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Design of steel and composite beams with web openings - Verification using finite element method

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Abstracts. This paper presents the findings of a design development project for perforated beams fully integrated with building services. A unified design approach for both steel and composite beams with large rectangular web openings is proposed which is based on plastic design methods and formulated in accordance with analytical structural design principles. Moreover, finite element models are established after careful calibration against test data, and comparison on the predicted ultimate loads of two composite beams with rectangular web openings from the finite element models and the proposed design method is also presented. It is demonstrated that the proposed design method is able to predict the ultimate loads of composite beams with rectangular web openings against 'Vierendeel' mechanism satisfactorily.

Key words: composite beams with web openings; perforated sections; integration with buildings services; 'Vierendeel' mechanism.

1. Composite beams integrated with building services

Composite construction in commercial buildings has been well established in many countries over the past few decades, and steel frames with hot rolled or fabricated sections and pre-cast concrete or composite slabs provide an effective means of fast-track construction with column-free office spaces. The benefits of composite action are increased strength and stiffness with 1.5 to 2.5 times of moment resistances and stiffnesses of steel sections, leading to economy in the size of the steel sections used. The design of composite beams is covered in BS5950: Part 3 (1990), Eurocode 4 (1994), and also in AS2327 (1996). Design handbooks for both steel and composite beams with profiled steel decking may also be found in the literature (Lawson and Chung 1994, Oehlers and Bradford 1995, AS2327.1 1997).

In modern commercial buildings with high specifications in building services, there is always a need to incorporate building services within floor zones while at the same time to minimize the depth of floor zones in order to reduce the overall height of the buildings. A common method of incorporating services within the floor-ceiling zone of buildings is to create large openings in the webs of beam members. The openings are most likely to be rectangular or circular, and may be in the form of discrete openings or a series of openings along the beam.

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The presence of web openings may have a severe penalty on the load carrying capacities of floor beams, depending on the shapes, the sizes, and the locations of the web openings. Due to the presence of web openings, three different modes of failure may take place at the perforated sections as follows:

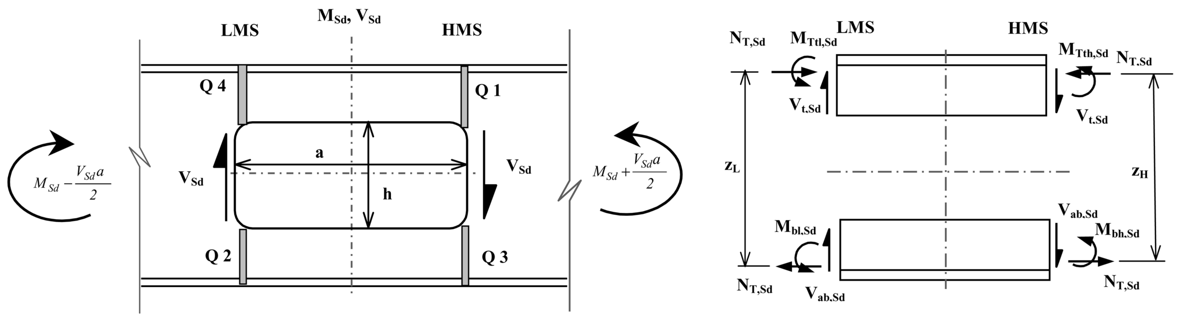
i) shear failure
ii) flexural failure, and
iii) 'Vierendeel' mechanism

2. Analytical design approach for beams with single web openings

An overall review on the design recommendations (Lawson 1987, Darwin 1990, Redwood and Cho 1993, Clawson and Darwin 1980, Chung and Lawson 2001) shows that in general, there are two design approaches (Ko and Chung 2002) in assessing the structural behaviour of beams with rectangular web openings:

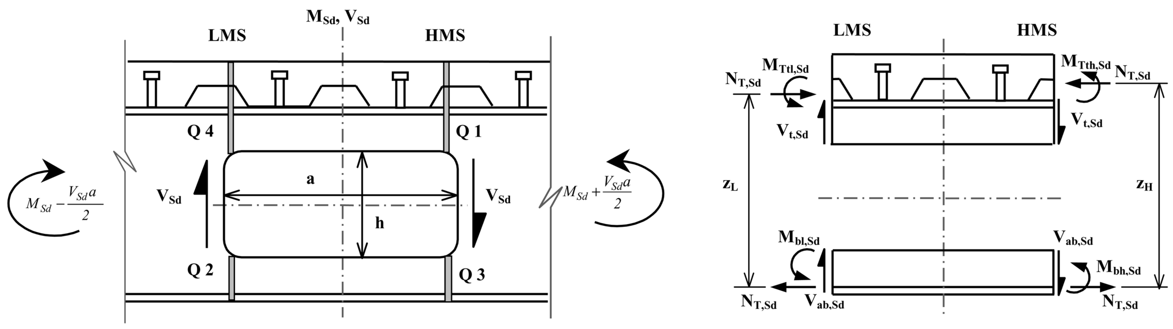
2.1. Tee section approach

In this approach, the structural adequacy of a beam with web openings depends on the section resistances of the tee sections above and below the web openings under co-existing axial forces N_T, shear forces V_T and local moments M_T, as shown in Figs. 1 and 2. All of these local forces and moments



a) Global forces b) Local forces

Fig. 1 Global and local actions at perforated section of a steel beam



a) Global forces b) Local forces

Fig. 2 Global and local actions at perforated section of a composite beam are due to global bending action. The accuracy of the design methods depends on the accuracy of a number of design rules against respective failure modes. Moreover, there are a number of different ways in allowing for the effects of co-existing axial and shear forces in assessing the moment resistances of tee sections. The calculation procedures are usually complicated and they differ significantly among each other, depending on the design methodology adopted, and also the calculation efforts involved. It should be noted that the design methods are often very general, and applicable in principle for beams with web openings of various sizes and shapes. However, due to the complexity of the problems, approximate design rules are often presented for practical design to reduce calculation effort, leading to conservative results.

2.2. Perforated section approach

In this approach, the perforated cross-sections are the critical sections to be considered in design. The structural adequacy of the beams depends on the section resistances of the perforated sections under co-existing global shear force, V_{Sd}, and bending moment, M_{Sd}. In general, the design procedures for both the shear and the moment resistances of perforated sections are relatively simple and similar among different design methods. However, the '*Vierendeel*' moment resistances of the perforated sections are evaluated implicitly based on various assumptions on the effects of co-existing shear forces and moments. Simplifications are usually made to those design rules derived from the *Tee section approach*, and thus, empirical global shear-moment (V_o-M_o) interaction curves are often provided to engineers for practical design. However, it is the simplification or the 'over-development' on the design rules that reduces the scope of applicability of the design rules.

For beams with multiple web openings, an additional failure mode, namely, buckling of web posts, may be critical when the openings are closely spaced. Moreover, additional deflection due to the presence of web openings should also be considered but the calculation is usually very laborious.

A detailed study (Chung and Ko 2002) on existing design rules for perforated beams shows that for beams with single web openings, many design rules are developed for small openings, i.e., the opening dimensions along section depth and member length are less than half of the section depth. Moreover, the design assumptions and methods are very different from one another, and in general, there is a lack of unified approach in both strength and stiffness assessments. In some cases, different sets of design rules are proposed by the same authors for openings of same shapes but different sizes. Furthermore, there is little guidance on the design of composite beams with perforated asymmetric fabricated I-sections in the literature.

3. Proposed design approach for perforated steel and composite beams

In order to rectify the deficiencies found in existing design methods, a unified design approach (Ko 2002, Chung and Ko 2003) is proposed, and the design formulation is presented according to the format of Eurcodes 3 and 4. Consider a steel beam and a composite beam each with a large rectangular web opening under global shear force, V_{Sd}, and global moment, M_{Sd}, as shown in Figs. 1(a) and 2(a). Local forces and moments are induced in the tee sections above and below the web opening, as shown in Figs. 1(b) and 2(b). Depending on the support condition and the position of the web opening along the beam, the interaction among co-existing axial forces N_T, shear forces V_T and moments M_T acting onto the tee sections vary significantly, leading to completely different structural behaviour of the beam.

With reference to the steel and the composite beams with rectangular web openings shown in Figs. 3 to 6, the general features of the proposed design method are summarised as follows:

- The section classification of all steel flanges and webs are either Class 1 plastic or Class 2 compact so that plastic stress blocks may be adopted in the section analysis.
- The axial resistances of composite and steel tee sections are evaluated according to the appropriate plastic stress blocks. Hence, the plastic axial centroids (PC) of both composite and steel tee sections are established.
- The plastic neutral axes (PNA) of both composite and steel tee sections are established through equilibrium consideration of horizontal forces, and the nominal plastic moment resistances are calculated by summing up the first moments of all the plastic stress blocks about the plastic neutral axes accordingly.
- In the presence of high shear force, the bending strengths of the steel webs in both the composite and steel tee sections are reduced throught von Mises yielding criterion.
- In the presence of applied axial forces due to global bending, the moment resistances of both the top composite and the bottom steel tee sections are reduced in general. The effect of applied axial forces is allowed through the introduction of additional plastic stress blocks, and thus, the plastic neutral axes of the tee sections are shifted. Hence, the modified plastic moment resistances of the tee sections are calculated by summing up the first moments of all the plastic stress blocks about the shifted plastic neutral axes.
- It is required to consider Vierendeel mechanism separately for the top composite and the bottom steel tee sections separately at follows:

a) Top composite tee-sections

$$M_{Tt1,Rd} + M_{Tth,Rd} + N_{T,Sd}(z_H - z_L) \geq (V_{at,Sd} + V_{c,Sd})a \tag{A1}$$

where $V_{at,Sd}$, $V_{c,Sd}$ are the shear forces resisted by the top steel tee-section and the concrete slab respectively

b) Bottom steel tee-sections

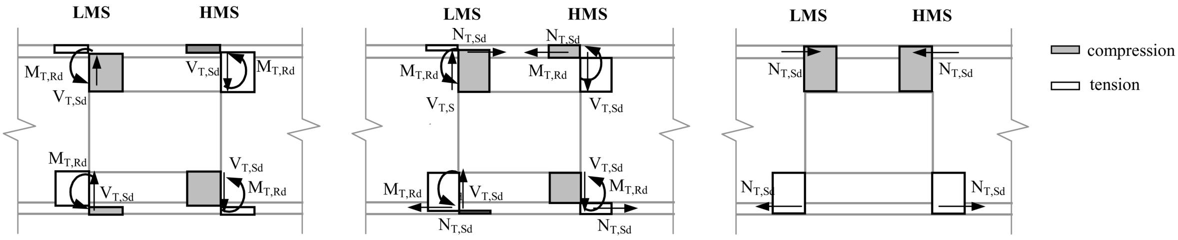
$$M_{Tb1,Rd} + M_{Tbh,Rd} \geq V_{ab,Sd}a \tag{A2}$$

where $V_{ab,Sd}$ is the shear force resisted by the bottom steel tee-section

If either of the above inequalities does not hold, then part of the shear force acting on the tee section should be distributed to the other tee section. Iterations should be performed until both inequalities do not hold which implies the occurrence of 'Vierendeel' mechanism.

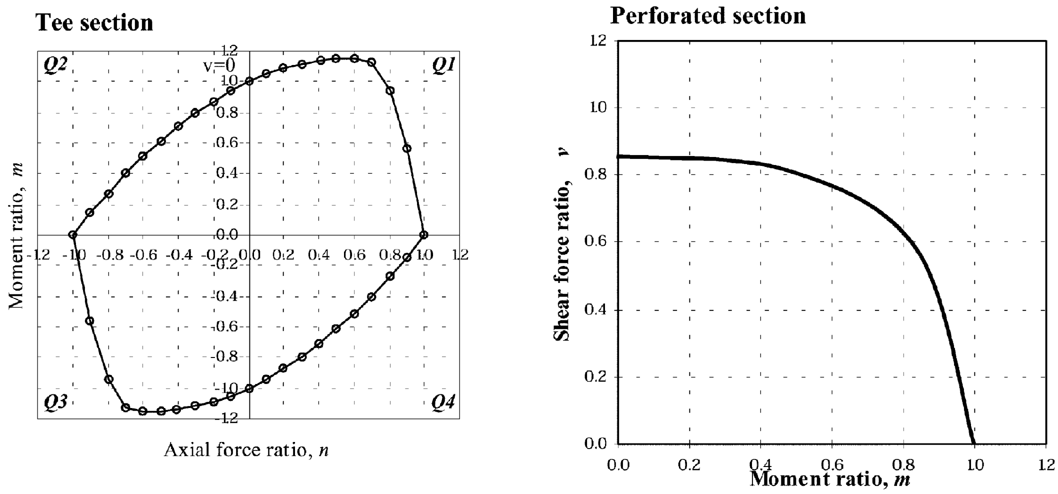
Refer to Appendix A for the derivation of the design expressions as well as the parameters of the proposed design method.

The plastic stress blocks acting onto the tee sections under different combinations of global shear forces and moments are shown in Figs. 3 and 5 for perforated steel and composite beams respectively. The moment-axial force interaction curves for both the top composite tee sections and the bottom steel tee sections are shown in Figs. 4(a) and 6(a) for perforated steel and composite beams respectively.



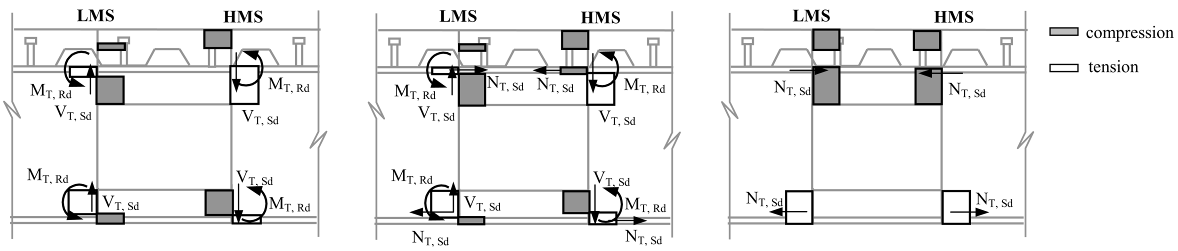
a) Global moment, $M_{Sd} = 0$
b) Global moment, $M_{Sd} \neq 0$
Global shear force, $V_{Sd} \neq 0$
c) Global shear force, $V_{Sd} = 0$

Fig. 3 Force distribution and plastic stress block patterns of a perforated steel beam



a) Local interaction curve
b) Global interaction curve

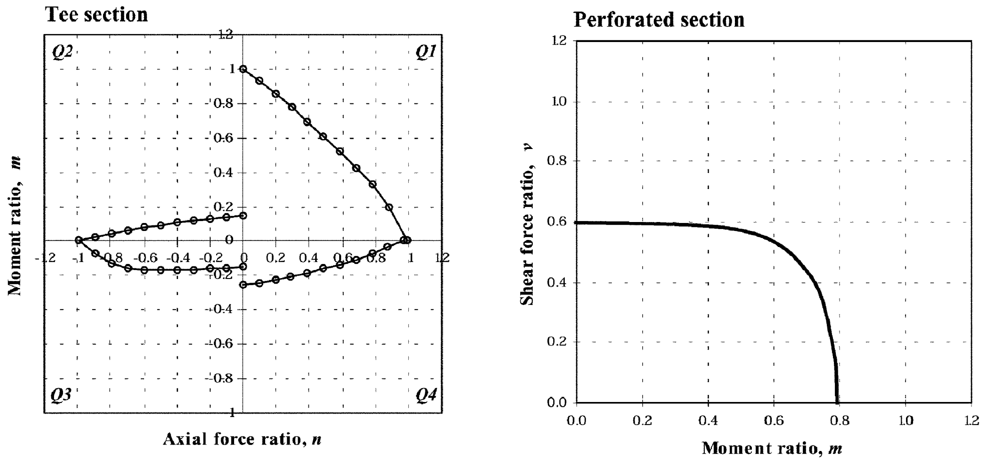
Fig. 4 Structural performance of steel beam with large rectangular web opening



a) Global moment, $M_{Sd} = 0$
b) Global moment, $M_{Sd} \neq 0$
Global shear force, $V_{Sd} \neq 0$
c) Global shear force, $V_{Sd} = 0$

Fig. 5 Force distribution and plastic stress block patterns of perforated section of a composite beam

Moreover, by equating the modified moment resistances of the four tee sections under co-existing axial forces, and shear forces if any, against the applied '*Vierendeel*' moment, $V_{Sd} \times a$, the ultimate loads of the perforated beams are obtained. This allows the generation of a global interaction curve between the applied shear force, V_{Sd}, and the applied moment, M_{Sd}, of the perforated section at the limit of failure as shown in Figs. 4(b) and 6(b) for perforated steel and composite beams respectively. The global interaction curves prescribe the maximum co-existing global shear force, V_{Sd}, and global moment, M_{Sd}, at



a) Local interaction curve b) Global interaction curve

Fig. 6 Structural performance of composite beam with large rectangular web opening

the perforated section of a beam located at various positions along the length of the beam member. Consequently, engineers may confirm the structural adequacy of the perforated sections of steel and composite beams easily with the aid of the global interaction curves. In case of structural inadequacy, engineers may readily relocate the web openings according to the global interaction curves. It should be noted that for steel beams with web openings of various shapes and sizes (Liu and Chung 2003, Chung and Liu 2003), a simple and yet effective empirical design method is proposed which is based on an extensive parametric study on hot rolled universal beams under practical loading and support conditions.

4. Finite element modeling

In order to verify the accuracy of the proposed design method, finite element method is employed and a finite element model for composite beams with large web openings is established. Comparison on the predicted ultimate loads of two composite beams with rectangular web openings from the finite element models and the proposed design method is presented after careful calibration against test data.

The general purpose finite element package ABAQUS (Version 6.3, 2002) was adopted for the numerical simulation of composite beams with large rectangular web openings. Since the principal mode of failure involves only in-plane deformation, a two-dimensional finite element model is adopted for simplicity together with the following features:

- Iso-parametric eight-noded two-dimensional plane stress elements (CPS4R) are used to model both the concrete slab and the steel beam. The stresses incorporated in each element include two in-plane direct stresses and one in-plane shear stress.
- A bi-linear stress-strain curve is adopted in the material model of steel together with the von Mises yielding criteria. The '* CONCRETE' option is used to model the reinforced concrete slab whose tensile strength is taken as 10% of its compressive strength.
- With geometric non-linearity incorporated into the finite element model, large deformation in the perforated section after yielding is predicted accurately to allow for load re-distribution within the

perforated section. Consequently, the 'Vierendeel' mechanism with the formation of plastic hinges in the top composite and the bottom steel tee sections above and below the web openings are investigated in details.

- Headed shear studs with 19 mm diameter are used in both tests. Every shear connector is modelled by one horizontal spring with a stiffness, K_h, for shear rigidity simulation and one vertical spring with a stiffness, K_v, for axial rigidity simulation. Moreover, the shear and the pull-out resistances of the connector are taken to be P_h and P_v respectively. Bi-linear force-deformation curves are adopted in both springs to allow for material yielding.
  It should be noted that both the stiffnesses and the resistances of the shear connectors depend not only on the steel and the concrete materials, but also on the configuration of the slab, i.e., either a solid concrete slab or a composite slab with profiled steel decking, and the arrangement of shear connectors, i.e., either one stud or two studs per trough.
- As concrete crushing is not incorporated in the concrete model, it is necessary to define a point of failure in the beam. A simple failure criterion is adopted where the beam is declared to be failed when the maximum deflection of the beam exceeds any one of the following:
  i) span over 90
  ii) overall beam depth over 30, or
  iii) 25 mm.

5. Calibration against experimental results

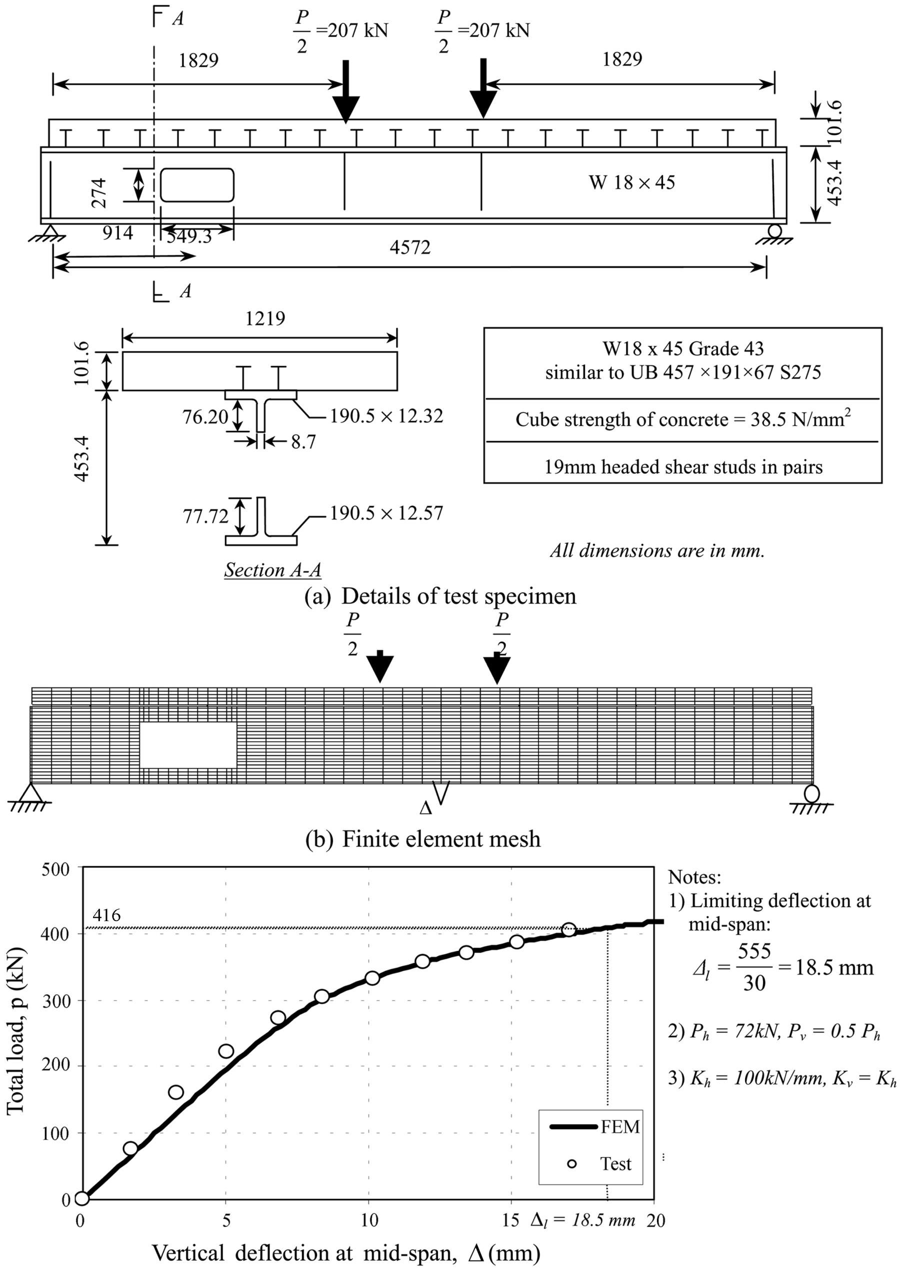
The finite element model was calibrated against data from two tests. The first test was a composite beam with a solid concrete slab which was conducted by Clawson and Darwin (1980), namely Test CD4. The second one was a composite beam with a composite slab using a trapezoidal decking which was conducted by Redwood and Poumbouras (1983), namely Test RP5. Table 1 presents the mechanical properties of the steel beams and the concrete slabs. Both the general test set-ups and the dimensions of the test specimens of Tests CD4 and RP5 are shown in Figs. 7(a) and 8(a) together with their finite element models in Figs. 7(b) and 8(b) respectively. Moreover, the predicted load deflection curves of the composite beams are plotted in Figs. 7(c) and 8(c) where the stiffnesses and the resistances of the shear connectors are also given. It should be noted that both the stiffnesses and the resistances of the shear connectors are estimated from typical push-out tests.

Table 1 Mechanical properties of test specimens

Tests	Failure load P (kN)	Failure mode	Measured yield strength (N/mm^2)			Cylinder strength (N/mm^2)	Cube strength (N/mm^2)
			Top flange	Bottom flange	Web		
CD4	207+207	Vierendeel	306.0	284.3	335.8	30.8	38.5
RP5	272	Vierendeel	301.5	301.5	325.4	18.8	23.5

Table 2 Comparison on ultimate loads

Test	Ultimate load (kN)		
	Experiment	Finite element method (FEM)	Proposed design method (PDM)
CD4	414	413	416
RP5	252	250	249



(c) Load-deflection curves

Fig. 7 Perforated composite beam with solid concrete slab - Test CD4

Table 2 summarizes the comparison on the ultimate loads obtained from the experiments and the finite element models, and the predicted load deflection curves are plotted in the same graphs of the measured data in Figs. 7(c) and 8(c) for easy comparison. It is shown that both the deformation characteristics and the ultimate loads obtained from the finite element models and the experiments agree well with one another.

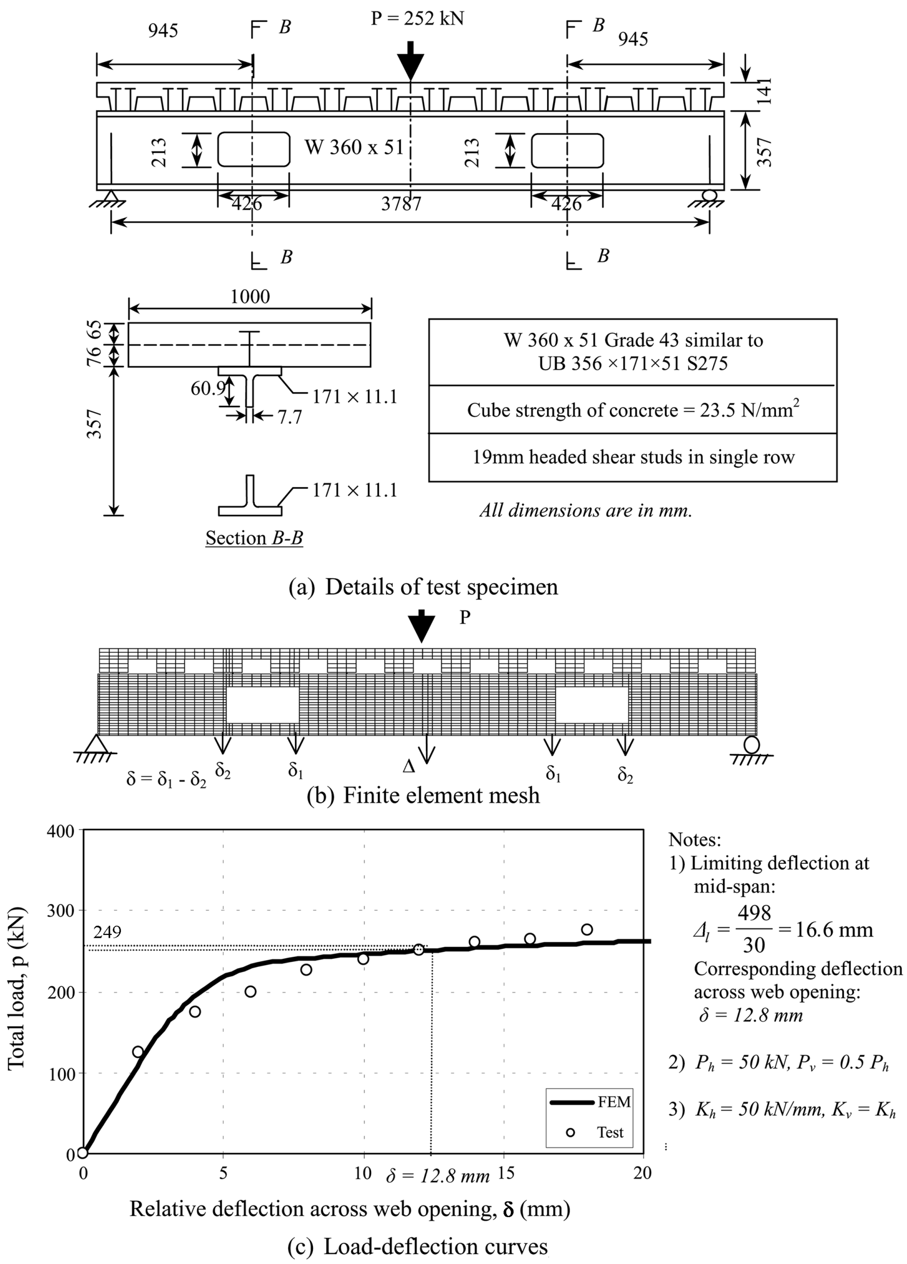


Fig. 8 Perforated composite beam and slab with profiled steel decking - Test RP5

6. Comparison with proposed design method

These two composite beams with large rectangular web openings are also analyzed by the proposed design method, and the results are compared with those obtained from the finite element models in terms of both ultimate loads and local force distributions. It is found that the ultimate loads obtained from both finite element models and the proposed design method agree very well with each other as shown in Table 2.

Table 3 Comparison on local moments and forces for test CD4

			FEM (P=416 kN)		PDM (P=416 kN)	
			LMS	HMS	LMS	HMS
‘Vierendeel’ moment (kNm)	Top tee		14.8	60.0	-5.5	84.8
			74.8		79.3	
	Bottom tee		6.9	15.0	8.5	14.3
			21.9		22.8	
	Σ		96.7		102.1	
Axial force (kN)	Top tee		363.8	369.8	320.5	320.5
	Bottom tee		364.7	359.4	320.5	320.5
Shear force (kN)	Top tee	Concrete	80	83	38	38
		Steel	75	77	131	131
			155	160	169	169
			208	212	210	210
	Bottom tee		53	52	41	41

Table 4 Comparison on local moments and forces for test RP5

			FEM (P=249 kN)		PDM (P=249 kN)	
			LMS	HMS	LMS	HMS
‘Vierendeel’ moment (kNm)	Top tee		-2.0	31.4	-6.2	39.1
			33.4		32.9	
	Bottom tee		3.9	8.0	5.0	8.1
			11.9		13.1	
	Σ		44.3		46.0	
Axial force (kN)	Top tee		268.9	272.7	256.6	256.6
	Bottom tee		276.9	274.1	256.6	256.6
Shear force (kN)	Top tee	Concrete	37	25	27	28
		Steel	47	63	67	66
			84	88	94	94
			122	126	124	124
	Bottom tee		38	38	30	30

The ‘Vierendeel’ moments obtained from the proposed design method and the finite element models are presented in Tables 3 and 4 for Tests CD4 and RP5 respectively. It is interesting to note that although there are minor discrepancies found in the local ‘Vienrendeel’ moment resistances between the HMS and the LHS at both the top composite and the bottom steel tee sections respectively, the total ‘Vierendeel’ moments for both the top composite tee sections and the bottom steel tee-sections from those two methods agree very well.

7. Conclusions

Based on plastic section analyses, a unified design approach for both steel and composite beams with large rectangular web openings is presented which is formulated in accordance with analytical structural design principles. It is demonstrated that the proposed design method is able to predict the ultimate loads of composite beams with rectangular web openings against ‘Vierendeel’ mechanism satisfactorily. It should be noted that the design approach also provides flexibility with rational basis for simplification, allowing calculation procedures of different levels of complexity suitable to manual or computer calculations, depending on the time constraint and the accuracy required.

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References

British Standards Institution, *BS5950: Structural Use of Steelwork in Building. Part 3 Section 3.1: Code of Practice for Design of Composite Beams*, BSI, London, 1990.

British Standards Institution, *DD ENV 1994-1-1 Eurocode 4: Design of Composite Steel and Concrete Structures, Part 1.1: General Rules and Rules for Buildings*, London, 1994.

Chung, K.F. and Lawson, R.M. (2001), "Simplified design of composite beams with large web openings to Eurocode 4", *J. Constructional Steel Research*, **57**(2), 135-163.

Chung, K.F. and Ko, C.H. (2002), "Harmonization on practical design of steel and composite beams with large web openings for full integration with building services", *Proc. of the Second Structural Engineering World Congress*, Yokohama, Japan, October (CD publication).

Chung, K.F. and Ko, C.H. (2003), "Steel and composite beams fully integrated with building services", *Proc. of Technical Seminar on Fire Engineering and Composite Construction*, 25 March, The Hong Kong Institute of Steel Construction, 4.1-4.25.

Chung, K.F. and Liu, T.C.H. (2003), "Steel beams with large web openings of various shapes: Empirical design method", *J. Constructional Steel Research*, **59**, 1177-1200.

Clawson, W.C. and Darwin, D. (1980), "Composite beams with web openings", SM Report No. 4, University of Kansas Center for Research, Lawrence, Kansas.

Composite Beam Design Handbook in accordance with AS2327.1 - 1996, jointly published by the Australian Institute of Steel Construction and Standards Australia, SAA HB91, 1997.

Darwin, D. (1990), "Steel and composite beams with web openings", *Steel Design Guide Series No. 2*, American Institute of Steel Construction, Chicago, IL, USA.

Ko, A.C.H. (2002), "A unified approach for steel and composite beams with web openings", MPhil Thesis, the Hong Kong Polytechnic University.

Ko, C.H. and Chung, K.F. (2002), "A review of recent developments on design of perforated beams", *Proc. of the Third Int. Conf. on Advances in Steel Structures*, Hong Kong, December, 121-128.

Lawson, R.M. (1987), "Design for openings in the webs of composite beams", *CIRIA Special Publication and SCI Publication 068,* CIRIA/The Steel Construction Institute, UK.

Lawson, R.M. and Chung, K.F. (1994), *Composite Beam Design to Eurocode 4*, The Steel Constitute Institute, SCI-P-**121**.

Liu, T.C.H. and Chung, K.F. (2003), "Steel beams with large web openings of various shapes: finite element study", *J. Constructional Steel Research*, **59**, 1159-1176.

Oehlers, D.J. and Bradford, M.A. (1995), *Composite Steel and Concrete Structural Members: Fundamental Behaviour*, Oxford: Pergamon Press.

Redwood, R.G. and Cho, S.H. (1993), "Design of steel and composite beams with web openings", *J. Constructional Steel Research*, **25**, 231-248.

Redwood, R.G. and Poumbouras, G. (1983), "Tests of composite beams with web openings", *Canadian J. Civil Engineering*, **10**(4), 713-721.

Standards Australia. *Composite Structures. Part 1: Simply Supported Beams.* Australian Standard AS2327.1 - 1996.

Appendix A

Design formulae for composite beams with rectangular web openings

A.1 Basic resistances of cross sections
A.1.1 Plastic axial resistances and location of plastic axial centroids
A.1.2 Plastic moment resistances and location of plastic neutral axis
a) Bottom steel tee-sections at HMS and LMS
b) Top composite tee-section at HMS
c) Top composite tee-section at LMS

A.2 Presence of axial forces
A.2.1 Effect of axial force on moment resistances of bottom steel tee-sections at HMS
A.2.2 Effect of axial force on moment resistances of bottom steel tee-sections at LMS
A.2.3 Effect of axial force on moment resistances of top composite tee-sections at HMS
a) Full shear connection and nominal PNA in concrete
b) Full shear connection and nominal PNA in steel flange
c) Full shear connection and nominal PNA in steel web
d) Partial shear connection and nominal PNA in steel flange or web
A.2.4 Effect of axial force on moment resistances of top composite tee-sections at LMS
a) Full shear connection and nominal PNA in concrete
b) Full shear connection and nominal PNA in steel web
c) Full shear connection and nominal PNA in steel flange
d) Partial shear connection and nominal PNA in steel flange or web

A.3 Presence of shear forces
A.3.1 Plastic shear resistances
A.3.2 Effect of shear forces on moment resistances of tee sections

A.4 Evaluation of axial forces acting on tee-sections

A.5 Equilibrium against 'Vierendeel' Mechanism

Consider a composite beam with a large rectangular web opening of $a \times h$ under a global moment M_{Sd} and a global shear force V_{Sd}, as shown in Fig. A1(a). After resolving the global forces and moments, Fig. A1(b) illustrates the co-existing local axial forces, shear forces and moments acting onto both the top composite and the bottom steel tee sections at both the HMS and the LMS of the web opening.

The design formulae for composite beams with large rectangular web openings are presented as follows.

A.1 Basic resistances of cross sections

A.1.1 Plastic axial resistances and location of plastic axial centroids

The cross-section of a composite perforated section is shown in Fig. A2(a) while the plastic stress blocks of both the top composite and the bottom steel tee-sections at the LMS and the HMS of the web

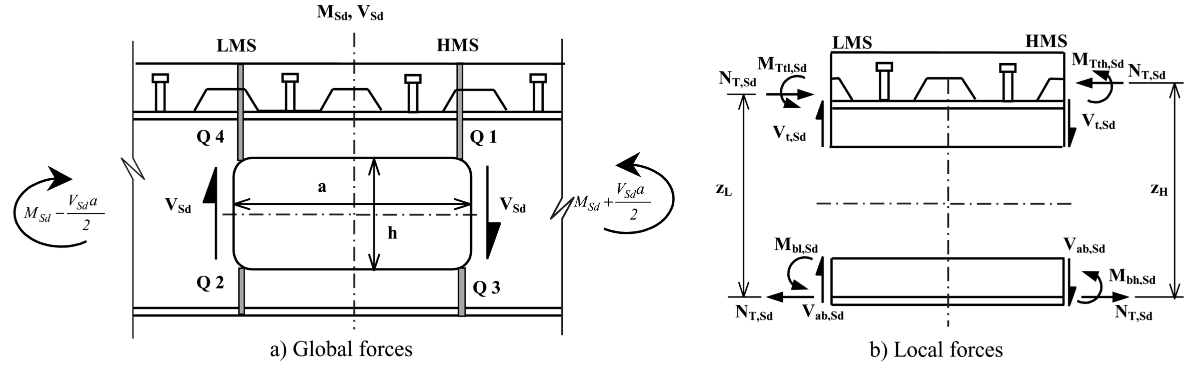


Fig. A1 Global and local actions at perforated section of a composite beam

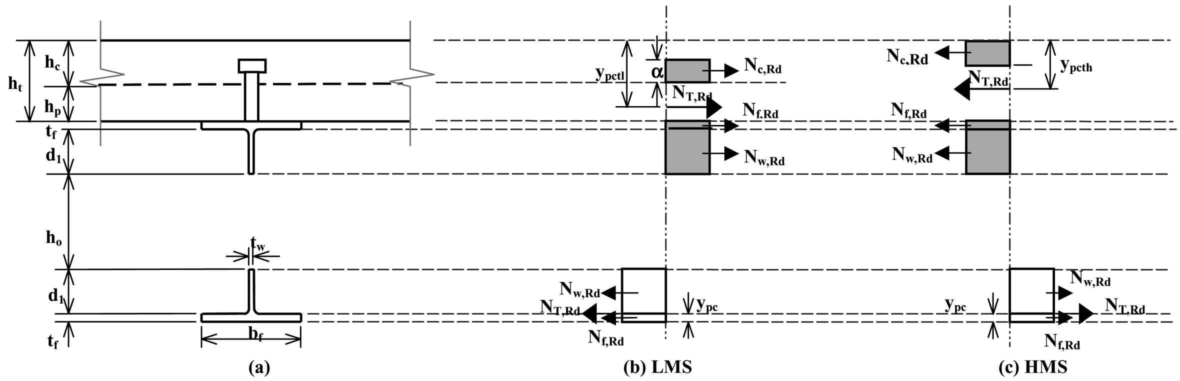


Fig. A2 Plastic axial centroids of tee-sections

opening are shown in Figs. A2(b) and A2(c) respectively. All the stress blocks in compression are shaded for easy reference.

The plastic axial resistance, $N_{T,Rd}$, of the bottom steel tee-sections is given by:

$$N_{T,Rd} = N_{f,Rd} + N_{w,Rd} \tag{A1}$$

where $N_{f,Rd}$ is the plastic axial resistance of the steel flange, and equal to $A_f f_{yf}$
$N_{w,Rd}$ is the plastic axial resistance of the steel web, and equal to $A_w f_{yw}$
f_{yf} is the design yield strength of the steel flange
f_{yw} is the design yield strength of the steel web
A_f is the area of the flange, and
A_w is the area of the web

When the tee-sections (steel or composite) are subjected to pure tension or compression, the axial force must be distributed over the sections in such a way that no moment is created, as shown in Figs. A2(b) and A2(c). In other words, the axial force resultant is considered as being applied at a specific point on the tee-section creating no moment. This point is referred as the plastic axial centroid (PC), and its distance from the bottom of the flange is designated as y_{pcb} for both bottom steel tee-sections, which is given as follows:

$$y_{pc} = \frac{N_{f,Rd}\left(\frac{t_f}{2}\right) + N_{w,Rd}\left(\frac{d_1}{2} + t_f\right)}{N_{f,Rd} + N_{w,Rd}} \tag{A2}$$

where $N_{f,Rd}$ and $N_{w,Rd}$ are the axial resistances of the flange and the web, respectively.

It should be noted that, as shown in Fig. A2, the PCs of the composite tee-sections at the LMS and the HMS are different because of the different stress blocks assumed. The concrete compressive force is assumed to act at the top and the bottom parts of the concrete slab at the HMS and the LMS, respectively. The position of the plastic axial centroid, y_{pct1}, of the top composite tee section from the top of the concrete slab at the LMS is given by:

$$y_{pct1} = \frac{N_{Rd}\left(d_e - \frac{\alpha}{2}\right) + N_{f,Rd}\left(\frac{t_f}{2} + h_t\right) + N_{w,Rd}\left(h_t + t_f + \frac{d_1}{2}\right)}{N_{Rd} + N_{f,Rd} + N_{w,Rd}} \tag{A3}$$

where N_{Rd} is equal to the minimum of $N_{sh,Rd}$ and $N_{c,Rd}$
$N_{sh,Rd}$ is the resistance of shear connection at the point under consideration
$N_{c,Rd}$ is the compressive resistance of the concrete slab
α is the depth of the plastic stress block of the concrete slab

The position of the plastic axial centroid, y_{pcth}, of the top composite tee section from the top of the concrete slab at the HMS is given by:

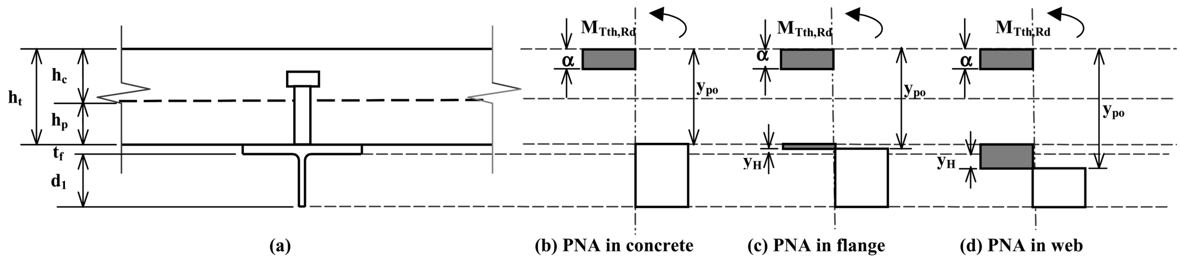


Fig. A3 Bending stress blocks of composite tee-section at HMS

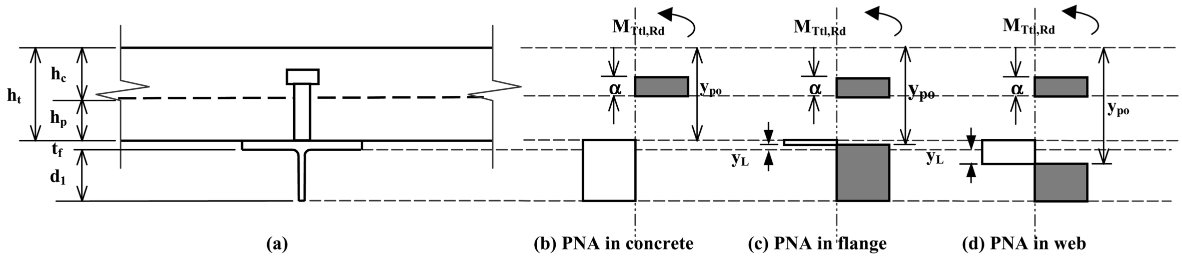


Fig. A4 Bending stress blocks of compostie tee-section at LMS

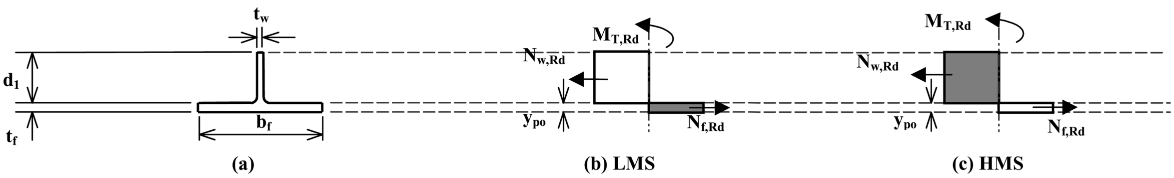


Fig. A5 Bending stress blocks of bottom steel tee-sections

$$y_{pcth} = \frac{N_{Rd}\left(\frac{\alpha}{2}\right) + N_{f,Rd}\left(\frac{t_f}{2} + h_t\right) + N_{w,Rd}\left(h_t + t_f + \frac{d_1}{2}\right)}{N_{Rd} + N_{f,Rd} + N_{w,Rd}} \tag{A4}$$

It should be noted that the PC is a point of significant importance since any effect of axial force on the moment resistances of the tee-sections is derived with reference to this point.

A.1.2 Plastic moment resistances and location of plastic neutral axis

a) Bottom steel tee-sections at HMS & LMS

As shown in Fig. A5, the plastic moment resistance of the tee-section, $M_{T,Rd}$, may be evaluated by taking moment of the forces about the PNA as follows:

$$M_{T,Rd} = N_{f,Rd}\left[\frac{y_{po}^2 + (t_f - y_{po})^2}{2t_f}\right] + N_{w,Rd}\left[\frac{d_1}{2} + t_f - y_{po}\right] \tag{A5}$$

where y_{po} is the distance of the plastic neutral axis (PNA) from the bottom of the flange

$$= \frac{N_{w,Rd} + N_{f,Rd}}{2N_{f,Rd}} t_f$$

For hot-rolled I beams with web openings of practical sizes, the PNA of the steel tee-sections always falls within the flanges, and thus only this case is considered.

All formulae and parameters required for the calculation of the nominal moment resistances of the bottom steel tee-sections are summarized in Table A1(a).

Table A1(a) Formulae in determining nominal moment resistance of steel tee section

Nominal PNA$_o$	$M_{T,Rd}$
Flange	$N_{f,Rd}\left[\frac{y_{po}^2 + (t_f - y_{po})^2}{2t_f}\right] + N_{w,Rd}\left[\frac{d_1}{2} + t_f - y_{po}\right]$
Web	$N_{f,Rd}\left[y_{po} - \frac{t_f}{2}\right] + N_{w,Rd}\left[\frac{(y_{po} - t_f)^2 + (d_1 - y_{po} - t_f)^2}{2d_1}\right]$

Note: $y_{po} = \frac{N_{w,Rd} + N_{f,Rd}}{2N_{f,Rd}} t_f$

Table A1(b) Parameters in determining moment resistance of steel tee section under axial force

	Nominal PNA$_o$	Shifted PNA	Criteria	α_f	α_w	β_f	β_w
HMS	Flange	Flange	$N_{TSd} - C_1 \leq 0$	$\dfrac{N_{TSd}}{2b_f f_{yf}}$	-	$y_{pc} - \left(y_{po} + \dfrac{\alpha_f}{2}\right)$	
		Web	$N_{TSd} - C_1 \leq C_2$	t_f - y_{po}	$\dfrac{N_{wSd}}{2t_w f_{yw}}$	$y_{pc} - \left(y_{po} + \dfrac{\alpha_f}{2}\right)$	$y_{pc} - \left(y_{po} + \alpha_f + \dfrac{\alpha_w}{2}\right)$
LMS	Flange	Flange	$N_{TSd} - C_3 \leq 0$	$\dfrac{N_{TSd}}{2b_f f_{yf}}$	-	$y_{pc} - \left(y_{po} - \dfrac{\alpha_f}{2}\right)$	

Notes:

$C_1 = 2(t_f - y_{po}) b_f f_{yf}$

$C_2 = 2 d_1 t_w f_{yw}$

$C_3 = 2 y_{po} b_f f_{yf}$

$N_{f,Sd} = 2\alpha_f b_f f_{yf}$

$N_{w,Sd} = 2\alpha_w t_w f_{yw}$

$M_{Tb1,Rd} = N_{T,Rd} - N_{f,Sd}\beta_f$

$M_{Tbh,Rd} = N_{T,Rd} + N_{f,Sd}\beta_f + N_{w,Sd}\beta_w$

$$y_{po} = \frac{N_{w,Rd} + N_{f,Rd}}{2N_{f,Rd}} t_f$$

$$y_{pc} = \frac{N_{f,Rd}\left(\dfrac{t_f}{2}\right) + N_{w,Rd}\left(\dfrac{d_1}{2} + t_f\right)}{N_{f,Rd} + N_{w,Rd}}$$

b) Top composite tee-section at HMS

The concept of plastic moment resistances of the composite tee-sections is more complicated as they are related also to the resistance of shear connection. Under pure bending, the nominal plastic neutral axis (PNA) of the composite tee-section at the HMS of the perforated section may locate within (i) the concrete slab, (ii) the steel flange, or (iii) the steel web, depending on the relative axial resistances of the steel tee-section, the concrete slab, and also the degree of shear connection. The axial resistances of the three components of the composite tee-section are given as follows:

- Concrete slab: $N_{c,Rd} = 0.85\ b_e d_e f_{ck}$ (A6a)
  where f_{ck} is the compressive cylinder strength of the concrete
  b_e is the effective width of the concrete slab, and
  d_e is the effective concrete depth
- Steel tee-section: $N_{a,Rd} = N_{w,Rd} + N_{f,Rd}$ (A6b)
  where $N_{w,Rd} = d_1\ t_w\ f_{yw}$
  $N_{f,Rd} = t_f b_f f_{yf}$
- Shear connectors: $N_{sh,Rd} = n_H\ P_{Rd}$ (A6c)
  where n_H is the number of shear connectors from the nearest support to the HMS of the perforated section, and
  P_{Rd} is the design strength of a shear connector

The stress blocks of all three cases are illustrated in Fig. A3. It is important to note that compressive force in the concrete is assumed to act at the top of the concrete slab while the tensile strength of the concrete is neglected.

Table A2(a) Formulae in determining nominal moment resistance of composite tee section at HMS

		Nominal PNA	y_H	y_{po}	$M_{Tth,Rd}$
Full shear connection	$N_{ShRd}>N_{cRd}>N_{a,Rd}$ or $N_{cRd}>N_{ShRd}>N_{a,rd}$	**Concrete**	t_f	$h_t+t_f-y_H$	$N_{Rd}\left(t_f+h_t-\frac{\alpha}{2}\right)-N_{f,Rd}\left(\frac{t_f}{2}\right)+N_{w,Rd}\left(\frac{d_1}{2}\right)$
	$N_{ShRd}>N_{a,Rd}>N_{cRd}$ or $N_{a,V,Rd}>N_{ShRd}>N_{cRd}$	**Flange** $N_{Rd}+N_{f,Rd}\geq N_{w,Rd}$	$\frac{N_{f,Rd}+N_{Rd}-N_{w,Rd}}{2b_f f_{yf}}$	$h_t+t_f-y_H$	$N_{Rd}\left(t_f+h_t-\frac{\alpha}{2}\right)+N_{f1,Rd}\left(\frac{t_f+y_H}{2}\right)-N_{f2,Rd}\left(\frac{y_H}{2}\right)+N_{w,Rd}\left(\frac{d_1}{2}\right)$
		Web $N_{Rd}+N_{f,Rd}<N_{w,Rd}$	$\frac{N_{f,Rd}-N_{Rd}-N_{f,Rd}}{2t_w f_{yw}}$	$h_t+t_f+y_H$	$N_{Rd}\left(t_f+h_t-\frac{\alpha}{2}\right)+N_{f,Rd}\left(\frac{t_f}{2}\right)-N_{w1,Rd}\left(\frac{y_H}{2}\right)+N_{w2,Rd}\left(\frac{d_1+y_H}{2}\right)$
Partial shear	$N_{a,V,Rd}>N_{cRd}>N_{shRd}$ or $N_{cRd}>N_{a,Rd}>N_{shRd}$	**Flange** $N_{Rd}+N_{f,Rd}\geq N_{w,Rd}$	$\frac{N_{w,Rd}+N_{Rd}-N_{w,Rd}}{2b_f f_{yf}}$	$h_t+t_f-y_H$	$N_{Rd}\left(t_f+h_t-\frac{\alpha}{2}\right)+N_{f1,Rd}\left(\frac{t_f+y_H}{2}\right)-N_{f2,Rd}\left(\frac{y_H}{2}\right)+N_{w,Rd}\left(\frac{d_1}{2}\right)$
		Web $N_{Rd}+N_{f,Rd}<N_{w,Rd}$	$\frac{N_{w,Rd}-N_{Rd}-N_{f,Rd}}{2t_w f_{yw}}$	$h_t+t_f+y_H$	$N_{Rd}\left(t_f+h_t-\frac{\alpha}{2}\right)+N_{f,Rd}\left(\frac{t_f}{2}\right)-N_{w1,Rd}\left(\frac{y_H}{2}\right)+N_{w2,Rd}\left(\frac{d_1+y_H}{2}\right)$

Notes:

$N_{f1,Rd} = (t_f-y_H)b_f f_{yf}$

$N_{f2,Rd} = y_H b_f f_{yf}$

$N_{w1,Rd} = y_H t_w f_{yw}$

$N_{w2,Rd} = (d_1-y_H)t_w f_{yw}$

$N_{Rd} = MIN(N_{ShRd}, N_{cRd}, N_{a,Rd})$

$\alpha = \frac{N_{Rd}}{N_{c,Rd}}(d_e)$

$d_e = h_t$ for soild slab

$d_e = h_c$ for profiled steel decking placed transversely to steel beam

$d_e = h_c+\frac{h_p}{2}$ for profiled steel decking placed parallel to steel beam

Table A2(b) Parameters in determining moment resistance of composite tee section under axial force at HMS

Nominal PNA	Shifted PNA	Criteria	α_c	α_f	α_w	β_c	β_f	β_w
Concrete	Concrete	$N_{TSd} - C_1 \le 0$	$\dfrac{N_{TSd}}{0.85 b_e f_{ck}}$		-	$y_{pcth} - \left(\alpha + \dfrac{\alpha_c}{2}\right)$	-	-
	Flange	$N_{TSd} - C_1 \le C_2$	$\dfrac{C_1}{0.85 b_e f_{ck}}$	$\dfrac{N_{TSd} - N_{cSd}}{2 b_f f_{yf}}$	-	$y_{pcth} - \left(\alpha + \dfrac{\alpha_c}{2}\right)$	$y_{pcth} - \left(y_{po} + \dfrac{\alpha_f}{2}\right)$	-
	Web	$N_{TSd} - C_1 - C_2 \le C_3$	$\dfrac{C_1}{0.85 b_e f_{ck}}$	t_f	$\dfrac{N_{TSd} - N_{cSd} - N_{fSd}}{2 t_w f_{yw}}$	$y_{pcth} - \left(\alpha + \dfrac{\alpha_c}{2}\right)$	$y_{pcth} - \left(y_{po} + \dfrac{\alpha_f}{2}\right)$	$y_{pcth} - \left(y_{po} + a_f + \dfrac{\alpha_w}{2}\right)$
Flange	Flange	$N_{TSd} - C_2 \le 0$	-	$\dfrac{N_{TSd}}{2 b_f f_{yf}}$	-	-	$y_{pcth} - \left(y_{po} + \dfrac{\alpha_f}{2}\right)$	-
	Web	$N_{TSd} - C_2 \le C_3$	-	y_H	$\dfrac{N_{TSd} - N_{fSd}}{2 t_w f_{yw}}$	-	$y_{pcth} - \left(y_{po} + \dfrac{\alpha_f}{2}\right)$	$y_{pcth} - \left(y_{po} + a_f + \dfrac{\alpha_w}{2}\right)$
Web	Web	$N_{TSd} - C_4 \le 0$	-	-	$\dfrac{N_{TSd}}{2 t_w f_{yw}}$	-	-	$y_{pcth} - \left(y_{po} + a_f + \dfrac{\alpha_w}{2}\right)$

Notes:

$$C_1 = MIN(N_{c,Rd}, N_{Sh,Rd}) - N_{a,Rd}$$
$$C_2 = 2 y_H b_f f_{yf}$$
$$C_3 = 2 d_1 t_w f_{yw}$$
$$C_4 = 2(d_1 - y_H) t_w f_{yw}$$

$$N_{c,Sd} = 0.85\,\alpha_c b_e f_{ck}$$
$$N_{f,Sd} = 2\,\alpha_f b_f f_{yf}$$
$$N_{w,Sd} = 2\,\alpha_w t_w f_{yw}$$

$$M_{Tth,Rd} = M_{Tth,Rd} + N_{c,Sd}\beta_c + N_{f,Sd}\beta_f + N_{w,Sd}\beta_w$$
$$y_{pcth} = \frac{N_{Rd}\left(\dfrac{\alpha}{2}\right) + N_{f,Rd}\left(\dfrac{t_f}{2} + h_t\right) + N_{w,Rd}\left(h_t + t + \dfrac{d_1}{2}\right)}{N_{Rd} + N_{f,Sd} + N_{w,Rd}}$$

In general, the depth of the compressive stress block in the concrete slab is given by:

$$\alpha = \frac{N_{Rd} d_e}{N_{c,Rd}} \tag{A7}$$

where N_{Rd} = Minimum ($N_{a,Rd}$, $N_{c,Rd}$, $N_{sh,Rd}$)

The plastic moment resistance, $M_{Tth,Rd}$, of the composite tee-section may be found by summing up the moments of all the resistances of both the concrete slab and the steel tee-section about the bottom of the steel flange.

However, if there is only partial shear connection, that is, $N_{Rd} = N_{sh,Rd}$, then the full compressive strength of the concrete is not attainable and the only possible stress blocks are shown in Figs. A3(c) and A3(d). The depth of the compressive stress block in the concrete slab may be found by using Eq. (A7). In general, Eq. (A7) is valid for both full and partial shear connection.

All formulae and parameters required for the calculation of the nominal moment resistances of the top composite tee-sections at HMS are summarized in Table A2(a).

c) Top composite tee-section at LMS

Whenever a composite beam with a perforated steel section is subjected to both moment and shear, tensile cracks may form at the top of the concrete slab at the LMS of the perforated section at failure, and at the same time the lower part of the concrete slab must be in compression in order to resist the thrust from the shear connectors. When global bending moment is increased, the axial force acting on the composite tee-section also increases, causing the cracks to close up, and hence larger compressive resistance may be developed in the concrete slab. Thus, it is reasonable to consider that the moment resistance of the composite tee-section at the LMS assumes the stress block as illustrated in Fig. A4, depending on the relative axial resistances of the concrete slab, the steel tee-section, and also the degree of shear connection. The axial resistances of the concrete slab and the steel tee-section are defined previously while the resistance of the shear connectors is equal to $n_L P_{Rd}$ where n_L is the number of shear connectors from the LMS of the perforated section to the nearest support.

The types of stress block for the composite tee-section at the LMS are similar to those for the cross-section at the HMS except that the resistance of shear connectors is smaller, i.e., the PNA may locate within (i) the concrete slab, (ii) the steel flange, or (iii) the steel web in the case of full shear connection while it may only locate within (iv) the steel flange or (v) the steel web in the case of partial shear connection.

The stress blocks shown in Fig. A4 are valid for composite tee-section with full or partial shear connection. The nominal moment resistance, $M_{Tt1,Rd}$, of the cross-section can be found by summing up the moments of each of the resistances of the concrete slab and the steel tee-section about the bottom of the flange.

All formulae and parameters required for the calculation of the nominal moment resistances of the top composite tee-sections at LMS are summarized in Table A3(a).

A.2. Presence of axial forces

Local axial forces arising from global bending action are assumed to act at the PCs of the tee-sections and will cause the plastic neutral axes to shift. Hence, the plastic moment resistances of the tee-sections are modified by the axial forces. The proposed method for allowing the effect of axial forces on the

Table A3(a) Formulae in determining nominal moment resistance of composite tee section at LMS

		Nominal PNA	y_L	y_{po}	$M_{Tt1,Rd}$
Full shear connection	$N_{ShRd}>N_{cRd}>N_{a,Rd}$ or $N_{cRd}>N_{ShRd}>N_{a,Rd}$	**Concrete**	t_f	$h_t+t_f-y_L$	$-N_{Rd}\left(t_f+h_p+\frac{\alpha}{2}\right)+N_{f,Rd}\left(\frac{t_f}{2}\right)-N_{w,Rd}\left(\frac{d_1}{2}\right)$
	$N_{ShRd}>N_{a,Rd}>N_{cRd}$ *or* $N_{a,V,Rd}>N_{ShRd}>N_{cRd}$	**Flange** $N_{Rd}+N_{w,Rd}<N_{f,Rd}$	$\dfrac{N_{f,Rd}-N_{Rd}-N_{w,Rd}}{2b_f f_{yf}}$	$h_t+t_f-y_L$	$-N_{Rd}\left(t_f+h_p+\frac{\alpha}{2}\right)+N_{f1,Rd}\left(\frac{t_f+y_L}{2}\right)-N_{f2,Rd}\left(\frac{y_L}{2}\right)+N_{w,Rd}\left(\frac{d_1}{2}\right)$
		Web $N_{Rd}+N_{w,Rd}\geq N_{f,Rd}$	$\dfrac{N_{w,Rd}+N_{Rd}-N_{f,Rd}}{2t_w f_{yw}}$	$h_t+t_f+y_L$	$-N_{Rd}\left(t_f+h_p+\frac{\alpha}{2}\right)+N_{f,Rd}\left(\frac{t_f}{2}\right)-N_{w1,Rd}\left(\frac{y_L}{2}\right)+N_{w2,Rd}\left(\frac{d_1+y_L}{2}\right)$
Partial shear	$N_{a,V,Rd}>N_{cRd}>N_{shRd}$ or $N_{cRd}>N_{a,V,Rd}>N_{shRd}$	**Flange** $N_{Rd}+N_{w,Rd}<N_{f,Rd}$	$\dfrac{N_{f,Rd}-N_{Rd}-N_{w,Rd}}{2b_f f_{yf}}$	$h_t+t_f-y_L$	$-N_{Rd}\left(t_f+h_p+\frac{\alpha}{2}\right)+N_{f1,Rd}\left(\frac{t_f+y_L}{2}\right)-N_{f2,Rd}\left(\frac{y_L}{2}\right)+N_{w,Rd}\left(\frac{d_1}{2}\right)$
		Web $N_{Rd}+N_{w,Rd}\geq N_{f,Rd}$	$\dfrac{N_{w,Rd}+N_{Rd}-N_{f,Rd}}{2t_w f_{yw}}$	$h_t+t_f+y_L$	$-N_{Rd}\left(t_f+h_p+\frac{\alpha}{2}\right)+N_{f,Rd}\left(\frac{t_f}{2}\right)-N_{w1,Rd}\left(\frac{y_L}{2}\right)+N_{w2,Rd}\left(\frac{d_1+y_L}{2}\right)$

Notes:

$N_{f1,Rd} = (t_f - y_H)b_f f_{yf}$

$N_{f2,Rd} = y_H b_f f_{yf}$

$N_{w1,Rd} = y_H t_w f_{yw}$

$N_{w2,Rd} = (d_1 - y_H)t_w f_{yw}$

$N_{Rd} = MIN(N_{ShRd}, N_{cRd}, N_{a,Rd})$

$\alpha = \dfrac{N_{Rd}}{N_{c,Rd}}(d_e)$

$d_e = h_t$ for soild slab

$d_e = h_c$ for profiled steel decking placed transversely to steel beam

$d_e = h_c + \dfrac{h_p}{2}$ for profiled steel decking placed parallel to steel beam

Table A3(b) Parameters in determining moment resistance of composite tee section under axial force at LMS

Nominal PNA	Shifted PNA	Criteria	α_c	α_f	α_w	β_c	β_f	β_w
Concrete	Concrete	$N_{TSd}-C_1\le 0$	$\frac{N_{TSd}}{0.85\,b_e f_{ck}}$	-	-	$y_{pct1}-\left(d_e-\alpha-\frac{\alpha_c}{2}\right)$	-	-
	Flange	$N_{TSd}-C_1\le C_2$	$\frac{C_1}{0.85\,b_e f_{ck}}$	$\frac{N_{TSd}-N_{cSd}}{2b_f f_{yf}}$	-	$y_{pct1}-\left(d_e-\alpha-\frac{\alpha_c}{2}\right)$	$y_{pct1}-\left(h_t+\frac{\alpha_f}{2}\right)$	-
	Web	$N_{TSd}-C_1-C_2\le C_3$	$\frac{C_1}{0.85\,b_e f_{ck}}$	t_f	$\frac{N_{TSd}-N_{cSd}-N_{fSd}}{2t_w f_{yw}}$	$y_{pct1}-\left(d_e-\alpha-\frac{\alpha_c}{2}\right)$	$y_{pct1}-\left(h_t+\frac{\alpha_f}{2}\right)$	$y_{pct1}-\left(h_t+a_f+\frac{\alpha_w}{2}\right)$
Web	Flange	$N_{TSd}-C_2\le 0$	-	$\frac{N_{TSd}}{2b_f f_{yf}}$	-	-	$y_{pct1}-\left(h_t+\frac{\alpha_f}{2}\right)$	-
	Web	$N_{TSd}-C_2\le C_4$	-	t_f	$\frac{N_{TSd}-N_{fSd}}{2t_w f_{yw}}$	-	$y_{pct1}-\left(h_t+\frac{\alpha_f}{2}\right)$	$y_{pct1}-\left(h_t+a_f+\frac{\alpha_w}{2}\right)$
Flange	Flange	$N_{TSd}-C_5\le 0$	-	$\frac{N_{TSd}}{2b_f f_{yf}}$	-	-	$y_{pct1}-\left(h_t+\frac{\alpha_f}{2}\right)$	-

Notes:

$C_1 = MIN(N_{c,Rd}, N_{Sh,Rd}) - N_{a,Rd}$

$C_2 = 2t_f b_f f_{yf}$

$C_3 = 2d_l t_w f_{yw}$

$C_4 = 2(d_l - y_L)t_w f_{yw}$

$C_5 = 2(t_f - y_L)b_f f_{yf}$

$N_{c,Sd} = 0.85\,\alpha_c b_e f_{ck}$

$N_{f,Sd} = 2\,\alpha_f b_f f_{yf}$

$N_{w,Sd} = 2\,\alpha_w t_w f_{yw}$

$M_{Tt1,Rd} = M_{Tt1,Rd} - N_{c,Sd}\beta_c - N_{f,Sd}\beta_f - N_{w,Sd}\beta_w$

$$y_{pct1} = \frac{N_{Rd}\left(d_e-\frac{\alpha}{2}\right)+N_{f,Rd}\left(\frac{t_f}{2}+h_t\right)+N_{w,Rd}\left(h_t+t_f+\frac{d_1}{2}\right)}{N_{Rd}+N_{f,Rd}+N_{w,Rd}}$$

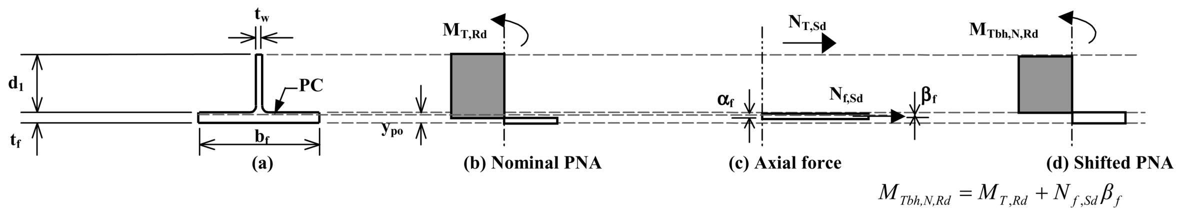


Fig. A6 Bending stress blocks of bottom steel tee-section at HMS under axial force (shifted PNA in flange)

plastic moment resistances of the tee-sections basically involves the subtraction or addition of the moments created by the axial forces from or to the nominal moment resistance of the tee-sections. Tee-sections at the HMS and the LMS of the perforated section are considered separately.

A.2.1 Effect of axial force on moment resistances of bottom steel tee-sections at HMS

Consider a tee-section subjected to a shear force and positive bending moment in Fig. A6, the addition of an axial force to the tee-section may cause the PNA to be located in the flange or the web depending on the magnitude of the force. Suppose an axial force of magnitude $N_{T,Sd}$ is applied to the tee-section in the same direction of the moment and assuming that $N_{T,Sd}$ is less than or equal to the residual compressive strength of the flange, 2 $(t_f - y_{po})\ b_f f_{yf}$, then the shifted PNA of the section will remain within the flange as shown in Fig. A6(d). The axial stress block can be assumed to act at a distance of β_f (see Fig. A6c) from the plastic axial centroid, introducing a beneficial effect to the plastic moment resistance. Thus, the modified moment resistance, $M_{Tbh,N,Rd}$, is larger than the nominal value of $M_{T,Rd}$ and is given by:

$$M_{Tbh,N,Rd} = M_{T,Rd} + N_{f,Sd}\beta_f \quad \text{(A8)}$$

where $N_{f,Sd} = 2\alpha_f t_f f_{yf}$

$$\beta_f = y_{pc} - \left(y_{po} + \frac{\alpha_f}{2}\right)$$

However, if the axial force continues to increase and exceeds the residual compressive strength of the flange, the axial stress block will extend to the web, which shifts the PNA down to the web as illustrated in Fig. A7(d). The axial force resisted by the web is then given by:

$$N_{w,Sd} = N_{T,Sd} - N_{f,Sd} \quad \text{(A9)}$$

where $N_{f,Sd} = 2\ (t_f - y_{po})\ b_f f_{yf}$

The modified plastic moment resistance is given by:

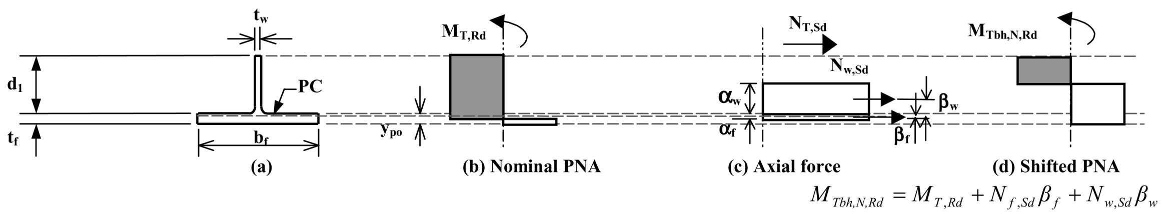


Fig. A7 Bending stress blocks of bottom steel tee-section at HMS under axial force (shifted PNA in web)

$$M_{Tbh,N,Rd} = M_{T,Rd} + N_{f,Sd}\beta_f + N_{w,Sd}\beta_w \tag{A10}$$

where β_w is the distance between the center of the axial stress block in the web and the plastic axial centroid

$$= y_{pc} - \left(y_{po} + \alpha_f + \frac{\alpha_w}{2}\right)$$

α_w is the depth of the axial stress block in the web

$$= \frac{N_{w,Sd}}{2t_w f_{yw}}$$

All formulae and parameters required for the calculation of the modified moment resistances of the bottom steel tee-sections under axial force at HMS are summarized in Table A1(b).

A.2.2 Effect of axial force on moment resistances of bottom steel tee-sections at LMS

Consider a steel tee-section shown in Fig. A8(a), the PNA of the section is located within the flange and the moment resistance of the section under zero axial force is equal to its nominal value $M_{T,Rd}$. The corresponding stress block is shown in Fig. A8(b). If an axial force, $N_{T,Sd}$, is applied to the tee-section, which is in a direction opposed to that of the moment resistance, a stress block is thus introduced and its depth, α_f, is given as follows:

$$\alpha_f = \frac{N_{T,Sd}}{2b_f f_{yf}} \tag{A11}$$

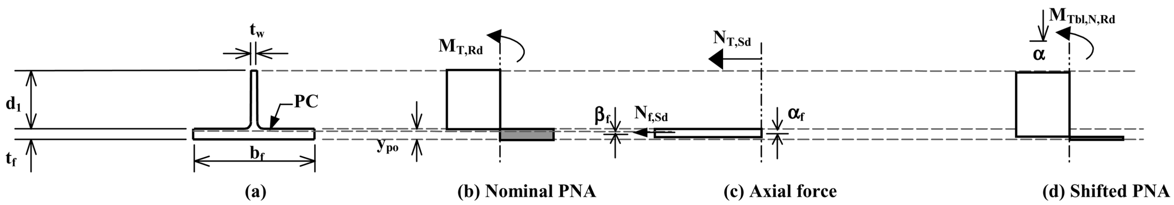
The stress blocks are shown in Fig. A8(c), and the PNA will shift towards the top of the flange to maintain equilibrium as shown in Fig. A8(d).

The application of the axial force, which is deviated from the plastic axial centroid (PC), causes a reduction in the plastic moment resistance and the modified moment resistance, $M_{Tb1,N,Rd}$, is given by:

$$M_{Tb1,N,Rd} = M_{T,Rd} - N_{f,Sd}\beta_f \tag{A12}$$

where β_f is the distance from the center of the axial stress block to the PC and

$$= y_{pc} - \left(y_{po} - \frac{\alpha_f}{2}\right)$$



$M_{Tb1,N,Rd} = M_{T,Rd} - N_{f,Sd}\beta_f$

Fig. A8 Bending stress blocks of bottom steel tee-section at LMS under axial force

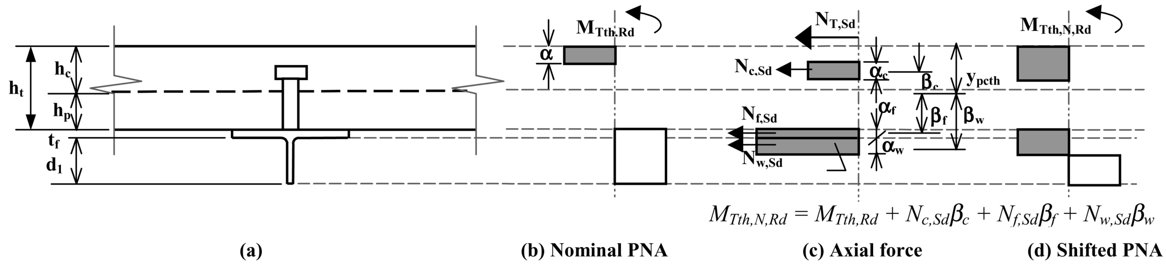


Fig. A9 Bending stress blocks of composite tee-section at HMS under axial force (nominal PNA in concreate)

All formulae and parameters required for the calculation of the modified moment resistances of the bottom steel tee-sections under axial force at LMS are summarized in Table A1(b).

A.2.3 Effect of axial force on moment resistances of top composite tee-sections at HMS

When the axial force arising from global bending action is applied to the composite tee-section at the HMS, its plastic moment resistance is modified in a way similar to that discussed for steel tee-section. The modification depends on the position of the nominal PNA, the magnitude of the axial force, and also the degree of shear connection in the composite tee-section.

a) Full shear connection and nominal PNA in concrete

Fig. A9(b) shows a composite tee-section together with its stress block when the nominal plastic neutral axis (PNA) locates within the concrete slab. The modified PNA may locate either in (i) the concrete slab, (ii) the steel flange, or (iii) the steel web, depending on the magnitude of the axial force.

Suppose the magnitude of the axial force is larger than the residual strength of the concrete, smaller of $N_{c,Rd}$ and $N_{sh,Rd}$ minus $N_{a,Rd}$, and is great enough to shift the nominal PNA into the web as shown in Fig. A9(d). The stress blocks have shown that the residual compressive strength of the concrete and the yield strength of the flange are reached such that a portion of the web is in compression while the remaining portion is in tension. As the stress blocks in the flange and the web due to the axial force are below the PC, the effect on moment resistance due to these two forces are non-beneficial. The magnitudes of the forces are given by:

$$N_{c,Sd} = \text{Minimum}\ (N_{c,Rd},\ N_{sh,Rd}) - N_{a,Rd} \quad \text{(A13a)}$$

$$N_{f,Sd} = 2\alpha_f b_f f_{yf} \quad \text{(A13b)}$$

$$N_{w,Sd} = 2\alpha_w t_w f_{yw} \quad \text{(A13c)}$$

where $\alpha_c = \dfrac{N_{c,Sd}}{0.85 f_{ck} b_e}$

$\alpha_f = t_f$

$\alpha_w = \dfrac{N_{T,Sd} - N_{c,Sd} - N_{f,Sd}}{2 t_w f_{yw}}$

The modified moment resistance of the composite tee-section is thus given by:

$$M_{Tth,N,Rd} = M_{Tth,Rd} + N_{c,Sd}\beta_c + N_{f,Sd}\beta_f + N_{w,Sd}\beta_w \tag{A14}$$

where $\beta_c = y_{pcth} - \left(\alpha + \frac{\alpha_c}{2}\right)$

$$\beta_f = y_{pcth} - \left(y_{po} + \frac{\alpha_f}{2}\right)$$

$$\beta_w = y_{pcth} - \left(y_{po} + \alpha_f + \frac{\alpha_w}{2}\right)$$

Other parameters when the shifted PNA remains in the flange or the concrete can be found similarly.

b) Full shear connection and nominal PNA in steel flange

When the axial resistance of the concrete slab is less than both the resistances of the shear connectors and the steel tee-section, the nominal PNA may locate within either the flange or the web as shown in Figs. A10 and A11, respectively. Suppose the nominal PNA locates in the flange, and an applied axial force is large enough to shift the nominal PNA into the web. Since the concrete slab is not able to take any applied axial force, it is distributed first to the steel flange, as shown in Fig. A10(d), and then resisted by the steel web as shown in Fig. A11(d) after the flange is fully yielded in compression.

Since the beneficial effect from the concrete stress block does not exist, there is always a reduction in the moment resistance due to the applied force resisted by both the flange and the web. The modified moment resistance, $M_{Tth,N,Rd}$, may be calculated using Eq. (14), with the second term on the right hand side of the equation set to zero as follows:

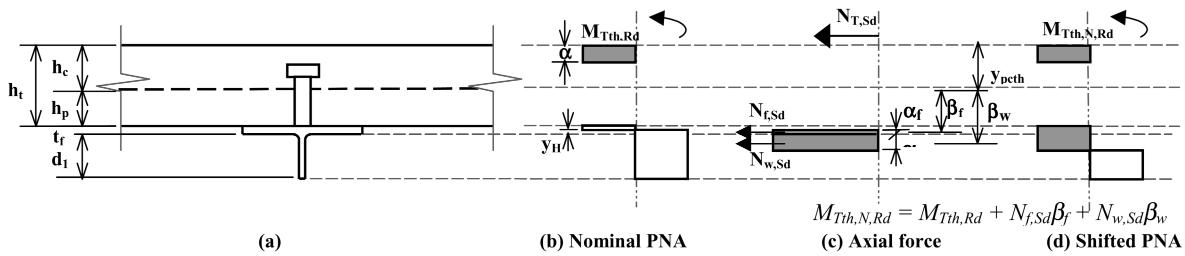


Fig. A10 Bending stress blocks of composite tee-section at HMS under axial force (nominal PNA in flange)

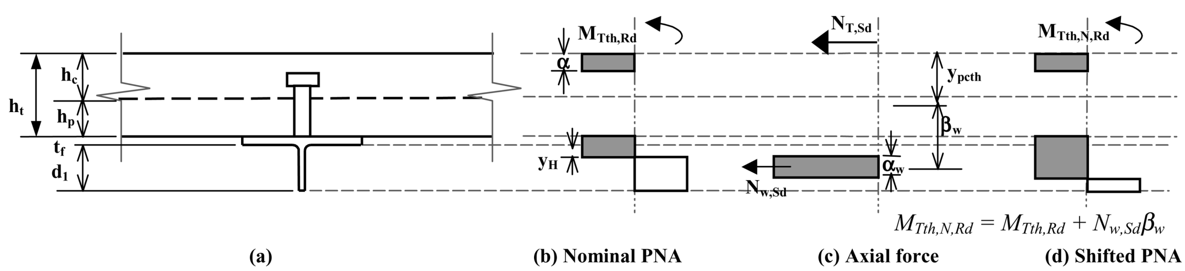


Fig. A11 Bending stress blocks of composite tee-section at HMS under axial force (nominal PNA in web)

$$M_{Tth,N,Rd} = M_{Tth,Rd} + N_{f,Sd}\beta_f + N_{w,Sd}\beta_w \tag{A15}$$

Other parameters when the shifted PNA remains in the flange can be found similarly.

c) Full shear connection and nominal PNA in steel web

In this case, $M_{Tth,N,Rd}$ can be found using Eq. (14) with the second and third terms being set to zero as follows:

$$M_{Tth,N,Rd} = M_{Tth,Rd} + N_{w,Sd}\beta_w \tag{A16}$$

As the force resisted by the web is equal to the applied axial force, the depth of the stress block in the web, α_w, is re-defined as follows:

$$\alpha_w = \frac{N_{T,Sd}}{2t_w f_{yw}} \tag{A17}$$

In practical situations, this case rarely happens.

d) Partial shear connection and nominal PNA in steel flange or web

For a composite tee-section under pure bending with partial shear connection, the concrete strength is always not fully utilized. As a result, there is always compression in the top steel tee-section in order to maintain equilibrium, and thus the nominal PNA may locate within the flange or the web. The effect of axial force on the moment resistance may be assessed using the method outlined previously for full shear connection. Figs. A10 and A11 also illustrate these two cases, respectively.

All formulae and parameters required for the calculation of the modified moment resistances of the top composite tee-sections under axial force at the HMS are summarized in Table A2(b).

A.2.4 Effect of axial force on moment resistances of top composite tee-sections at LMS

The effect of axial force on the moment resistance of the composite tee-section at the LMS follows the same philosophy as presented in A.2.1 for composite tee-section at the HMS. The axial force arising from the global bending action is assumed to act at the plastic axial centroid of the composite tee-section. The stress blocks for pure bending are modified by shifting the PNA to a new position allowing for the presence of the applied axial force. The modified moment resistance may then be found by adding or subtracting the moments contributed by the axial forces resisted by the concrete, the flange, and the steel web as appropriate.

a) Full shear connection and nominal PNA in concrete

Consider a case where there is full shear connection and $N_{c,Rd}$ is larger than $N_{a,Rd}$, the composite tee-section at the LMS under pure bending may assume a stress block as shown in Fig. A12. The location of the shifted PNA will depend on the magnitude of the axial force and may be located in (i) the concrete, (ii) the steel flange or (iii) the steel web. Suppose an axial force which is large enough to shift the nominal PNA into the web is applied to the composite tee-section, it is assumed to be distributed to the concrete first, then to the flange, and finally to the web. The force resisted by each element is shown in Fig. A12(c) and the final stress blocks are shown in Fig. A12(d) where the PNA is shifted to be located within the web.

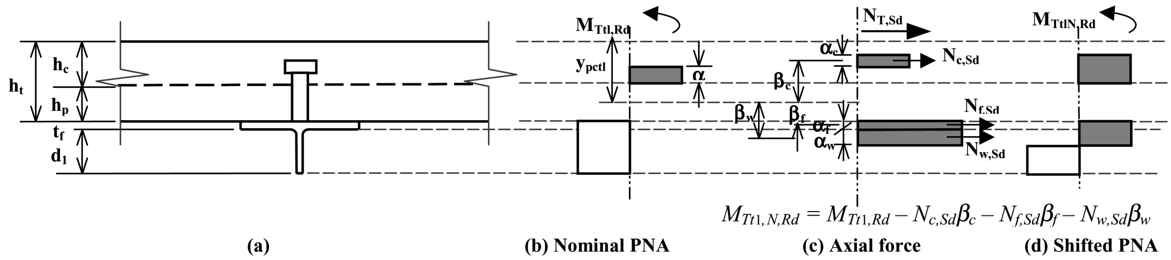


Fig. A12 Bending stress blocks of composite tee-section at LMS under axial force (nominal PNA in concreate)

The concrete slab can resist an axial force which is equal to its residual strength, the smaller of $N_{c,Rd}$ and $N_{sh,Rd}$ minus $N_{a,Rd}$, and hence, the depth of the stress block, α_c is re-defined as follows:

$$\alpha_c = \frac{\text{Minimum}(N_{c,Rd}, N_{sh,Rd}) - N_{a,Rd}}{0.85 b_e f_{ck}} \tag{A18}$$

This concrete force is acting opposite to the positive moment resistance with a lever arm, β_c, measured from the PC. Thus, it has a non-beneficial effect and the moment resistance of the composite tee-section is reduced. The remaining applied axial force is then distributed to the flange and also to the web if the resistance of the former is fully utilized. As the forces in the flange and the web are located below the PC, and act in the direction of positive moment resistance, they are considered to have beneficial effects on the moment resistance of the composite tee-section. The modified moment resistance due to the additional moments created by all these three forces can be calculated as follows:

$$M_{Tt1,N,Rd} = M_{Tt1,Rd} - N_{c,Sd}\beta_c - N_{f,Sd}\beta_f - N_{w,Sd}\beta_w \tag{A19}$$

where $\beta_c = y_{pct1} - \left(d_e - \alpha - \frac{\alpha_c}{2}\right)$

$$\beta_f = y_{pct1} - \left(h_t + \frac{\alpha_f}{2}\right)$$

$$\beta_w = y_{pct1} - \left(h_t + 2\lambda + \frac{\alpha_w}{2}\right)$$

α_f is the depth of the stress block in the steel flange

$= t_f$

α_w is the depth of the stress block in the steel web

$$= \frac{N_{T,Sd} - N_{c,Sd} - N_{f,Sd}}{2 t_w f_{yw}}$$

$N_{f,Sd} = 2\ \alpha_f\ b_f f_{yf}$

$N_{c,Sd} = 0.85 \alpha_c b_e f_{ck}$

$N_{w,Sd} = 2\ \alpha_w\ t_w f_{yw}$

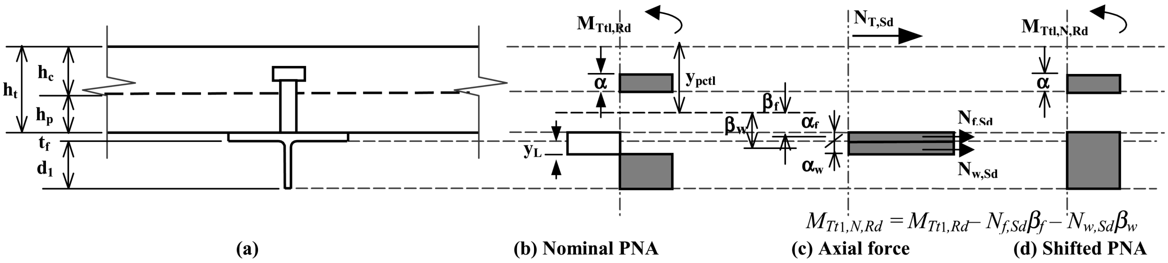


Fig. A13 Bending stress blocks of composite tee-section at LMS under axial force (nominal PNA in web)

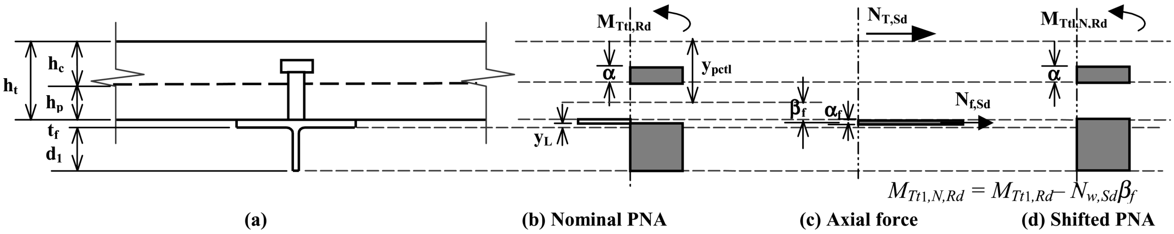


Fig. A14 Bending stress blocks of composite tee-section at LMS under axial force (nominal PNA in flange)

The modified moment resistances for other cases where the shifted PNA in the steel flange or the concrete slab can also be found similarly.

b) Full shear connection and nominal PNA in steel web

Similar to the case of composite tee-section at the HMS, the nominal PNA locates within the flange, as shown in Fig. A13(b), or within the web, as shown in Fig. 14(b), when the axial plastic resistance of the concrete is less than that of the top steel tee-section. In this case, only the part of the steel tee-section originally under tension is able to resist any applied axial compressive force.

Suppose the nominal PNA locates in the web and that the axial force is large enough to shift the nominal PNA into the web as shown in Fig. A13(d). The axial resistance of the flange is fully utilized and the applied axial force resisted by the web, $N_{w,Sd}$, is given by:

$$N_{w,Sd} = N_{T,Sd} - T_{f,Sd} \tag{A20}$$

where $N_{f,Sd} = 2\alpha_f b_f f_{yf}$

$\alpha_f = t_f$

The modified moment resistance of the composite tee-section is given by:

$$M_{Tt1,N,Rd} = M_{Tt1,Rd} - N_{f,Sd}\beta_f - N_{w,Sd}\beta_w \tag{A21}$$

where $\beta_f = y_{pct1} - \left(h_t + \dfrac{\alpha_f}{2}\right)$

$\beta_w = y_{pct1} - \left(h_t + \alpha_f + \dfrac{\alpha_w}{2}\right)$

α_w is the depth of the stress block in the steel web

$$= \frac{N_{w,Sd}}{2t_w f_{yw}}$$

When the axial force is not large enough to shift the PNA to be located within the web, only part of the flange is resisting the axial force and the parameters required to evaluate the modified moment resistance are found similarly.

c) Full shear connection and nominal PNA in steel flange

When the nominal PNA is located within the steel flange as shown in Fig. A14(b), then the shifted PNA always locates within the flange. The modified moment resistance of the composite tee-section is given by:

$$M_{Tt1,N,Rd} = M_{Tt1,Rd} - N_{f,Sd}\beta_f \tag{A22}$$

where $\beta_f = y_{pct1} - \left(h_t + \frac{\alpha_f}{2}\right)$

α_f is the depth of part of the flange resisting the axial force

$$= \frac{N_{T,Sd}}{2b_f f_{yf}}$$

d) Partial shear connection and nominal PNA in steel flange or web

For those composite tee-sections with partial shear connection, the nominal PNA falls either in the flange or the web. The procedures for assessing the effect of the axial force on the moment resistance are exactly the same as discussed previously for full shear connection. The stress blocks are shown in Figs. A13 and A14.

All formulae and parameters required for the calculation of the modified moment resistances of the top composite tee-sections under axial force at LMS are summarized in Table A3(b).

A.3 Presence of shear force

A.3.1 Plastic shear resistances

According to Eurocode 3, the plastic shear resistance, $V_{T,Rd}$, of a steel tee-section is limited to that contributed by the full depth of the web and a portion of the flange, and is given by:

$$V_{T,Rd} = V_{f,Rd} + V_{w,Rd} \tag{A23}$$

where $V_{f,Rd}$ is the shear resistance of the flange, and equal to $A_{vf}\left(\frac{f_{yf}}{\sqrt{3}}\right)$

$V_{w,Rd}$ is the shear resistance of the web, and equal to $A_{vw}\left(\frac{f_{yw}}{\sqrt{3}}\right)$

A_{vf} is the shear area of the flange, and equal to $(0.75t_f + t_w)t_f$

A_{vw} is the shear area of the web, and equal to $d_1\, t_w$

It should be noted that the shear area of the flange given above has been carefully calibrated in an extensive finite element study on steel beams with large web openings.

A.3.2 Effect of shear forces on moment resistances of tee sections

The effect of shear force is then taken into account by reducing the bending strengths of the flange and the web through von Mises yielding criterion as follows:

$$f_{vw} = \sqrt{f_{yw}^2 - 3\,\tau_w^2} \tag{A24a}$$

$$f_{vf1} = \sqrt{f_{yf}^2 - 3\,\tau_f^2} \tag{A24b}$$

where $\tau_w = \dfrac{V_{w,Sd}}{A_{vw}}$

$\tau_f = \dfrac{V_{f,Sd}}{A_{vf}}$

$V_{w,Sd} = V_{T,Sd}\left(\dfrac{V_{w,Rd}}{V_{T,Rd}}\right)$

$V_{f,Sd} = V_{T,Sd}\left(\dfrac{V_{f,Rd}}{V_{T,Rd}}\right)$

$V_{f,Sd}$, $V_{w,Sd}$ are the shear forces resisted by the shear areas of the web and the flange, respectively

$V_{T,Rd}$ is the applied shear resistance of the tee-section $= V_{f,Rd} + V_{w,Rd}$

$V_{T,Sd}$ is the applied shear force on the tee-section

f_{vw}, f_{vf1} are the reduced bending strengths of the shear areas of the web and the flange, respectively

Since the bending strength of a significant area of the flange is not reduced, an average reduced bending strength, f_{vf}, is calculated for the whole flange as follows:

$$f_{vf} = \frac{f_{yf}\,(A_f - A_{vf}) + f_{yf1} A_{vf}}{A_f} \tag{A25}$$

where A_f is the total area of the flange, and

A_{vf} is the shear area of the flange

After the reduced bending strengths of both the web and the flange are found, the basic axial and the moment resistances of a tee-section are then calculated using f_{vw} and f_{vf} accordingly.

A.4 Evaluation of axial forces acting on tee-sections

In order to design against the 'Vierendeel' mechanism of composite beams with web openings, consider the equilibrium at the HMS of the perforated section. The global moment at the centerline of the opening, M_{Sd}, may be written as follows:

$$M_{Sd} = N_{T,Sd} z_H + M_{Tth,Sd} + M_{Tbh,Sd} - \frac{V_{Sd} a}{2} \tag{A26}$$

where z_H is the moment arm between the axial forces acting on the top composite and the bottom steel tee-sections at the HMS of the perforated section.

Similarly, the global moment may also be obtained by considering the equilibrium at the LMS of the perforated section as follows:

$$M_{Sd} = N_{T,Sd} z_L - M_{Tt1,Sd} - M_{Tb1,Sd} + \frac{V_{Sd} a}{2} \quad \text{(A27)}$$

where z_L is the moment arm between the axial forces acting on the top composite and the bottom steel tee-sections at the LMS.

By summing the above two equations to eliminate the shear force, the global moment can be expressed in terms of only the local axial forces and moments as follows:

$$M_{Sd} = N_{T,Sd}\left(\frac{z_H + z_L}{2}\right) + \frac{M_{Tth,Sd} + M_{Tbh,Sd} - M_{Tt1,Sd} - M_{Tb1,Sd}}{2} \quad \text{(A28)}$$

For any given global moment at the centerline of the perforated section, the above equation is re-arranged to give the axial force acting on the top composite and the bottom steel tee-sections as follows:

$$N_{T,Sd} = \frac{2M_{Sd} - (M_{Tth,Sd} + M_{Tbh,Sd} - M_{Tt1,Sd} - M_{Tb1,Sd})}{z_H + z_L} \quad \text{(A29)}$$

Once the axial force is known, the moment resistances of the composite and the steel tee-sections may be evaluated through the use of the formulae given above. However, since the axial force also depends on the magnitudes of the moment resistances, iterations are required. The moment resistances found for the tee-sections are then used to check against the 'Vierendeel' mechanism by satisfying the equilibrium condition of both the top composite and the bottom steel tee-sections.

A.5 Equilibrium against Vierendeel Mechanism

It is required to consider 'Vierendeel' mechanism separately for the top composite and the bottom steel tee sections as follows:

a) Top composite tee-sections

$$M_{Tt1,Rd} + M_{Tth,Rd} + N_{T,Sd}(z_H - z_L) \geq (V_{at,Sd} + V_{c,Sd})a \quad \text{(A30a)}$$

where $V_{at,Sd}$, $V_{c,Sd}$ are the shear forces resisted by the top steel tee-section and the concrete slab respectively

b) Bottom steel tee-sections

$$M_{Tb1,Rd} + M_{Tbh,Rd} \geq V_{ab,Sd} a \quad \text{(A30b)}$$

where $V_{ab,Sd}$ is the shear force applied to the bottom steel tee-section

If either of the above inequalities does not hold, then part of the shear force acting on the tee section should be distributed to the other tee section. Iterations should be performed until both inequalities do not hold which implies the occurrence of 'Vierendeel' mechanism.

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