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Limitation of effective length method and codified second-order analysis and design

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Abstract. The effective length method for flexural (column) buckling has been used for many decades but its use is somewhat limited in various contemporary design codes to moderately slender structures with elastic critical load factor (λ_{cr}) less than 3 to 5. In pace with the use of higher grade steel in recent years, the influence of buckling in axial buckling resistance of a column becomes more important and the over-simplified assumption of effective length factor can lead to an unsafe, an uneconomical or a both unsafe and uneconomical solution when some members are over-designed while key elements are under-designed. Effective length should not normally be taken as the distance between nodes multiplied by an arbitrary factor like 0.85, 1.0, 2.0 etc. Further, the classification of non-sway and sway-sensitive frames makes the conventional design procedure tedious to use and, more importantly, limited to simple regular frames. This paper describes the practical use of second-order analysis with section capacity check allowing for P-δ and P-Δ effects together with member and system imperfections. Most commercial software considers only the P-Δ effect, but not member and frame imperfections nor P-δ effect, and engineers must be very careful in their uses. A verification problem is also given for validation of software for this type of powerful second-order analysis and design. It is a trend for popular and advanced national design codes in using the second-order analysis as a norm for analysis and design of steel structures while linear analysis may only be used in very simple structures.

Key words: buckling; steel structures; imperfections.

1. Introduction

Recent national codes in various countries highlight the preference of using the second-order analysis in place of the effective length method for design of slender and moderately slender structures with elastic critical load factor (λ_{cr}, defined as the ratio of elastic critical load causing the structure to buckling elastically to the design load) less than 5 for AS4100 (1998), 4 for BS5950 (2000) or 3 for Eurocode-3 (2003). This demonstrates that these codes do not consider the effective length method as a reliable

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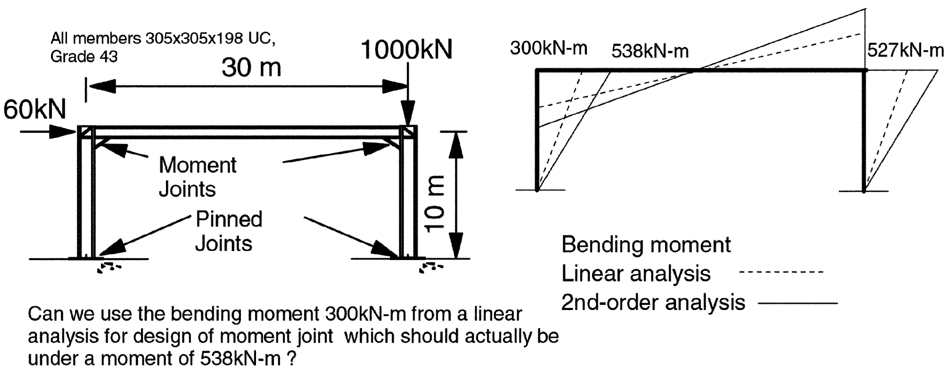


Fig. 1 A simple portal

method for slender structures, although they vary in the critical load factor for definition of slenderness beyond which the manual assessment of effective length method is no longer acceptable.

It is not difficult to note among many practical frames that the effective length method is not applicable. The simplest case will be the ignorance of the P-Δ effect in the connection design of a portal shown in Fig. 1. It can be seen that when the column is subjected to a concentrated point load at its top, the linear analysis does not provide information on the P-Δ moment due to sway and vertical force for connection design.

2. Linear vs. second-order nonlinear analysis and design method

In linear analysis, displacement and stress are assumed to be linearly proportional to loads because the stiffness is assumed constant. For stocky structures, this assumption is correct and direct computation of stress can be used to confirm structural adequacy.

Unlike a tie in tension, stress across a section in a column under axial compression cannot be calculated as the axial force divided by the cross sectional area. As shown in Fig. 2, the maximum stress

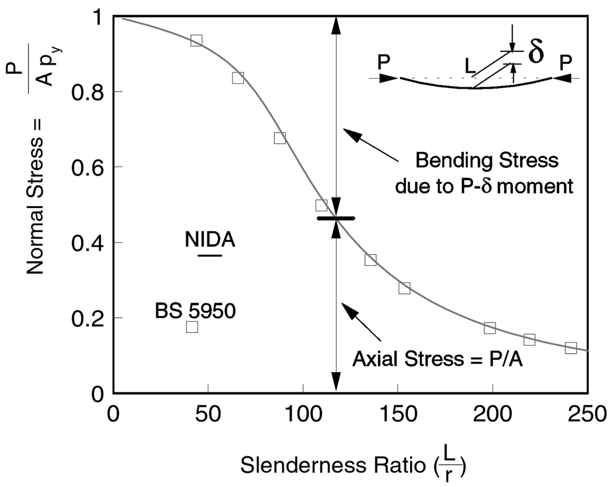


Fig. 2 Reduction of buckling resistance due to member slenderness

Table 1 Uses of initial imperfections in Eurocode 3 (2003) "Design of steel structures" PrEN1993-1-1-2003

Buckling curve acc. to Table 6.1	Elastic analysis e_0/L	Plastic analysis e_0/L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

is made up of two parts, being due to the axial compression force and due to P-δ moment which depends on member slenderness (L/r) and Young's modulus of elasticity (E). To compensate for this, we therefore need to reduce the design stress using Table 1 in BS5950 (2000). The limitations of the linear analysis used with the effective length method include the followings.

1. Design of sloped and inclined members are not acceptable by this approach (see clause 2.4.2.6, p.14, BS5950 (2000)).
2. Interactive behaviour of a group of members cannot be considered reliably that the effective length L_e is normally assumed inaccurately.
3. Effect of change of stiffness in the presence of axial force is not considered (i.e., a compressive brace takes a much lower force than a tension brace in a cross bracing which cannot be considered in the analysis)
4. End moments and loads along a member increase the initial deflection of a member which leads to a larger P-δ moment, and thus a reduction in its load capacity. This effect cannot be considered in the effective length method.
5. Leaning columns (columns with both end pinned) create an additional horizontal force which weakens a structure sway stability. This effect cannot be considered in a linear analysis.
6. When the elastic critical load factor λ_{cr} is less than 4 in BS5950 (2000) or 5 in AS4100 (1998), the method is not allowed for uses.

In a second-order elastic or first plastic hinge analysis, a structure is assumed to reach its design capacity when it starts to yield or when the first plastic hinge is formed, with full allowance of the nonlinear effects such as member imperfections, effects of loads on the buckling capacity such as reduced effective length due to load acting along the members, residual stresses and changes of member stiffness. Unlike the effective length method, this approach considers a structure as a system instead of a group of individual members behaving independently.

In line with the ever increasing use of high strength steel which has a higher yield stress but same Young modulus (E) as those of low carbon steel, structures to date are more prone to instability than those in the past, and collapse is more controlled by Young's modulus rather than by yield stress. A proper consideration of buckling is becoming more important. Therefore more rigorous methods are derived for better estimation of the load capacities of slender structures.

3. Second-order P-Δ-only analysis

The second-order analysis is a method recommended by many steel design codes including AS4100 (1998), BS5950 (2000) and Eurocode-3 (2003).

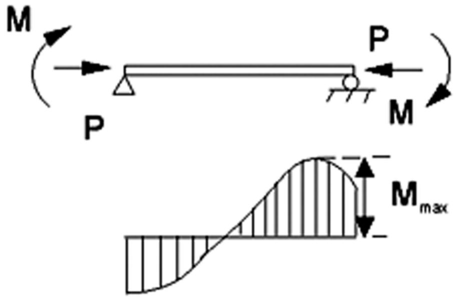


Fig. 3 Maximum moment may occur at a location not at ends nor mid-span

The users must be careful in using a second-order analysis which can be P-Δ-only or P-Δ-δ analysis. Some computer programs consider only the P-Δ effect by adding displacements to geometry, and therefore we need to include the P-δ effect in the design process in which the P-Δ effect is accounted for in the analysis part.

The P-Δ-only analysis does not consider changes of member stiffness under axial forces and displacements along the member length. Thus, it cannot predict the buckling strength or design strength of a pinned-pinned or pinned-fixed column under an axial force. For non-sway frames, this second-order analysis of plotting the bending moment diagram allowing for sway displacement becomes irrelevant.

The P-Δ-only method must allow for frame imperfections which can be simulated by application of notional force. In BS5950 (2000), a notional force of 0.5% of the factored vertical dead and live loads is used for building frames. For temporary structures like scaffolding, this percentage should be increased to 1% in accordance with BS5973 (1993).

Checking member strength at two ends is inadequate because of missing of P-δ moment. This is why some software cannot be used for direct design by the second-order analysis. The proposed method locates the maximum P-δ-Δ moment and thus it overcomes the problem.

4. Second-order P-Δ-δ analysis

Second-order analysis with section capacity check as a completed design procedure must allow for both the P-Δ and the P-δ effects as well as member and frame imperfections.

P-Δ-δ analysis can solve both sway and non-sway problems since it considers moments induced by change of geometry in a structure and by deflection along members.

If we consider both the P-δ and the P-Δ effects, we then need not assume an effective length and the load capacity of a structure can then be determined by checking the section strength of the member. For example, we can obtain the same buckling load as Table 2 of BS5950 (2000) for columns with any boundary condition WITHOUT assuming an effective length which, in general, is unknown. A complete description on the element formulation for the present analysis can be found in Chan and Zhou (1994).

5. The theory

Member curvature under an axial load must be considered in determination of the member buckling capacity. The imperfections and the Perry constant are used not only to account for geometrical

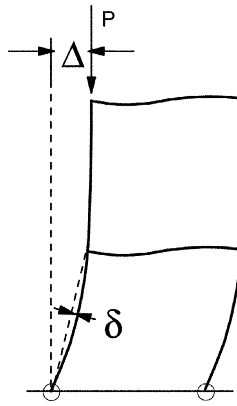


Fig. 4 P-Δ and P-δ effects

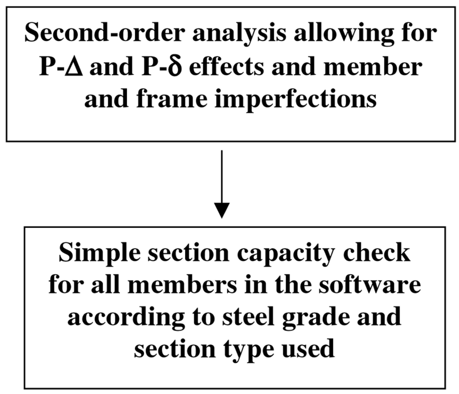


Fig. 5 Proposed simple design

Table 2 Proposed initial imperfections calibrated by Chan and Cho (2002)

Type of section	Axis of bending			
	x-x		*y-y*	
	Section	$\delta_0/L\cdot 1000$	Section	$\delta_0/L\cdot 1000$
UB	305×165×40	1.697	127×76×13	1.685
UC	356×368×129	3.076	356×406×634	2.860
CHS	508.0×10.0	1.389	-	-
SHS	300×300×6.3	1.598	-	-
RHS	300×200×6.3	1.513	500×200×8.0	1.732
Channel	Any axis:		152×89	4.474

imperfections, but also the effect of residual stresses etc., and therefore, these imperfection parameters should be considered as equivalent imperfections which give lower bound fit to the buckling curves for various types of columns. Curves “a” to “d” in BS5950 (2000) are used to consider this effect for different types of sections and buckling axes.

The concept of the Perry-Robertson formula can be summarized as follows.

A column fails when the sum of the axial stress and the P-δ bending stress equals to the material

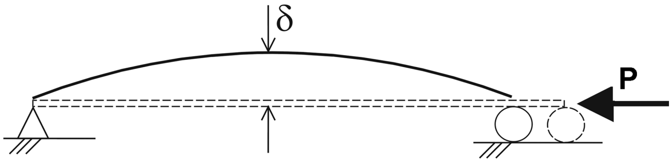


Fig. 6 Buckling of a simple column

design strength as,

$$\frac{P}{A} + \frac{P(\delta + \delta_o)}{Z} = p_y \tag{1}$$

$$\frac{P}{A} = p_c = p_y \Big/ \left(1 + \frac{(\delta + \delta_o)A}{Z}\right) \tag{2}$$

p_c is the permissible buckling stress from Table 2 in BS5950 (2000). A is the cross sectional area, Z is the elastic modulus, p_y is the design strength, δ will be equal to the load-induced mid-span lateral deflection plus the initial imperfection.

The P-δ moment can be shown to be,

$$P\delta = \frac{\delta_o P P_{cr}}{P_{cr} - P} \tag{3}$$

in which δ_o is the initial imperfection, and P_E is the elastic Euler's critical load for a pinned-pinned column given by $P_E = \frac{\pi^2 EI}{L_e^2}$. We can see that the P-δ moment depends on the member effective length, L_e, imperfection parameter, and the flexural rigidity, EI.

If one uses a curved element with an initial imperfection, and determines accurately the P-δ moment and also the P-Δ moment by using deformed geometry in the analysis, it can then design without assuming an effective length. The output will be more accurate and reliable since erroneous assumption on the effective length L_e can be avoided.

To extend further the concept, if we include P-Δ moment, end moments and lateral load along a member of using in place of in Eq. (3), all these instability effects can be considered in the analysis. It is interesting to note that additional moment due to an axial force, and deflection of the member caused by end moments or load along the members is not included in the effective length method in the design code.

6. Section capacity check in a second-order analysis

With other terms readily obtained from a linear analysis, the proposed method checks the strength of every member by the following section capacity equation.

$$\frac{P}{Ap_y} + \frac{M_x + P(\Delta_x + \Delta_{0x}) + P(\delta_x + \delta_{0x})}{M_{cx}} + \frac{M_y + P(\Delta_y + \Delta_{0y}) + P(\delta_y + \delta_{0y})}{M_{cy}} = \varphi \le 1 \tag{4}$$

where

P is the axial force in member

p_y is the design strength

M_{cx} is the moment capacity about the major x axis. If lateral-torsional buckling is considered, the smaller of buckling moment, M_b, and the plastic moment M_{cx} divided by an equivalent moment factor m_{LT} should be used.

Z_y is the elastic modulus about the minor axis

M_y, M_z are the moments about the major and the minor axes obtained from a linear analysis.

φ is the material consumption factor. If $\varphi > 1$, a member fails in design strength check and if $\varphi << 1$, the member section can be reduced in size.

As the effective length is to account for the P-Δ and the P-δ effects, the above inclusion of the P-Δ and the P-δ effects will automatically consider the effects due to buckling.

7. Member imperfections

Imperfections refer to out-of-straightness in a supposed straight member and lack of verticality in a frame. All practical structures contain imperfections, and both linear and second-order nonlinear analyses must consider these effects either implicitly or explicitly. In conventional design methods, the effective length is varied by judgement to cater for the buckling effect, and different buckling curves are then used. The Perry factor is a parameter to generate the sharpness of a curve for best fitting of the buckling curve over a wide range of slenderness as shown in Fig. 7. The Perry factor η includes not only the effect of geometrical imperfection, but also residual stress etc. such that their weakening effects are replaced by an equivalent imperfection as indicated in Eq. (5).

$$(p_E - p_c)(p_y - p_c) = \eta p_E p_c \tag{5}$$

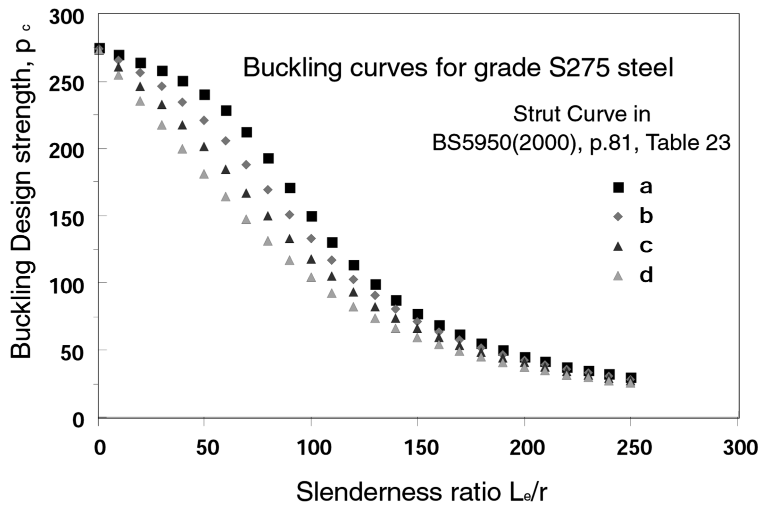


Fig. 7 Buckling curves for different types of sections

in which p_E is the Euler buckling stress given by $\frac{\pi^2 E}{(L_E/r)^2}$, η is the Perry factor which is used to control the sharpness of the buckling curve, p_y is the material design strength and p_c is the compressive strength allowing for the effect of buckling.

Table 5.1 of Eurocode 3 (2003) is re-presented in Table 1, and Chan and Cho (2002) propose a set of member initial imperfections in Table 2 for direct computer application. These imperfections should then be used for second-order analysis and design of steel structures.

8. Local plate and lateral-torsional buckling

The effects of local plate buckling and lateral-torsional buckling of beams must be considered in the design checking for sections with very thin plates as well as for beams with open sections and bent about its major axis with intermediate (non-continuous) lateral restraints. However, their behaviour is more localised that the effect of stiffness of the complete frame and of members far away from the concerned member is minimal. Therefore, the design codes do not require relating frame stability to beam buckling, of which the consideration is different from the flexural (Euler) buckling. To this, it is adequate to directly use the design code formulae for checking. In general, the process becomes trivial in most cases, and hence, and will not be further elaborated here.

9. Verifications through examples

As shown in Fig. 8, the results of a sub-assemblage in Appendix E of BS5950 (2000) is checked and compared. By varying the length of the horizontal beam, we can then check the results from Appendix E against the proposed method for verification. For sway sensitive, non-sway and partially sway cases, different spring stiffness "k" can be assigned appropriately. For example, k is very large of say 10^6 of column stiffness for non-sway case while 0 for sway sensitive case.

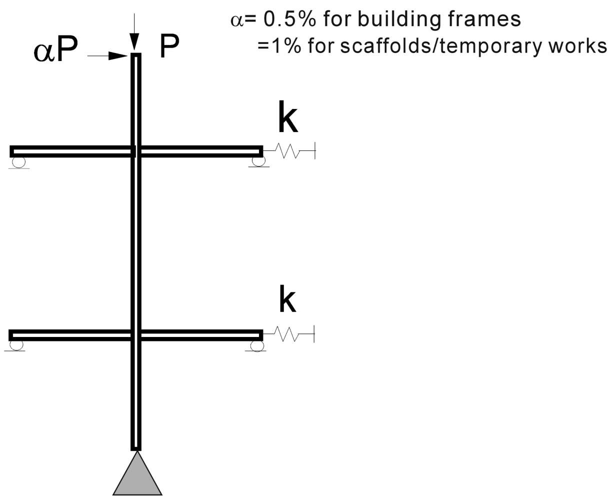


Fig. 8 Checking of design strength of a column in a sub-frame in a sway and non-sway frame in Fig. E.3 BS5950 (2000)

Two examples are chosen with an objective of testing the performance of an element for second-order analysis and applying the second-order analysis to solve a practical problem.

9.1. Example 1: Benchmark and testing example for verification of P-δ effect

The example is suggested to be a benchmark problem for testing the proficiency of a second-order analysis and design software where the elastic buckling load and the design load carrying capacity of a fixed-pinned (prop-cantilever) column by 1 element per member is computed as shown in Fig. 9, and compared against the analytical method. CHS88.9×3.2 is used and its properties are L=5 m, A=8.62 cm^2, I=79.2 cm^4, Z=17.8 cm^3, p_y=275 N/mm^2, and E=2.05×10^5 N/mm^2.

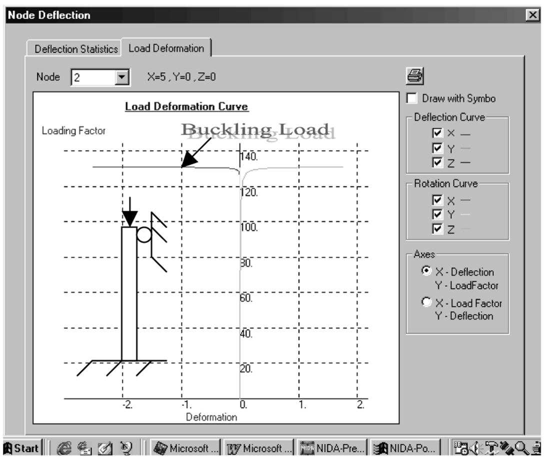
We use 1 element per member since using 2 or more elements per member is too complicated for practical structures which includes determination of mid-span nodal coordinates for the direction of member imperfection. Also, the most critical section is not at either end nor at mid-span, and thus using 2 straight elements to model a member is inadequate. In the proposed design method, the effective length is not assumed nor calculated.

The analytical buckling load is $=\dfrac{\pi^2 EI}{(0.7L)^2}$=130.8 kN. When using Fig. E.4 of Appendix E, BS5950 (2000) with k_1=0, k_2=1, the slenderness ratio is L_e/L=0.7, and the design load is therefore 108 kN.

In the tabulated results, we can see that the proposed method gives a very close solution to both the analytical method and the BS5950 (2000) result. However, for practical structures where the effective length is not simple to determine, both the hand and the code methods can hardly be applied effectively whilst the present approach experiences no difficulty.

This problem is possibly the most simple bench mark example for testing the reliability and accuracy of a software. This benchmark example assists us to check the validity of a software for second-order analysis. If the software cannot give you a right answer here, we cannot expect it to give you a correct answer for more complex practical structures.

It can be seen when using different values of member imperfections and specified notional forces, one can generate a similar buckling curves as in the code, and thus using this value of member imperfection for design of a complex structure made of thousands of members.



Methods	Elastic buckling load (kN)	Design Load Capacity (kN)
Proposed method by 1 element/member	131	108
Analytical	130.8	-
BS5950(2000)	-	108.6

Fig. 9 Results of second-order analysis against design code

9.2. Example 2: Design of a roof truss without intermediate restraints

The roof truss shown in Fig. 10 is a roof structure in a shopping mall with a span of 23.5 m and a depth of 1.77 m. The top chords of the truss are made of 300×200×10 RHS, the bottom chords 250×150×12.5 RHS, the web members 100×100×8 RHS and the transoms 200×100×10 RHS. The steel grade is S355 with a design strength for sections of wall thickness below 16 mm equal to 355 N/mm^2. The wind loads are assumed to act one way onto the transoms and then transferred to the chords. The critical load case will be the wind uplift making the bottom chord in compression. The effective length of the member is found to be unrealistically determined as 23.5 m when using BS5950 (2000) which states that "the out-of-plane lengths as the distance between purlins or longitudinal ties, provided that such ties are properly connected to an adequate restraint systems".

A simple truss of a 4.8 m span truss was studied and tested by Chan *et al*. (2002) who used a second-

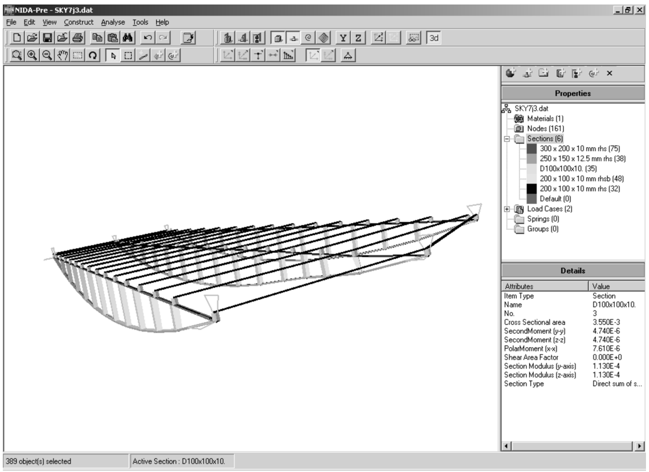
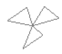


Fig. 10 2-bay of the roof truss.  is pinned support

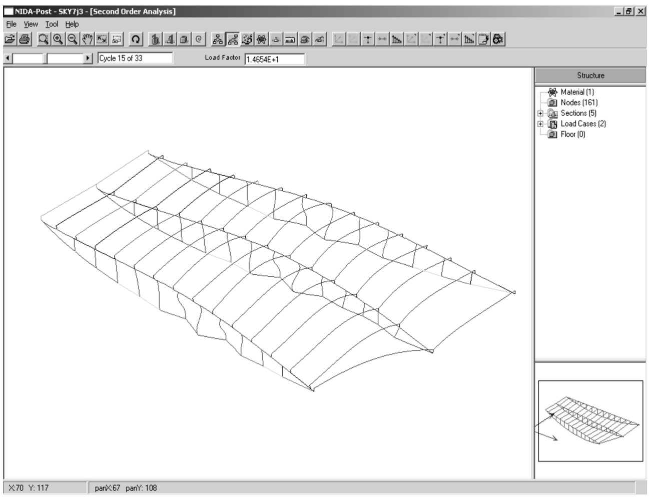


Fig. 11 Deformed and buckled shape of the roof truss

order analysis for the prediction of the design capacity of a tested truss without the use of any effective length. Using a similar concept but applied to a practical structure, this example serves to illustrate how the method is employed for solving a problem which is designed in the past by unreliable judgment with results vary from one engineer to another.

Unlike the linear analysis which is only used once for determination of bending moment, the present method can achieve two objectives. Firstly, the structure is checked against a fixed set of design load to see if any member is likely to be failed by buckling. Secondly, the structure is loaded by a series of incremental loads until failure of a member at which the structural design load is considered to have been reached. The first method is termed as "Capacity check at fixed design load" while the second method is termed as "Maximum design load determination". Both methods use the same theory and their difference is only on the numerical procedure of applying a constant or an incremental load, the second approach is used in this example.

In the analysis, the system fails under a wind pressure of 2.7 kPa which is just above the wind pressure of $2.2\times1.2=2.64$ kPa. The complete design process using a computer of Pentinum III of 800 MHz takes only 4 seconds with 5 equilibrium iterations. The buckled shape of the structure is shown in Fig. 11, and it can be seen that the bottom chord buckles in the form of multiple curves. The guess of the effective length is, hence shown to unreliable here.

10. Conclusions

The use of second-order analysis for design of steel structures has been allowed in many modern design codes, and its taken as a more reliable design approach than the effective length method which is based on educated guess on buckled mode shapes. Structures with elastic critical load factors less than 3 to 5, depending on the code being used, can only be designed by this new method. The authors believe that it is a matter of time for the method to be widely used as an alternative or even a replacement of the effective length method. To remain competitive, engineers should not hesitate to employ the proposed method for checking structural stability and strength which normally lead to a safer and a more economical structure since some members will not be over-designed whilst key members are under-designed.

Acknowledgements

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