Steel and Composite Structures, Vol. 6, No. 3 (2006) 183-202
DOI: http://dx.doi.org/10.12989/scs.2006.6.3.183

Finite element response sensitivity analysis of continuous steel-concrete composite girders

Alessandro Zona†

Department PROCAM, University of Camerino, Viale della Rimembranza, Ascoli Piceno 63100, Italy

Michele Barbato‡ and Joel P. Conte‡†

Department of Structural Engineering, University of California, San Diego,
9500 Gilman Drive, La Jolla, CA 92093-0085, USA

(Received December 30, 2004, Accepted November 30, 2005)

Abstract. The behavior of steel-concrete composite beams is strongly influenced by the type of shear connection between the steel beam and the concrete slab. For accurate analytical predictions, the structural model must account for the interlayer slip between these two components. This paper focuses on a procedure for response sensitivity analysis using state-of-the-art finite elements for composite beams with deformable shear connection. Monotonic and cyclic loading cases are considered. Realistic cyclic uniaxial constitutive laws are adopted for the steel and concrete materials as well as for the shear connection. The finite element response sensitivity analysis is performed according to the Direct Differentiation Method (DDM); its analytical derivation and computer implementation are validated through Forward Finite Difference (FFD) analysis. Sensitivity analysis results are used to gain insight into the effect and relative importance of the various material parameters in regards to the nonlinear monotonic and cyclic response of continuous composite beams, which are commonly used in bridge construction.

Keywords: steel-concrete composite beams; beam elements; finite element method; nonlinear analysis; response sensitivity analysis; material parameters.

1. Introduction

The last ten years have shown a growing interest in finite element modeling and analysis of steel-concrete composite structures, with applications to seismic resistant frames and bridges (Spacone and El-Tawil 2004). The behavior of composite beams, made of two components connected through shear connectors to form an interacting unit, is significantly influenced by the type of connection between the steel beam and the concrete slab. Flexible shear connectors allow development of partial composite action (Oehlers and Brandford 2000) and, for accurate analytical predictions, structural models of composite structures must account for the interlayer slip between the steel and concrete components. Thus, a

†Assistant Professor, E-mail: alessandro.zona@unicam.it
‡Graduate Student, E-mail: mbarbato@ucsd.edu
‡†Professor, Corresponding author, E-mail: jpconte@ucsd.edu

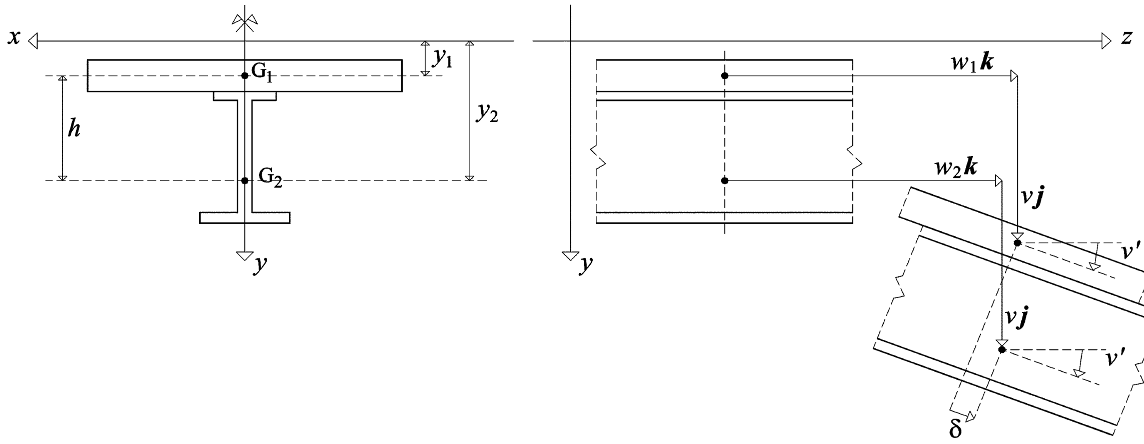

Fig. 1 Kinematics of 2D composite beam model

composite beam finite element able to capture the interface slip is an essential tool. The three-dimensional model under general state of stress (Dall'Asta 2001) simplifies to the model introduced by Newmark *et al.* (1951) if only the in-plane bending behavior is considered. In the Newmark's model, the geometrically linear Euler-Bernoulli beam theory (i.e., small displacements, rotations and strains) is used to model the two parts of the composite beam; the effects of the deformable shear connection are accounted for by using an interface model with distributed bond, and the contact between the steel and concrete components is enforced (Fig. 1). The interface slip is small since computed from the difference in longitudinal displacements of the steel and composite fibers at the steel-concrete interface.

Compared to common monolithic beams, composite beams with deformable shear connection present additional difficulties. Even in very simple structural systems (e.g., simply supported beams), complex distributions of the interface slip and force can develop; furthermore these distributions can be strongly influenced by the shear connection properties. Different finite elements for composite beams with deformable shear connection have been proposed (e.g., Ayoub and Filippou 2000, Salari and Spacone 2001, Dall'Asta and Zona 2004a). Despite the difficulties of the problem in the nonlinear range, refined locking-free displacement-based elements (such as the one used in this study) produce accurate global and local results provided that the structure is properly discretized (Dall'Asta and Zona 2002, 2004a, 2004b, 2004c).

In the last few years, in addition to model-based response simulation of structures, growing attention has been given to the analysis of structural response sensitivity to various geometric, mechanical, and material properties defining the structure, and to loading parameters. This increasing interest is due in part to the fact that finite element response sensitivities represent an essential ingredient for gradient-based optimization methods needed in structural optimization, structural reliability analysis, and finite element model updating (Ditlevsen and Madsen 1996, Kleiber *et al.* 1997, Melchers 1999). In addition, finite element response sensitivities are invaluable for gaining deeper insight into the effect and relative importance of the various geometric, material, and loading parameters defining the structure and its loading environment.

This paper focuses on the materially-nonlinear-only static response sensitivity computation using displacement-based, locking-free finite elements for composite beams with deformable shear connection

(Dall'Asta and Zona 2002). Realistic uniaxial cyclic constitutive laws are adopted for the steel and concrete materials of the beam and for the shear connection. The monotonic and cyclic responses of these material and finite element models were already validated previously (Zona *et al.* 2004, 2005) through comparison with experimental data available in the literature (Ansourian 1981, Bursi and Gramola 2000). The sensitivity analysis is performed following the Direct Differentiation Method (DDM), the derivation and computer implementation of which are validated through Forward Finite Difference (FFD) analysis (Conte 2001, Conte *et al.* 2003). Results of sensitivity analysis are used to investigate and quantify the effect and relative importance of the various material parameters in regards to the monotonic and cyclic nonlinear response of continuous composite beams, which are commonly used in bridge construction.

2. Finite element response analysis

After spatial discretization using the finite element method, the equilibrium equations of a materially-nonlinear-only model of a structural system under quasi-static loading condition can be expressed as

$$\mathbf{R}(\mathbf{u}(t,\theta),\theta) = \mathbf{F}(t,\theta) \tag{1}$$

where t = pseudo-time, θ = scalar sensitivity parameter (material or loading variable), $\mathbf{u}$ = vector of nodal displacements, $\mathbf{R}$ = history dependent internal resisting force vector, $\mathbf{F}$ = applied quasi-static load vector. The solution $\mathbf{u}_{n+1}$ of Eq. (1) at discrete time t_{n+1} is obtained through the Newton-Raphson iterative procedure, which consists of solving the following linearized system of equations at each iteration:

$$\left.\frac{\partial \mathbf{R}(\mathbf{u}(\theta),\theta)}{\partial \mathbf{u}}\right|_{\mathbf{u}_{n+1}^{i}} (\mathbf{u}_{n+1}^{i+1} - \mathbf{u}_{n+1}^{i}) = \mathbf{F}_{n+1} - \mathbf{R}(\mathbf{u}_{n+1}^{i}(\theta),\theta) \tag{2}$$

until

$$\psi(\mathbf{u}_{n+1}) = \mathbf{F}_{n+1} - \mathbf{R}(\mathbf{u}_{n+1}) = \mathbf{0} \tag{3}$$

is satisfied within an assigned tolerance. In the above equations, the subscript $(...)_{n+1}$ indicates that the quantity to which it is attached is evaluated at time t_{n+1}, while the superscript i refers to the i-th iteration. The problem can be easily extended to the dynamic load case by adding inertial and damping forces in Eq. (1) and integrating the resulting equations of motion (or equations of dynamic equilibrium) using the Newmark-β time-stepping scheme (Chopra 2001), yielding a linearized equation similar to Eq. (2) (Conte 2001, Conte *et al.* 2003, 2004). The matrix operator on the left-hand-side (LHS) of Eq. (2) is the structure consistent tangent stiffness matrix:

$$(\mathbf{K}_T^{stat})_{n+1}^{i} = \left.\frac{\partial \mathbf{R}(\mathbf{u}(\theta),\theta)}{\partial \mathbf{u}}\right|_{\mathbf{u}_{n+1}^{i}} \tag{4}$$

The second term on the right-hand-side (RHS) of Eq. (2) represents the internal resisting force vector, which is obtained by assembling, at the structure level, the vectors of elemental internal resisting forces, $\mathbf{Q}^{(e)}$, as

$$\mathbf{R}(\mathbf{u}(\theta),\, \theta) \;=\; \underset{e=1}{\overset{Nel}{\mathbf{A}}} \{\mathbf{Q}^{(e)}(\mathbf{q}^{(e)}(\theta),\theta)\} \tag{5}$$

where $\mathbf{A}_{e=1}^{Nel}\{\cdots\}$ denotes the direct stiffness assembly operator from the element level (in local element coordinates) to the structure level in global reference coordinates; *Nel* represents the total number of finite elements in the model; and $\mathbf{q}^{(e)}$ is the vector of element nodal displacements in local coordinates. In the displacement-based finite element methodology, the element internal resisting force vector is obtained as

$$\mathbf{Q}(\mathbf{q}(\theta),\theta) \;=\; \int_0^L \mathbf{B}^T(z)\mathbf{D}(\mathbf{B}(z)\mathbf{q}(\theta),\theta)dz \tag{6}$$

where $\mathbf{B}$ is the transformation matrix between the vector of element nodal displacements $\mathbf{q}$ and the vector of generalized section deformations $\mathbf{d}$, i.e.,

$$\mathbf{d}(z,\, \theta) \;=\; \mathbf{B}(z)\mathbf{q}(\theta) \tag{7}$$

and $\mathbf{D}$ denotes the vector of active stress resultants at the section level. In the case of a composite beam with deformable shear connection (Dall'Asta and Zona 2002), the vector of generalized section deformations is defined as

$$\mathbf{d}^T(z,\theta) \;=\; [\varepsilon_1(z,\theta) \quad \varepsilon_2(z,\theta) \quad \chi(z,\theta) \quad \delta(z,\theta)] \tag{8}$$

where ε_1 and ε_2 are the axial strains at the reference points G_1 (concrete slab) and G_2 (steel beam), respectively, χ is the curvature (same for concrete slab and steel beam) and δ is the slip at the interface between the concrete slab and the steel beam (Fig. 1). The vector of section stress resultants is defined as

$$\mathbf{D}^T(z,\theta) \;=\; [N_1(z,\theta) \quad N_2(z,\theta) \quad M_{12}(z,\theta) \quad f_s(z,\theta)] \tag{9}$$

where N_1 is the axial force in the concrete slab, N_2 is the axial force in the steel beam, M_{12} is the summation of the bending moments in the concrete slab and steel beam, and f_s is the interface shear force per unit length (Dall'Asta and Zona 2002). The stress resultants N_1, N_2 and M_{12} are calculated by numerical integration over the concrete and steel parts of the beam cross-section, which are discretized using a fiber model, i.e.,

$$N_\alpha(z,\theta) \;=\; \int_{A_\alpha} \sigma(y,z,\mathbf{q}(\theta),\theta)\, dA, \quad \alpha = 1,2 \tag{10}$$

$$M_{12}(z,\theta) \;=\; \sum_{\alpha=1}^{2}\left(\int_{A_\alpha}(y-y_\alpha)\sigma(y,z,\mathbf{q}(\theta),\theta)dA\right) \tag{11}$$

where $\sigma(y,z,\theta)$ is the normal stress; y_1 and y_2 are the reference points of the two components of the composite beam (Fig. 1) (usually, but not necessarily, taken as the centroids of the concrete slab and the steel beam, respectively); and A_1 and A_2 are the cross-sections of the concrete slab and steel beam, respectively. The calculation of the stress on the RHS of Eqs. (10) and (11) and of f_s is

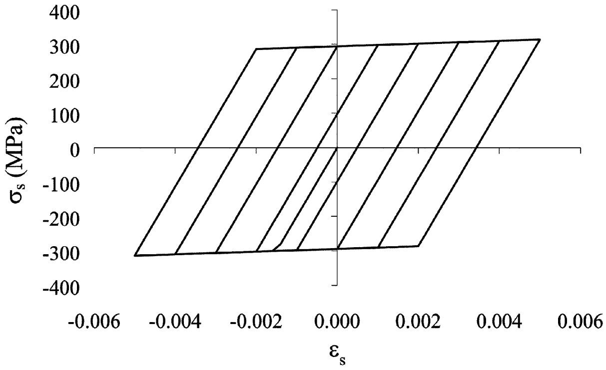

Fig. 2 Typical cyclic response of steel material model

performed at the material level, thus the nonlinear constitutive laws adopted in this study are defined next.

The constitutive law used for the steel of the beam is a uniaxial cyclic plasticity model with the von Mises yield criterion in conjunction with linear kinematic and isotropic hardening laws. The detailed formulation of the steel constitutive law together with all material parameters defining it can be found in (Conte *et al.* 2003). The same constitutive law is also used for the reinforcing steel in the concrete slab. A typical cyclic response of the steel material model adopted in this study is shown in Fig. 2.

The selected constitutive law for the concrete material is a uniaxial cyclic law with monotonic envelope given by the Popovics-Saenz law (Balan *et al.* 1997, 2001, Kwon and Spacone 2002). The details of the formulation of this constitutive law and related material parameters can be found in (Zona *et al.* 2004). A typical cyclic response of the concrete material model adopted herein is given in Fig. 3.

The constitutive law used for the shear connectors is a slip-force cyclic law with monotonic envelope given by the Ollgaard *et al.* law (1971). The cyclic response of the shear connectors is a modified version of the model proposed by Eligenhausen *et al.* (1983) and is similar to the model used by Salari and Spacone (2001). The details of the formulation of this constitutive law and related material parameters can be found in (Zona *et al.* 2004). A typical cyclic response of the constitutive model for the shear connectors used in this study is shown in Fig. 4.

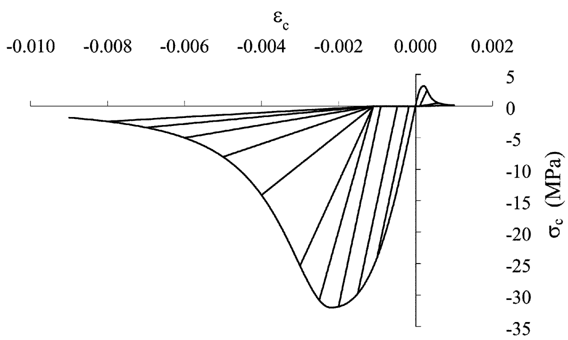

Fig. 3 Typical cyclic response of concrete material

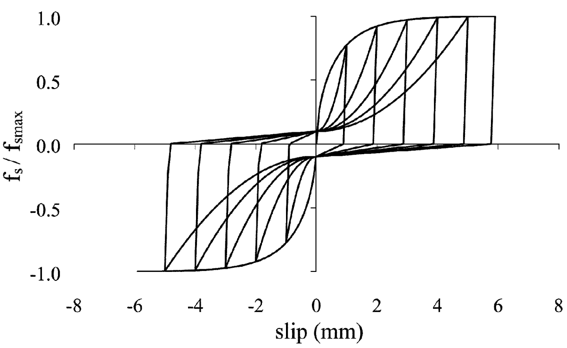

Fig. 4 Typical cyclic response of constitutive model for shear connectors

3. Finite element response sensitivity analysis by the direct differentiation method

If r denotes a generic scalar response quantity (e.g., displacement, strain, stress), then by definition, the sensitivity of r with respect to the material or loading parameter θ is expressed mathematically as the absolute partial derivative of r with respect to the variable θ, $\left.\frac{\partial r}{\partial \theta}\right|_{\theta = \theta_0}$ where θ_0 denotes the nominal value taken by the sensitivity parameter θ for the finite element response analysis.

In the sequel, following the notation proposed by Kleiber *et al.* (1997), the scalar response quantity $r(\boldsymbol{\vartheta}) = r(\mathbf{f}(\boldsymbol{\vartheta}), \boldsymbol{\vartheta})$ depends on the parameter vector $\boldsymbol{\vartheta}$ (defined by n time-independent sensitivity parameters, i.e., $\boldsymbol{\vartheta} = [\theta_1 \cdots \theta_n]^T$), both explicitly and implicitly through the vector function $\mathbf{f}(\boldsymbol{\vartheta})$. It is assumed that $\frac{dr}{d\boldsymbol{\vartheta}}$ denotes the gradient or total derivative of r with respect to $\boldsymbol{\vartheta}$, $\frac{dr}{d\theta_i}$ is the absolute partial derivative of the argument r with respect to the scalar variable θ_i, $i = 1, \ldots, n$, (i.e., the derivative of the quantity r with respect to the parameter θ_i considering explicit and implicit dependencies), and $\left.\frac{\partial r}{\partial \theta_i}\right|_z$ is the partial derivative of r with respect to parameter θ_i when the vector of variables $\mathbf{z}$ is kept constant/fixed. In the particular and important case in which $\mathbf{z} = \mathbf{f}(\boldsymbol{\vartheta})$, the expression $\left.\frac{\partial r}{\partial \theta_i}\right|_z$ reduces to the partial derivative of r considering only the explicit dependency of r on parameter θ_i. For $\boldsymbol{\vartheta} = \theta = \theta_1$ (case of single sensitivity parameter), the adopted notation reduces to the usual elementary calculus notation. The derivations in the sequel consider the case of a single (scalar) sensitivity parameter θ without loss of generality, due to the uncoupled nature of the sensitivity equations with respect to multiple sensitivity parameters.

Following the Direct Differentiation Method (DDM) (Conte 2001, Conte *et al.* 2003), the consistent response sensitivities are computed at each time step, after convergence of the response computation. This requires an exact differentiation of the finite element algorithm for the response calculation (including the numerical integration scheme for the material constitutive law) with respect to the sensitivity parameter θ. Consequently the response sensitivity calculation algorithm affects the various hierarchical layers of finite element response calculation, namely: (1) structure level, (2) element level, (3) section level and (4) material level.

Assuming that $\mathbf{u}_{n+1}$ is the converged solution (up to some iteration residuals satisfying a specified tolerance) at discrete time t_{n+1}, and differentiating Eq. (3) with respect to θ using the chain rule of differentiation, yields

$$\left.\frac{\partial \mathbf{R}(\mathbf{u}_{n+1}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{u}_{n+1}} + \left.\frac{\partial \mathbf{R}(\mathbf{u}_{n+1}(\theta),\theta)}{\partial \mathbf{u}_{n+1}}\right|_{\theta} \frac{d\mathbf{u}_{n+1}}{d\theta} = \frac{d\mathbf{F}_{n+1}}{d\theta} \tag{12}$$

and the following response sensitivity equation at the structure level is obtained:

$$(\mathbf{K}_T^{stat})_{n+1} \frac{d\mathbf{u}_{n+1}}{d\theta} = \frac{d\mathbf{F}_{n+1}}{d\theta} - \left.\frac{\partial \mathbf{R}(\mathbf{u}_{n+1}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{u}_{n+1}} \tag{13}$$

The second term on the RHS of Eq. (13) represents the partial derivative of the internal resisting force vector, $\mathbf{R}(\mathbf{u}_{n+1})$, with respect to sensitivity parameter θ under the condition that the displacement vector $\mathbf{u}_{n+1}$ remains fixed (conditional derivatives). It can be expressed as

$$\left.\frac{\partial \mathbf{R}(\mathbf{u}_{n+1}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{u}_{n+1}} = \mathop{\mathbf{A}}_{e=1}^{Nel} \left\{ \left.\frac{\partial \mathbf{Q}^{(e)}(\mathbf{q}^{(e)}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}^{(e)}} \right\} \tag{14}$$

The calculation of the conditional derivative on the RHS of Eq. (14) is performed at the element level, computing the conditional derivative of the element internal resisting force vector as

$$\left.\frac{\partial \mathbf{Q}(\mathbf{q}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}} = \int_0^L \mathbf{B}^T(z)\left.\frac{\partial \mathbf{D}(\mathbf{B}(z)\mathbf{q}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}} dz \tag{15}$$

The finite element used in this study is a displacement-based shear locking free composite beam element with 10 degrees of freedom, two of which are internal. For computational efficiency, these internal degrees of freedom are not assembled at the structure level, but are taken into account using a static condensation procedure (Bathe 1995). The algorithm for response sensitivity computation in the presence of static condensation has been derived by the authors and presented elsewhere (Zona *et al.* 2005).

The calculation of the conditional derivative on the RHS of Eq. (15) is carried out at the section level as

$$\left.\frac{\partial N_\alpha(z,\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}} = \int_{A_\alpha}\left.\frac{\partial \sigma(y,z,\mathbf{q}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}} dA, \qquad \alpha = 1,2 \tag{16}$$

$$\left.\frac{\partial M_{12}(z,\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}} = \sum_{\alpha=1}^{2}\left(\int_{A_\alpha}(y-y_\alpha)\left.\frac{\partial \sigma(y,z,\mathbf{q}(\theta),\theta)}{\partial \theta}\right|_{\mathbf{q}_{n+1}} dA\right) \tag{17}$$

The conditional derivative on the RHS of Eqs. (16) and (17) and the conditional derivative of f_s (given $\mathbf{q}_{n+1}$) are calculated at the material level, hence the (discretized) material constitutive equations must be differentiated analytically with respect to the material parameters (Zona *et al.* 2004).

Once the RHS of Eq. (13) has been formed, the nodal displacement response sensitivities $\frac{d\mathbf{u}_{n+1}}{d\theta}$ can be solved for and subsequently the unconditional derivatives of all history/state variables at all integration points are updated. The nodal displacement sensitivities at the structural level are transformed into local coordinates for each element, thus yielding the element nodal displacement sensitivities, $\frac{d\mathbf{q}_{n+1}}{d\theta}$. Then, using the compatibility relations, the sensitivities of the generalized section deformations are obtained as

$$\frac{d\mathbf{d}_{n+1}(\theta)}{d\theta} = \mathbf{B}(z)\frac{d\mathbf{q}_{n+1}(\theta)}{d\theta} \tag{18}$$

and the sensitivities of the section stress resultants are evaluated using the unconditional derivatives (with respect to the sensitivity parameter θ) of the material constitutive relations as

$$\frac{dN_\alpha(z,\theta)}{d\theta} = \int_{A_\alpha}\frac{d\sigma(y,z,\mathbf{q}(\theta),\theta)}{d\theta}dA, \qquad \alpha = 1,2 \tag{19}$$

$$\frac{dM_{12}(z,\theta)}{d\theta} = \sum_{\alpha=1}^{2}\left(\int_{A_\alpha}(y-y_\alpha)\frac{d\sigma(y,z,\mathbf{q}(\theta),\theta)}{d\theta}dA\right) \tag{20}$$

Notice that, once the numerical response of the structural system is known at time t_{n+1}, the matrix sensitivity equation, Eq. (13), is linear and has the same LHS matrix operator (consistent tangent stiffness matrix) as the consistently linearized global equilibrium equations, Eq. (2), at the end of the iteration which satisfies convergence for the response calculation at discrete time t_{n+1}. Therefore, only the RHS of Eq. (13) needs to be recomputed and since the factorization of the consistent tangent stiffness matrix is already available at the converged time step t_{n+1}, solution of the response sensitivity Eq. (13) is computationally efficient.

4. Finite element response sensitivity analysis by the forward finite difference method

The Forward Finite Difference (FFD) method is used to validate the derivation and computer implementation of the Direct Differentiation Method (DDM) (Conte 2001, Conte *et al.* 2003, 2004) as applied to composite frame structures. The FFD method consists in estimating the derivative of the response by finite difference approximations, i.e., imposing a perturbation of the sensitivity parameter θ and using a first-order approximation (Kleiber *et al.* 1997). For example, the following expression:

$$\frac{d\mathbf{u}_{n+1}}{d\theta} \cong \frac{\mathbf{u}_{n+1}(\theta+\Delta\theta) - \mathbf{u}_{n+1}(\theta)}{\Delta\theta} \tag{21}$$

where $\Delta\theta$ is an assigned small perturbation of the sensitivity parameter, is used for estimating the sensitivities of the element nodal displacements to parameter θ. The optimum value of $\Delta\theta$ is determined by studying the convergence of the finite difference approximation of the response derivatives with decreasing values of $\Delta\theta$. It has been observed that there is an optimum value of $\Delta\theta/\theta$ that makes the finite difference result closest to the DDM result. However, this optimum value of $\Delta\theta/\theta$ usually depends on both the loading case and the sensitivity parameter θ itself. When the relative sensitivity parameter increment grows above this optimum value, the finite difference results worsens due to truncation error (i.e., effects of higher order terms in Taylor series expansion of response parameter). If the relative sensitivity parameter increment is decreased below this optimum value, so as to reduce the truncation error, we have an excessive condition error. The latter is due to round-off errors in the computer (i.e., finite precision arithmetic) or occurs if the response is calculated by an iterative process which is terminated early (as is the case for the incremental-iterative Newton-Raphson method typically used in nonlinear finite element analysis). In some cases, there may not be any sensitivity parameter increment $\Delta\theta$ which yields an acceptable error. This is the so-called "step-size dilemma" (Haftka and Gürdal 1992, Gu and Conte 2003).

The first-order FFD method requires a repeat of the entire analysis at least once for each desired sensitivity parameter. Thus, FFD analysis is computationally significantly more expensive than the DDM, especially when dealing with a large number of sensitivity parameters as in finite element reliability analysis (Li and Der Kiureghian 1995, Vijalapura *et al.* 2000).

5. Finite element response sensitivity analysis for gradient-based optimization methods

Optimization problems are very common in many engineering disciplines such as structural optimization, structural reliability, structural identification and finite element model updating (Kleiber *et al.* 1997). These problems can be stated as follows. Given an objective function $g(\mathbf{x})$ of a vector of variables **x**, find the optimal values $\mathbf{x}^*$ which minimize $g(\mathbf{x})$ with **x** subjected to the constraints $\mathbf{c}^{(e)}(\mathbf{x})=\mathbf{0}$ and $\mathbf{c}^{(i)}(\mathbf{x})\geq\mathbf{0}$, where $\mathbf{c}^{(e)}(\mathbf{x})$ and $\mathbf{c}^{(i)}(\mathbf{x})$ are vector valued functions of **x** describing equality and inequality constraints, respectively.

In structural optimization, for example, the objective function $g(\mathbf{x})$ is a cost function used to measure the soundness of the design; the variables **x** are design parameters describing the system; the constraints $\mathbf{c}^{(e)}(\mathbf{x})$ and $\mathbf{c}^{(i)}(\mathbf{x})$ are limits on the variability of the design parameters assuring the safety of the system, equilibrium conditions, technical availability of materials, construction procedures, etc. (Kleiber *et al.* 1997). Another example is in structural reliability analysis, where the optimization problem consists of finding the design point, defined as the most likely failure point in the standard normal space in which the calculation of the failure probability is performed (Ditlevsen and Madsen 1996, Melchers 1999).

All classical optimization methods (Newton's method, quasi-Newton methods, linear programming, quadratic programming, penalty function methods, etc.) are gradient based, i.e., they require the evaluation of the gradient of the objective function. In general, the most computationally intensive task in solving an optimization problem in structural engineering is the calculation of the response sensitivities to material, geometric and loading parameters. For this reason, accurate and efficient calculation of finite element response sensitivities is crucial.

6. Application examples

The above framework for finite element response and response sensitivity analysis is illustrated using a testbed structure consisting of a two-span continuous beam shown in Fig. 5. The two spans are of equal length (30 m) and the cross-section and material properties are assumed to remain constant along the beam. The connection between concrete slab and steel beam was designed in order to achieve full shear connection (Oehlers and Bradford 2000, CEN 1997a,b) using a uniform distribution of shear connectors along the beam with strength f_{smax} = 1225 kN/m.

A vertical force F is applied at the mid-point of each span as shown in Fig. 5. Both the structure and the loading are symmetric; thus, only half of the structure is modeled and analyzed. Two load cases are considered: (a) monotonic increase of the magnitude of the two concentrated loads; and (b) cyclic application of the two concentrated loads. The nonlinear analyses were performed until collapse, which is defined as the point at which the ultimate strain or the ultimate slip was reached for the first time along any of the material fibers or along the shear connection, respectively.

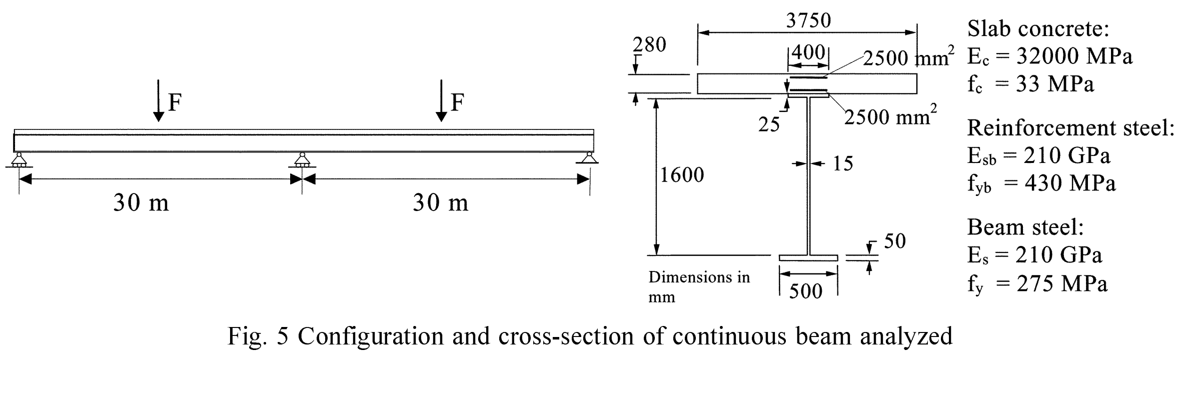

Fig. 5 Configuration and cross-section of continuous beam analyzed

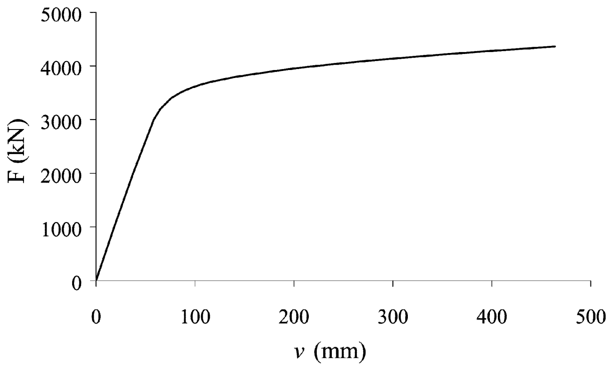

Fig. 6 Computed load-deflection curve for monotonic load case

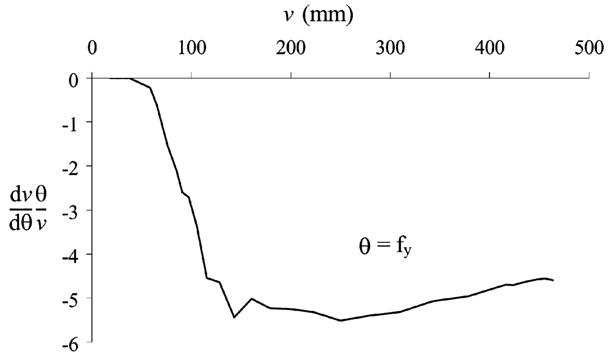

Fig. 7 Sensitivity of mid-span deflection, v, to f_y (monotonic load case)

This example represents a structural system commonly used in bridge construction and presents all the typical difficulties encountered in nonlinear analysis of composite structures, e.g., cracking of concrete in traction, softening of concrete under compression, wide spread yielding in the steel beam and in the slab steel reinforcements, high gradients of slip and shear force along the connection.

Nine material parameters are considered for the sensitivity analysis presented below: the yield stress (f_y), modulus of elasticity (E_0) and kinematic hardening modulus (H_k) of the steel beam material, the strength (f_{smax}) of the shear connection, the compressive strength (f_c) and modulus of elasticity (E_c) of the concrete, the yield stress ($f_{y,reb}$), modulus of elasticity ($E_{0,reb}$) and kinematic hardening modulus ($H_{k,reb}$) of the steel reinforcements of the concrete slab.

The sensitivity results are presented in normalized form, i.e., multiplied by the value of the parameter and divided by the value of the response quantity, the sensitivity of which is considered. Consequently, the normalized sensitivities represent the percent variation of the subject response quantity for a unitary percent variation of the sensitivity parameter. In this way, the normalized response sensitivities reveal directly the relative importance of all the material parameters considered in regards to a given response quantity at various loading stages of the structure.

All the results shown below have been computed using the DDM and validated by the FFD method using increasingly small perturbations of the sensitivity parameter. For the sake of brevity, the comparison between DDM and FFD results is shown only for a few cases.

6.1. Monotonic load case

The load-deflection curve under monotonic loading is plotted in Fig. 6. Collapse as defined above is reached at the load $F = 4360$ kN with a mid-span deflection of 463 mm, when the ultimate slip (8 mm) in the shear connection is attained for the first time (at 18.75 m from the moment-free end of the beam).

The sensitivity of the mid-span deflection, v, to the yield stress of the steel beam ($\theta = f_y$) as a function of the mid-span deflection itself is shown in Fig. 7. The sensitivity is zero before the steel beam yields for the first time, since prior to first yield, f_y does not affect the response. Once the yielding of the steel beam initiates and propagates, the sensitivity to f_y increases in magnitude. For example, when the structure is in its initial linear elastic range, a 1% variation of the yield stress f_y does not produce any variation of the mid-span deflection, since the sensitivity is zero. When the structure is in the yielding plateau, a 1% increase of the yield stress f_y produces an approximately 5% reduction in the mid-span deflection.

The sensitivities of the mid-span deflection, v, to (i) the modulus of elasticity of the steel beam material ($\theta = E_0$), (ii) the hardening modulus of the steel beam material ($\theta = H_k$), (iii) the shear connection strength ($\theta = f_{smax}$), and (iv) the yield stress of the steel reinforcements ($\theta = f_{y,reb}$), are plotted in Fig. 8. The sensitivities of the mid-span deflection, v, to (i) the concrete modulus of elasticity

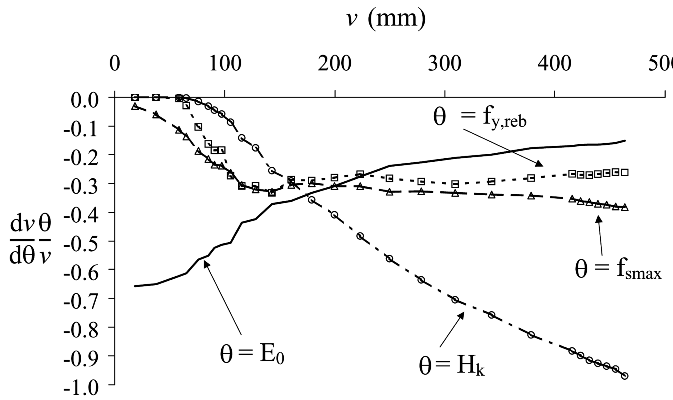

Fig. 8 Sensitivities of mid-span deflection, v, to E_0, H_k, f_{smax} and $f_{y,reb}$ (monotonic load case)

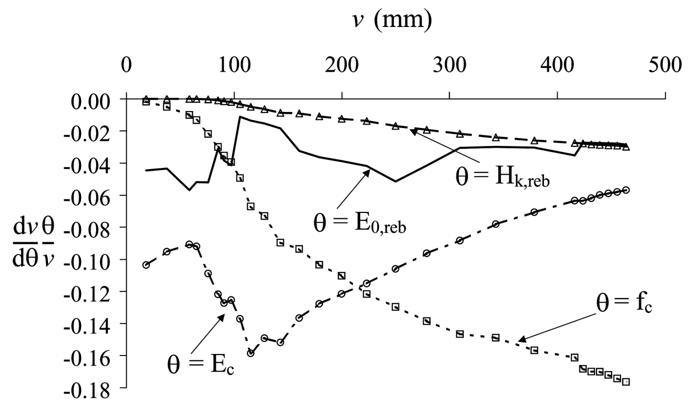

Fig. 9 Sensitivities of mid-span deflection, v, to E_c, f_c, $E_{0,reb}$ and $H_{k,reb}$ (monotonic load case)

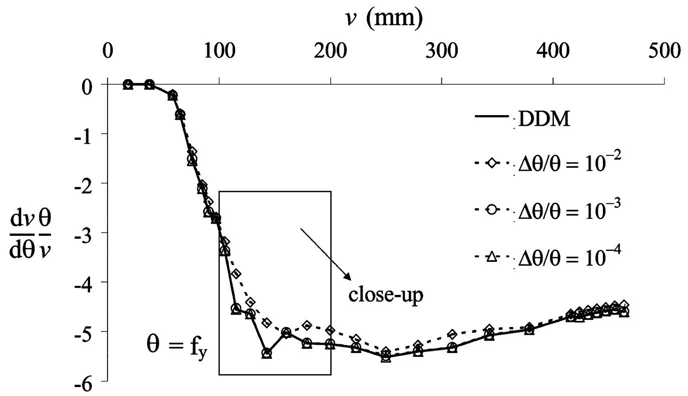

Fig. 10 Sensitivity of mid-span deflection, v, to f_y using DDM and FFD (monotonic load case)

($\theta = E_c$), (ii) the concrete compressive strength ($\theta = f_c$), (iii) the modulus of elasticity of the steel reinforcements ($\theta = E_{0,reb}$), and (iv) the hardening modulus of the steel reinforcements ($\theta = H_{k,reb}$), are plotted in Fig. 9.

The sensitivities in Fig. 8 and Fig. 9 assume smaller values than the sensitivity to f_y shown in Fig. 7. Thus, f_y is the material parameter that influences the most significantly the nonlinear global response as measured by the mid-span deflection v. The sensitivity to H_k increases considerably after the first yielding in the steel beam and becomes the second highest sensitivity at collapse. The sensitivity to E_0, even though it is the highest during the early stage of loading when the structure is still mostly in its linear elastic range, decreases steadily as the structure evolves toward collapse, when the material strength parameters become more important than the elastic ones. The sensitivity to the strength f_{smax} of the shear connection largely increases as the structure develops inelastic deformations and becomes the third highest sensitivity at collapse. For example, a 1% increase in the shear connection strength produces a reduction of about 0.4% in the mid-span deflection at collapse. The sensitivities to the concrete material parameters behave qualitatively as those to the steel beam material parameters, i.e., the sensitivity to the modulus of elasticity E_c decreases, while the sensitivity to f_c increases as the structure evolves in its inelastic range towards collapse. However, quantitatively, the response sensitivities to the steel beam material parameters are higher than those to the concrete material parameters.

The higher response sensitivities to the steel beam material parameters are a consequence of the fact that the steel beam is carrying the largest part of the applied loads. Thus, even a small variation of the material properties of the steel beam can significantly affect the structural response. Moreover, the response sensitivity computation proves the intuitive fact that the most important material parameters for serviceability (deflection-controlled) limit-states are stiffness related (e.g., E_0 and E_c), while strength related parameters (e.g., f_y, f_c, f_{smax}) become predominant for collapse limit-states. An exception is given by the hardening moduli of the steel beam and steel reinforcement materials: even if they are stiffness related quantities, they may become important only when hardening of the material develops significantly, thus when large plastic deformations are attained.

Sensitivities of local response quantities (e.g., beam internal forces, shear connection force, local plastic deformations) are also computed using the DDM, and are not reported here for the sake of brevity. Examples of sensitivities of local response quantities for steel-concrete composite beams can be found in (Zona *et al.* 2004, 2005).

For validation purposes, the sensitivity results obtained by the DDM have been compared to their

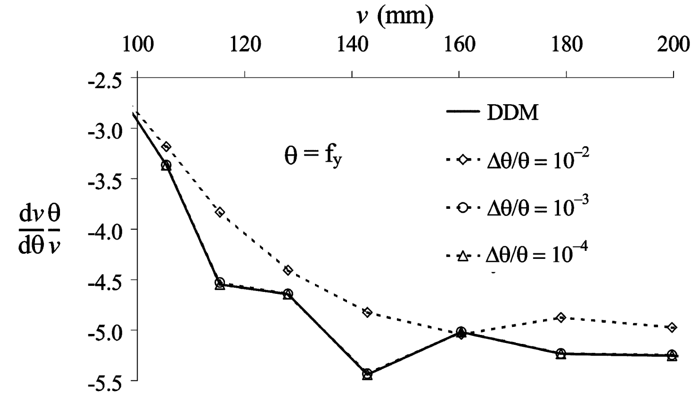

Fig. 11 Sensitivity of mid-span deflection, v, to f_y using DDM and FFD (monotonic load case)

counterparts obtained via FFD analysis. Only comparisons between the sensitivities to f_y computed by the DDM and by FFD analysis are shown here (Figs. 10 and 11). Three levels of perturbation of parameter f_y have been considered ($\Delta\theta/\theta = 10^{-2}, 10^{-3}, 10^{-4}$). Fig. 10 and a close-up in Fig. 11 show that the FFD results converge asymptotically to the DDM results as $\Delta\theta/\theta$ becomes increasingly small. In this particular case, the FFD results are already converged for $\Delta\theta/\theta = 10^{-3}$.

6.2. Cyclic load case

In the first load cycle, the two applied forces F simultaneously increase from zero to 3500 kN, then decrease back to zero. In the second load cycle, the forces F increase from zero to 4000 kN and then revert back to zero. In the third load cycle, the forces F increase from zero to 4250 kN and then are reduced back to zero. Finally, the forces F increase from zero until collapse (as defined above) is reached. The computed load-deflection curve obtained for the cyclic load case is given in Fig. 12. Collapse is reached under the load F = 4360 kN with a mid-span deflection of 464 mm, when the ultimate slip (8 mm) in the shear connection is attained for the first time (at 18.75 m from the moment-free end of the beam).

The collapse point nearly coincides with that of the monotonic load case, since the structure unloads

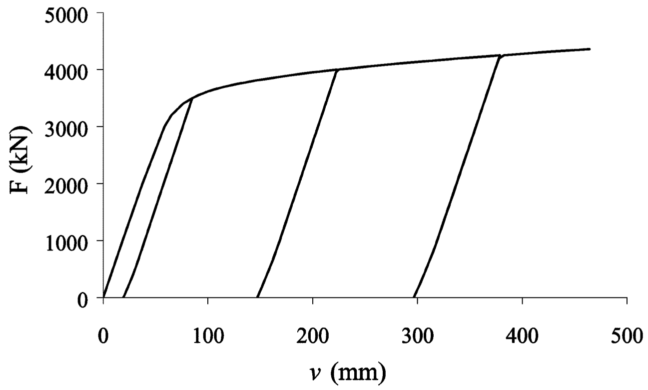

Fig. 12 Computed load-deflection curve for cyclic load case

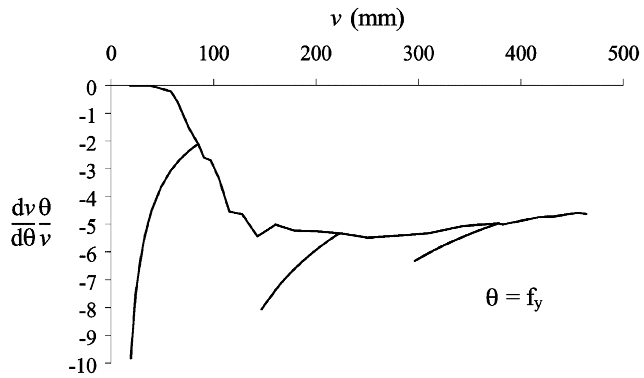

Fig. 13 Sensitivity of mid-span deflection, v, to f_y (cyclic load case)

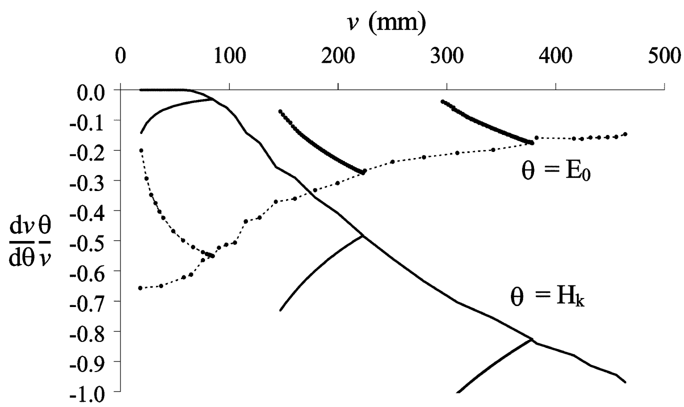

Fig. 14 Sensitivities of mid-span deflection, v, to E_0 and H_k (cyclic load case)

and reloads elastically without hysteresis, and once back on the monotonic envelope, it behaves as if it had never unloaded before. This is a consequence of the material models used in this study (no damage models have been included in the material constitutive laws) and of the loading history without load reversals (the materials unload partially along linear paths without reaching the loading branches in the reversal direction and reload along the same linear paths). The cyclic loading behavior analyzed herein is not of the most general form. Examples of more complex cyclic behavior (with load reversals) are given in (Zona *et al.* 2004, 2005).

The sensitivity of the mid-span deflection, v, to the yield stress of the steel beam material ($\theta = f_y$) is shown as a function of the mid-span deflection itself in Fig. 13. It is observed that the response sensitivity along the unloading and reloading paths is significantly higher than that along the loading path. The sensitivities of the mid-span deflection, v, to the modulus of elasticity ($\theta = E_0$) and kinematic hardening modulus ($\theta = H_k$) of the steel beam material are plotted in Fig. 14; the sensitivity to the shear connection strength ($\theta = f_{smax}$) is plotted in Fig. 15; the sensitivity to the yield stress of the steel reinforcements ($\theta = f_{y,reb}$) is shown in Fig. 16, while the sensitivities to the concrete modulus of elasticity ($\theta = E_c$) and to the modulus of elasticity of the steel reinforcements ($\theta = E_{0,reb}$) are plotted in Fig. 17; finally the sensitivity to the concrete compressive strength ($\theta = f_c$) and the sensitivity to the kinematic hardening modulus of the steel reinforcements ($\theta = H_{k,reb}$) are shown in Fig. 18 and Fig. 19, respectively.

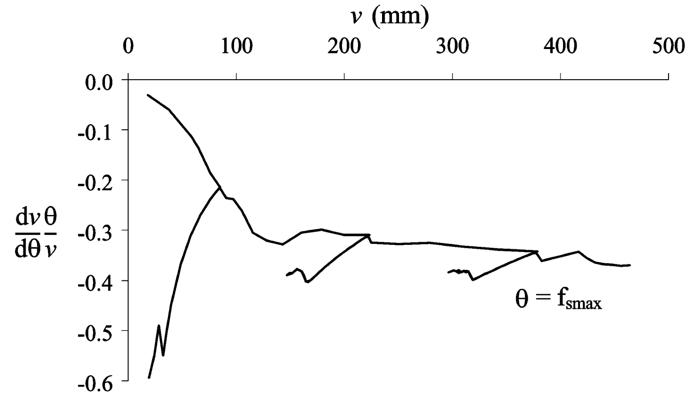

Fig. 15 Sensitivity of mid-span deflection, v, to f_{smax} (cyclic load case)

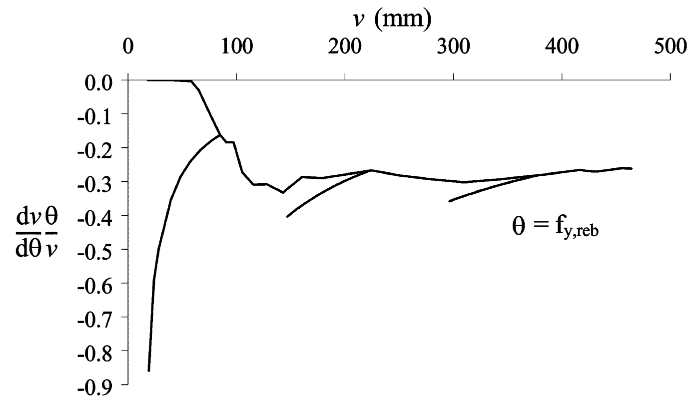

Fig. 16 Sensitivity of mid-span deflection, v, to $f_{y,reb}$ (cyclic load case)

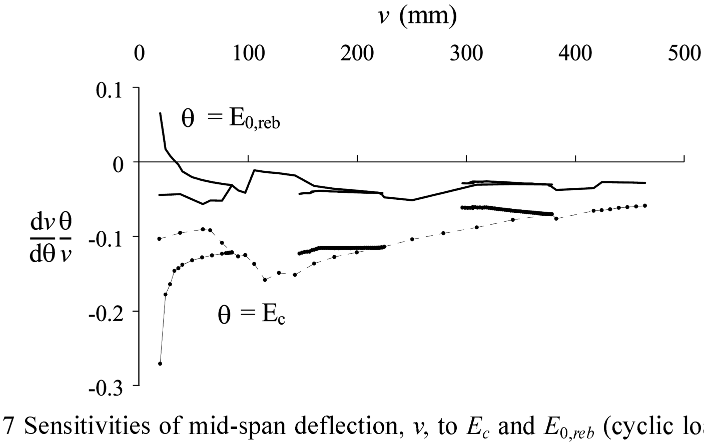

Fig. 17 Sensitivities of mid-span deflection, v, to E_c and $E_{0,reb}$ (cyclic load case)

As in the monotonic load case, the sensitivities to parameters E_0, H_k, f_{smax}, f_c, E_c, $f_{y,reb}$, $E_{0,reb}$, and $H_{k,reb}$ are significantly smaller than the sensitivity to f_y. Moreover, it is observed that the sensitivities along the unloading and reloading paths are systematically higher than the ones along the loading path if the response sensitivity considered increases in magnitude along the loading path towards collapse (e.g.,

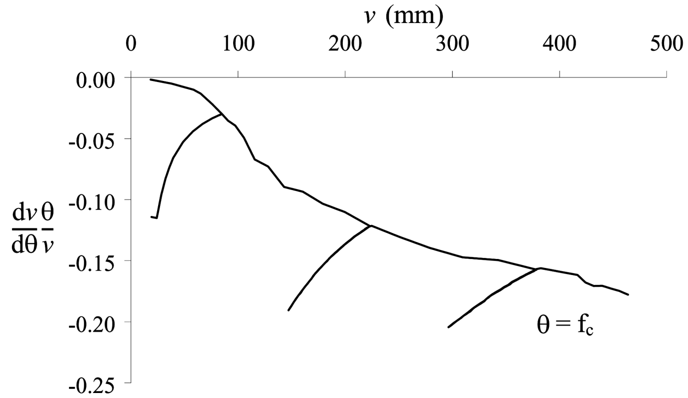

Fig. 18 Sensitivity of mid-span deflection, v, to f_c (cyclic load case)

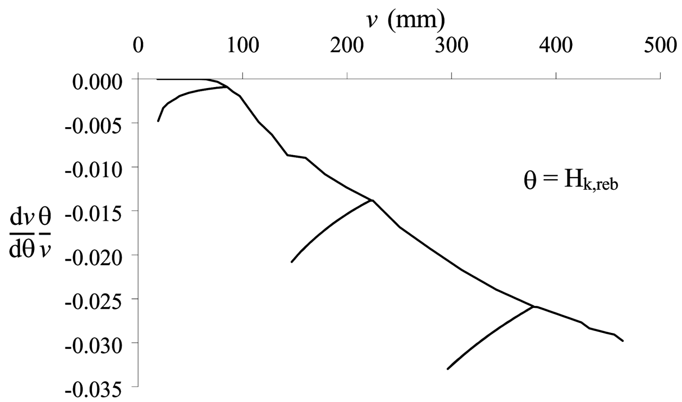

Fig. 19 Sensitivities of mid-span deflection, v, to $H_{k,reb}$ (cyclic load case)

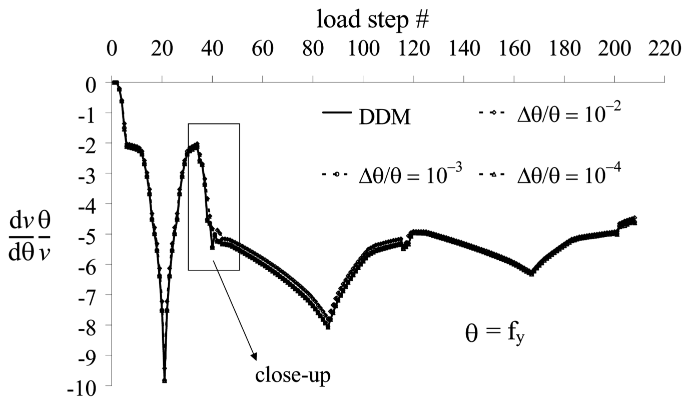

Fig. 20 Sensitivity of mid-span deflection, v, to f_y using DDM and FFD (cyclic load case)

case of f_{smax}, f_c and of first unloading-reloading cycle for E_c). The opposite behavior appears if the response sensitivity decreases in magnitude along the loading path (e.g., case of E_0 and of the second and third cycles for E_c).

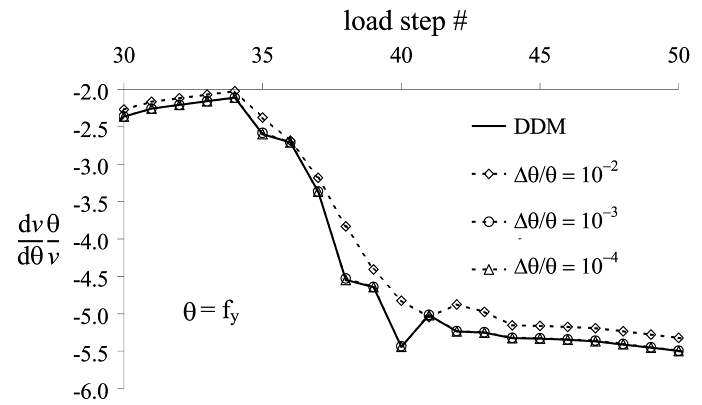

Fig. 21 Sensitivity of mid-span deflection, v, to f_y using DDM and FFD (cyclic load case)

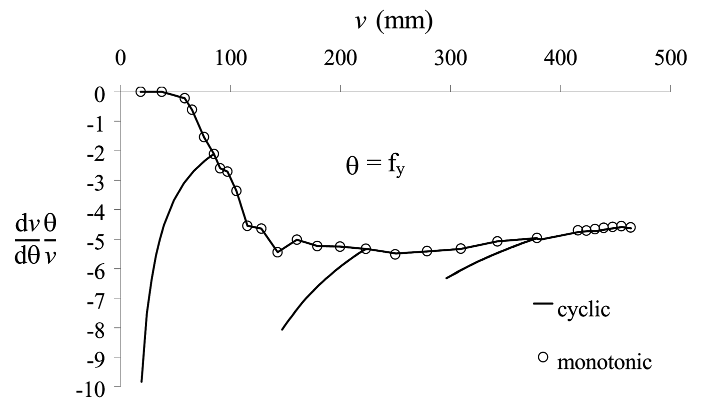

Fig. 22 Comparison of monotonic and cyclic response sensitivities to f_y

Validation through FFD analysis of the sensitivity results obtained using the DDM is shown below in Figs. 20 and 21 for sensitivity parameter $\theta = f_y$. Three levels of perturbation of material parameter f_y have been considered, namely $\Delta\theta/\theta = 10^{-2}$, 10^{-3}, 10^{-4}. Fig. 20 and a close-up in Fig. 21 show that the FFD results converge asymptotically to the DDM results as $\Delta\theta/\theta$ becomes increasingly small. As in the monotonic load case, the FFD results are already converged for $\Delta\theta/\theta = 10^{-3}$.

The continuous beam considered has a very similar response behavior under the monotonic and cyclic load cases, except for the unloading and reloading paths. During loading, the load-deflection curves are practically the same, due to the material models selected and the loading protocol. For the same reasons, the sensitivities of the mid-span deflection to the nine material parameters considered herein are practically the same for the monotonic and cyclic load cases, as shown in Fig. 22 for example.

7. Conclusions

This paper deals with materially-nonlinear-only analytical response sensitivity analysis, using displacement-based finite elements, of composite beams with deformable shear connection under quasi-static monotonic and cyclic load conditions. Computing analytical finite element response sensitivities has two main advantages: (i) computational efficiency as compared to finite difference methods for estimating sensitivities, especially in the presence of a large number of sensitivity parameters as in finite element reliability analysis, and (ii) circumventing the step size dilemma.

A two-span continuous composite beam is considered as an application example representative of a structural system commonly used in bridge construction. This example presents the main difficulties typically encountered in nonlinear analysis of composite structures, e.g., cracking of concrete in traction and softening under compression, wide spread yielding in the steel beam and in the slab steel reinforcements, high gradients of deformation and force along the shear connection. Results of sensitivity analysis, performed following the Direct Differentiation Method (DDM) and validated by means of Forward Finite Difference (FFD) analysis, are shown to gain insight into the effect and relative importance of the various material parameters. The computed sensitivities confirm the general intuition that the most important material parameters for serviceability (deflection-controlled) limit-states are stiffness related, while strength related parameters become predominant for collapse limit-states. Not only can response sensitivity analysis confirm or disaffirm engineering intuition about the relative importance of various material parameters, but also quantify these sensitivities in the various phases of loading/unloading/reloading at moderate extra computational cost. In the application example considered in this study, the inelastic global response of the structure near collapse is most sensitive to the yield stress of the steel beam and then to the shear connection strength. The results presented here can be generalized in the future to steel-concrete composite structures of various overall geometric configurations and member cross-section properties, and to different material properties and shear connector distributions.

Future applications may involve the use of response sensitivity analysis of steel-concrete composite structures for solving gradient-based optimization problems such as those encountered in structural optimization (e.g., optimum design of shear connectors), structural reliability analysis (e.g., evaluation of safety improvement in support of retrofit design), structural identification (e.g., nonlinear finite element model updating).

Acknowledgements

Partial support of this research by the National Science Foundation under Grant No. CMS-0010112 is gratefully acknowledged. The authors wish to thank Prof. Filip C. Filippou at the University of California, Berkeley, and Dr. Paolo Franchin at the University of Rome “La Sapienza”, Italy, for providing us with the Matlab-based nonlinear structural analysis framework, FEDEASLab, used in this study together with the added framework for finite element response sensitivity analysis. The authors also want to acknowledge Prof. Enrico Spacone at the University “G. D’Annunzio” of Chieti, Italy, and Dr. Reza Salari at RockSol Consulting Group, Inc., Boulder, Colorado, for very useful discussions and suggestions regarding the finite element modeling part of this work. In addition, the first author wishes to thank Prof. Luigino Dezi at the Università Politecnica delle Marche, Italy, and Prof. Andrea Dall’Asta at the University of Camerino, Italy, for their personal and financial support.

References

Ansourian, P. (1981), “Experiments on continuous composite beams”, *Proc. Instn. Civ. Engrs*. Part 2, **71**(Dec.), 25-51.

Ayoub, A. and Filippou, F. C. (2000), "Mixed formulation of nonlinear steel-concrete composite beam element", *J. Struct. Eng.*, ASCE, **126**(3), 371-381.

Balan, T. A., Filippou, F. C. and Popov, E. P., (1997), "Constitutive model for 3D cyclic analysis of concrete structures", *J. Eng. Mech.*, ASCE, **123**(2), 143-153.

Balan, T. A., Spacone, E. and Kwon, M. (2001), "A 3D hypoplastic model for cyclic analysis of concrete structures", *Eng. Struct.*, **23**(4), 333-342.

Bathe, K. J. (1995), *Finite Element Procedures*. Prentice Hall.

Bursi, O. S. and Gramola, G. (2000), "Behaviour of composite substructures with full and partial shear connection under quasi-static cyclic and pseudo-dynamic displacements". *Material and Structures*, RILEM, **33**, 154-163.

CEN, Comité Européen de Normalization (1997a), *Eurocode 4: Design of composite steel and concrete structures - Part 1.1: General - Common rules and ruled for buildings*, ENV 1994-1, Brussels.

CEN, Comité Européen de Normalization (1997b), *Eurocode 4: Design of composite steel and concrete structures - Part 2: Bridges*, ENV 1994-2, Brussels.

Chopra, A. K. (2001), *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, Second Edition, Prentice Hall.

Conte, J. P. (2001), "Finite element response sensitivity analysis in earthquake engineering", *Earthquake Engineering Frontiers in the New Millennium*. Spencer & Hu, Swets & Zeitlinger, 395-401.

Conte, J. P., Vijalapura, P. K. and Meghella, M. (2003), "Consistent finite element response sensitivities in seismic reliability analysis", *J. Eng. Mech.*, ASCE, **129**(12), 1380-1393.

Conte, J. P., Barbato, M. and Spacone, E. (2004), "Finite element response sensitivity analysis using force-based frame models", *Int. J. Numer. Methods Eng.*, **59**(13), 1781-1820.

Dall'Asta, A. (2001), "Composite beams with weak shear connection", *Int. J. Solids Struct.,* **38**(32-33), 5605-5624.

Dall'Asta, A. and Zona, A. (2002), "Non-linear analysis of composite beams by a displacement approach", *Comput. Struct.*, **80**(27-30), 2217-2228.

Dall'Asta, A. and Zona, A. (2004a), "Three-field mixed formulation for the non-linear analysis of composite beams with deformable shear connection", *Finite Elements in Analysis and Design*, **40**(4), 425-448.

Dall'Asta, A. and Zona, A. (2004b), "Slip-locking in finite elements for composite beams with deformable shear connection", *Finite Elements in Analysis and Design*, **40**(13-14), 1907-1930.

Dall'Asta, A. and Zona, A. (2004c), "Comparison and validation of displacement and mixed elements for the non-linear analysis of continuous composite beams", *Comput. Struct.*, **82**(23-26), 2117-2130.

Ditlevsen, O. and Madsen, H. O. (1996), *Structural Reliability Methods*. Wiley.

Eligenhausen, R., Popov, E. P. and Bertero, V. V. (1983), "Local bond stress-slip relationships of deformed bars under generalized excitations", Report No. 83/23*,* EERC Earthquake Engineering Research Center, University of California, Berkeley, p.162.

Gu, Q. and Conte, J. P. (2003), "Convergence studies in nonlinear finite element response sensitivity analysis", *Proc. of the 9th Int. Conf. on Applications of Statistics and Probability in Civil Engineering*, San Francisco, CA, USA.

Haftka, R. T. and Gürdal, Z. (1992), *Elements of Structural Optimization*. Kluwer Academic Publishers.

Kleiber, M., Antunez, H., Hien, T. D. and Kowalczyk, P. (1997), *Parameter Sensitivity in Nonlinear Mechanics: Theory and Finite Element Computations*. Wiley.

Kwon, M. and Spacone, E. (2002), "Three-dimensional finite element analyses of reinforced concrete columns", *Comput. Struct.,* **80**(2), 199-212.

Li, C. C. and Der Kiureghian, A. (1995), "Mean out-crossing rate of nonlinear response to stochastic input", *Proc. of the 7th Int. Conf. on Applications of Statistics and Probability (ICASP7)*, **1**, 295-302.

Melchers, R. M. (1999), *Structural Reliability Analysis and Prediction*, Second edition, Wiley.

Newmark, N. M., Siess, C. P. and Viest, I. M. (1951), "Tests and analysis of composite beams with incomplete interaction", *Proc. Soc. Exp. Stress Anal.*, **9**(1), 75-92.

Oehlers, D. J. and Bradford, M. A. (2000), *Elementary Behaviour of Composite Steel and Concrete Structural Members*, Butterworth-Heinemann.

Ollgaard, J. G., Slutter, R. G. and Fisher, J. W. (1971), "Shear strength of stud connectors in lightweight and normal weight concrete", *AISC Eng. J.*, 55-64.

Salari, M. R. and Spacone, E. (2001), "Analysis of steel-concrete composite frames with bond-slip", *J. Struct. Eng.*, ASCE, **127**(11), 1243-1250.

Spacone, E. and El-Tawil, S. (2004), "Nonlinear analysis of steel-concrete composite structures: state of the art", *J. Struct. Eng.*, ASCE, **130**(2), 159-168.

Vijalapura, P. K., Conte, J. P. and Meghella, M. (2000), "Time-variant reliability analysis of hysteretic SDOF systems with uncertain parameters and subjected to stochastic loading", *Proc. of the 8th Int. Conf. on Applications of Statistics and Probability in Civil Engineering (ICASP8)*, Sydney, Australia, 827-834.

Zona, A., Barbato, M. and Conte, J. P. (2004), "Finite element response sensitivity analysis of steel-concrete composite structures", Report SSRP-04/02, Department of Structural Engineering, University of California, San Diego.

Zona, A., Barbato, M. and Conte, J. P. (2005), "Finite element response sensitivity analysis of steel-concrete composite beams with deformable shear connection", *J. Eng. Mech.*, ASCE, **131**(11), 1126-1139.

CC