{3}$$

Position of the centroid (the point O) is in the centre of the web. Moments of inertia of the cross section area with respect to the y and z axes are as follows:

$$J_y = \frac{1}{2}tb^2\left(\frac{1}{2}b + c\right), \quad J_z = 2t\left(a^2b + (a-t)^2\left(\frac{1}{2}b + \frac{1}{3}(a-t)\right) + \frac{1}{3}\left(a^3 - (a-c)^3\right)\right) \tag{4}$$

The product of inertia is as follows:

$$J_{yz} = tb\left[\frac{1}{4}b(a-t) + t\left(a - \frac{1}{2}t\right) - c\left(a - \frac{1}{2}c\right)\right] \tag{5}$$

The shear center of this section is located in the middle of the web of an I-section. Therefore the sectorial functions in the selected points of the profile (see Fig. 4) are of the following forms:

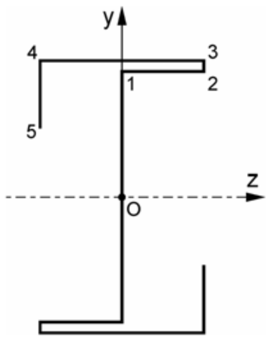


Fig. 4 Scheme of the lipped I-section (Model 2)

$$\omega_1 = -bt - \tilde{\omega}_0, \qquad \omega_2 = -\frac{b}{2}(a+t) - \tilde{\omega}_0, \qquad \omega_3 = -\frac{b}{2}a - \tilde{\omega}_0,$$

$$\omega_4 = \frac{b}{2}a - \tilde{\omega}_0, \qquad \omega_5 = \frac{b}{2}(a+c) - \tilde{\omega}_0,$$

where $\tilde{\omega}_0 = \dfrac{b[8t(a-t) + b(a+3t) - 2c(2a+c)]}{4(2a+3b+2c-2t)}$.

The sectorial moment of inertia of the cross section has the form:

$$J_{\varpi} = 2t\left[\omega_1^2(a-t) + \frac{1}{6}(\omega_1^2 + \omega_1\omega_2 + \omega_2^2)b + \frac{1}{3}(\omega_3^2 + \omega_3\omega_4 + \omega_4^2) + \frac{1}{3}(\omega_4^2 + \omega_4\omega_5 + \omega_5^2)c\right] \tag{6}$$

Geometric properties of particular cross sections are also discussed by Magnucki and Ostwald (2005).

The strength and stability conditions of the thin-walled beams are based on classical Vlasov theory. Detailed descriptions of this theory have been presented by many authors. Strength condition for the beam in a pure bending state is of the following form:

$$\frac{MH}{J_z 2} \le \sigma_{allow},$$

where M – bending moment kN · m, σ_{allow} – allowable stress MPa (yield stress divided by safety factor), from which

$$M \le M_1, \quad M_1 = M = 2\frac{J_z}{H}\sigma_{allow} \tag{7}$$

Critical elastic moment of lateral buckling M_{cr} is of the following form (Magnucki and Ostwald 2005):

$$M_{CR} = \frac{\pi E}{n_b L}\sqrt{\frac{J_y J_t}{2(1+v)}\left(1 + 2(1+v)\frac{\pi^2 J_\omega}{L^2 J_t}\right)} \tag{8}$$

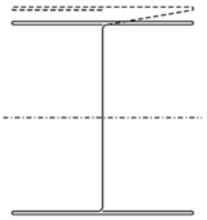
where: J_y, J_t, J_ω – moment of inertia, moment of inertia for torsion, sectorial moment of inertia, respectively,
E, ν – Young's modulus, Poisson ratio,
n_b – factor of safety.

The condition of general stability as lateral buckling condition is in the form:

$$M \le M_2 = M_{CR} \tag{9}$$

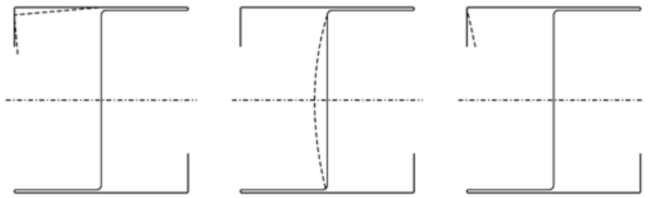
Thin-walled cold-formed beams are very susceptible for local buckling of individual elements of the cross section (web, flanges and lip). The forms of local buckling are presented in Figs. 5 and 6.

The first form of local buckling is based on the assumption that the flange is a beam supported on an elastic foundation and that the flange is relocated only in parallel upward direction, locally receding from the web. This assumption was confirmed by Magnucki and Ostwald's experiment (2005).



1. Local bending buckling of the flange
2. Local bending buckling of the web

Fig. 5 The two forms of local buckling for the double flange I-section (Model 1)



1. Local bending buckling of the flange
2. Local bending buckling of the web
3. Local bending buckling of the lip

Fig. 6 The three forms of local buckling for the lipped I-section (Model 2)

The first condition of local buckling for flange may be written in the form (Fig. 5, form 1):

$$M \le M_3, \text{ where } M_3 = \frac{J_z}{n_{bl}(a-y_p)}\sigma_{CR}^{(flange)} \tag{10}$$

where: n_{bl} – factor of safety for local buckling,
y_p – centroid of the flange cross section.

For the Model 1 the critical stress of the beam flange under pure bending is in the form (Magnucki and Ostwald 2005):

$$\sigma_{CR}^{(flange)} = 4\sqrt{2}E\left(\frac{t}{b}\right)^2$$

For the Model 2 (Fig. 6, form 1) the critical stress is in the form (Magnucki and Ostwald 2005):

$$\sigma_{CR}^{(flange)} = \frac{12}{b^2 t\left[\frac{1}{2}b+3c\right]}\left[GJ_{tp}+\frac{\pi^2}{4}\left(\frac{b}{L}\right)^2 EJ_{zp}\right],$$

where $y_p = \dfrac{c^2}{b+2c}$, $J_{zp} = tc^3\left(\dfrac{1}{3}-\dfrac{1}{2}\dfrac{c}{b+2c}\right)$, $J_{tp} = \dfrac{1}{6}t^3(b+2c)$.

The second condition of local buckling (Figs. 5 and 6, form 2) for web may be written in the form:

$$M \le M_4, \text{ where } M_4 = \frac{J_z}{n_{bl}(a-t)}\sigma_{CR}^{(web)} \tag{11}$$

For the Model 1 and 2 the critical stress is in the form (Magnucki and Ostwald 2005):

$$\sigma_{CR}^{(web)} = \frac{\pi^2 E}{2(1-\nu^2)}\left(\frac{t}{(a-t)}\right)^2 \tag{12}$$

For the Model 2 the third condition of local buckling for lip may be written in the form (Fig. 6, form 3):

$$M \le M_5, \text{ where } M_5 = \frac{J_z}{n_{bl} a} \sigma_{CR}^{(bent)} \tag{13}$$

The critical stress of the beam lip under pure bending is in the form (Magnucki and Ostwald 2005):

$$\sigma_{CR}^{(bent)} = \frac{2}{(1+v)\left(4 - 3\frac{c}{a}\right)} E\left(\frac{t}{c}\right)^2$$

In above equations $n_{bl} = 1.5 \cdot n_b$ is the factor of safety with respect to local buckling. This factor was assumed to exceed the safety factor by 50% for the case of general buckling. Values of both factors are different in order to avoid interactions between both buckling forms. In the authors' opinion, the safety factor equal to 50% will be sufficient in the context of presented optimal design problem, when different forms of buckling and their possible interactions are treated as constraints. If such interaction arises, a critical load will be significantly lower compared to the determined value.

3. Mathematical model of the optimization problem

3.1 Optimization procedure

Multicriteria optimization problems in engineering practice are solved generally with the application of minimum in the Pareto sense. In the presented paper, the Pareto frontier was generated with the help of the normalized normal constraint (NC) method (Messac 2003). In comparison with classical procedure based on weighting coefficients approach (the weighted sum method WS), this allows us to obtain all Pareto-optimal points, which are uniformly located on the Pareto frontier. The normalized normal constraint method NC, which solves bicriteria optimization problems, is in the form:

$$\min_{x \in \Re^n} \{\overline{Q_2}(x) \in \Re : x \in X\} \tag{14}$$

subject to

$$\overline{N}_1^T(\overline{Q}(x) - \overline{X}_{pj}) \le 0 \tag{15}$$

The feasible domain $X \subset \Re^n$ is determined in the following way

$$X = \{x \in \Re^n : g_j(x) \le 0,\, h_k(x) = 0,\, x_{li} \le x_i \le x_{ui},\, 1 \le j \le r,\, 1 \le k \le s,\, 1 \le i \le n\},$$

where $x = [x_1, x_2, \ldots, x_n]^T$ is vector of design variables, n the number of variables, $g_j(x)$ and $h_k(x)$ are the inequality and equality constraint respectively, r, s the number of the inequality and equality constraints, $x_{li} \in (\Re \cup \{-\infty\})$, $x_{ui} \in (\Re \cup \{\infty\})$. The Eq. (15) is an additional constraint.

The following auxiliary definitions were taken into account (see Figs. 7 and 8):

$\overline{N}_1 = \overline{Q}(x^{2*}) - \overline{Q}(x^{1*})$ – definition of the auxiliary vector on the utopia line,

$\overline{Q}(x) = [\overline{Q}_1(x), \overline{Q}_2(x)]^T$ – normalized vector of optimality criteria,

$\overline{X}_{pj} = (1 - w)\overline{Q}(x^{1*}) + w\overline{Q}(x^{2*})$ – generic point on the utopia line,

where

$$\overline{Q}_1(x) = \frac{Q_1(x) - Q_1(x^{1*})}{Q_1(x^{2*}) - Q_1(x^{1*})}, \quad \overline{Q}_2(x) = \frac{Q_2(x) - Q_2(x^{2*})}{Q_2(x^{1*}) - Q_2(x^{2*})}, \tag{15a}$$

$$Q_1(x^{1*}) = \min_x \{Q_1(x) \in \Re : x \in X\}, \quad Q_2(x^{2*}) = \min_x \{Q_2(x) \in \Re : x \in X\},$$

$Q(x) = [Q_1(x), Q_2(x)]^T$ – vector of optimality criteria,

$w \in \langle 0, 1 \rangle$ – increment generating point on the utopia line.

3.2 Design variables and optimization criteria

The optimization problems have been solved with the use of continuous and discrete sets of decision variables. The applied approach was based on the combination of the normal constraint method with the classical systematic search method. Because the number of design variables was not significant, use of such procedure enhances the applicability of numerical calculations and interpretation of results. In the case of continuous set of design variables the analysis of the solutions enables defining active constraints and selecting appropriate actions aimed at improving effectiveness of the cross section. The solutions obtained with the help of discrete set of decision variables may be applied in engineering practice.

In the presented paper the parameters a, b and t were taken as the decision variables (see Figs. 2 and 3).

In presented work it is assumed that the first optimization criterion is the weight of the beam, simply expressed by the area of the beam cross section.

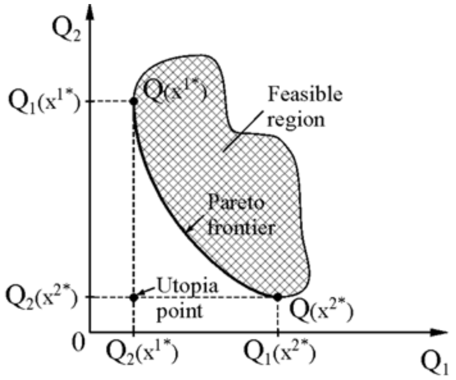


Fig. 7 Non-normalized design space and the Pareto frontier for bicriteria problem (Mesac 2003)

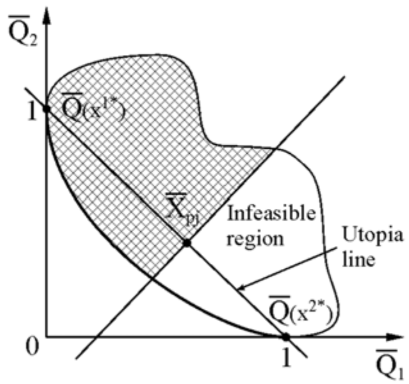


Fig. 8 Normalized design space and graphical representation of NC procedure (Mesac 2003)

$$Q_1(x) = A\ [\mathrm{mm}^2] \rightarrow \min \tag{16}$$

For the Model 1 the first criterion is expressed in the form

$$Q_1 = 2t(a + 2b),$$

and for the Model 2

$$Q_1 = 2t\left(a + \frac{3}{2}b + c\right).$$

The second criterion is deflection of the centre of beam, expressed as follows

$$Q_2(x) = \frac{M \cdot L^2}{8EJ_z}[\mathrm{mm}] \rightarrow \min \tag{17}$$

In optimization procedure the both criteria are normalized according to Eq. 15(a). The results of numerical calculation are presented in original forms.

3.3 Constraints

For each of considered cross sections the sets of geometric constraints have been determined, including the following conditions.

1. The values of all decision variables > 0.
2. For thin-wall feature of beam, the constraint $H/t \geq 10$ must be fulfilled for the elements of cross section.
3. A set of constructional and technological conditions must be determined for each of the cross sections. These conditions are related to standard requirements (Eurocode 3, AISC and AISI, for example) and to manufacturer feasibility.

The set of constraints and conditions is defined as follows:

- geometric constraints and thin-walled conditions (conditions number 1 ÷ 6)
  $a - t > 0$ (1), $b - t > 0$ (2), $t > 0$ (3),
  $H = 2a + t \leq H_{\max}$ (4), $b + t \leq H_{\max}$ (5), $b + 3t \leq H$ (5, for Model 2)
  $t \leq t_{\max}$ (6),
- the strength condition $M \leq M_1$ (7),
- the condition of general stability $M \leq M_1$ (8),
- the conditions of local and distortional stability
- $M \leq M_3$ (9), $M \leq M_4$ (10) $M \leq M_5$ (11), only for Model 2,
- the condition of maximum deflection of the beam $v_{\max} \leq L/250$ (12), arbitrary limit,
- the condition connected with application of NC optimization procedure (15).

For the Model 2 the equality constraint $J_{yz} = 0$ was also taken into account. This approach simplified mathematical model, based on the assumption, that Y, Z axes are the principal moment of inertia axes (see Fig. 3). When $c \leq a$, from Eq. (5) the following equality is obtained

$$c = a\left(1 - \sqrt{1 - \frac{k}{a^2}}\right), \quad \text{where} \quad k = \frac{1}{2}b(a-t) + t(2a-t).$$

Based on this equation, it can be notified that c is defined with the help of decision variables a, b and t. Additionally, the condition presented below must be taken into account

$$b \leq 2(a-t)\,.$$

Finally, the fifth constraint for the Model 2 has the following form

$$b + 3t \leq H.$$

The above constraints and conditions numbering is used in Table 7, in which the active constraints are shown.

4. Numerical calculations

The numerical calculations were performed based on the following input data:
- beam depth $H_{\max}$ = 200 mm,
- beam length L = 1, 2, 3, 4, 5 m,
- sheet thickness t = 1–16 mm (with 1 mm interval),
- properties of the material $E = 2.05 \cdot 10^5$ MPa, $\nu = 0.3$, $\sigma_{allow} = 307.5$ MPa,
- loads $M = 10$ kN·m,
- safety coefficients $n_b = 1.8$, $n_{bL} = 1.5 \cdot n_b = 2.7$.

Results of numerical calculations are presented in the form of diagrams, figures and tables.

The results for the beam under pure bending with cross section defined by Model 1 and Model 2 are presented. In Table 1 and 2 the sets of Pareto-optimal solutions for increment values w = <0, 1> are presented (11 compromise solutions in the Pareto sense). The solution for the increment w = 0 corresponds to scalar optimization with the cross section area considered as the criterion, while the increment w = 1.0 leads to scalar optimization with the deflection as the criterion. The cases for w = <0.1, 0.9> correspond to optimal bicriteria solutions with different degrees of compromise. For each increment w = <0, 1> two optimal solutions are presented. The upper rows show the solutions based on the discrete set of design variables – these are so called standard solutions, which are in agreement with the national standards. The lower rows show the exact solutions, based on the continuous set of design variables.

The sets of Pareto-optimal solutions (Pareto frontiers) obtained with discrete set of design variables, for beams with length L = 2 and 5 m, are presented in Figs. 9 and 10. In these figures the minimum and maximum values of particular criterion are marked, with indication of the ideal solution. In Figs. 9 and 10, the points representing the scalar optimization solution with w = 0 (optimization of the first criterion) and with w = 1.0 (optimization of the second criterion) and bicriteria optimization with w = 0.5 are marked. An additional calculation shows that the increment w used in the NC procedure may be considered as the weighting coefficient used in classical multicriteria procedure based on the weighting function method. Results of the calculation indicates, that w = 0.5 means the equal importance the both optimization criteria.

Table 1 Pareto-optimal solutions set for Model 1, M = 10 kN·m, L = 2 m

w	Decision variables mm			H	Q_1	Q_2
	a	b	t	mm	mm^2	mm
0	66.0	91.0	2.0	134.0	992.0	7.119
	88.41	87.68	1.72	178.54	906.11	4.547
0.1	73.0	90.0	2.0	148.0	1012.0	5.793
	97.10	83.87	1.79	195.99	946.78	3.705
0.2	84.0	87.0	2.0	170.0	1032.0	4.410
	99.11	106.97	1.77	200.00	1109.61	2.901
0.3	99.0	87.0	2.0	200.0	1092.0	3.087
	99.03	136.14	1.94	200.00	1441.19	2.149
0.4	99.0	147.0	2.0	200.0	1572.0	1.949
	98.92	183.64	2.17	200.00	2020.27	1.474
0.5	98.0	177.0	3.0	199.0	2712.0	1.134
	98.41	196.82	3.18	200.00	3130.87	0.963
0.6	97.0	180.0	5.0	199.0	4570.0	0.702
	97.51	195.01	4.99	200.00	4865.10	0.645
0.7	96.0	193.0	7.0	199.0	6748.0	0.492
	96.34	192.67	7.33	200.00	7057.30	0.469
0.8	95.0	188.0	10.0	200.0	9420.0	0.374
	95.00	190.00	10.00	200.00	9501.28	0.370
0.9	93.0	185.0	13.0	199.0	12038.0	0.317
	93.55	187.09	12.91	200.00	12075.08	0.312
1.0	92.0	184.0	16.0	200.0	14720.0	0.276
	92.00	184.00	16.00	200.00	14720.00	0.276

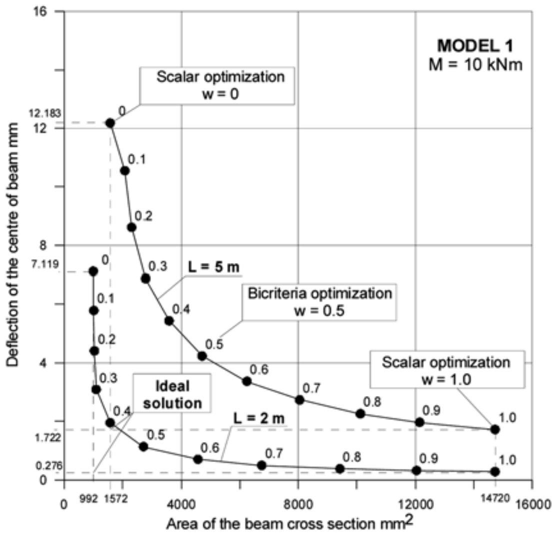


Fig. 9 Discrete Pareto-optimal solutions set for the Model 1

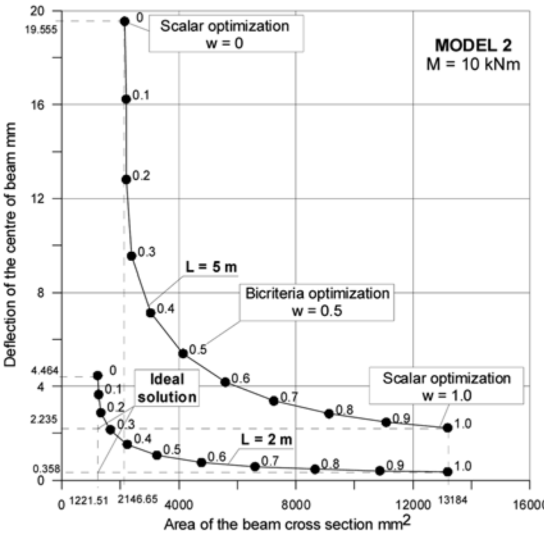


Fig. 10 Discrete Pareto-optimal solutions set for the Model 2

Table 2 Pareto-optimal solutions set for Model 2, M = 10 kN·m, L = 2 m

w	Decision variables mm			c	H	Q_1	Q_2
	a	b	t	mm	mm	mm^2	mm
0	81.0	67.0	3.0	22.1	165.0	1221.51	4.464
	98.64	66.13	2.72	20.99	200.00	1189.21	3.193
0.1	89.0	66.0	3.0	21.5	181.0	1256.92	3.658
	98.58	79.19	2.83	25.25	200.00	1374.23	2.669
0.2	98.0	68.0	3.0	21.9	199.0	1331.25	2.878
	98.93	143.29	2.14	49.60	200.00	1554.53	2.145
0.3	98.0	98.0	3.0	31.9	199.0	1661.36	2.155
	98.80	168.54	2.39	64.59	200.00	1989.59	1.681
0.4	98.0	148.0	3.0	53.3	199.0	2240.01	1.529
	98.41	169.26	3.19	66.65	200.00	2670.88	1.275
0.5	98.0	163.0	4.0	63.7	200.0	3249.77	1.065
	97.81	169.70	4.37	69.48	200.00	3688.96	0.948
0.6	97.0	158.0	6.0	64.0	200.0	4775.45	0.757
	96.99	169.23	6.01	72.92	200.00	5094.36	0.714
0.7	96.0	163.0	8.0	72.1	200.0	6601.34	0.575
	95.96	167.21	8.08	76.56	200.00	6837.11	0.559
0.8	95.0	168.0	10.0	85.8	200.0	8655.61	0.467
	94.75	163.30	10.50	79.96	200.00	8816.70	0.459
0.9	93.0	148.0	14.0	73.1	200.0	10867.51	0.401
	93.36	157.43	13.27	82.87	200.00	10947.97	0.396
1.0	92.0	152.0	16.0	92.0	200.0	13184.00	0.358
	92.00	152.00	16.00	92.00	200.00	13183.95	0.358

Table 3 Optimal parameters of a cross section [mm] for a beam under pure bending M = 10 kN·m (Model 1)

	Scalar optimization w = 0					Bicriteria optimization w = 0.5					Scalar optimization w = 1.0
L	1	2	3	4	5	1	2	3	4	5	
H	128.0	134.0	158.0	180.0	200.0	199.0	199.0	200.0	199.0	199.0	200.0
a	63.0	66.0	78.0	89.0	99.0	98.0	98.0	98.0	97.0	97.0	92.00
b	59.0	91.0	113.0	131.0	147.0	134.0	177.0	166.0	160.0	187.0	184.00
t	2.0	2.0	2.0	2.0	2.0	3.0	3.0	4.0	5.0	5.0	16.00

The optimal parameters of a cross section for the beam under pure bending M = 10 kN·m are shown in Table 3 and 5. In these tables the parameters obtained from the optimization procedure with discrete set of decision variables are presented. The optimal shapes of the beam with cross section defined by Model 1 and 2 are presented in Tables 4 and 6.

Tables 3 and 5 present optimal discrete parameters of the cross section. In Table 5 some changes in the trend are noticeable with increase of beam length L. These changes are connected with skip of the sheet thickness t.

In Table 7 the numbers of active constraints are presented. The active constraints were specified with the help of continuous set of decision variables (the lower rows in Table 1 and 2 for different w). For both models, in the case of scalar optimization with the first criterion expressed by the area of cross section (w = 0), the condition connected with global buckling (8) and local buckling (9, 10) were active. The constraint limiting the depth of the cross section to H_{max} = 200 mm (constraint 4) were active for the beam with length L = 3–5 m for Model 1 and for L = 1–5 m for Model 2.

In the case of scalar optimization with the second criterion expressed by the deflection of beam (w = 1.0) only the geometric constraints (4, 5, 6) were active.

Table 4 Optimal shapes of beam under M = 10 kN · m (Model 1)

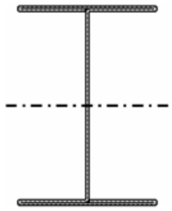
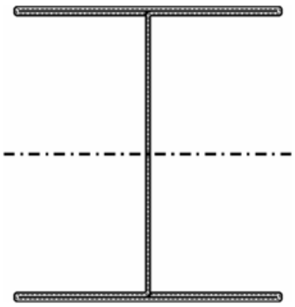
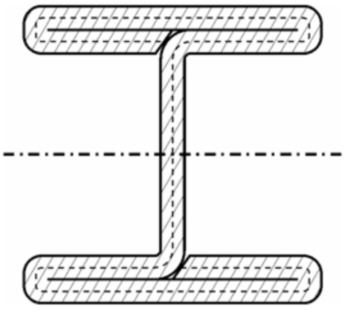
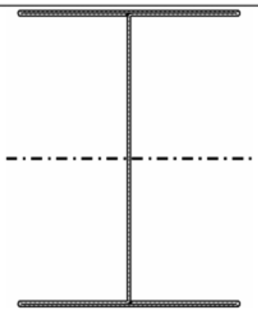
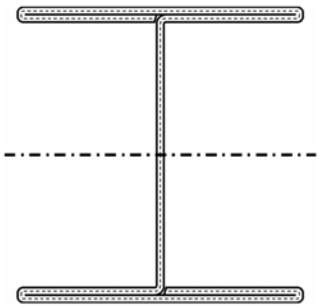
	w = 0	w = 0.5	w = 1.0
L = 2 m	 t = 2.0 mm	 t = 2.0 mm	 t = 16 mm
L = 5 m	 t = 2.0 mm	 t = 5.0 mm	

Table 5 Optimal parameters of a cross section [mm] for a beam under pure bending M = 10 kN · m (Model no 2)

	Scalar optimization w = 0					Bicriteria optimization w = 0.5					Scalar optimiza-tion w = 1.0
L	1	2	3	4	5	1	2	3	4	5	
H	119.0	165.0	199.0	138.0	146.0	199.0	200.0	199.0	199.0	199.0	200.0
a	58.0	81.0	98.0	67.0	71.0	98.0	98.0	97.0	97.0	97.0	92.00
b	47.0	67.0	83.0	93.0	105.0	139.0	163.0	169.0	149.0	166.0	152.00
c	16.38	22.08	26.71	34.76	39.83	48.78	63.72	70.73	56.88	68.23	92.00
t	3.0	3.0	3.0	4.0	4.0	3.0	4.0	5.0	5.0	5.0	16.00

Table 6. Optimal shapes of beam under M = 10 kN · m (Model 2)

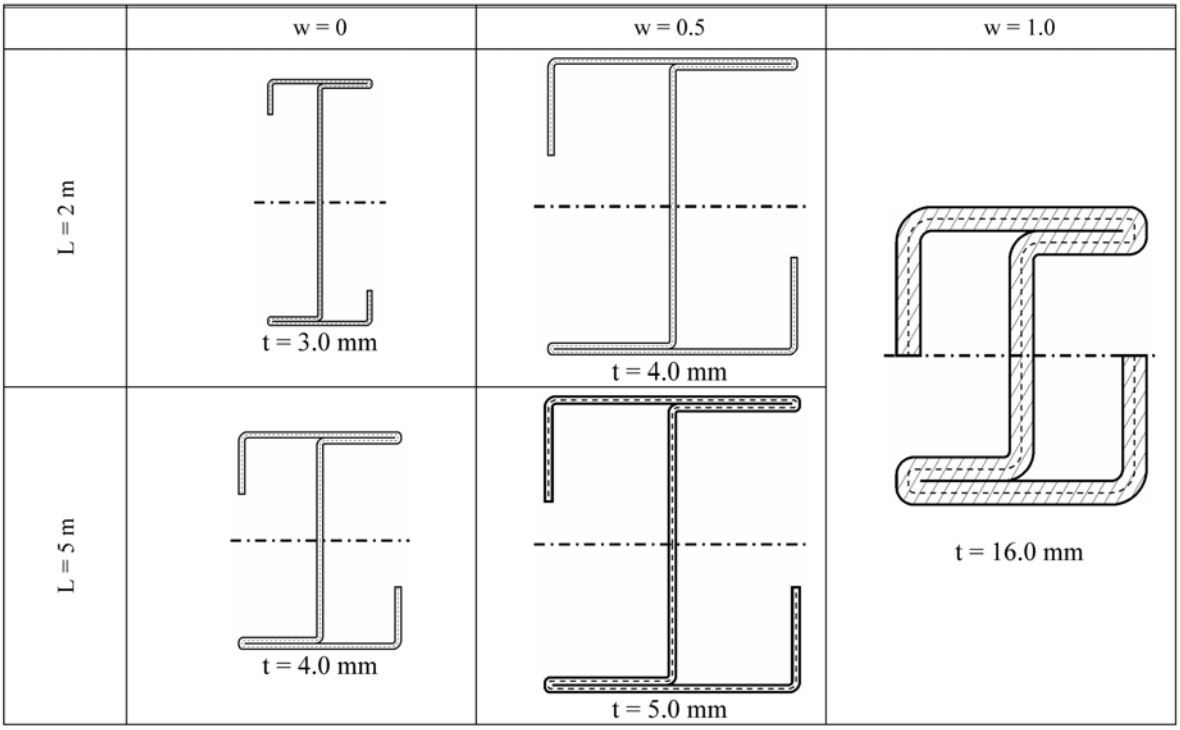


Table 7 Numbers of active constraints

<table>
<tr><th>Model</th><th>w</th><th>L = 1 m</th><th>L = 2 m</th><th>L = 3 m</th><th>L = 4 m</th><th>L = 5 m</th></tr>
<tr><td rowspan="3">1</td><td>0</td><td>7, 8, 10</td><td>8, 9, 10</td><td>4, 8, 9</td><td>4, 8, 9</td><td>4, 8, 9</td></tr>
<tr><td>0.5</td><td>4, 9</td><td colspan="4">4, 5</td></tr>
<tr><td>1.0</td><td colspan="5">4, 5, 6</td></tr>
<tr><td rowspan="3">2</td><td>0</td><td colspan="5">4, 8, 9</td></tr>
<tr><td>0.5</td><td colspan="5">4</td></tr>
<tr><td>1.0</td><td colspan="5">4, 5, 6</td></tr>
</table>

In the case of bicriteria optimization (w = 0.5) the geometric constraints (4, 5) were active. For the Model 1, with L = 1 m, the local buckling condition for the flange (9) was active (in this case H = 199 mm < H_{max} = 200 mm.

Analysis of the results of numerical calculations shows that the geometric condition imposing the limit of the cross section depth H is the most important constraint.

The values of the first criterion for beams L = 1–5 m are presented in Table 8, while the values of the second criterion are presented in Table 9. The parameters w = 0 and w = 1.0 refer to scalar optimization with first and second optimization criterion respectively, the parameter w = 0.5 refers to bicriteria optimization. The values of both criteria are also presented in Figs. 9–11.

Table 8 First criterion – beam cross section area [mm^2] (Model 1 and 2)

w	Model no	L = 1 m	2 m	3 m	4 m	5 m
0	1	724.0	992.0	1216.0	1404.0	1572.0
	2	869.26	1221.51	1495.25	1930.07	2146.65
0.5	1	2196.0	2712.0	3440.0	4170.0	4710.0
	2	2131.69	3249.77	4212.32	3773.75	4142.25
1.0	1			14720.00		
	2			13184.00		

Table 9 Second criterion – beam deflection [mm] (Model 1 and 2)

w	Model no	L = 1 m	2 m	3 m	4 m	5 m
0	1	2.880	7.119	9.229	10.846	12.183
	2	3.121	4.464	5.542	15.746	19.555
0.5	1	0.365	1.134	2.055	3.128	4.232
	2	0.403	1.065	1.917	3.789	5.405
1.0	1	0.069	0.276	0.620	1.102	1.722
	2	0.089	0.358	0.805	1.430	2.235

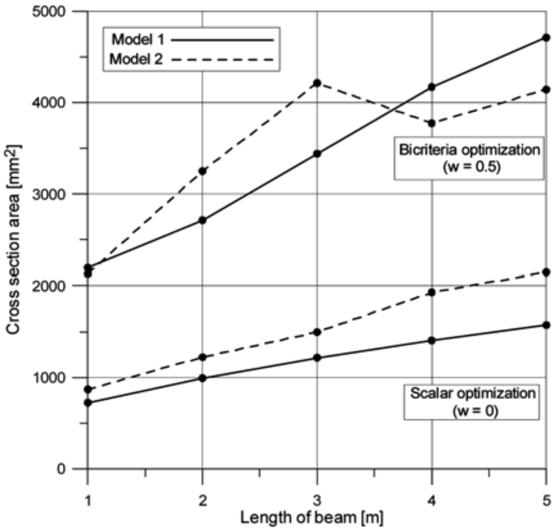


Fig. 11 The first criterion for beams with different length (scalar and bicriteria solutions)

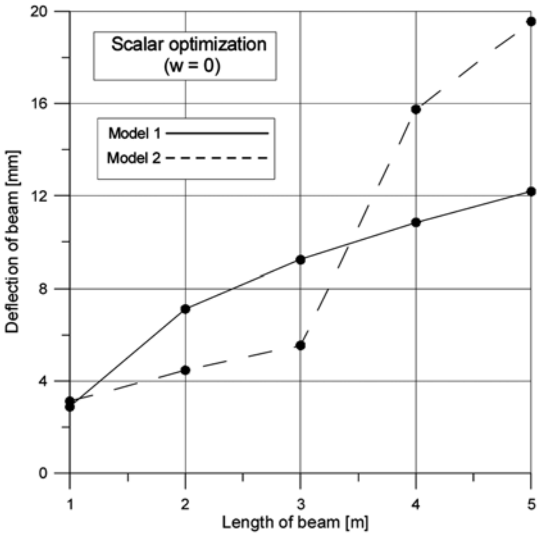


Fig. 12 The second criterion for beams with different length (scalar solutions)

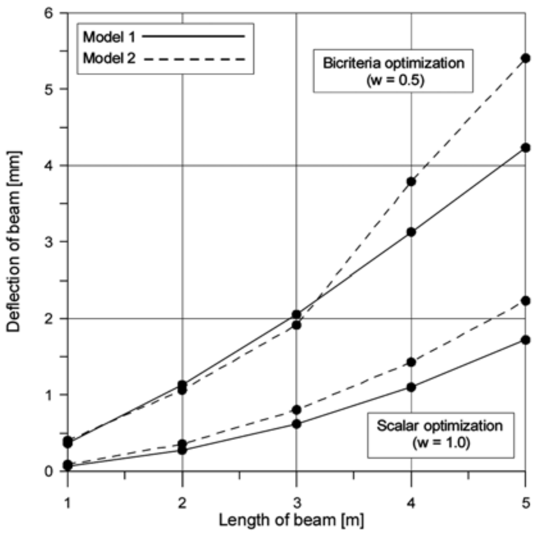


Fig. 13 The second criterion for beams with different length (bicriteria solutions)

5. Conclusions

Bicriteria optimization gives the designer new possibilities during decision-making process. A designer has at their disposal the set of so-called compromise solutions, with a different ratio between the two criteria. The result of scalar optimization is a single element of this set. In such a situation the designer may better assess and understand the thin-walled structure behavior. Bicriteria optimization is an approach requiring a designer to formulate not only appropriate optimization criteria, but first of all a suitable set of constraints. For the scalar optimization with $w = 0$ (the weight of the beam, represented by area of the cross section as an objective function), the most important are the general and local conditions of stability. In all cases the strength condition was not active. With increase of the w increment, that controls the rate of both criteria, the significance of geometric constraints increases. In the case of scalar optimization, with $w = 1.0$ (the beam deflection as an objective function), only the geometric constraints are decisive – this statement is obvious.

Proper formulation of the geometric constraints is one of the most important requirements for the results of bicriteria optimization. The set of constraints should be formulated with consideration of the requirements of a manufacturer (manufacturing technology), a designer (formulation of mathematical model) and users (cost and application of the structure).

Further research on the optimal design of cold-formed thin-walled beams will focus on beams with different cross sections loaded by uniform loads as concentrated forces as well.

Acknowledgement

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