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Practical second-order analysis and design of single angle trusses by an equivalent imperfection approach

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Abstract. Steel angles are widely used in roof trusses as web and chord members and in lattice towers. Very often angle members are connected eccentrically. As a result, not only an angle member is under an axial force, but it is also subject to a pair of end eccentric moments. Moreover, the connection at each end provides some fixity so neither pinned nor the fixed end represents the reality. Many national design codes allow for the effects due to eccentricities by modifying the slenderness ratio and reducing the compressive strength of the member. However, in practice, it is difficult to determine accurately the effective length. The concept behind this method is inconsistent with strength design of members of other cross-sectional types such as I or box sections of which the buckling strength is controlled by the Perry constant or the initial imperfection parameters. This paper proposes a method for design of angle frames and trusses by the second-order analysis. The equivalent initial imperfection-to-length ratios for equal and unequal angles to compensate the negligence of initial curvatures, load eccentricities and residual stresses are determined in this paper. From the obtained results, the values of imperfection-to-length ratios are suggested for design and analysis of angle steel trusses allowing for member buckling strength based on the Perry-Robertson formula.

Key words: angles; buckling; eccentricities; initial imperfections.

1. Introduction

Single angle members have a broad range of applications, such as chord members in trusses and members in latticed towers. Single angles are widely used because of their ease of fabrication, erection and transportation. However, the analysis of single angle members is complex. It is not uncommon to connect an angle member to another by bolting directly at its end through their legs. Therefore, in practice, an angle member is loaded eccentrically. As a result, not only is an angle member under an axial force, it is also subjected to a pair of end moments at its ends. Also, since the location of the centroid does not coincide with the shear centre, twisting about the shear centre occurs simultaneously. Meanwhile, the connection at each end provides a restraining force which is beneficial to the compression capacity of the angle members. Moreover, as shown in Fig. 1, since equal angles, which are referred to as equal-leg angles, are monosymmetric and unequal angles, which are referred to as unequal-leg angles, are asymmetric, their principal planes are almost always inclined to the loading planes. This further complicates the analysis of angle members. The above-mentioned features are

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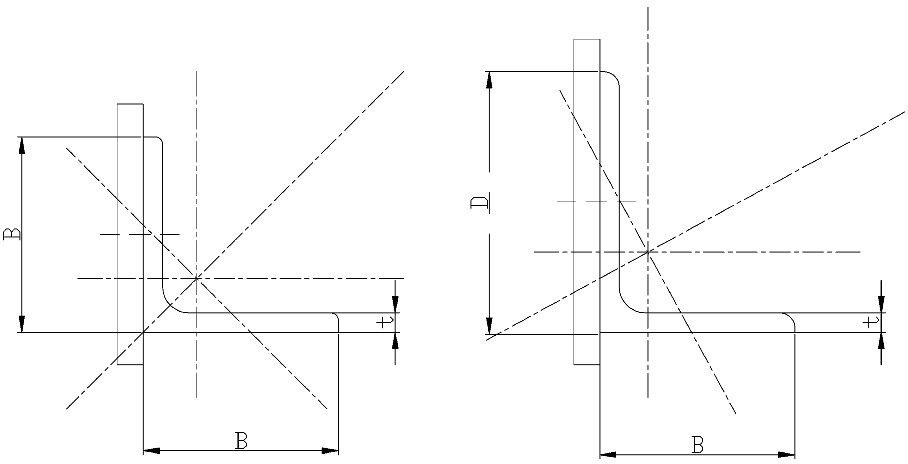


Fig. 1 Geometry of the equal and unequal angle sections

unique to angle sections making the design of single angle members controversial for some time. In a rational design procedure, the adverse effect of the end eccentricity and the beneficial effect due to the end restraint on the compression capacity should be considered.

In reality, angle members are subjected to axial compression, biaxial bending as well as twisting. Interestingly, depending on the slenderness, angle members under axial load may fail by flexural buckling about weak axis or flexural-torsional buckling (Kitipornchai 1983). For slender members, design should be controlled by flexural buckling about the weak axis; for stocky members, design should be governed by flexural-torsional buckling; for members having intermediate slenderness, both failure modes should be considered in the design. Kitipornchai (1983) carried out a parametric study on flexural-torsional buckling of single angle compression members. The elastic buckling loads are calculated and expressed in terms of dimensionless parameters. It can be seen that the flexural-torsional buckling mode is the most significant for stocky columns. Kitipornchai and Lee (1986) expanded the research to inelastic analysis by the finite element method using the tangent modulus concept. Idealized residual stress distribution is assumed in the analysis. Results reveal that for equal angles the inelastic flexural buckling curve is always lower than the inelastic flexural-torsional curve while for unequal angles the inelastic flexural-torsional buckling curve is lower than the inelastic flexural buckling curve. Since angles are thin-walled sections, local plate buckling may take place which further reduces the compressive strength. Effective width/section or effective stress methods can be used to account for the effect of local plate buckling. Nevertheless, most practical hot-rolled angle sections are compact, making the local buckling less important in practical application.

To achieve the exact elastic solution of an angle with end restraints and eccentric compressive load, close-form solutions are not always possible and numerical methods must be adopted. Trahair (1969) carried out a theoretical investigation on elastic biaxial bending and torsion of beam-columns with symmetrical end loading and restraints. The theory is based on differential equilibrium equations for major axis bending, minor axis bending and torsion which have been given by Goodier (1941, 1942). The finite integral method was used to solve the problem. The behaviour of eccentrically loaded end restrained single angle struts is studied and compared with experimental data reported by Foehl (1948). It is shown that the first yield loads are quite close to the failure load and may be used as conservative estimates of the ultimate strengths of the single angle struts. Kitipornchai and Chan (1987) employed another approach to solve the problem of elastic behaviour of restrained beam-columns. The element geometric stiffness matrix for angles is derived. The equilibrium paths are determined from the incremental and the total-force-deformation equilibrium equations. The results are compared with those reported by Trahair (1969). When the geometry is not updated, the results are found to agree well with the finite integral solutions (Trahair 1969). However, when the geometry is updated, the influence of the prebuckling deformations is apparent. In other words, the conventional numerical procedure can grossly overestimate the actual member capacity.

While the structural behaviour of single angle struts is complicated, the analysis and the design appear to be over-simplified. In current practice, the internal forces of a structural system are calculated based on first-order analysis and then checked against resistance according to design codes. In LRFD (AISC 1999), the global second-order (P-Δ) effect due to the change of structural geometry is considered by the amplification factors B_1 and B_2, the complicated situation encountered by single angle struts is allowed for by the interaction equations which are basically derived for doubly symmetric sections. Flexural-torsional buckling is considered by reducing the compressive strength of these struts using design equations which are developed on the basis of a reasonable conversion of research data. However, many design codes seem to be over-simplified and the design rules were developed based on flexural buckling. In BS5950 (BSI 2000) and Eurocode 3 (CEN 2003), angle struts may be treated as concentrically loaded, the adverse effect due to load eccentricity and the beneficial effect due to end connection are allowed for by modifying the effective slenderness ratio. The P-Δ effect is compensated by assuming an effective length. Although the design procedure is simplified, it may not be able to allow for flexural-torsional buckling for stocky and intermediate slenderness range.

Adluri and Madugula (1992) compared results of experimental data on eccentrically loaded single angle members free to rotate in any directions at the ends with the available literature with AISC LRFD (1986) and AISC ASD (1989) specifications. The experimental investigations, which cover a wide spectrum of single angle struts, were carried out by Wakabayashi and Nonaka (1965), Mueller and Erzurumlu (1983), and Ishida (1968). Adluri and Madugula (1992) summarized these results and concluded that the interaction formulas given in AISC LRFD (1986) and AISC ASD (1989) are highly conservative when applied to eccentrically loaded single angle members. It is because these interaction formulas were derived primarily for doubly symmetric sections and the moment ratios in these formulas are evaluated for the case of maximum stresses about each principal axis. This practice does not pose a problem on doubly symmetric sections such as I sections because the four corners are critical for moments about both principal axes simultaneously. However, for angle sections, as they are monosymmetric or asymmetric, the points having maximum bending stress about both principal axes usually do not coincide. As a consequence, the load capacities of the sections calculated from these interaction equations are underestimated (Adluri and Madugula 1992). Similar conclusions can be drawn when using other design codes. Bathon *et al.* (1993) carried out 75 full-scale tests which cover a slenderness ratio ranging from 60 to 210. The tested specimens were unrestrained against rotation and twisting at the end supports. It was noted that the ASCE Manual 52 (1988) under-predicted the capacities of single angle struts. The above-mentioned research appears to ignore the effect due to end connection details, which may also affect the buckling resistances of the angle struts. Elgaaly *et al.* (1992) conducted an experimental program to investigate the structural behaviour of non-slender single angle struts as part of three-dimensional trusses. The specimens cover a range of slenderness ratio from 60 to 120. Results show that both the ASCE Manual 52 (1988) and AISC LRFD (1986) are inadequate for single angle members with low slenderness ratio.

In order to eliminate the unnecessary discrepancy between the actual failure load and the design load, Adluri and Madugula (1992) suggested that the moment interaction factors given in AISC LRFD (1986) should be revised. Woolcock and Kitipornchai (1986) proposed a design method based on experimental observation. However, this method is limited to single web compression members in plane trusses. In the proposed method, not only the effect of load eccentricity is taken into account, but cases where web members are all on one side or on opposite side are also considered. This method was considered simpler and less conservative than the conventional axial force-biaxial bending interaction approach. For those design codes which do not consider flexural-torsional buckling, Kitipornchai (1983) introduced an equivalent slenderness ratio which is defined as the slenderness ratio of a member having the same cross-section as the actual member, which, when buckling in a pure flexural mode about the minor axis, has the same buckling load as the actual member buckling in a flexural-torsional mode. The proposed expressions for equivalent slenderness ratio are simple. With this slight modification on slenderness ratio, the existing design rules can still be employed.

So far, most research is focused on individual element behaviour and design which have been used traditionally in the past decades and can hardly be incorporated into second-order analysis software for practical design. This paper proposes a bow element with initial and varying curvature to simulate the buckling strength of an angle strut. Similar to the design of symmetrical hot-rolled sections, the bow element replaces the typical load eccentricity and asymmetrical bending by an equivalent imperfection with the additional formulae to consider flexural-torsional and torsional buckling loads, the element can readily be used for practical design of angle trusses by the second-order analysis, of which the research appears to be not yet completed or reported. The work covers a wide range of equal and unequal angles which are listed in the Steelwork Design Guide to BS5950: Part 1 (SCI 2000).

2. Description of model

The model used in this study is a column with initial imperfection to simulate the effect due to eccentricity as shown in Fig. 2. The lower end is a pin support and the upper end is a roller support. The

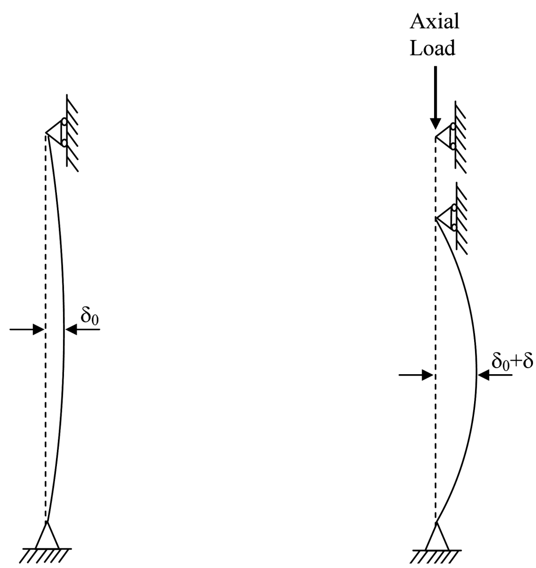


Fig. 2 A simple model for calibration studies (before and after deformation)

buckling length, or the effective length, is equal to the member length. Because of the presence of geometric imperfection, the column will be bent when a load is applied before the Euler load is reached. The values of imperfection-to-length ratio (δ_0/L) of both equal and unequal angles are determined so that the same buckling design curve given in BS5950 (BSI 2000) and Eurocode 3 (CEN 2003) can be obtained with these values of imperfections. The buckling design curves of these sections of slenderness ratios ranging from 10 to 350 are generated and incorporated into a computer programme for second-order analysis (Nida 2002) by inserting the member imperfections into the member curvature. Comparisons are made between these curves and those given in BS5950 (BSI 2000) and Eurocode 3 (CEN 2003) to ensure they are close but conservative to the code requirements in order that the result can be safely used in practical design.

3. Model assumptions

The assumed failure mode is flexural buckling about the principal minor axis. Therefore, the initial imperfection is imposed about the principal minor axis so that bending about the principal minor axis is activated. This assumption is mostly true for slender members. In addition, compression members containing thin-plate elements such as angles are vulnerable to local buckling of the cross-section which reduces the buckling strengths of the members. In this paper, the effect of local buckling is ignored. In other words, only Class 3 semi-compact sections in which as defined by BS5950 (BSI 2000) only the extreme fibre in compression can be loaded to the design strength, or above angle sections are considered in this paper. However, if slender sections are used, the equivalent plate slenderness can be used to determine the equivalent sectional properties. Other failure modes such as flexural buckling about geometric axis and flexural-torsional buckling which are uncommon on slender members are not directly considered in the investigation but they may be considered implicitly in the analysis by using the proposed equivalent imperfection.

4. Theoretical analysis of imperfect columns

The hypothetic perfectly straight compression member remains undeflected when a concentric load is applied. When the load reaches the Euler load, or the elastic buckling load, the member may buckle under a slight lateral disturbance. However, there is no perfectly straight member in reality; real members shown in Fig. 2 possess some initial curvatures which cause members to bend at the instant when the load is applied. This increases the maximum stress of the member. In this case, the member is failed by the first yield instead of elastic buckling. If the initial curvature is such that:

$$u_0 = \delta_0 \sin\frac{\pi x}{L} \tag{1}$$

the theoretical compressive stress can be calculated by the Perry-Robertson formula (Heyman 1998) as:

$$\sigma^2 - \sigma[\sigma_0 + (1 + \eta)\sigma_E] + \sigma_0\sigma_E = 0 \tag{2}$$

in which σ is the critical stress, σ_0 is the yield stress, σ_E is the Euler stress, and $\eta = \frac{\delta_0 \cdot y}{r^2}$ where δ_0 is the maximum initial curvature at the mid-length and y is the distance from the extreme fibre in compression to the neutral axis of the cross section. However, this formula is questionable when the slenderness ratio of a member is low because stocky members, which more likely fail at squash load, are not sensitive to initial imperfections (SCI 1988). Besides, this formula does not take into consideration the effects due to load eccentricity and residual stresses.

5. Buckling strengths for imperfect columns

In addition to initial imperfection, the strength of a real member may be reduced because of the presence of residual stresses, eccentricities and material nonlinearity. A more accurate analysis can be done by computer modelling in which the initial curvature, load eccentricities, residual stresses and material nonlinearity are specified. This method is seldom used in practical design. Instead, simplified design rules are developed from the results of computer modelling and other test data available in literature. Qualitatively, these imperfections can be represented by a magnified initial curvature which has an equivalent effect on the behaviour of the member as a collective effect of all these defects (Trahair *et al*. 2001). This equivalent imperfection method is currently adopted by BS5950 (BSI 2000) and Eurocode 3 (CEN 2003).

In BS5950 (BSI 2000), in order to account for the effects due to the above-mentioned imperfections, the original Perry factor η in Eq. (2) is replaced by a modified one as,

$$\eta = 0.001a(\lambda - \lambda_0) \geq 0 \tag{3}$$

where a is the Robertson constant
λ is the slenderness ratio
λ_0 is the limiting slenderness and should be taken as $0.2\sqrt{\frac{\pi^2 E}{\sigma_0}}$

The special features of this modification include the allowance of load eccentricity, residual stresses and the inclusion of various sectional shapes (e.g. universal beams, universal columns and hollows) through the Robertson constant a of four design buckling curves. In addition, the modified Perry factor allows for the stocky column effect by setting a limiting slenderness. The Robertson constant a was developed from the results of computer modelling and correlations with available experimental results. The value of a as 5.5 for equal and unequal angles bent about any axis provides an accurate curve-fit result for a wide range of slenderness ratio.

In Eurocode 3 (CEN 2003), design rules were developed in a similar fashion as in BS5950 (BSI 2000) with the Perry factor written as:

$$\eta = \alpha(\bar{\lambda} - 0.2) \geq 0 \tag{4}$$

in which α is the imperfection factor analogous to the Robertson constant a in BS5950 (BSI 2000) which shifts the buckling design curves for various section shapes and $\bar{\lambda}$ is the non-dimensional slenderness given by:

$$\bar{\lambda} = \sqrt{\frac{A \cdot f_y}{N_{cr}}} \tag{5}$$

where N_{cr} is the elastic critical force for the relevant buckling mode based on A which is the cross-sectional area.

The limiting non-dimensional slenderness of 0.2 accounts for the stocky column effect. For equal and unequal angles bending about any axis, the imperfection factor α is 0.34.

6. Equivalent imperfection

6.1. Equivalent imperfection to BS5950 (BSI 2000)

When calculating compressive strength of an eccentrically loaded member, a second-order analysis program takes the eccentricities, residual stresses and initial imperfections into account by means of initial imperfection at mid-span of the member. Therefore, these effects are controlled by the value of δ_0 / L which is calculated as follows.

Theoretically, the expression of the Perry factor η is given by:

$$\eta = \frac{\delta_0 \cdot y}{r^2} = \frac{\delta_0}{L} \cdot \frac{y}{r} \cdot \frac{L}{r} = \frac{\delta_0}{L} \cdot \frac{y}{r} \cdot \lambda \tag{6}$$

The modified Perry factor η from BS5950 (BSI 2000) is given in Eq. (3). To determine the equivalent value of initial imperfection, we have,

$$\eta = \frac{\delta_0}{L} \cdot \frac{y}{r} \cdot \lambda = 0.001a(\lambda - \lambda_0) \tag{7}$$

Therefore,

$$\frac{\delta_0}{L} = \frac{0.001a(\lambda - \lambda_0)}{y/r \cdot \lambda} \tag{8}$$

Eq. (8) shows that the initial imperfection is a function of member slenderness and the dimensionless term *y/r* from the cross-sectional properties. For most practical purposes, angles of intermediate and slender range are commonly used and shorter angle columns can be designed by simple formula based on a linear analysis. Considering angle members of slenderness ratio ranging from 120 to 350, i.e., slender members, the ratio of $\lambda - \lambda_0$ to λ ranges from 0.86 to 0.95 which is close to unity, so λ_0 is small compared with λ and may be neglected. Therefore, the expression of δ_0 / L can be simplified to:

$$\frac{\delta_0}{L} = \frac{0.001a}{y/r} \tag{9}$$

6.2. Equivalent imperfection to Eurocode 3 (CEN 2003)

The equivalent amplified initial imperfection to Eurocode 3 (CEN 2003) can be worked out by a similar approach using Eq. (4) and Eq. (6) as,

$$\eta = \frac{\delta_0}{L} \cdot \frac{y}{r} \cdot \lambda = \alpha(\bar{\lambda} - 0.2) \tag{10}$$

Rearranging terms, we have,

$$\frac{\delta_0}{L} = \frac{\alpha(\bar{\lambda} - 0.2)}{y/r \cdot \lambda} \tag{11}$$

Neglecting the limiting slenderness term, Eq. (11) can be simplified to:

$$\frac{\delta_0}{L} = \frac{\alpha}{y/r}\sqrt{\frac{f_y}{\pi^2 E}} \tag{12}$$

7. Nonlinear analysis using the Newton-Raphson method

7.1. The physical concept of the Newton-Raphson method

To trace the load-deflection path, or the equilibrium path, of a structure, the stiffness matrix for an element which is derived from the energy principle (Chan and Zhou 1995) should be formed first and then transformed and assembled to form the global stiffness matrix for the complete structure for the finite element analysis. To solve the displacements of the nodes, a non-linear solution technique is required. The Newton-Raphson method is probably the most common technique used today. The essence of this method is its incremental-iterative scheme with efficiency and accuracy. It performs iterations until an equilibrium point is reached in each incremental load cycle.

The key steps of the Newton-Raphson procedure of the present analysis are as follows:

1. Apply incremental force vector $[\Delta F]$
2. Calculate the incremental global displacements $[\Delta u]$ for the structure

$$[\Delta u] = [K_T]^{-1}[\Delta F] \tag{13}$$

where $[K_T]$ is the global stiffness matrix

3. Update geometry $[x]_{i+1}$ and accumulate displacements $[u]_{i+1}$

$$[x]_{i+1} = [x]_i + [\Delta u] \tag{14}$$

$$[u]_{i+1} = [u]_i + [\Delta u] \tag{15}$$

4. Extract element displacement vector $[u_e]$ from global displacement vector and transform to local displacement vector $[u_l]$

$$[u_l] = [L][u_e] \tag{16}$$

where $[L]$ is the transformation matrix

5. Calculate the element resistance force vector $[F_l]$ and transform to global axes $[F_e]$

$$[F_l] = [k][u_l] \tag{17}$$

$$[F_e] = [L]^T[F_l] \tag{18}$$

where $[k]$ is the element stiffness matrix

6. Add up all element resistance force vectors to global resistance force vector $[R]$

$$[R] = \sum[F_e] \tag{19}$$

7. Compute the unbalanced forces $[\Delta R]$

$$[\Delta R] = [F] - [R] \tag{20}$$

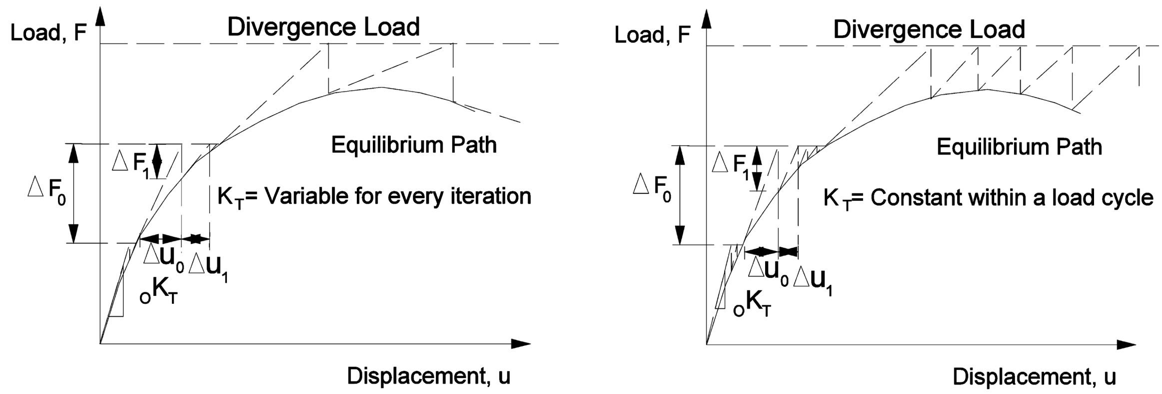
where $[F]$ is the external force vector

This process is repeated until the unbalanced forces are eliminated to a sufficiently small error. To achieve this, it is necessary to revise the displacements and re-check equilibrium condition. The convergence check is carried out using the following conditions:

$$[\Delta u]^T[\Delta u] < 0.1\% \cdot [u]^T[u] \tag{21}$$

$$[\Delta F]^T[\Delta F] < 0.1\% \cdot [F]^T[F] \tag{22}$$

After the convergence requirements are satisfied, a new load cycle begins with an updated tangent stiffness matrix and the whole previous process repeats. The whole incremental-iterative process is presented graphically in Fig. 3. In the conventional Newton-Raphson method, the tangent stiffness is updated every iteration which makes the procedure take fewer iterations to achieve convergence; in the modified Newton-Raphson method, the tangent stiffness is kept constant within each load cycle which makes the computer time shorter as it is time-consuming to form and solve for the tangent stiffness matrix. A compromise of both methods may achieve the most optimal number of iterations. However, both methods may encounter divergence problem which is referred to as when the applied incremental load approaches the limit point, the equilibrium errors in Eqs. (21) and (22) increase with the number of iterations until over-flow as shown in Fig. 3.



(a) Conventional Newton-Raphson method (b) Modified Newton-Raphson method

Fig. 3 The Newton-Raphson method

Table 1 Summary of δ_0/L

	Equal angle		Unequal angle	
Section	60×60×5	25×25×5	150×75×10	65×50×8
y/r	1.978	2.355	1.822	2.112
1000 δ_0/L	2.781	2.335	3.018	2.604

(a) BS5950

	Equal angle		Unequal angle	
Section	40×40×4	25×25×5	200×100×15	65×50×8
y/r	2.028	2.355	1.860	2.112
1000 δ_0/L	1.931	1.663	2.105	1.854

(b) Eurocode 3

To improve this defect of divergence, the Newton-Raphson method is revised to be subjected to an additional constraint such as the arc-length distance or the minimum residual displacement norm. Further details can be found in Chan and Chui (2000).

7.2 Buckling design curves to BS5950 (BSI 2000) and Eurocode 3 (CEN 2003)

With the Newton-Raphson procedure, the buckling design curves to BS5950 (2000) and Eurocode 3 (2003) can be traced using the equivalent initial imperfection. As can be seen from Eq. (9) and Eq. (12), the value of δ_0/L is inversely proportional to the value of y/r. For the worst scenario, δ_0/L can be obtained when the minimum value of y/r is substituted into Eq. (9) for BS5950 (2000) and Eq. (12) for Eurocode 3 (2003). For similar geometrical cross-sections, the values of y/r are roughly constant; so are the values of δ_0/L. Therefore, the values of δ_0/L for other angle sections will only slightly vary. To demonstrate the likeness of y/r of similar geometric cross-sections, the sections having the smallest and largest values of y/r for equal and unequal angle sections and their corresponding δ_0/L values are summarized in Tables 1(a) and 1(b). The next step is to generate the design curves using second-order analysis software (Nida 2002) by inputting these values of δ_0/L for the worst cases. Figs. 4(a) and 4(b) show the buckling curves of equal and unequal angles of steel grade S275 derived from the basis of the equivalent initial imperfections in Tables 1(a) and 1(b). Generally, the results generated by the proposed method agree well with the design code BS5950 (BSI 2000) and Eurocode (CEN 2003). The discrepancies between the present curves and the design code curves will be less than 9% for the BS5950 (BSI 2000) and the Eurocode 3 (CEN 2003), the curves tend to converge to the same value when the slenderness ratio is high. It is because when the member is slender, the effect of neglecting the limiting slenderness is insignificant.

8. Applications

So far, the proposed method is only suitable for angle trusses with single-bolted connection. As assumed, the failure mode is flexural buckling about the principal minor axis. In BS5950 (BSI 2000), the compression resistance of an angle member with single bolted connection should be taken as 80% of the compression resistance of a concentrically loaded member. However, such simplified approach is not allowed in Eurocode 3 (CEN 2003) as when only one single bolt is used, the end moments induced

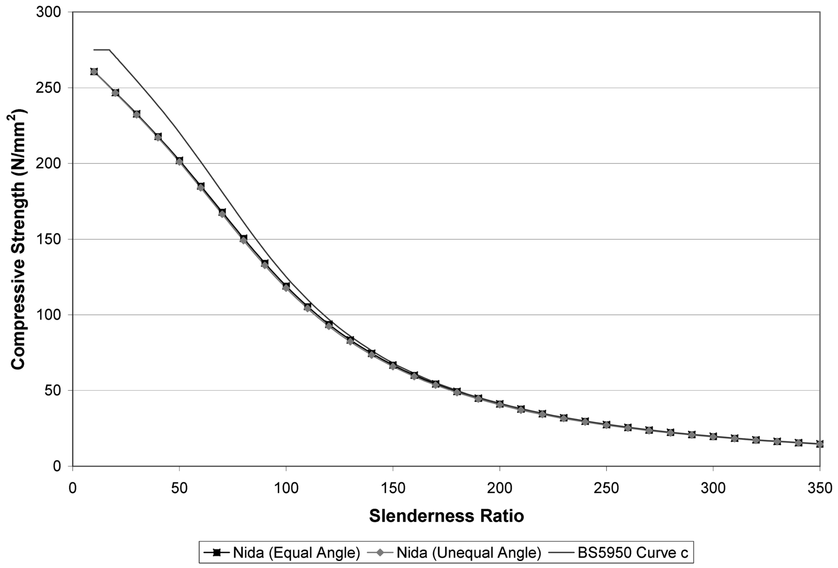


Fig. 4(a) Compressive strength curves to BS5950

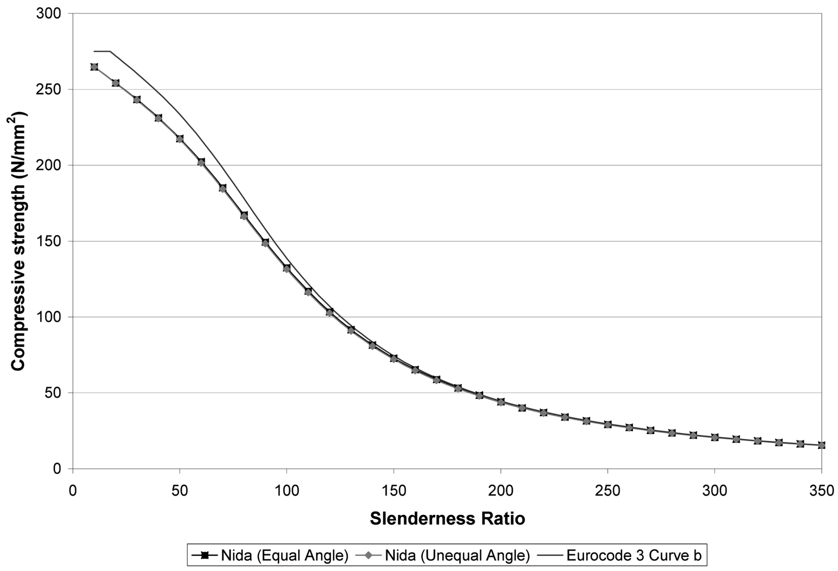


Fig. 4(b) Compressive strength curves to Eurocode 3

by the load eccentricities should be considered. When designing trusses of various serial sizes of angles with single-bolted connection to BS5950 (2000) by the proposed method, it is suggested the initial imperfection per member length tabulated in Table 1(a) should be adopted to compensate for the effect due to load eccentricity and other imperfections such as initial curvature, residual stresses and material non-linearity. The failure load of the truss calculated should be reduced to 80%. However, since the effect of local plate buckling is neglected, this method can be used for Class 4 slender sections in which elements are subject to the compression that does not meet the limits for class 3 semi-compact sections as defined in BS5950 (BSI 2000) only with appropriate reduction of section properties or design strength to design codes by the effective width or effective stress methods.

8.1 Example 1

Fig. 5 shows a typical roof truss. Members in compression are shown in thick lines. In this example, the roof truss is of span 9.0 m and maximum height 2.6 m. Load per joint is 10.0 kN. The material used is grade S275 steel with Young's modulus of 205 kN/mm^2. Assume that rafters will be 100×75×8 angle (I_v=34.7 cm^4 and A=13.5 cm^2), main ties will be 80×60×8 angle (I_v=17.1 cm^4 and A = 10.6 cm^2), and internal bracing members will be 65×50×6 angle (I_v=7.49 cm^4 and A=6.59 cm^2). In the conventional approach, to facilitate calculation, two assumptions have been usually made. First, loads are applied through the centroid. Second, all joints are pin-jointed so no moments are produced. In fact, these assumptions are hardly true because the joints are almost always bolted or welded away from the centroid. However, in general, the error due to these assumptions will not cause the structure to have a lower than expected safety margin. In this method, a linear analysis is carried out to determine the internal forces of the truss. The buckling resistance of individual members is then determined using formulae given in BS5950 (2000) in a separated step. Results show that the rafter will yield first and the load factor, which is the ratio of failure load to design load is 2.16. When using the second-order analysis, the internal web members are assumed to be pin-jointed and the other components are assumed to be continuous, the initial imperfection-to-length ratio is amplified to 3.018×10^{-3} for all unequal angles. It is found that the strut will yield first and the load factor is 4.54 which is more than double of the load by BS5950 (2000). It can be observed that for different connection assumptions yielding of material will start at different members which will affect the load capacity of the structure. In other words, with the aid of computer analysis, the proposed method can achieve a more rigorous design than the BS5950 (2000). Since the P-Δ and the P-δ effects which are referred to as the second-order effects due to the change of structural geometry and the deflection along a member respectively after loads (Chan and Chui 2000) are already included during the analysis via geometry update and the use of initial imperfection, the section capacity is adequate for strength design which can be completed at the same time when the analysis is completed. Obviously, the present method is much more efficient since separated member check is not needed.

As the assumption of a conventional analysis and design assumes, all connections are pinned, which is in contrast with the actual scenario. The present second-order analysis assumes that the rafter to be

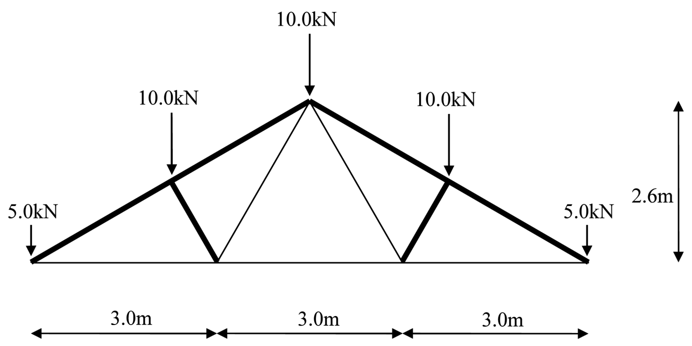


Fig. 5 Roof truss

continuous and the web members are pinned which more closely reflect the actual structure and therefore the present result shows this error in assumption is significant.

8.2 Example 2

The planar 1.0 m×3.0 m tower truss as part of a braced web portal frame is subject to two vertical forces shown in Fig. 6. The internal segment is designed based on an assumed effective length. According to BS5950 (2000), in order to allow for the effects due to lack of verticality, a notional horizontal force of 0.5% of the vertical load is applied at the same level. The design load, F, of the truss is determined in this example. The material design strength is 275 N/mm^2 and Young's modulus is 205 kN/mm^2, and the angle is of size 40×40×4 (I_v = 1.89 cm^4 and A = 3.09 cm^2). When using the BS5950 (2000), different engineers may make different assumptions on effective length and varied results will be obtained. As summarized in Table 2, if L_E = 1.0 L, where L is the distance between bracings, the failure load will be 52.8 kN; if, L_E = 0.85 L, the failure load will be 67.2 kN; and if L_E = 0.7 L, the failure design load will be 85.2 kN. When using the second-order analysis, the initial imperfection-to-length ratio is amplified to 2.781×10^{-3} for equal angles and the failure load is 83.4 kN. Without assuming any effective length, the failure load can be computed as the P-Δ effect is automatically considered by geometry update and the P-δ effect by member bowing.

Comparing the BS5950 (2000) design method and the proposed method, if the effective length is taken as 1.0 L, the failure load calculated according to BS5950 (2000) is about one-third lower than that calculated by the proposed method; if the effective length is taken as 0.85 L, the failure load calculated by the BS5950 (2000) approach is still about 20% lower than that calculated by the proposed method.

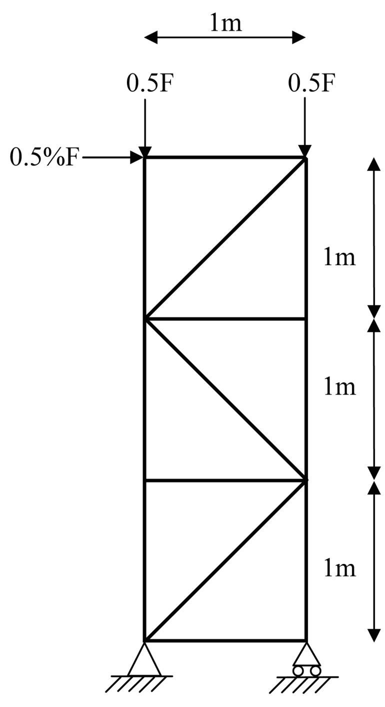


Fig. 6 Tower truss

Table 2 Comparison between BS5950 and the proposed method

Effective Length	BS5950 Load (kN)	$\frac{P}{P^*}$ (1)
1.0L	52.8	0.633
0.85L	67.2	0.806
0.7L	85.2	1.022

(1) $\frac{P}{P^*}$ refers to the ratio of the BS5950 load to the load calculated by the proposed method.

But, when the effective length is taken as 0.7 L, the BS5950 (2000) method and the proposed method will give almost the same result. In this example, it is shown that the conventional design method appears to be unreliable for the reason that a small difference in effective length can lead to a sizeable difference in design load. On the other hand, the second-order analysis avoids the error associated with the assumption of effective length.

9. Conclusions

This paper proposes a new design method for angle trusses and frames. When designing steel frames and trusses of various section sizes of angles by the proposed method, the initial imperfection per member length tabulated in Tables 1(a) and 1(b) should be used to symbolize the effects due to initial curvatures load eccentricities, residual stresses and material nonlinearities. These values will always give a reasonably conservative estimate of compressive strength of angle members which is at most 9% lower than the design compressive strength given in BS5950 (BSI 2000). This method is appropriate to angle members having higher slenderness range. Simple formulae can be used for designing stocky angle columns of which flexural buckling is unlikely and uncertainty in effective length approximation can be reduced. Another limitation is this method does not consider the beneficial effects which may be induced due to the eccentric connection. The present second-order analysis allowing both the P-Δ and the P-δ effects represents a new concept for the design of angle trusses and frames which is more reliable and efficient than the current prescriptive design method.

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Notation

α : Imperfection factor
δ_0 : Initial centre imperfection
η : Perry factor
λ : Slenderness ratio
λ_0 : Limiting slenderness
$\bar{\lambda}$: Non-dimension slenderness
σ : Critical stress
σ_0 : Yield stress
σ_E : Euler stress
a : Robertson constant
A : Gross cross-sectional area
E : Modulus of elasticity
f_y : Yield strength
$[F]$: External force vector
$[\Delta F]$: Incremental force vector
$[F_e]$: Element resistance vector in global system
$[F_l]$: Element resistance vector in local system
$[k]$: Element stiffness matrix
$[K_T]$: Global tangent stiffness matrix
L : Member length
$[L]$: Transformation matrix
N_{cr} : Elastic critical force for the relevant buckling mode based on A
r : Radius of gyration
$[R]$: Global resistance vector
$[\Delta R]$: Unbalanced force vector
$[x]$: Global coordinates
y : Distance from the extreme fibre in compression to the neutral axis
$[u]$: Global displacement vector
$[\Delta u]$: Incremental global displacement vector
$[u_e]$: Element displacement vector in global system
$[u_l]$: Element displacement vector in local system

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