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Transient memory response of a thermoelectric half-space with temperature-dependent thermal conductivity and exponentially graded modulii

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Abstract. In this work, we consider a problem in the context of thermoelectric materials with memory-dependent derivative for a half space which is assumed to have variable thermal conductivity depending on the temperature. The Lamé's modulii of the half space material is taken as a function of the vertical distance from the surface of the medium. The surface is traction free and subjected to a time dependent thermal shock. The problem was solved by using the Laplace transform method together with the perturbation technique. The obtained results are discussed and compared with the solution when Lamé's modulii are constants. Numerical results are computed and represented graphically for the temperature, displacement and stress distributions. Affectability investigation is performed to explore the thermal impacts of a kernel function and a time-delay parameter that are characteristic of memory dependent derivative heat transfer in the behavior of tissue temperature. The correlations are made with the results obtained in the case of the absence of memory-dependent derivative parameters.

Keywords: thermoelectric materials; fractional order theory; variable of thermal conductivity; variable Lamé's moduli; perturbation method; numerical results

1. Introduction

Heat exchange keeps on being a field of real enthusiasm for building and logical analysts, just as originators, designers, and producers. Impressive exertion has been given to explore in customary applications, for example, substance preparation, general assembling, and vitality gadgets, including general power frameworks, heat exchangers, and superior gas turbines (Goldstein *et al.* 2005).

Many authors have formulated new theories of thermoelasticity to replace the coupled theory introduced by Biot (1956). The heat equations associated with these theories are hyperbolic and hence automatically eliminate the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity.

Two generalizations introduced to the coupled theory. The first generalization to coupled thermoelasticity is due to Lord and Shulman (1967), who introduced the theory of generalized thermoelasticity with one relaxation time. The second generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times proposed by Green and Lindsay (1972). One can refer to Chandrashekhariah (1998) and Hetnarski and Ignaczak (2000) for a review, presentation of generalized theories. Within the theoretical contributions to the subject are the proofs of uniqueness theorems under different conditions by Sherief (1986) and Ezzat and El-Karamany (2002). Among the contributions to the subject of generalized thermoelasticity are the works of Marin (1995) and Sharma and Marin (2014). A couple of examinations subject to these generalized theories were researched in Refs. Othman *et al.* (2002), Ezzat (2006), Mukhopadhyay and Kumar (2009), Lata *et al.* (2016), Lata and Kaur (2019), Lata and Singh (2019), Zenkour (2017), Zenkour and Abouelregal (2019), Daikh *et al.* (2020), Kumar *et al.* (2016; 2017), Zenkour and Alghanmi *et al.* (2019) and Sobhy and Zenkour (2020).

Direct conversion between electricity and heat by using thermoelectric materials has attracted much attention because of their potential applications in Peltier coolers and thermoelectric power generators (See Ref. Rowe, 1995). Thermoelectric gadgets have numerous alluring highlights contrasted and the customary liquid based coolers and power age innovations, for example, long life, no moving part, any commotion, and simple support also, high unwavering quality. Nevertheless, their utilization has been constrained by the moderately low execution of present thermoelectric materials. The productivity of a thermoelectric material is identified with the supposed dimensionless thermoelectric figure-of-merit ZT by Tritt (2000). The expansion in ZT drives specifically to change in the vitality transformation productivity of thermoelectric generators and in the cooling proficiency of Peltier modules (See Ref. Tritt 2000). Much exertion has been made to raise the ZT of thermoelectric bulk materials for vitality transformation productivity, so there have been a few changes in ZT. The thermoelectric figure of legitimacy gives a measure of the nature of such materials for

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applications and is characterized by Hiroshige *et al.* (2007), $ZT = (\sigma_o S^2 / k)T$ with a specific end goal to accomplish a high figure of legitimacy; one requires a high thermopower *S*. Among the commitments in continuum mechanics of thermoelectric materials are crafted by Shercliff (1979) and Ezzat and Youssef (2010).

Over the latest couple of years, fractional examination was associated viably in various areas to modify many existing models of physical methodology, e.g., science, demonstrating and ID, and ID, equipment, wave expansion and viscoelasticity (Bagley and Torvik, 1986). Povstenko (2009) investigated new thermoelasticity models that use fractional derivative. The fractional order theory of thermoelasticity was derived by Sherief *et al.* (2010). As of late, Ezzat (2011) set up another model of fractional heat conduction equation utilizing the Taylor-Riemann series expansion of time- fractional order. Kothari and Mukhopadhyay (2011), Sherief and Abd El-Latief (2013), and Ezzat and El-Bary (2016) solved some problems in the context of this theory. Bo *et al.* (2015) displayed a conservative numerical strategy for tackling the two-dimensional non-straight fractional reaction-sub diffusion equations, while Zhang *et al.* (2018) presented a period space ghastly technique for the time-space fragmentary Fokker-Planck condition and its contrary issue. Yu *et al.* (2013, 2020) and Mashat and Zenkour (2020) solved some problems in fractional order generalized thermoelasticity.

The memory-dependent derivative is defined in an integral form of a common derivative with a Kernel function on a slipping interval. Thus, this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). Its definition is more intuitionistic for understanding the physical meaning and the corresponding memory dependent differential equation has more expressive force (Yu *et al.* 2014). One can allude to Ezzat and El-Bary (2017), Lotfy and Sarkar (2017), Tiwari and Mukhopadhyay (2018), Xue *et al.* (2018) and Biswas (2019) for an overview of utilizations of memory-dependent derivative analytics.

It is well known that, for the weakly nonlinear systems, the internal resonances can be investigated by the perturbation method (See Refs. Hu *et al.* 2013, 2020a). But the perturbation method will show some limitations in the investigation of the internal resonance phenomena for the strongly nonlinear systems. Thus, several approaches those are expected to replace the perturbation method, including the geometric method, the numerical approximation method and the improved analytical approximation method, were proposed to investigate the internal resonance phenomena for the strongly nonlinear systems. Among which, the method of multiple scales (Hu *et al.* 2020 b,c,d,e) is one of relatively developed approaches for the strongly nonlinear systems.

The reason for this work is that tackle an issue with regards to MDD thermoelasticity hypothesis for a practically reviewed thermoelectric half space in which Lamé's modulii and attractive porousness are taken as elements of the vertical separation from the outside of the half-space, while the thermal conductivity is taken as a component of temperature. The Laplace change system is utilized to take care of the issue. The reversal of the Laplace changes is completed utilizing a numerical methodology proposed by Honig and Hirdes (1984). The game plans are addressed graphically for various estimations of the thermoelectric power, MDD parameters and the magnetic number on all considered fields.

2. Mathematical model

The governing equations represent entire device of generalized thermoelasticity with memory-dependent derivative heat transfer of thermoelectric material in the presence of a constant magnetic field consists of:

1. The figure-of-merit ZT_o at some reference temperature T_o (Hiroshige *et al.* 2007)

$$ZT_o = \frac{\sigma_o s_o^2}{k} T_o \tag{1}$$

where s_o is Seebeck coefficient at T_o.

2- The first Thomson relation at T_o (Morelli 1997)

$$\pi_o = s_o T_o \tag{2}$$

where π_o is the Peltier coefficient at T_o .

3-The the displacement equation

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2} + \left(\boldsymbol{J} \wedge \boldsymbol{B} \right)_i \tag{3}$$

where $\boldsymbol{B}$ is the magnetic induction vector given by $\boldsymbol{B} = \mu_o \boldsymbol{H}$

4-Modified Ohm's law is defined as (Ezzat and Youssef 2010)

$$J_i = \sigma_o \left(E_i + \dot{u}_k \wedge B_j - s_o T_{,i} \right). \tag{4}$$

5- The constitutive equations

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - (3\lambda + 2\mu) \alpha_T \theta. \tag{5}$$

6-Heat equation with memory-dependent derivative heat transfer (Ezzat *et al.* 2016)

$$\begin{aligned} \operatorname{div}\left(k \operatorname{grad}\theta - \Pi J_j\right) &= \rho C_E \frac{\partial \theta}{\partial t} + T_o (3\lambda + 2\mu)\alpha_T \frac{\partial u_{i,i}}{\partial t} \\ &+ \int_{t-\omega}^{t} K(t-\xi)\left(\rho C_E \frac{\partial^2 \vartheta(x,\xi)}{\partial \xi^2} + T_o (3\lambda + 2\mu)\alpha_T \frac{\partial^2 u_{i,i}}{\partial \xi^2}\right) d\xi, \end{aligned} \tag{6}$$

where ω is the time delay and $K(t-\omega)$ is the kernel function in which can be picked unreservedly as

$$K(t-\xi) = 1 - \frac{2n}{\omega}(t-\xi) + \frac{m^2 (t-\xi)^2}{\omega^2} = \begin{cases} 1 & if \;\; m = n = 0 \\ 1 - \dfrac{(t-\xi)}{\omega} & if \;\; m = 0, n = 1/2 \\ 1 - (t-\xi) & if \;\; m = 0 \;\; n = \omega/2 \\ (1 - \dfrac{t-\xi}{\omega})^2 & if \;\; m = n = 1, \end{cases}$$

7-Kinematic relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{7}$$

where $\theta = |T - T_o|$ and $\dfrac{\theta}{T_o} << 1$.

In the above equations a dot denotes differentiation with respect to time while a comma denotes material derivatives. The summation convention is used.

3. One dimensional physical problem

Now, we shall consider thermoelectric solid of finite conductivity σ_o occupying the region $x \geq 0$ composed of a material whose Lamé's parameters depend on the vertical distance x from the surface while the thermal conductivity is taken as a function of temperature. The surface of the half-space is taken to be traction free and is subjected to both a thermal shock that is a function of time and a constant magnetic field with components (0, H_o, 0) in the absence of an external electric field. Now for the one-dimensional problems, all the viewed facets will depend only on the space variables x and time t.

The displacement vector has aspects

$$u_x = u(x,t),\ u_y = 0,\ u_z = 0 \tag{8}$$

The strain-displacement relation

$$e = \frac{\partial u}{\partial x} \tag{9}$$

The components of the electromagnetic induction vector are $B_x = B_z = 0,\quad B_y = \mu_o H_o = B_o$, while the components of the Lorentz force appearing in Eq. (3) are given by

$$F_x = -\sigma_o B_o^2 \frac{\partial u}{\partial t},\ F_y = F_z = 0., \tag{10}$$

The components of current density vector are

$$J_x = -\sigma_o s_o \frac{\partial \theta}{\partial x}. \tag{11}$$

The constitutive relation

$$\sigma = \sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - (3\lambda + 2\mu)\alpha_T\,\theta \tag{12}$$

The equation of motion

$$\begin{aligned}\rho\frac{\partial^2 u}{\partial t^2} + \sigma_o B_o^2\frac{\partial u}{\partial t} &= (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - (3\lambda + 2\mu)\alpha_T\frac{\partial \theta}{\partial x}\\ &+\left(\frac{\partial \lambda}{\partial x} + 2\frac{\partial \mu}{\partial x}\right)\frac{\partial u}{\partial x} - \alpha_T\left(3\frac{\partial \lambda}{\partial x} + 2\frac{\partial \mu}{\partial x}\right)\theta.\end{aligned} \tag{13}$$

The generalized energy equation with memory-dependent derivative (Ezzat *et al*. 2016)

$$\begin{aligned}\frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right) + \sigma_o s_o \pi_o\frac{\partial^2 \theta}{\partial x^2} &= \rho C_E\frac{\partial \theta}{\partial t} + T_o(3\lambda + 2\mu)\alpha_T\frac{\partial^2 u}{\partial t\,\partial x}\\ &+\int_{t-\omega}^{t} K(t-\xi)\left(\rho C_E\frac{\partial^2 \theta}{\partial \xi^2} + T_o(3\lambda + 2\mu)\alpha_T\frac{\partial^3 u}{\partial x\,\partial \xi^2}\right)d\xi.\end{aligned} \tag{14}$$

We expect that the thermal conductivity k and the specific heat C_E are elements of in some scope of the temperature with the end goal that

$$k = k(\theta) = k_o(1 + K_1\theta),\quad \rho C_E = \kappa^{-1}k \tag{15}$$

where K_1, k_o and κ are a constants such that when $K_1 \to 0$ we get a material with constant thermal conductivity k_o. K_1 is small quantity and κ is called the thermal diffusivity.

We consider the mapping (similar to Kirchhoff's transformation) (Sherief and Abd El-Latief 2013)

$$\varphi = \frac{1}{k_o}\int_0^{\theta} k(\theta)\,\mathrm{d}\theta = \int_0^{\theta}(1 + K_1\theta)\,\mathrm{d}\theta = \theta + \frac{K_1}{2}\theta^2 \tag{16}$$

Using Eqs. (15) and (16) neglecting small quantities of the second order and higher, Eqs. (11)-(14) yield

$$e = \frac{\partial u}{\partial x} \tag{17}$$

$$\sigma = (\lambda + 2\mu)\frac{\partial u}{\partial x} - (3\lambda + 2\mu)\alpha_T\,\varphi, \tag{18}$$

$$\begin{aligned}\rho\frac{\partial^2 u(x,t)}{\partial t^2} + \sigma_o\mu_o^2 H_o^2\frac{\partial u(x,t)}{\partial t} &= (\lambda + 2\mu)\frac{\partial^2 u(x,t)}{\partial x^2} - (3\lambda + 2\mu)\alpha_T\frac{\partial \varphi(x,t)}{\partial x}\\ &+\left(\frac{\mathrm{d}\lambda(x)}{\mathrm{d}x} + 2\frac{\mathrm{d}\mu(x)}{\mathrm{d}x}\right)\frac{\partial u(x,t)}{\partial x} - \alpha_T\left(3\frac{\mathrm{d}\lambda(x)}{\mathrm{d}x} + 2\frac{\mathrm{d}\mu(x)}{\mathrm{d}x}\right)\varphi(x,t),\end{aligned} \tag{19}$$

$$\begin{aligned}k_o(1 + ZT_o)\frac{\partial^2 \varphi}{\partial x^2} &= \frac{\rho C_E}{\eta}\left(\frac{\partial \varphi}{\partial t}\right) + \frac{T_o(3\lambda + 2\mu)\alpha_T}{k_o}\left(\frac{\partial^2 u}{\partial t\,\partial x}\right)\\ &+\int_{t-\omega}^{t} K(t-\xi)\left(\frac{\rho C_E}{\eta}\left[\frac{\partial^2 \varphi}{\partial \xi^2}\right] + \frac{T_o(3\lambda + 2\mu)\alpha_T}{k_o}\left[\frac{\partial^3 u}{\partial x\,\partial \xi^2}\right]\right)d\xi,\end{aligned} \tag{20}$$

From now on, we shall take λ and μ in the form

$$\lambda = \lambda_o e^{-ax},\qquad \mu = \mu_o e^{-ax} \tag{21}$$

where λ_o, μ_o and α are constants. Thus Eqs. (18)-(20) take the form

$$\sigma = (\lambda_o + 2\mu_o)e^{-\alpha x}\frac{\partial u}{\partial x} - (3\lambda_o + 2\mu_o)\alpha_T\,e^{-\alpha x}\varphi \tag{22}$$

$$\rho\frac{\partial^2 u}{\partial t^2} + \sigma_o B_o^2\frac{\partial u}{\partial t} = (\lambda_o + 2\mu_o)\frac{\partial}{\partial x}\left(e^{-\alpha x}\frac{\partial u}{\partial x}\right) - (3\lambda_o + 2\mu_o)\alpha_T\frac{\partial}{\partial x}\left(e^{-\alpha x}\varphi\right), \tag{23}$$

$$k_o\,(1+ZT_o)\frac{\partial^2\varphi}{\partial x^2}=\frac{\rho C_E}{\kappa}\left(\frac{\partial\varphi}{\partial t}\right)+\frac{T_o(3\lambda_o+2\mu_o)\alpha_T}{k_o}e^{-ax}\left(\frac{\partial^2 u}{\partial t\,\partial x}\right)$$
$$+\int_{t-\omega}^{t}K(t-\xi)\left(\frac{\rho C_E}{\kappa}\left[\frac{\partial^2\varphi}{\partial\xi^2}\right]+\frac{T_o(3\lambda_o+2\mu_o)\alpha_T}{k_o}e^{-ax}\left[\frac{\partial^3 u}{\partial x\,\partial\xi^2}\right]\right)d\xi. \tag{24}$$

After acquiring φ, the temperature increase θ can be gotten by illuminating Eq. (16) to give

$$\theta=\frac{-1+\sqrt{1+2k_1\varphi}}{k_1} \tag{25}$$

We will utilize the accompanying non-dimensional factors

$$x'=c\,\eta x,\ u'=c\eta u,\ t'=c^2\eta t,\ \sigma'=\frac{\sigma}{\lambda_o+2\mu_o},\ a'=\frac{a}{c\eta},\ \eta=\frac{\rho C_E}{k},$$
$$\varphi'=\frac{(3\lambda_o+2\mu_o)\alpha_T}{\lambda_o+2\mu_o}\varphi,\ J'=\frac{J}{H_o c\,\eta}.$$

Utilizing the above non-dimensional factors, Eqs. (22)-(24) take the form

$$\sigma=\mathrm{e}^{-ax}\left(\frac{\partial u}{\partial x}-\varphi\right) \tag{26}$$

$$\frac{\partial^2 u}{\partial t^2}+M\,\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[e^{-ax}\left(\frac{\partial u}{\partial x}-\varphi\right)\right] \tag{27}$$

$$\left(1+ZT_o\right)\frac{\partial^2\varphi}{\partial x^2}=\left(1+\omega D_\omega\right)\left(\frac{\partial\varphi}{\partial t}+\varepsilon e^{-ax}\,\frac{\partial^2 u}{\partial x\,\partial t}\right) \tag{28}$$

where

$$D_\omega f(t)=\frac{1}{\omega}\int_{t-\omega}^{t}K(t-\xi)\,f'(\xi)\,\mathrm{d}\xi\,,$$

where $M=\sigma_o B_o^2/\rho c^2\eta$ is magnetic number and $\varepsilon=T_o(3\lambda_o+2\mu_o)^2\,\alpha_T^2\,/(\lambda_o+2\mu_o)k_o\eta$.

We assume that the initial conditions take the form

$$u(x,0)=\dot{u}(x,0)=\sigma(x,0)=\dot{\sigma}(x,0)=\varphi(x,0)=\dot{\varphi}(x,0)=0 \tag{29}$$

while the boundary conditions consist of:

(1) A thermal shock is applied to the boundary plane $x=0$ in the form

$$\theta(0,t)=f\,(t),\quad \theta(\infty,t)=0,\quad t>0 \tag{30}$$

where $f\,(t)$ is a known function of t.

(2) Mechanical boundary condition

The bounding plane $x=0$ is taken to be traction-free

$$\sigma(0,t)=0,\quad \sigma(t,\infty)=0,\quad t>0. \tag{31}$$

From now on, the kernel function form $K(t-\xi)$ can be chosen freely as

$$K(t-\xi)=1-\frac{2n}{\omega}(t-\xi)+\frac{m^2(t-\xi)^2}{\omega^2}=\begin{cases}1 \quad if\ \ m=n=0\\ 1-\dfrac{(t-\xi)}{\omega}\quad if\quad ,m=0,n=\dfrac{1}{2}\\ 1-(t-\xi)\quad if\ \ m=0\ \ n=\omega/2\\ (1-\dfrac{t-\xi}{\omega})^2\quad if\ \ m=n=1,\end{cases} \tag{32}$$

where m and n are constants.

4. The analytical solutions in the Laplace-transform domain

Performing the Laplace transform with parameter s defined by the relation

$$\mathrm{L}\{\psi(x,t)\}=\bar{\psi}(x,s)=\int_0^\infty \mathrm{e}^{-st}\psi(x,t)\,\mathrm{d}t\,,$$

of both sides Eqs. (26)-(28), we get a coupled system of the following equations

$$\mathscr{D}\left[e^{-ax}\left(\mathscr{D}\bar{u}-\varphi\right)\right]=s\,(s+M)\bar{u} \tag{33}$$

$$\mathscr{D}^2\bar{\varphi}=s\varpi\left(\bar{\varphi}+\varepsilon e^{-ax}\,\mathscr{D}\bar{u}\right) \tag{34}$$

$$\bar{\sigma}=\mathrm{e}^{-ax}\left(\mathscr{D}\bar{u}-\bar{\varphi}\right) \tag{35}$$

where $\mathscr{D}=\dfrac{\partial}{\partial x}$, $\varpi(s)=\dfrac{1+\Omega}{1+ZT_o}$,

$$G(s)=(1-e^{-s\omega})(1-\frac{2n}{\omega s}+\frac{2m^2}{\omega^2 s^2})-(m^2-2n+\frac{2m^2}{\omega s})e^{-s\omega}$$

$$L\{\omega D_\omega f\,(t)\}=F(s)\begin{cases}[(1-e^{-s\omega})],\quad m=n=0\\ [1-\dfrac{1}{\omega s}(1-e^{-s\omega})],\quad m=0,n=\dfrac{1}{2}\\ [(1-e^{-s\omega})-\dfrac{1}{s}(1-e^{-s\omega})+\omega e^{-s\omega}],\ m=0,n=\dfrac{\omega}{2}\\ [(1-\dfrac{2}{\omega s})+\dfrac{2}{\omega^2 s^2}(1-e^{-s\omega})],\quad m=n=1,\end{cases} \tag{36}$$

and

$$F(s)=L\{(\frac{\partial\varphi}{\partial t}+\varepsilon e^{-ax}\,\frac{\partial^2 u}{\partial x\,\partial t})\}=s(\bar{\varphi}+\varepsilon e^{-ax}D\bar{u})\,.$$

The boundary conditions (30) and (31) become

$$\bar{\theta}(0,s)=\bar{f}\,(s) \tag{37}$$

$$\bar{\sigma}(0,s)=0 \tag{38}$$

We shall use the perturbation method to solve the above equations (Nayfeh 1973). By expanding the temperature, displacement and stress functions as follows:

$$\bar{\varphi}=\bar{\varphi}^{(0)}+a\,\bar{\varphi}^{(1)}+a^2\,\bar{\varphi}^{(2)}+\cdots\cdots,$$

$$\bar{u} = \bar{u}^{(0)} + a\bar{u}^{(1)} + a^2\bar{u}^{(2)} + \cdots\cdots,$$
$$\bar{\sigma} = \bar{\sigma}^{(0)} + a\bar{\sigma}^{(1)} + a^2\bar{\sigma}^{(2)} + \cdots\cdots,$$

where $\bar{\varphi}^{(i)}$ and $\bar{u}^{(i)}$, $i = 1, 2$ are functions to be determined.

Eqs. (33) and (34) gives, upon equating the coefficients of α in both sides up to order 1

$$\mathscr{D}^2\bar{u}^{(0)} - \mathscr{D}\bar{\varphi}^{(0)} = s(s+M)\bar{u}^{(0)} \tag{39}$$

$$\mathscr{D}^2\bar{u}^{(1)} - \mathscr{D}\bar{\varphi}^{(1)} - s(s+M)\bar{u}^{(1)} = x\,(s+M)\bar{u}^{(0)} + \mathscr{D}\bar{u}^{(0)} - \bar{\varphi}^{(0)} \tag{40}$$

$$\mathscr{D}^2\bar{\varphi}^{(0)} = s\varpi\left(\bar{\varphi}^{(0)} + \varepsilon\,\mathscr{D}\bar{u}^{(0)}\right) \tag{41}$$

$$\mathscr{D}^2\bar{\varphi}^{(1)} - s\varpi\left(\bar{\varphi}^{(1)} + \varepsilon\,\mathscr{D}\bar{u}^{(1)}\right) = -x\,\varepsilon s\varpi\mathscr{D}\bar{u}^{(0)} \tag{42}$$

Utilizing development of condition (35), we acquire

$$\bar{\sigma}^{(0)} = \mathscr{D}\bar{u}^{(0)} - \bar{\varphi}^{(0)} \tag{43}$$

$$\bar{\sigma}^{(1)} = \mathscr{D}\bar{u}^{(1)} - \bar{\varphi}^{(1)} - x\,\bar{\sigma}^{(0)} \tag{44}$$

Dispensing $\bar{\varphi}^{(0)}$ with between conditions (39) and (41), we get

$$\left\{\mathscr{D}^4 - \left[s\,(s+M) + s\,\varpi\,(\varepsilon+1)\right]\mathscr{D}^2 + \varpi s^2(s+M)\right\}\bar{u}^{(0)} = 0 \tag{45}$$

The general solution of Eq. (45) which is bounded for x ≥ 0 has the form

$$\bar{u}^{(0)}(x,s) = -\sum_{i=1}^{2} A_i\,k_i e^{-k_i x}, \tag{46}$$

where $k_i, i = 1, 2$ are the roots of the characteristic equation with positive real parts of
$k^4 - \left[s\,(s+M) + s\,\varpi\,(\varepsilon+1)\right]k^2 + \varpi s^2(s+M) = 0$,
satisfying the relations

$$\begin{aligned} k_1^2 + k_2^2 &= s\,(s+M) + s\,\varpi\,(\varepsilon+1), \\ k_1^2 k_2^2 &= \varpi\, s^2(s+M), \end{aligned} \tag{47}$$

and $A_i, i = 1, 2$ are parameters depending on *s* to be determined from the boundary conditions of the problem.
Substitution from Eq. (46) into Eq. (41), we get

$$\bar{\varphi}^{(0)}(x,s) = \sum_{i=1}^{2} A_i\left[k_i^2 - s(s+M)\right]e^{-k_i x}, \tag{48}$$

The boundary conditions (37) and (39) become

$$\bar{\varphi}^{(0)}(0,s) = \bar{g}(s) \tag{49a}$$

$$\bar{\varphi}^{(1)}(0,s) = 0 \tag{49b}$$

$$\bar{\sigma}^{(0)}(0,s) = \frac{\partial \bar{u}^{(0)}}{\partial x} - \bar{\varphi}^{(0)} = 0, \quad x = 0 \tag{49c}$$

$$\bar{\sigma}^{(1)}(0,s) = \frac{\partial \bar{u}^{(1)}}{\partial x} - \bar{\varphi}^{(1)} = 0, \quad x = 0 \tag{49d}$$

where
$\bar{g}(s) = \bar{f}(s) + (k_1/2)\bar{f}^{\,2}(s)$.

In order to determine $A_i, i = 1, 2$ we shall use the boundary conditions (49a) and (40c) to obtain
$A_1 = -A_2 = \dfrac{\bar{g}(s)}{(k_1^2 - k_2^2)}$.

Conditions (46) and (48) become

$$\bar{u}^{(0)}(x,s) = -\frac{\left(k_1 e^{-k_1 x} - k_2 e^{-k_2 x}\right)}{k_1^2 - k_1^2}\bar{g}(s) \tag{50}$$

$$\bar{\varphi}^{(0)}(x,s) = \frac{\left(\left[k_1^2 - s(s+M)\right]e^{-k_1 x} - \left[k_2^2 - s(s+M)\right]e^{-k_2 x}\right)}{k_1^2 - k_2^2}\bar{g}(s) \tag{51}$$

Dispensing $\bar{u}^{(1)}$ with between conditions (40) and (41), we get non homogenous differential equation

$$\begin{aligned} &\left\{\mathscr{D}^4 - \left[s\,(s+M) + s\,\varpi\,(\varepsilon+1)\right]\mathscr{D}^2 + \varpi s^2(s+M)\right\}\bar{\varphi}^{(1)} \\ &\qquad = \varpi s\varepsilon\left(-2\mathscr{D}\bar{\varphi}^{(0)} + x\,[2s(s+M) - \mathscr{D}^2]\mathscr{D}\bar{u}^{(0)}\right), \end{aligned} \tag{52}$$

which has a general solution in the form

$$\bar{\varphi}^{(1)} = \left(B_1 + \ell_1 x^2 + \tau_1 x\right)e^{-k_1 x} + \left(B_2 + \ell_2 x^2 + \tau_2 x\right)e^{-k_2 x} \tag{53}$$

where

$$\ell_1 = -\frac{a_1}{4k_1(k_1^2 - k_2^2)}, \quad \ell_2 = \frac{a_2}{4k_2(k_1^2 - k_2^2)},$$

$$\tau_1 = -\frac{1}{2k_1(k_1^2 - k_2^2)}\left[b_1 + \frac{5k_1^2 - k_2^2}{2k_1(k_1^2 - k_2^2)}a_1\right], \tau_2 = \frac{1}{2k_1(k_1^2 - k_2^2)}\left[b_2 - \frac{5k_1^2 - k_2^2}{2k_1(k_1^2 - k_2^2)}a_2\right],$$

$$a_1 = s\varpi\varepsilon k_1^2[2s(s+M) - k_1^2]A_1, \quad a_2 = -\,s\varpi\varepsilon k_2^2[2s(s+M) - k_2^2]A_1,$$
$$b_1 = 2s\varpi\varepsilon k_1[k_1^2 - s(s+M)]A_1, \quad b_2 = -\,2s\varpi\varepsilon k_2[k_2^2 - s(s+M)]A_1$$

In the same manner the displacement distribution $\bar{u}^{(1)}$ satisfies the differential equation

$$\left(\mathscr{D}^2 - k_1^2\right)\left(\mathscr{D}^2 - k_2^2\right)\bar{u}^{(1)} = (m_1 x + m_2)e^{-k_1 x} + (m_3 x + m_4)e^{-k_2 x} \tag{54}$$

and has the general solution in the form

$$\bar{u}^{(1)} = \left(B_3 + M_1 x^2 + M_2 x\right) e^{-k_1 x} + \left(B_4 + M_3 x^2 + M_4 x\right) e^{-k_2 x} \quad (55)$$

where

$$m_1 = \left[(s+M)(s\varpi - 1) + \varepsilon\varpi k_1^2)\right] s k_1 A_1, \quad m_2 = \left[(s+M)(3k_1^2 - 1) - \varepsilon\varpi\right] s A_1,$$
$$m_3 = \left[(s+M)(s\varpi - 1) + \varepsilon\varpi k_2^2)\right] s k_2 A_1, \quad m_4 = \left[(s+M)(3k_2^2 - 1) - \varepsilon\varpi\right] s A_1,$$
$$M_1 = \frac{-m_1}{4k_1(k_1^2 - k_2^2)}, \quad M_2 = \frac{-1}{2k_1(k_1^2 - k_2^2)}\left[m_2 + \frac{5k_1^2 - k_2^2}{2k_1(k_1^2 - k_2^2)} m_1\right],$$
$$M_3 = \frac{m_3}{4k_2(k_1^2 - k_2^2)}, \quad M_4 = \frac{1}{2k_1(k_1^2 - k_2^2)}\left[m_4 - \frac{5k_1^2 - k_2^2}{2k_1(k_1^2 - k_2^2)} m_3\right].$$

Substituting structure Eqs. (44), (53) and (55) into Eq. (42), looking at the coefficient of the exponentials in the subsequent condition, we get

$$B_3 = \frac{1}{\varepsilon k_1 s\varpi}\left[\varepsilon s\varpi M_2 + s(\varpi - k_1^2) B_1 - \ell_2\right] \quad (56a)$$

$$B_4 = \frac{1}{\varepsilon k_2 s\varpi}\left[\varepsilon s\varpi M_4 + s(\varpi - k_2^2) B_2 - \ell_4\right] \quad (56b)$$

By using the boundary conditions (49b) and (49d) into Eqs. (53) and (55), we have

$$B_1 = -B_2 \quad (56c)$$

$$k_1 B_3 + k_2 B_4 = M_2 + M_4 \quad (56d)$$

Solving the above system, we have

$$B_1 = -\frac{\tau_1 + \tau_2}{k_1^2 - k_2^2}.$$

The other constants can be easily obtained from Eqs. (56(a)-56(c)).

This completes the solution in the Laplace transform domain.

5. Numerical inversion of the Laplace transforms

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transforms can be written as Honig and Hirdes (1984)

$$f(t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\, \bar{f}(d + iy)\, \mathrm{d}y,$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$.

Expanding the function $h(t) = \exp(-dt) f(t)$ in a Fourier series in the interval [0, $2L$], we obtain the approximate formula

$$f(t) \approx f_N(t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k, \quad \text{for } 0 \le t \le 2L \quad (57)$$

where

$$c_k = \frac{e^{dt}}{L}\mathrm{Re}\left[e^{ik\pi t/L}\, \bar{f}\left(d + ik\pi/L\right)\right] \quad (58)$$

Two methods are used to reduce the total error. First, the ‘Korrektur’ method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur-method uses the following formula to evaluate the function $f(t)$

$$f(t) = f_{NK}(t) = f_N(t) - e^{-2dL} f_{N'}(2L + t) \quad (59)$$

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (57). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$, be the sequence of partial sums of (57).We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \varepsilon_{1,m} = s_m, \quad m = 1, 2, 3, \ldots$$

and

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + 1/\left(\varepsilon_{n,m+1} - \varepsilon_{n,m}\right), \quad n, m = 1, 2, 3, \ldots.$$

It can be shown from Honig and Hirdes (1984) and Durbin (1973) that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1}, \ldots$ converges to $f(t) - c_0/2$ faster than the sequence of partial sums.

6. Numerical results

The technique dependent on a Fourier arrangement extension proposed by Honig and Hirdes (1984) is received to alter the Laplace change in the previous section. The numerical code has been readied utilizing Fortran 77 programming language. The amount of calculation (and hence the execution time) depends on several parameters within the program. First, there is a parameter “nsig” which is the number of significant digits defining the relative error as $(10)^{-\mathrm{nsig}}$. We usually take nsig=5. Near points of discontinuity of the function, the program might fail to converge, and we have to decrease nsig. Another parameter is the maximum number of terms in the Fourier series to be added within one saw-tooth of the ε-algorithm. This is taken as 10000. The last parameter is the number of saw-teeth of the ε-algorithm to be considered. This is taken as 50. All in all; the program evaluates the value of any function at 50 points in less than 2 minutes (Sherief and Hussein 2018).

The function $f(t)$ can be picked as

$$f(t) = \begin{cases} \sin\left(\dfrac{\pi t}{\beta}\right) & 0 \le t \le \beta \\ 0 & \text{otherwise} \end{cases}.$$

Table 1 Values of the constants (Ezzat et al. 2016, Shereif and Abd El-Latief, 2016)

$\rho = 8954 kg / m^3$	$\kappa = 1.13\,(10)^{-4} m^2 / s$	$E = 525x10^7 N / m^2$
$C_E = 381 J / (kg.K)$	$\lambda^o = 7.76 \times (10)^{10} kg / (ms^2)$	$\alpha_T = 1.78(10)^{-5} K^{-1}$
$\mu^o = 3.86 \times (10)^{10} kg / (ms^2)$	$\eta = 8886.73 sec/ m^2$	$T_o = 293 K$
$\varepsilon = 0.0168$	$a = 0.3\, m^{-1}$	$H_o = 1.0 C / m.sec$

Hence

$$\bar{f}(s) = \frac{\pi\beta\left(1-e^{-\beta s}\right)}{\beta^2 s^2 + \pi^2}, \quad \bar{f}^2(s) = \frac{2\pi^2\beta\left(1-e^{-\beta s}\right)}{s\left(\beta^2 s^2 + 4\pi^2\right)} \quad \text{and}$$

$$\bar{g}(s) = \bar{f}(s) + (k_1/2)\bar{f}^2(s).$$

So as to translate the numerical calculations, we consider material properties of copper-like material, whose physical information is given in Table 1:

The numerical system laid out above was utilized to get the temperature θ, displacement u, stress σ and current density J in various cases. The outcomes are appeared in Figs. 1(a)-1(e) for temperature distribution and in Figs. 2(a)-2(d) for displacement distribution, while in Figs. 3(a)-3(d) for stress distribution are represented to graphically at various places of x. In the present area, we have endeavored to demonstrate the impact of the kernel function and additionally the time-delay among the idea of different physical fields.

In order to study the effect of different values of time-delay on the physical quantities of thermoelectric materials, Figs. 1(a), 2(a) and 3(a) are plotted.

Fig. 1(a) shows the space assortment of the temperature spread. In this figure, solid line addresses the course of action procured in the packaging of dynamic coupled theory (Biot theory, $\omega = 0$) and various lines address the game plans got for the circumstance $\omega = 0.0009, 0.009, 0.09$. We saw that the temperature fields have been affected when postponement ω, where the growing of the estimation of the parameter causes decreasing in temperature fields. The thermal waves are steady limits, smooth and reach to resolute state dependent upon the estimation of time-delay ω, which suggests that the particles transport the glow to various particles viably and this makes the lessening rate of the temperature more important than various ones. Furthermore, the warm waves cut x-center point even more immediately when increases.

Figs. 2(a) and 3(a) display the displacement and stress distributions with distance for two different theories; Biot theory, $\omega = 0$ and MDD theory, $\omega > 0$ when the magnetic number has two values M $(M = 0)$, absent of the magnetic field and in the present of the magnetic field, $M > 0$). We find that the attractive field acts to diminish the displacement and stress fields. This is generally known as attractive damping.

Figs. 1(b), 2(b) and 3(b) show the variety of temperature, displacement, stress and current density circulations in thermoelectric circular depression with spiral separation x for three values of figure-of-merit at room temperature ZT_o, namely, ZT_o = 1, 3 and 5. We noticed that the stress and displacement field has been affected by the figure-of-merit values, where the expanding of the estimation of figure-of-merit causes decreasing in the magnitude of the stress and displacement field while causes increasing in the temperature and current density. From these figures, we learn that the efficiency of thermoelectric figure-of-merit is inversely proportional to the temperature of the solid particles (Nolas *et al.* 2001). These results agree with the expectation by the relation $ZT = \left(\sigma_o S^2 / k\right)T$.

Figs. 1(c), 2(c) and 3(c) depict the space variation of the temperature, displacement and stress distributions. In these figures the effect of the Lamé's moduli on these distributions are studied. We note that changing of Lamé's modulii has very small effect on the temperature profile, an increase in the values of Lamé's modulii results in an increase in the magnitude values of displacement and stress fields.

Figs. 1(d), 2(d) and 3(d) individually, demonstrate the temperature, removal and stress conveyances for $k_1 = 0, -0.25$. These figures demonstrate the contrasts between the speculations of memory-subordinate subsidiaries practically reviewed thermoelectric materials with consistent thermal conductivity and those of variable conductivity. It was discovered that the difference in the thermal conductivity significantly affects the every single thought about capacity. Its impact on increase in the conductivity will in general increment the outright estimation all things considered.

The impact of various types of portion work $K(t-\xi)$ on the temperature conveyance, Fig. 2(b) has been plotted. This figure addresses the dimensionless estimation of temperature for wide extent of extended partition x ($0 \le x \le 1$) and for various types of piece work. We gained from this assumes indispensable miracle found in these expect that the game plan of any of the considered limit in the new model is kept in a restricted region. Past this region, the assortments of these assignments do whatever it takes not to happen. This infers to the game plans agreeing the new summed up theory demonstrate the direct of constrained rates of wave spread.

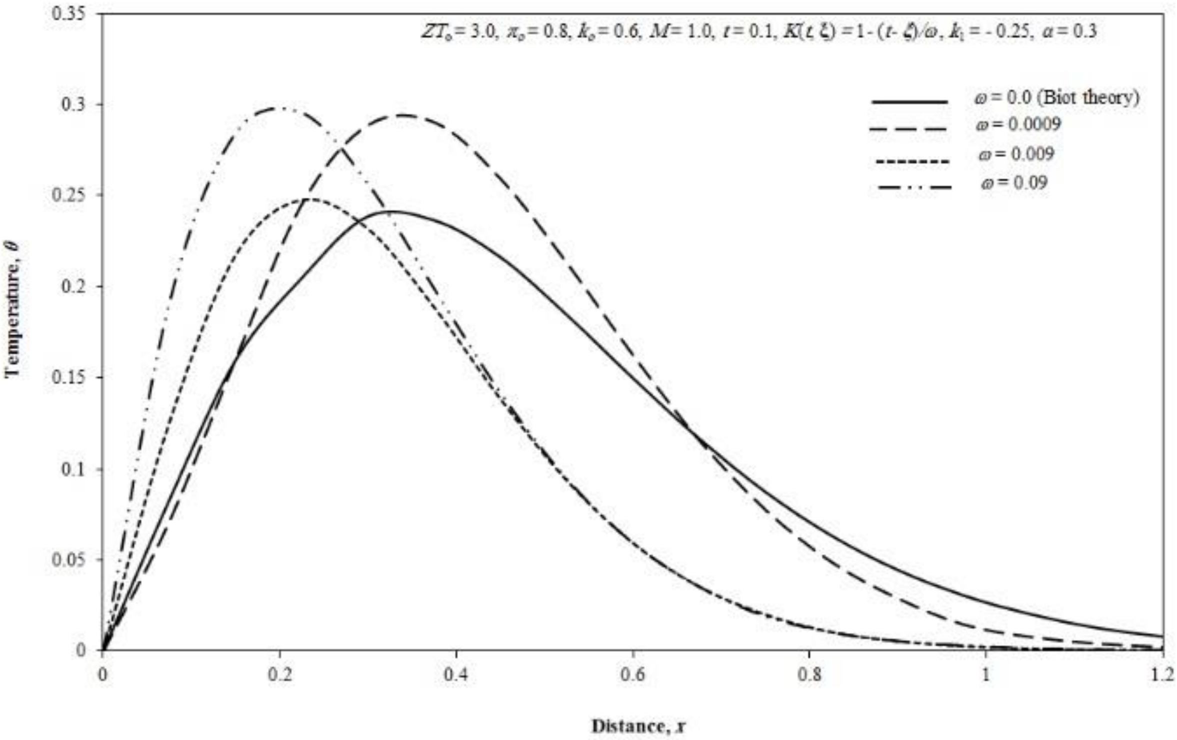

Fig. 1(a) The variation of temperature for different values of time-delay ω

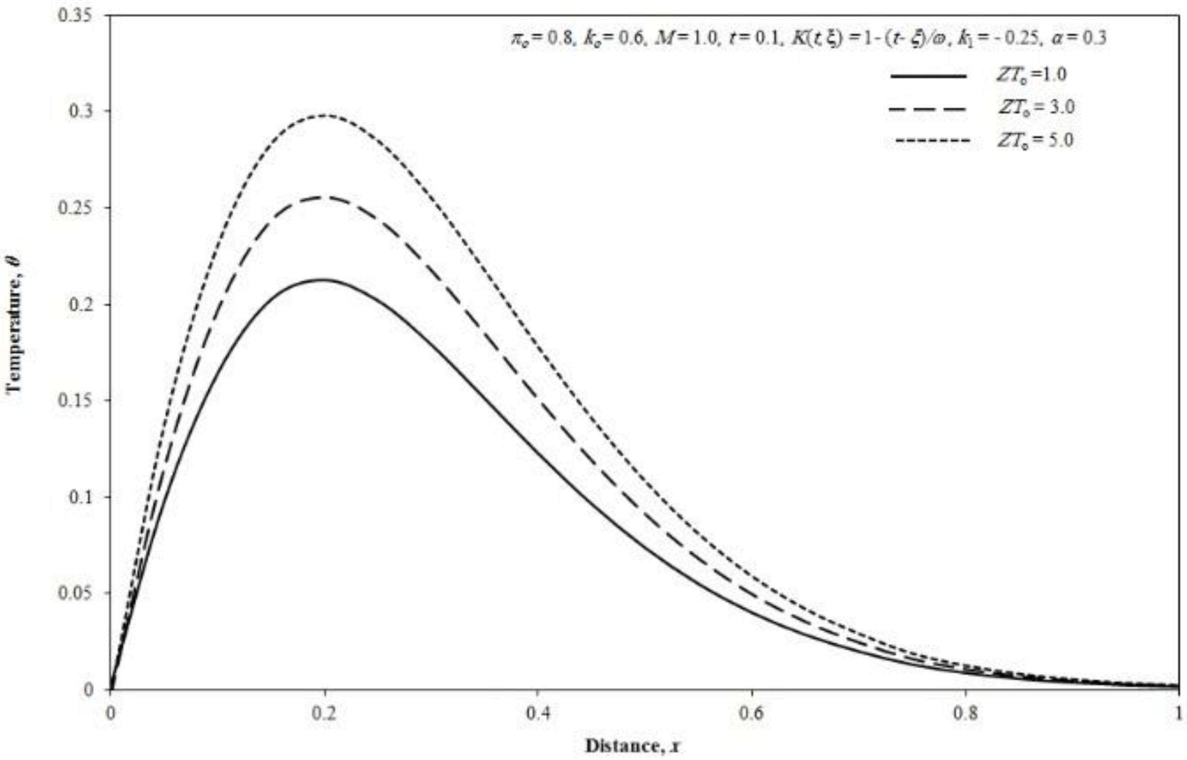

Fig. 1(b) The variation of temperature for different values of figure-of-merit ZT_o

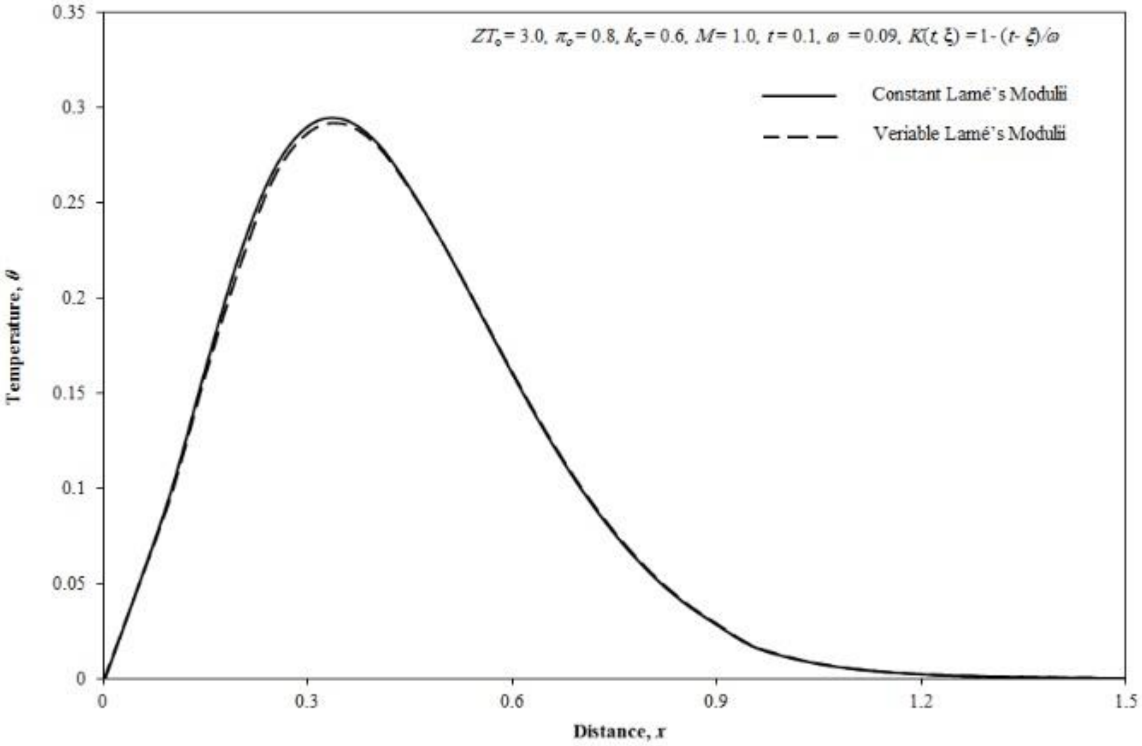

Fig. 1(c) The variation of temperature for different cases of Lamé's moduli

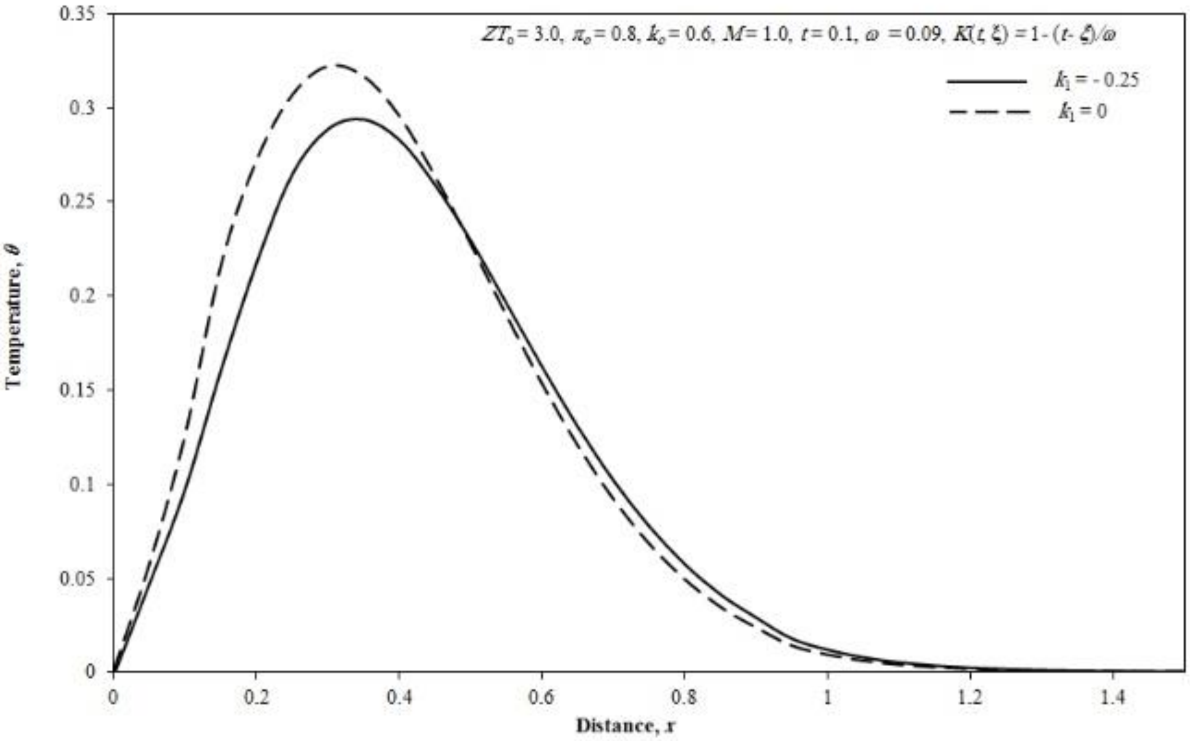

Fig. 1(d) The variation of temperature for different values of thermal conductivity

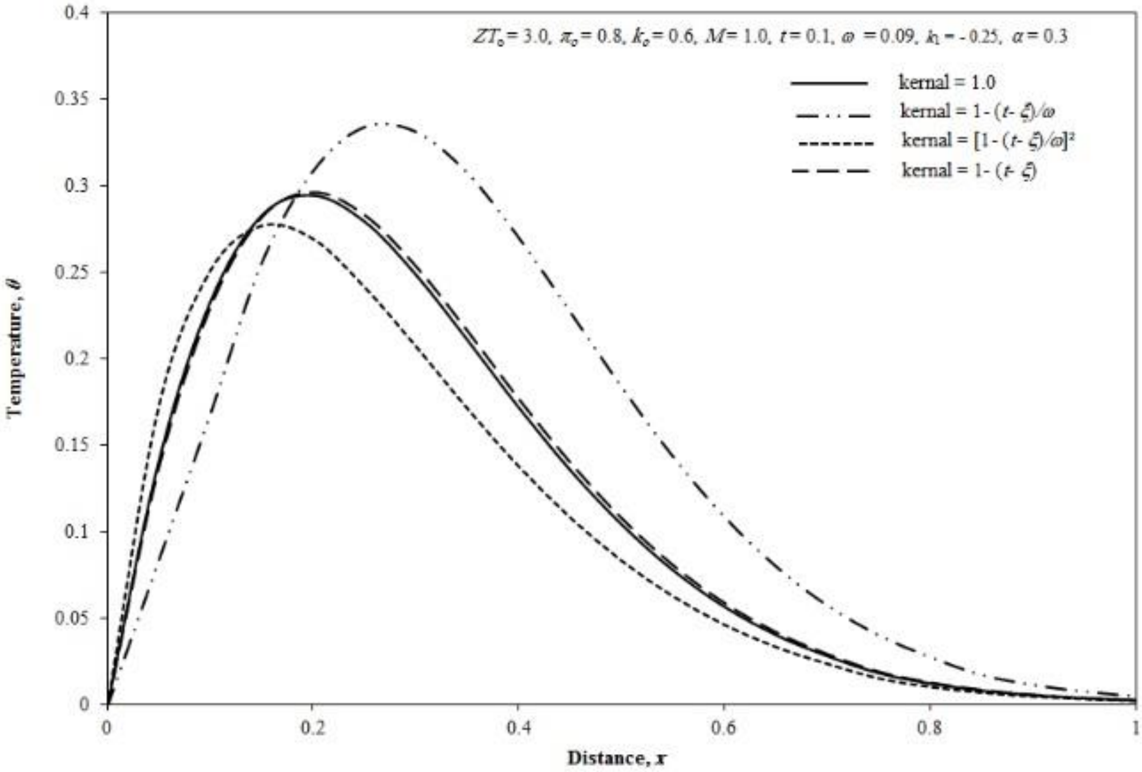

Fig. 1(e) The variation of temperature for different forms of kernal function $K(t, \xi)$

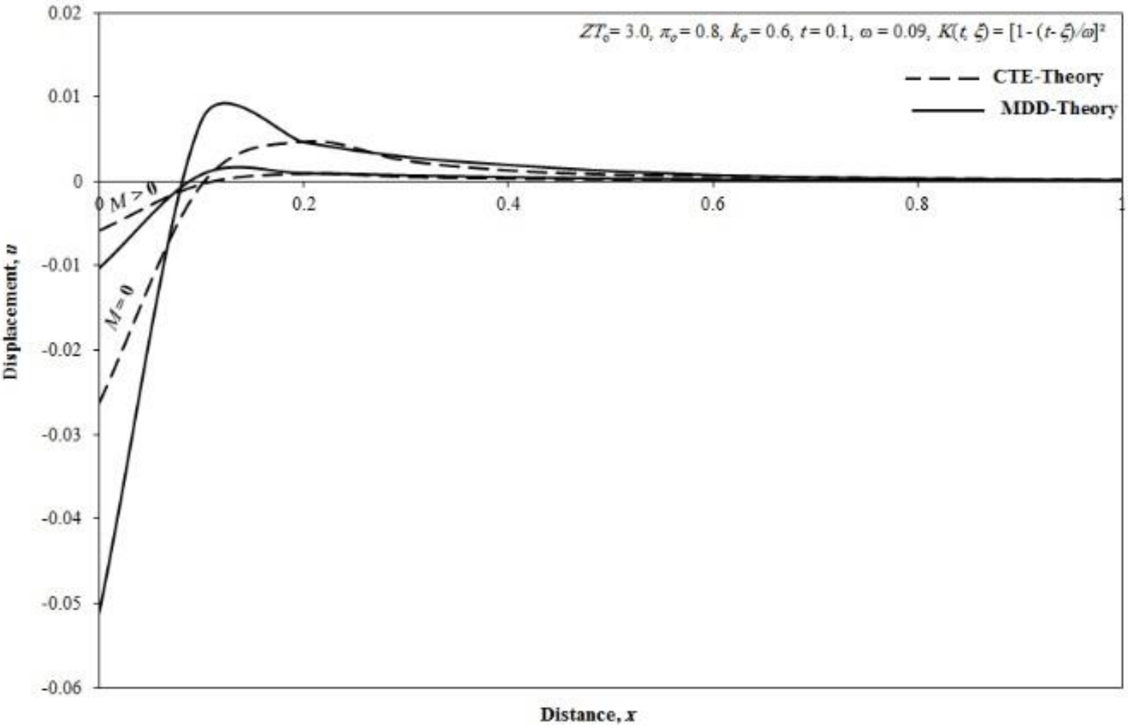

Fig. 2(a) The variation of temperature for different values of magnetic number M for different theories

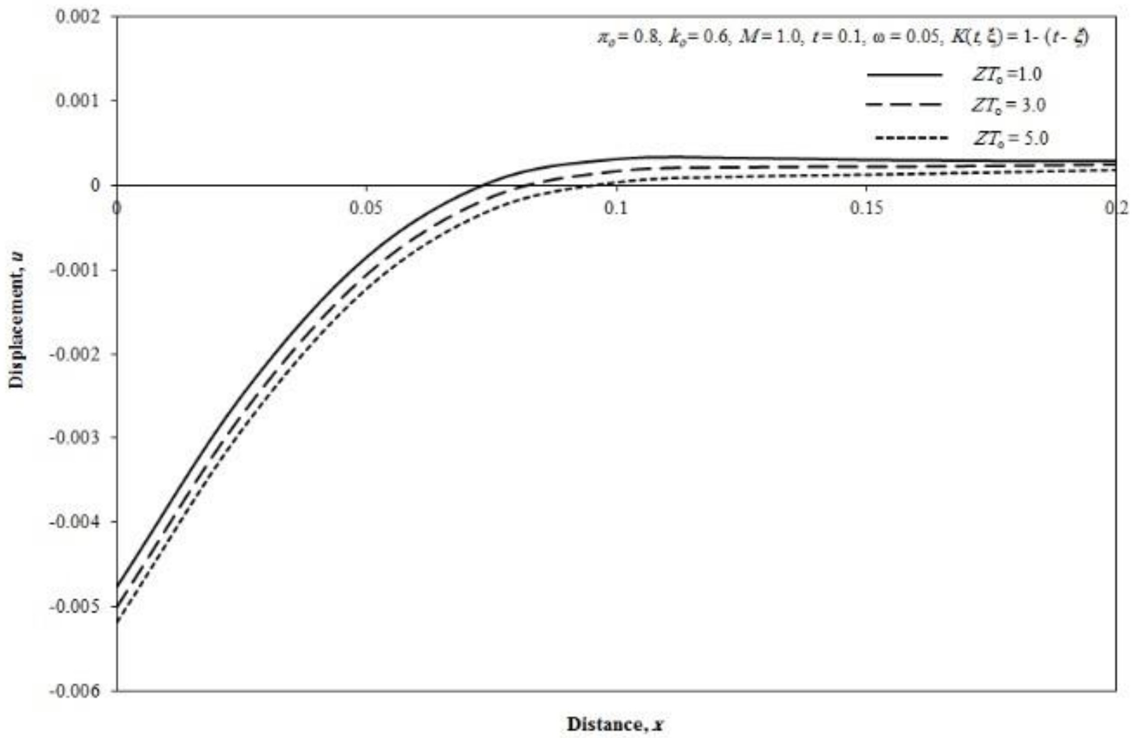

Fig. 2(b) The variation of displacement for different values of figur-of-miret at room temperature ZT_o

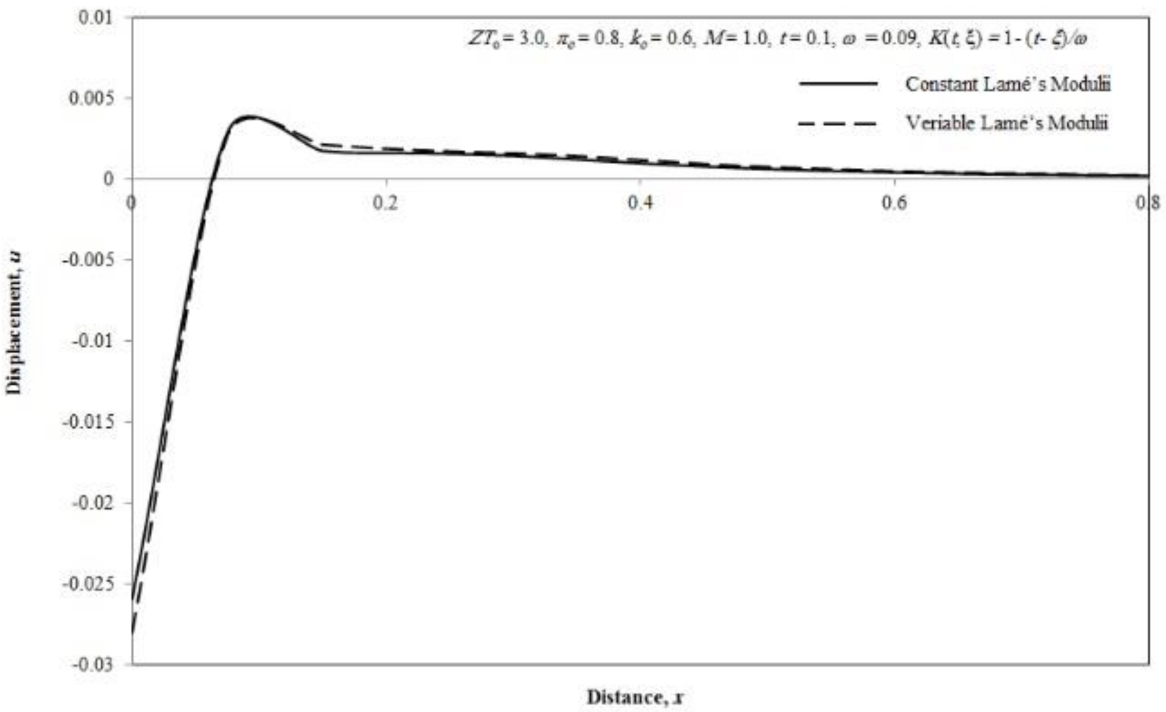

Fig. 2(c) The variation of displacement for different cases of Lamé's moduli

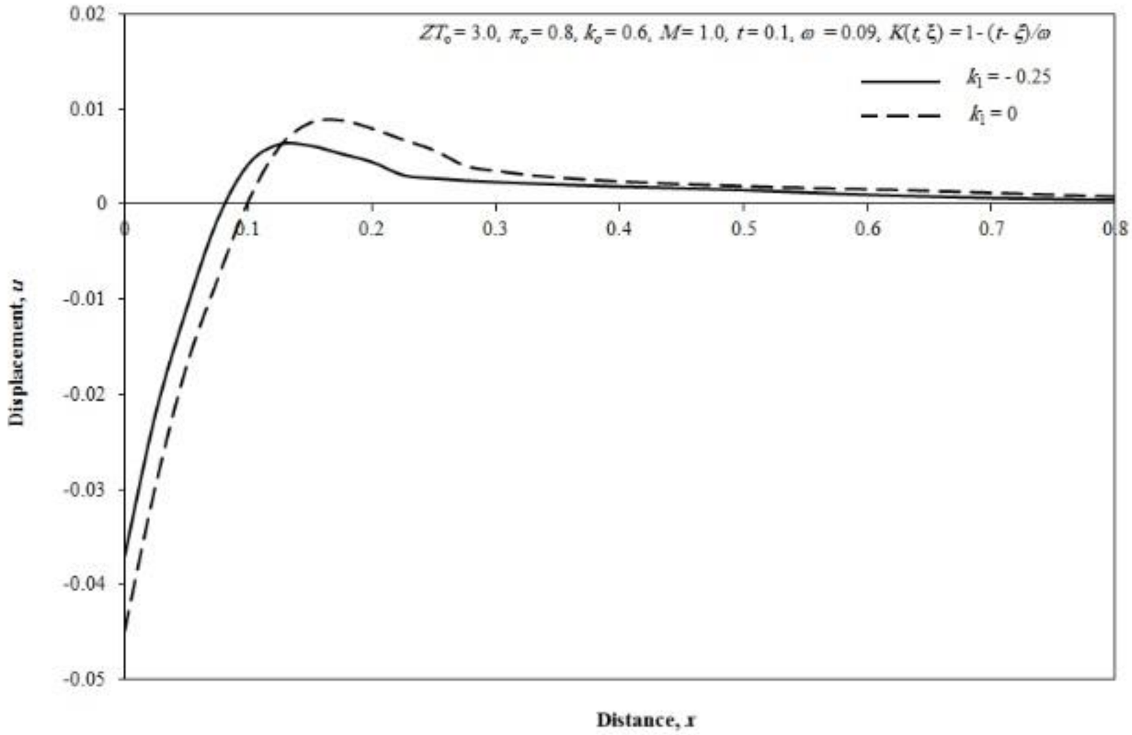

Fig. 2(d) The variation of displacement for different values thermal conductivity

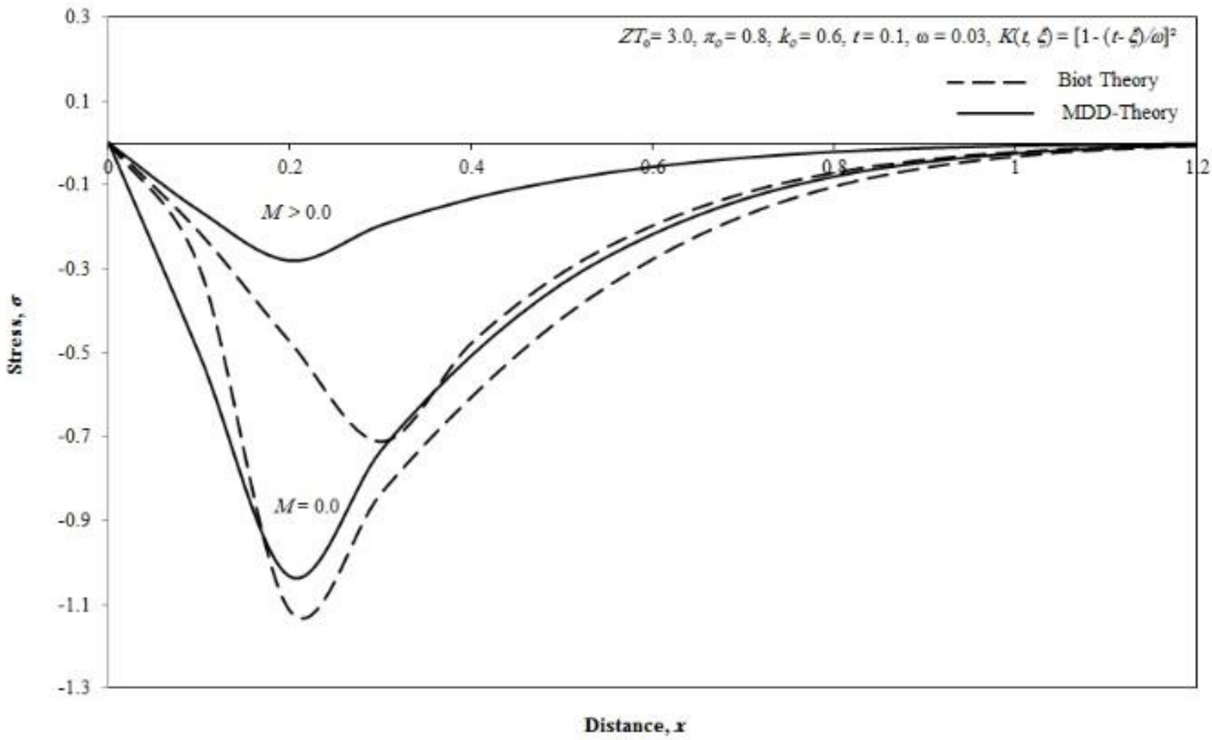

Fig. 3(a) The variation of temperature for different values of magnetic number *M* for different theories

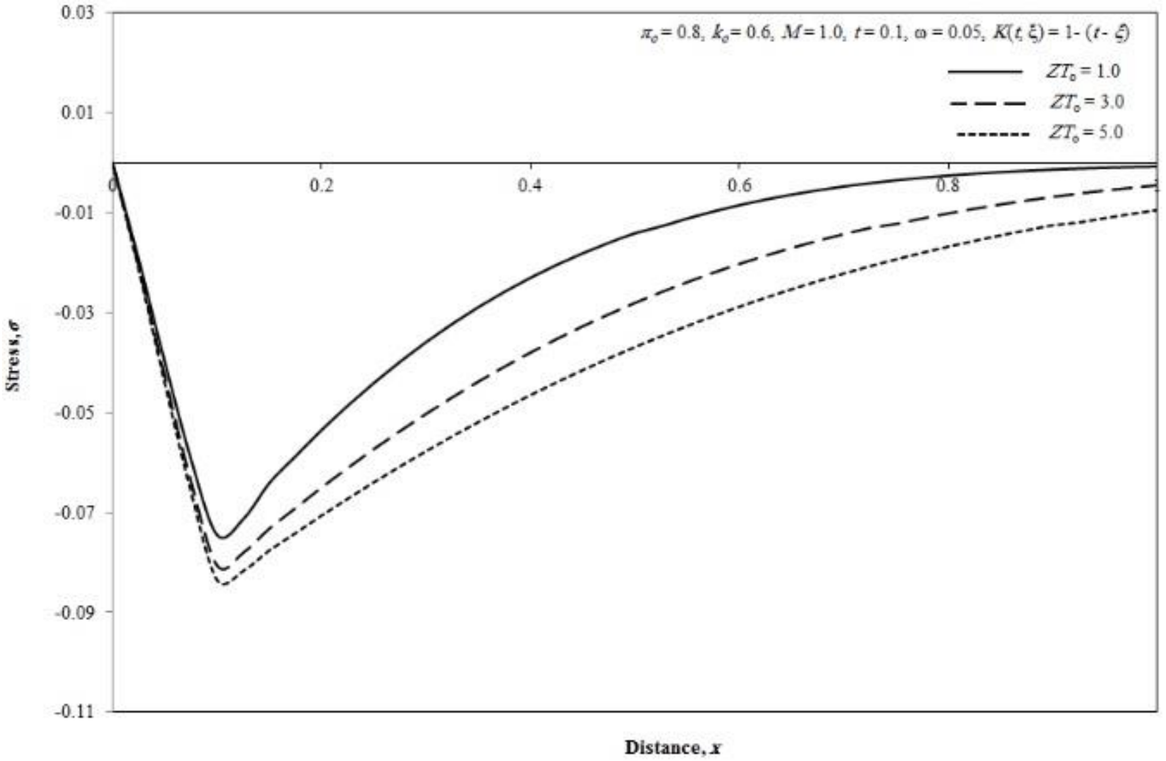

Fig. 3(b) The variation of stress vs. distance for different values of figur-of-miret at room temperature ZT_o

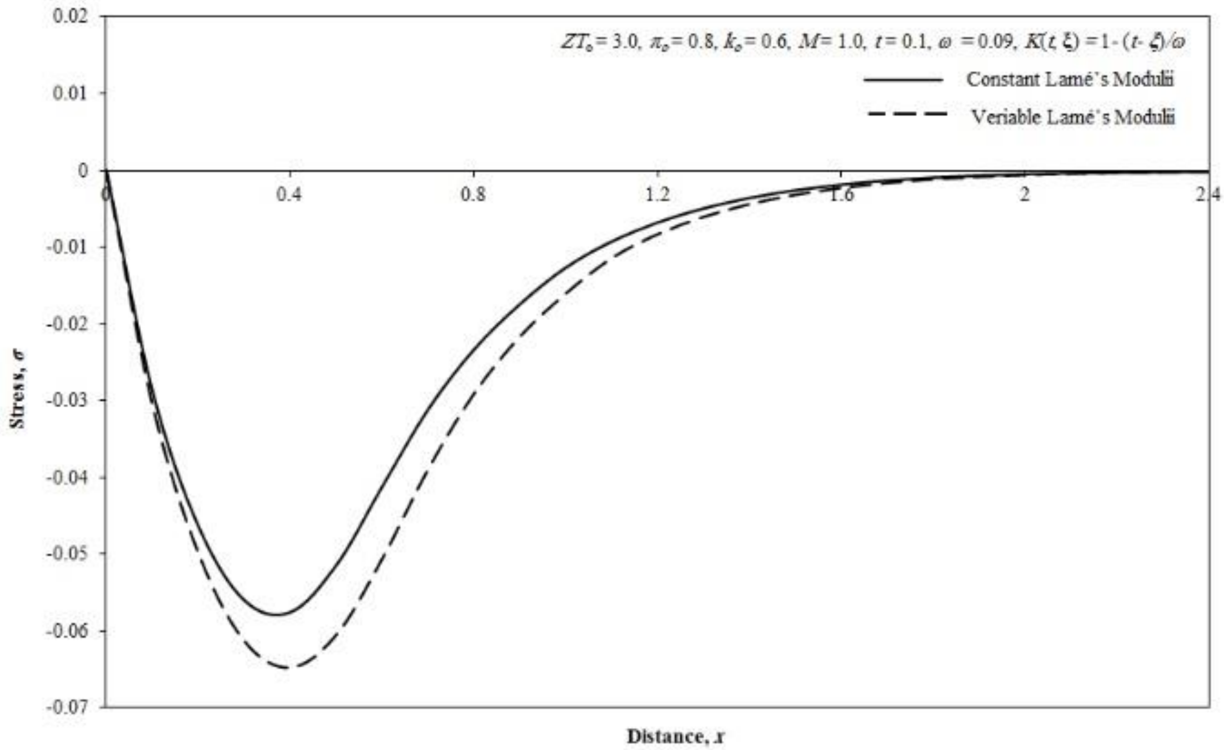

Fig. 3(c) The variation of stress for different cases of Lamé's moduli

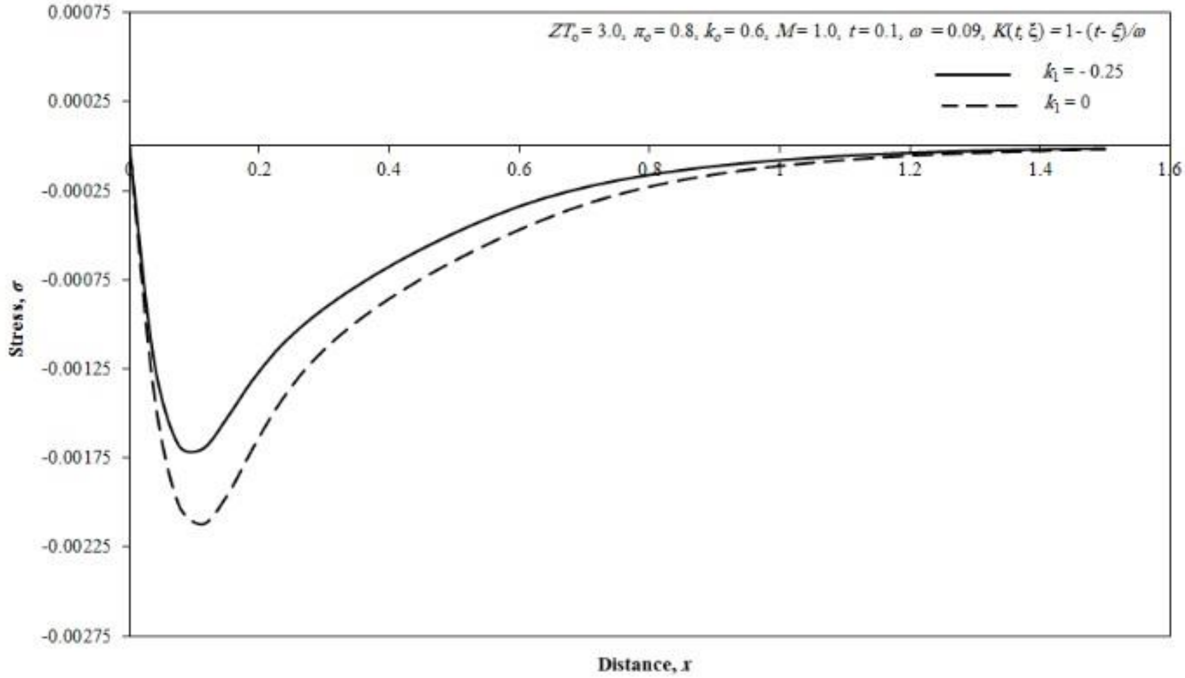

Fig. 3(d) The variation of displacement for different values of thermal conductivity

7. Conclusions

- The primary objective of this work is to present a scientific model in MDD electro-thermoelasticity when the Lamé's modulii is taken as elements of the vertical separation from the outside of thermoelectric materials with variable thermal conductivity within the sight of a uniform attractive field.
- This model enables us to improve the efficiency of a thermoelectric material figure-of-merit $ZT = \left(\sigma_o S^2 / k\right)T$., where S is the thermoelectric power or Seebeck coefficient (Hicks 1993, Hiroshige *et al*. 2007). It is known that in order to achieve a high thermoelectric material figure-of-merit, one requires low thermal conductivity. This can occur according to the choice of suitable values of time-delay ω.
- It is necessary to take into account that the coefficient of thermal conductivity has to be dependent on the absolute temperature because most real materials with increased conductivity tend to increase the absolute value of all the functions considered (Sherief *et al*. 2016). This situation is usually ignored by most researchers who consider a constant thermal conductivity.
- The changes of magnetic parameters $M = \sigma_o B_o^2 / \rho c^2 \eta$, where σ_o is the electric conductivity caused by different technological processes have been tested in more laboratories, which led to effects on stress and deformation for the thermoelastic materials (Tumanski 1999). The nearness of an attractive field impacts to decrease the extents of the profiles of the thermophysical amounts.
- The memory-dependent derivative is defined in an integral form of a common derivative with a kernel function on a slipping interval. So this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). Its definition is more intuitionistic for understanding the physical meaning and the corresponding memory dependent differential equation has more expressive force
- As per this new hypothesis, we need to build another arrangement for FGM_s materials as indicated by their time-postpone where this parameter turns into another marker of its capacity to lead heat in directing medium.

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