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Dual-phase-lag model on thermo-microstretch elastic solid Under the effect of initial stress and temperature-dependent

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Abstract. The present paper attempts to investigate the propagation of plane waves in an isotropic elastic medium under the effect of initial stress and temperature-dependent properties. The modulus of elasticity is taken as a linear function of the reference temperature. The formulation is applied under the thermoelasticity theory with dual-phase-lag; the normal mode analysis is used to obtain the expressions for the displacement components, the temperature, the stress, and the strain components. Numerical results for the field quantities are given in the physical domain and illustrated graphically. Comparisons are made with the results predicted by different theories (Lord–Shulman theory, the classical coupled theory of thermoelasticity and the dual-phase-lag model) in the absence and presence of the initial stress as well as the case where the modulus of elasticity is independent of temperature.

Keywords: thermoelasticity; initial stress; microstretch; dual-phase-lag; normal mode analysis; temperature-dependent

1. Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g., concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the polymer type, e.g., polymethyl-methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves i.e. for the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant, this influence of microstructure results in the development of new type of waves are not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, and concrete are typical media with micro-structures. More generally, most of the natural and manmade materials including engineering, geological and biological media possess a microstructure. In the classical thermoelasticity (CT) theory due to Biot (1956), the equation of the heat conduction is a parabolic type. It could predict the infinite speed of the heat propagation in elastic media, but it was inconsistent with experimental observation. With this motivation, Lord and Shulman (1967) conduction equation, with the heat flux and its time derivative is taken into account. The heat equation associated with this theory is essential of a hyperbolic type. In the Green and Lindsay theory (1972), established the (L-S) and (G-L) generalized thermoelasticity theories respectively. In the (L-S) theory, a relaxation time parameter introduced into the Fourier heat constitutive equations were modified by introducing two relaxation time parameters. Both the equations of motion and heat conduction are of the hyperbolic type. The two theories can better characterize thermal disturbances with a limited speed of the wave propagation and exhibit the so-called second sound effect in solids. The other hyperbolic thermo-elasticity theory was proposed by Tzou (1995a), called the dual-phase-lag model, in which Fourier's law is replaced by an approximation to a modification of Fourier's law with two different time translations for the heat flux and the temperature gradient. The temperature dependence is an important physical property of materials reflecting the elastic deformation capacity of the material when subjected to an applied external load. Most of the investigations were done under the assumption of the temperature-independent material properties, which limit the applicability of the obtained solutions to certain ranges of temperature. At high temperature, the material characteristics such as the modulus of elasticity, Poisson's ratio, the coefficient of thermal expansion and the thermal conductivity are no longer constants as Lomarkin (1976). In recent years due to the progress in various fields in science and technology the necessity of taking into consideration the real behavior of the material characteristics prosperities as the temperature dependent measurements. In the classical theory of elasticity, the effect of gravity neglected in a general manner. Bromwich (1898) in particular on an elastic globe, was the first to study the effect of gravity of the problem of

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propagation of waves in solids. If the temperature dependence of material properties is neglected, this is due to significant errors as discussed by Noda (1986). The analytical solution of thermoelastic interaction in a half-space by pulsed laser heating was studied by Othman and Said (2014); Abbas and Marin (2017). Extensive studies by Ezzat and El-Bary (2017), Abd-Elaziz *et al*. (2019), Lata and Singh (2019), Kumar *et al*. (2019), Othman and Marin (2017), Othman (2003), Othman (2011), Othman *et al*. (2013), Marin *et al*. (2017), Marin (1997), Marin (1999), Said and Othman (2016), Lata *et al*. (2020), Medani *et al*. (2019), Mirzaei *et al*. (2019), Arefi (2016a,b), Arefi and Zenkour (2016, 2019); Arefi *et al*. (2016, 2017a,b); Loghman *et al.* (2017), Itu *et al.* (2019) have discussed the temperature-dependence of material properties.

The purpose of the present paper is to obtain the displacement, the temperature, the microrotation, normal force stress, and tangential couple stress in a microstretch thermoelastic medium under the effect of initial stress and temperature dependent. The normal mode analysis is used to solve this problem. The distribution of the considered variables is represented graphically. A comparison is carried out between the temperature, stresses, micro-rotation and displacements as calculated on the generalized microstretch thermoelastic medium for (CT), (L-S) theories, and (DPL) model for the propagation of the waves in a semi-infinite generalized thermal microstretch elastic solid.

2. Basic equations

Following Eringen (1999) and Tzou (1995b) the equations of motion and the constitutive relations in a homogeneous isotropic microstretch Thermoelastic solid in the absence of body forces, and in the presence of body couples, stretch force, and heat sources, with a dual-phase-lag are given by

$$\sigma_{ji,j} = \rho u_{i,tt} \tag{1}$$

$$\varepsilon_{ijr}\sigma_{jr} + m_{ji,j} = \rho j \phi_{i,tt} \tag{2}$$

$$\alpha_0 \nabla^2 \phi^* + \frac{1}{3}\nu T - \frac{1}{3}\lambda_1 \phi^* - \frac{1}{3}\lambda_0 \nabla.\boldsymbol{u} = \frac{3}{2}\rho j \phi^*_{,tt}, \tag{3}$$

$$\begin{aligned} k^*(1+\tau_\theta \frac{\partial}{\partial t})\nabla^2 T &= (1+\tau_q \frac{\partial}{\partial t})(\beta_1 T_0 \nabla.\boldsymbol{u}_{,t} \\ &+ \nu T_0 \phi^*_{,t} + \rho C_E T_{,t}). \end{aligned} \tag{4}$$

And the constitutive relations are

$$\begin{aligned} \sigma_{ij} &= (\lambda u_{r,r} + \lambda_o \phi^* - \beta_1 T)\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \\ &+ k(u_{j,i} - \varepsilon_{ijr}\phi_r) - p(\delta_{ij} + \omega_{ij}), \end{aligned} \tag{5}$$

$$m_{ij} = \alpha \phi_{r,r}\delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i}, \tag{6}$$

$$\lambda_i^* = \alpha_0 \phi^*_{,i}. \tag{7}$$

Where $\lambda, \mu, \alpha, \beta, \gamma, \alpha_0, \lambda_0, \lambda_1, k$ are material constants ρ is the mass density $u = (u_1, u_2, u_3)$ is the displacement vector and $\phi = (\phi_1, \phi_2, \phi_3)$ is the microrotation vector, ϕ^* is the scalar microstretch function, T, T_0 are the small temperature increment and the reference temperature of the body chosen such that $|T - T_0/T_0| << 1$, k^* is the coefficient of the thermal conductivity, C_E the specific heat at constant strain, $\beta_1 = (3\lambda + 2\mu + k)\alpha_{t_1}$, $\nu = (3\lambda + 2\mu + k)\alpha_{t_2}$, j expansion thermal linear of coefficients the are $\alpha_{t_1}, \alpha_{t_2}$ is the coefficients of micro-inertia, σ_{ij}, m_{ij} are the components of stress and couple stress tensors respectively, δ_{ij} is the Kronecker delta, τ_θ is the phase-lag of the gradient of temperature, τ_q is the phase-lag of heat flux where $0 \le \tau_\theta \le \tau_q$.

3. Formulation of the problem

We consider a homogeneous isotropic microstretch generalized thermoelastic half-space. All the field quantities will be functions of the time variable t and of the coordinates x and z. For the two-dimensional problems, we take $\boldsymbol{u}(x,z,t) = (u(x,z,t), 0, w(x,z,t))$, $\boldsymbol{\phi} = (0, \phi_2(x,z,t), 0)$, $\phi^*(x,z,t)$, $T(x,z,t)$,

Then the equations of motion under the influence of temperature-dependent investigation Eqs. (1)-(4), and we define the following dimensionless, quantities

$$\begin{gathered} \bar{x}_i = \frac{\omega^*}{C_1}x_i, \quad \bar{u}_i = \frac{\rho C_1 \omega^*}{\beta_1 T_0}u_i, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{C} = \frac{\beta_2}{\rho^2 C_1^2}C, \\ (\bar{\phi}^*, \bar{\phi}_2) = \frac{\rho C_1^2}{\beta_1 T_0}(\phi^*, \phi_2), \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0}, \end{gathered} \tag{8}$$

$$(\bar{t}, \bar{\tau}_\theta, \bar{\tau}_q, \bar{\tau}^0) = \omega^*(t, \tau_\theta, \tau_q, \tau^0), \quad (\bar{\lambda}_i^*, \bar{m}_{ij}) = \frac{\omega^*}{C_1 \beta_1 T_0}(\lambda_i^*, m_{ij}).$$

Where, $\omega^* = \dfrac{\rho C^* C_1^2}{k^*}$, $C_1^2 = \dfrac{\lambda + 2\mu + k}{\rho}$ and ω^* is the characteristic frequency of the medium. From the dimensionless quantities in the basic governing equations we obtain, (dropping the dashed for convenience), and we assume that

$\lambda = \lambda_2 f(T)$, $\mu = \mu_0 f(T)$, $\gamma = \gamma_0 f(T)$, $\lambda_0 = \lambda_0^* f(T)$, $k = k_0 f(T)$, $\beta = \beta_0 f(T)$, $\alpha = \alpha_1 f(T)$, $\beta_1 = \beta_{10} f(T)$, $\alpha_0 = \alpha_{01} f(T)$, $\lambda_1 = \lambda_3 f(T)$, $\upsilon = \upsilon_0 f(T)$.

Where $f(T)$ is a non-dimensional function depending on temperature, during the case of temperature-independent modulus of elasticity $f(T)$ can be taken in the form $f(T) = 1 - \beta^* T_0$, where β^* is an empirical material constant and we introduce the potential functions $q(x,z,t)$, $\psi(x,z,t)$ such that

$$u = q_{,x} + \psi_{,z}, \quad w = q_{,z} - \psi_{,x}. \tag{9}$$

From (9) in (1)–(4) we get

$$(a_6 \nabla^2 - \frac{\partial^2}{\partial t^2})q + a_3 \phi^* - T = 0, \quad (10)$$

$$(a_7 \nabla^2 - \frac{\partial^2}{\partial t^2})\psi - a_4 \phi_2 = 0, \quad (11)$$

$$(a_{12} \nabla^2 - a_{14} - \frac{\partial^2}{\partial t^2})\phi_2 + a_{13} \nabla^2 \psi = 0, \quad (12)$$

$$(a_8 \nabla^2 - a_{10} - \frac{\partial^2}{\partial t^2})\phi^* - a_{11} \nabla^2 q + a_9 T = 0, \quad (13)$$

$$[k^*(1+\tau_\theta \frac{\partial}{\partial t})\nabla^2 - (\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2})a_{17}]T - (\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2})a_{15} \nabla^2 q - (\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2})a_{16}\phi^* = 0. \quad (14)$$

The constitutive relations are

$$\sigma_{xx} = a_{18}\, e + a_3\phi^* - T + a_{20}\frac{\partial u}{\partial x} - p, \quad (15)$$

$$\sigma_{zz} = a_{18}\, e + a_3\phi^* - T + a_{19}\frac{\partial w}{\partial z} - p, \quad (16)$$

$$\sigma_{yy} = a_{18}\, e + a_3\phi^* - T - p, \quad (17)$$

$$\sigma_{xz} = a_1 \frac{\partial w}{\partial x} + a_{20}\frac{\partial u}{\partial z} + a_5(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) + a_4 \phi_2, \quad (18)$$

$$\sigma_{zx} = a_1 \frac{\partial u}{\partial z} + a_{20}\frac{\partial w}{\partial x} - a_5(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) - a_4 \phi_2, \quad (19)$$

$$m_{xy} = a_{21}\frac{\partial \phi_2}{\partial x}, \quad (20)$$

$$m_{zy} = a_{21}\frac{\partial \phi_2}{\partial z}, \quad (21)$$

$$\lambda_x^* = a_{22}\frac{\partial \phi^*}{\partial x}, \quad (22)$$

$$\lambda_z^* = a_{22}\frac{\partial \phi^*}{\partial z}. \quad (23)$$

The constants a_i, $i = 1 \to 26$ are defined in Appendix A.

4. solution of the problem

The solution of the considered physical variable can be decomposed in terms of the normal modes in the following

$$\{\phi^*, \phi_2, q, T, \psi, \sigma_{ij}, m_{ij}\}(x,z,t) = \{\bar{\phi}^*, \bar{\phi}_2, \bar{q}, \bar{T}, \bar{\psi}, \bar{\sigma}_{ij}, \bar{m}_{ij}\}(z) e^{(\omega t + ibx)}. \quad (24)$$

Where, b is the wave number and ω is the frequency. From (24) in Eqs. (10)-(14) we obtained

$$(a_6 D^2 - A_1)\bar{q} + a_3\bar{\phi}^* - \bar{T} = 0, \quad (25)$$

$$(a_8 D^2 - A_2)\bar{\phi}^* - (a_{11}D^2 - A_3)\bar{q} + a_9\bar{T} = 0, \quad (26)$$

$$(A_4 D^2 - A_5)\bar{T} - (A_6 D^2 - A_7)\bar{q} - A_8\bar{\phi}^* = 0, \quad (27)$$

$$(a_7 D^2 - A_9)\bar{\psi} - a_4\bar{\phi}_2 = 0, \quad (28)$$

$$(a_{12}D^2 - A_{10})\bar{\phi}_2 + (a_3 D^2 - A_{11})\bar{\psi} = 0. \quad (29)$$

Eliminating $\bar{q}, \bar{\phi}^*$ and $\bar{T}$ between Eqs. (25)-(27), we get

$$(D^6 - AD^4 + BD^2 - C)\{\bar{q}(z), \bar{\phi}^*(z), \bar{T}(z)\} = 0, \quad (30)$$

Equation (29) can be factored as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)\{\bar{q}(z), \bar{\phi}^*(z), \bar{T}(z)\} = 0, \quad (31)$$

Where $k_n^2 (n = 1,2,3)$ are the roots of the characteristic equation of the homogeneous equation of Eqs. (25)-(27).

The general solution of Eqs. (25)-(27) is given by

$$q(x,z,t) = \sum_{n=1}^{3} M_n \exp(-k_n z + \omega t + ibx), \quad (32)$$

$$\phi^*(x,z,t) = \sum_{n=1}^{3} H_{1n} M_n \exp(-k_n z + \omega t + ibx), \quad (33)$$

$$T(x,z,t) = \sum_{n=1}^{3} H_{2n} M_n \exp(-k_n z + \omega t + ibx). \quad (34)$$

In a similar manner eliminating $\bar{\psi}, \bar{\phi}_2$ between Eqs. (28) and (29), we obtain the differential equation

$$(D^4 - E\, D^2 + F)\{\bar{\psi}(z), \bar{\phi}_2(z)\} = 0, \quad (35)$$

Eq. (35) can be factored as

$$(D^2 - k_1^2)(D^2 - k_2^2)\{\bar{\psi}(z), \bar{\phi}_2(z)\} = 0, \quad (36)$$

Where k_m^2 (m = 4,5) are the roots of the characteristic equation of the homogeneous equation of Eqs. (28) and (29). The general solution of Eqs. (28) and (29) is given by

$$\psi(x,z,t) = \sum_{m=4}^{5} M_m \exp(-k_m z + \omega t + ibx), \quad (37)$$

$$\phi_2(x,z,t) = \sum_{m=4}^{5} H_{1m} M_m \exp(-k_m z + \omega t + ibx). \quad (38)$$

To obtain the components of the displacement vector, from (32) and (37) in (9)

$$u(x,z,t) = [\sum_{n=1}^{3} ibM_n \exp(-k_n z) - \sum_{m=4}^{5} k_m M_m \exp(-k_m z)]\exp(\omega t + ibx), \quad (39)$$

$$w(x,z,t) = [\sum_{n=1}^{3} -k_n M_n \exp(-k_n z) - \sum_{m=4}^{5} ibM_m \exp(-k_m z)]\exp(\omega t + ibx), \quad (40)$$

$$e(x,z,t) = \sum_{n=1}^{3} (k_n - b^2) M_n \exp(-k_n z + \omega t + ibx). \quad (41)$$

The stress tensor and the couple stress tensor and the micro-rotation from (15)-(23), dimensionless quantities are given by

$$\sigma_{xx}(x,z,t) = -P + [\sum_{n=1}^{3} H_{3n} M_n \exp(-k_n z) + \sum_{m=4}^{5} H_{3m} M_m \exp(-k_m z)]\exp(\omega t + ibx), \quad (42)$$

$$\sigma_{yy}(x,z,t) = \sum_{n=1}^{3} H_{4n} M_n \exp(-k_n z + \omega t + ibx) - p, \quad (43)$$

$$\sigma_{zz}(x,z,t)=\sum_{n=1}^{3}H_{5n}M_n\exp(-k_nz+\omega t+ibx),$$
$$+\sum_{m=4}^{5}H_{5m}M_m\exp(-k_mz+\omega t+ibx)-\mathrm{p}, \quad (44)$$

$$\sigma_{xz}(x,z,t)=[\sum_{n=1}^{3}H_{7n}M_n\exp(-k_nz)$$
$$+\sum_{m=4}^{5}H_{7m}M_m\exp(-k_mz)]\exp(\omega t+ibx), \quad (45)$$

$$\sigma_{zx}(x,z,t)=[\sum_{n=1}^{3}H_{6n}M_n\exp(-k_nz)$$
$$+\sum_{m=4}^{5}H_{6m}M_m\exp(-k_mz)]\exp(\omega t+ibx), \quad (46)$$

$$m_{zy}(x,z,t)=\sum_{m=4}^{5}H_{9m}M_m\exp(-k_mz+\omega t+ibx), \quad (47)$$

$$m_{xy}(x,z,t)=\sum_{m=4}^{5}H_{8m}M_m\exp(-k_mz+\omega t+ibx), \quad (48)$$

$$\lambda_x^*(x,z,t)=\sum_{n=1}^{3}H_{8n}M_n\exp(-k_nz+\omega t+ibx), \quad (49)$$

$$\lambda_z^*(x,z,t)=\sum_{n=1}^{3}H_{9n}M_n\exp(-k_nz+\omega t+ibx). \quad (50)$$

Where H_{in}, $(i=1,\ldots,9)$, are defined in Appendix B.

5. Boundary conditions

In order to determine the parameters M_j, $(j=1,2,3,4,5)$ we consider the boundary conditions as follows

$$\sigma_{xx}=-p,\quad T=f_1e^{(\omega t+ibx)},\quad \lambda_z^*=\sigma_{xz}=m_{zy}=0. \quad (51)$$

From (51) in (34), (42), (45), (48), (49) and solving these equations for $M_j,(j=1,\ldots,5)$

by using the inverse of the matrix method as follows

$$\begin{pmatrix}M_1\\M_2\\M_3\\M_4\\M_5\end{pmatrix}=\begin{pmatrix}\mathrm{H}_{21}&\mathrm{H}_{22}&\mathrm{H}_{23}&0&0\\H_{31}&H_{32}&H_{33}&H_{34}&H_{35}\\H_{71}&H_{72}&H_{73}&H_{74}&H_{75}\\0&0&0&H_{94}&H_{95}\\H_{91}&H_{92}&H_{93}&0&0\end{pmatrix}^{-1}\begin{pmatrix}f_1\\0\\0\\0\\0\end{pmatrix}. \quad (52)$$

6. Special cases of microstretch thermoelastic theory and particular cases

(i) Equations of the microstretch thermoelastic with DPL theory when $\tau_\theta>\tau_q>0$.

(ii) Equations of the microstretch thermoelastic with the L-S theory when $\tau_\theta=0,\quad \tau_q>0$

(iii) Equations of the microstretch thermoelastic with CT theory when $\tau_\theta=\tau_q=0$

7. Numerical results and discussions

The analysis is conducted for a magnesium crystal-like material. Following Eringen (1984), the values of micropolar parameters are,

$\lambda=9.4\times10^{10}Nm^{-2}$, $\mu=4.0\times10^{10}Nm^{-2}$, $k=1.0\times10^{10}Nm^{-2}$,

$\rho=1.74\times10^{3}kgm^{-3}$, $j=0.2\times10^{-19}m^{-2}$, $\gamma=0.779\times10^{-9}N$, $x=1$.

Thermal parameters are given by,

$C_E=1.04\times10^{3}jkg^{-1}k^{-1}$, $k^*=1.7\times10^{6}Jm^{-1}s^{-1}k^{-1}$, $\alpha_{t_1}=2.33\times10^{-5}k^{-1}$,

$\alpha_{t_2}=2.48\times10^{-5}k^{-1}$, $T_0=298k$.

The microstretch parameters are taken as,

$\alpha_0=0.779\times10^{-9}N$, $\lambda_0=0.5\times10^{10}Nm^{-2}$, $\lambda_1=0.5\times10^{10}Nm^{-2}$.

By using the numerical technique, the distribution of the real part of the displacement component w, the temperature T, the scalar microstretch function ϕ^*, the stress component σ_{xx}, the stress component σ_{xz}, and the microrotation comparisons the 31-2s in Fig given is λ_x^*, have established for the theories CT, DPL, and L-S in two cases:

(i) With and without initial stress $[p=0,\quad p=0.001,$ $\beta^*=0.001517]$ as shown in the Figs. 2- 7.

(ii) For two different values of β^* $(\beta^*=0,\ \beta^*=0.001517)$ as shown in the Figs. 8-13.

Fig. 2 shows that the displacement component w in the context of the three theories (CT, DPL, and L-S) at $p=0.001$ begins from a negative value and increases to the maximum value in the range $0<z<0.82$. It is clear from this figure that, the values of w at $p=0$ are greater compared to those for the values of w at $p=0.001$ for three theories, while the values are the same in the two cases in the range $1.4<z$. Fig. 3 depicts that the variation of the temperature T begins from negative value and increases to the maximum value in the range $0<z<0.55$, then decreases to the minimum value in the range $0.55<z<2$. All the theories have the same behavior, the values of are higher than $p=0.001$ at T that of $p=0$, while the values are the same in the two cases in the range $2<z$. Fig. 4 explains the distribution of the scalar micro stretch function ϕ^* in the context of the three theories. The values of σ_{xx} decrease to minimum values in the range $0.33<z<1.36$, σ_{xx} but the values of in $p=0$ at σ_{xx} of that than higher are $p=0.001$, at σ_{xx} the range $0<z<1.36$, while the values are the same in the two cases in the range $1.36<z$. Fig. 6 depicts the distribution of the stress component σ_{xz} in the context of the three theories. The values of and zero from begin σ_{xz} satisfy the boundary condition and all the theories have the same behavior.

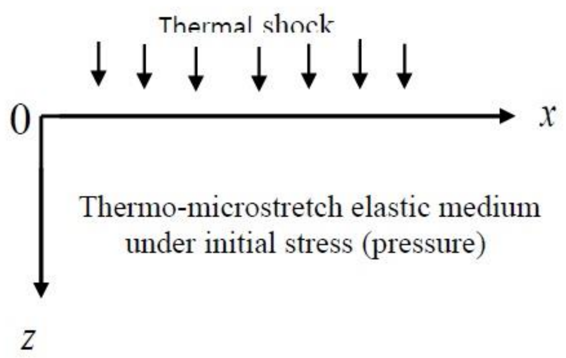

Fig. 1 Geometry of the problem

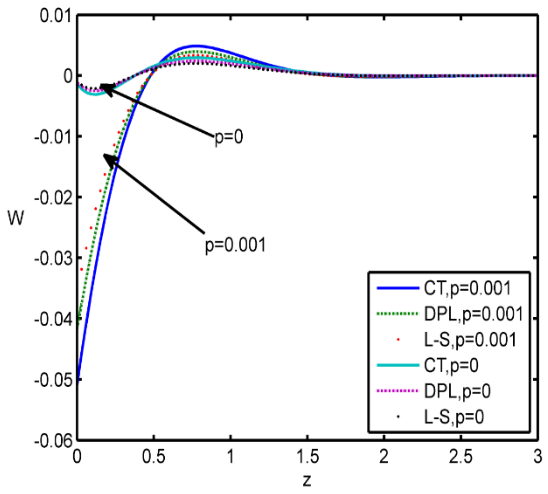

Fig. 2 Distribution of the displacement component w in the presence and absence of initial stress

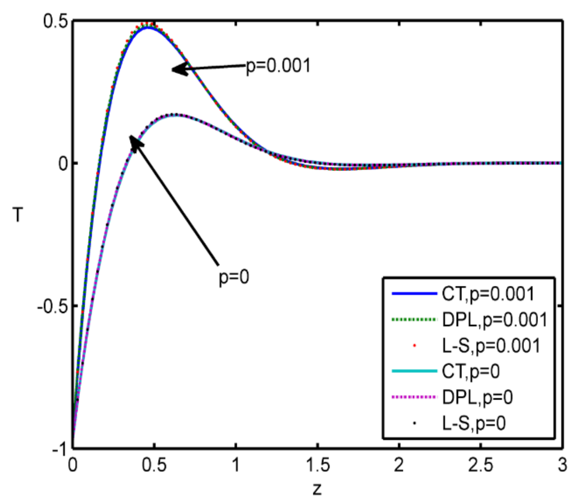

Fig. 3 Distribution of the temperaturein the presence T and absence of initial stress

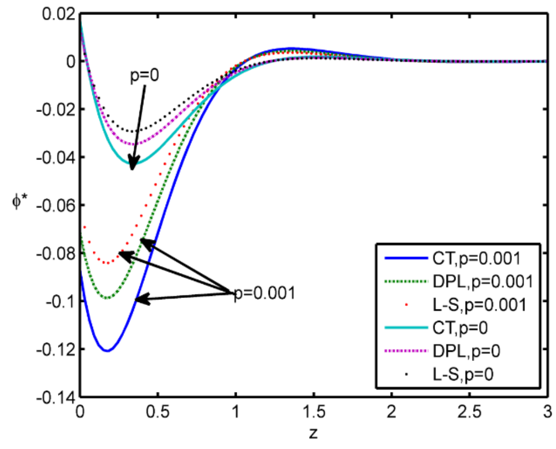

Fig. 4 Distribution of the scalar micro-stretch function ϕ^* in the presence and absence of initial stress

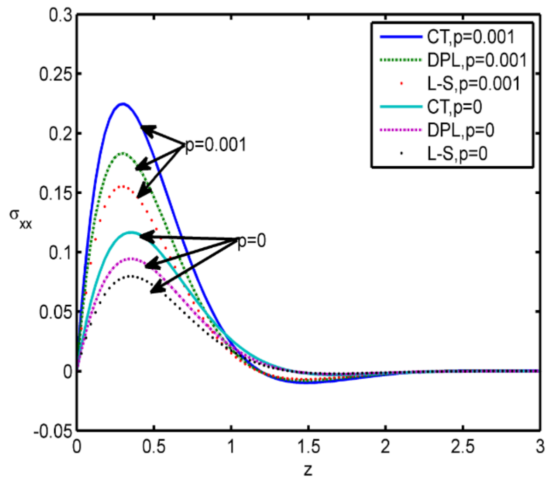

Fig. 5 Distribution of the stress component σ_{xx} in the presence and absence of initial stress

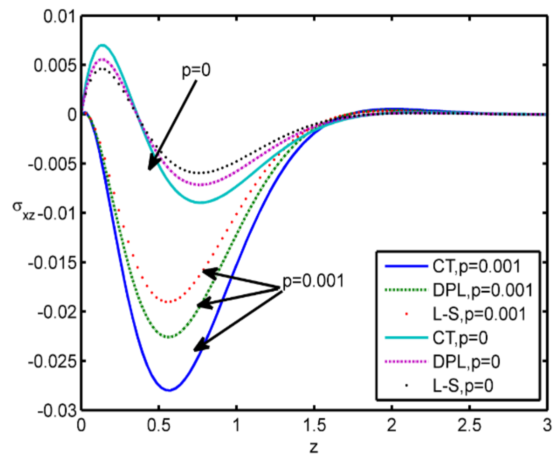

Fig. 6 Distribution of the stress component σ_{xz} in the presence and absence of initial stress

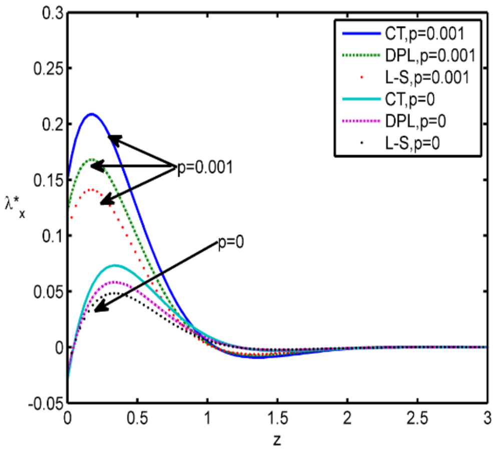

Fig. 7 Distribution of the microstress λ_x^* in the presence and absence of initial stress

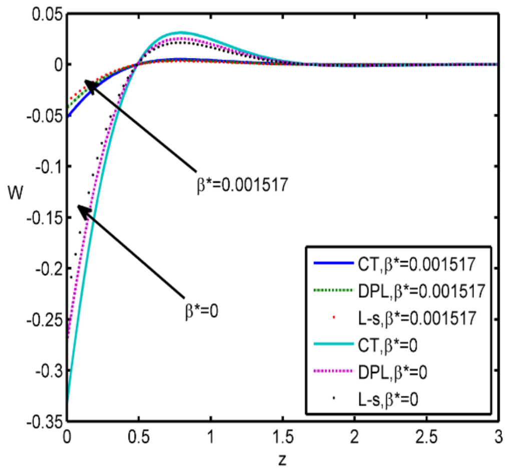

Fig. 8 Distribution of the displacement component w at two values of β^*.

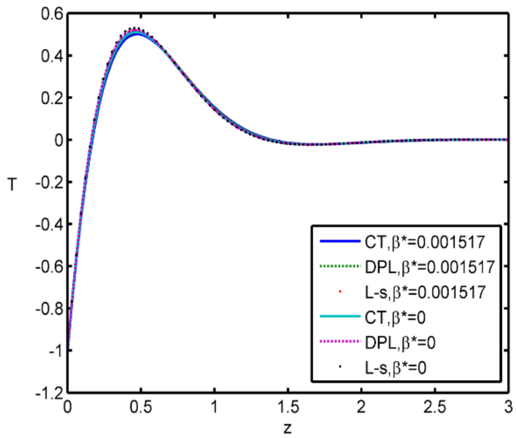

Fig. 9 Distribution of the temperature T at two values of β^*.

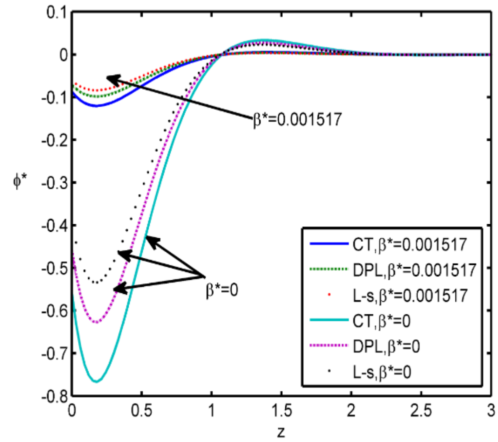

Fig. 10 Distribution of the scalar micro-stretch function β^*. at two values of ϕ^*

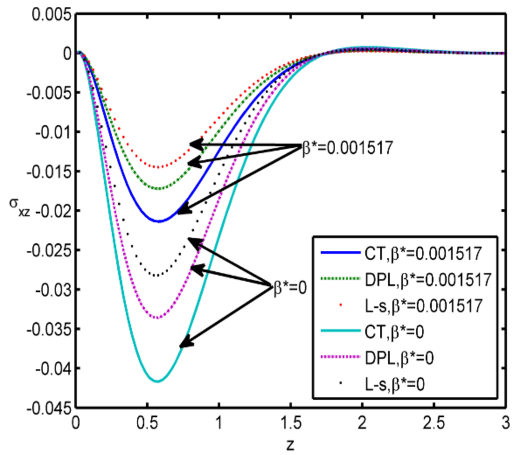

Fig. 12 Distribution of the stress component σ_{xz} at two values of β^*.

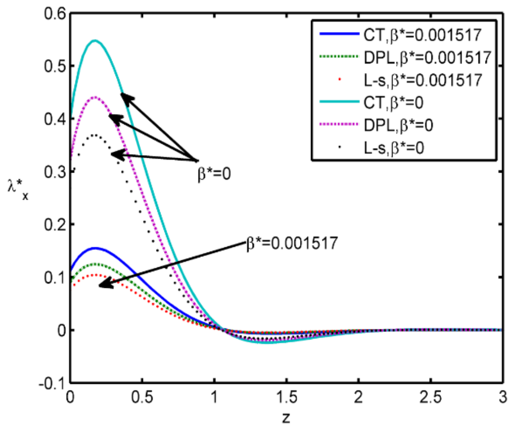

Fig. 13 Distribution of the microstress λ_x^* at two values of β^*.

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Appendix A

$$a_1=\frac{\mathrm{f}(T)(\mu_0+k_0)}{\rho c_1^2},\qquad a_2=\frac{\mathrm{f}(T)(\lambda_2+\mu_0)}{\rho c_1^2},\qquad a_3=\frac{\mathrm{f}(T)\lambda_0}{\rho c_1^2},$$

$$a_4=\frac{\mathrm{f}(T)k_0}{\rho c_1^2},\ a_5=\frac{p\,\mathrm{f}(T)T_0\beta_{10}}{2\rho c_1^2},\ a_6=a_1+\mathrm{a}_2,\quad a_7=a_1-\mathrm{a}_5,$$

$$a_8=\frac{2\,\mathrm{f}(T)\alpha_{01}}{3j\,\rho c_1^2},\qquad a_9=\frac{2\nu_0 c_1^2}{9j\,\rho\,\omega^{*2}\beta_{10}},\qquad a_{10}=\frac{2\lambda_3\,\mathrm{f}(T)}{9j\,\rho\,\omega^{*2}},$$

$$a_{11}=\frac{2\lambda_0^*\,\mathrm{f}(T)}{9j\,\rho\,\omega^{*2}},\quad a_{12}=\frac{\gamma_0\,\mathrm{f}(T)}{j\,\rho c_1^2},\qquad a_{13}=\frac{(k_0-\beta_{10}T_0 p)\,\mathrm{f}(T)}{\rho j\,\omega^{*2}},$$

$$a_{14}=\frac{2k_0\,\mathrm{f}(T)}{j\,\rho\,\omega^{*2}},\ a_{15}=\frac{\beta_{10}^2T_0[\mathrm{f}(T)]^2}{\rho\omega^*},\ a_{16}=\frac{\nu_0\beta_{10}T_0[\mathrm{f}(T)]^2}{\rho\omega^*},$$

$$a_{17}=\frac{\rho c_E c_1^2}{\omega^*},\qquad a_{18}=\frac{\lambda_2\,\mathrm{f}(T)}{\rho c_1^2},\qquad a_{19}=\frac{\mathrm{f}(T)(2\mu_0+k_0)}{\rho c_1^2},$$

$$a_{20}=\frac{\mathrm{f}(T)\mu_0}{\rho c_1^2},\qquad a_{21}=\frac{\gamma_0\,\mathrm{f}(T)\omega^{*2}}{\rho c_1^4},\qquad a_{22}=\frac{\alpha_{01}\,\mathrm{f}(T)\omega^{*2}}{\rho c_1^4},$$

$$\delta_1=1+\tau_\theta\omega,\ \delta_2=\omega+\tau_q\omega^2,\ A_1=a_2b^2+\omega^2,\ A_2=a_8b^2+a_{10}+\omega^2,$$

$$A_3=a_{11}b^2,\quad A_4=k^*\delta_1,\quad A_5=k^*\delta_1b^2+a_{17}\delta_2,\quad A_6=a_{15}\delta_2,$$

$$A_7=a_{15}\delta_2b^2,\qquad A_8=a_{16}\delta_2,\qquad A_9=a_7b^2+\omega^2,$$

$$A_{10}=a_{12}b^2+a_{14}+\omega^2,\qquad A_{11}=a_{13}b^2,\qquad A_{12}=a_8a_6A_4,$$

$$A_{13}=a_6a_8A_5+a_6A_2A_4-a_{11}a_3A_4+a_8A_6-\mathrm{a}_8\mathrm{A}_1\mathrm{A}_4,$$

$$A_{14}=a_6A_2A_5+a_6a_9A_8+a_8A_1A_5+A_1A_2A_4-\mathrm{a}_3\,a_{11}A_5+A_2A_6+a_8A_7+a_{11}A_8,$$

$$A_{15}=A_1A_2A_5+a_9A_1A_8-a_3A_3A_5-a_3a_9A_7+A_2A_7-A_3A_8,$$

$$A=\frac{A_{13}}{A_{12}},\quad B=\frac{A_{14}}{A_{12}},\quad \mathrm{C}=\frac{A_{15}}{A_{12}},\quad E=\frac{a_7A_{10}+a_{12}A_9-a_3a_4}{a_7a_{12}},$$

$$F=\frac{A_9A_{10}-a_4A_{11}}{a_7a_{12}}.$$

Appendix B

$$H_{1n}=\frac{(a_{11}-a_6a_9)k_n^2+a_9A_1-A_3}{a_8k_n^2-A_2-a_3a_9},\quad (n=1,2,3),$$

$$H_{1m}=\frac{(a_3-a_7)k_m^2+A_9-A_{11}}{a_{12}k_m^2-a_4-A_{10}},\quad (\mathrm{m}=4,5),$$

$$H_{2n}=\frac{A_6k_n^2-A_8H_{1n}-A_7}{A_4k_n^2-A_5},(n=1,2,3),\quad H_{3m}=-iba_{19}k_m,\quad (\mathrm{m}=4,5),$$

$$H_{3n}=a_{18}(k_n^2-b^2)+a_3H_{1n}-H_{2n}-a_{19}b^2,(\mathrm{n}=1,2,3),$$

$$H_{4n}=a_{18}(k_n^2-b^2)+a_3H_{1n}-H_{2n},\quad (n=1,2,3),$$

$$H_{5n}=a_{18}(k_n^2-b^2)+a_3H_{1n}-H_{2n}+a_{19}k_n^2,\quad (n=1,2,3),$$

$$H_{5m}=iba_{19}k_m,(\mathrm{m}=4,5),H_{6n}=-ibk_n(a_1+a_{20}),(n=1,2,3),$$

$$H_{6m}=a_1k_m^2+a_{20}b^2-a_5(k_m^2+b^2)-\mathrm{a}_4H_{1m},\quad (\mathrm{m}=4,5),$$

$$H_{7n}=H_{6n},\quad (n=1,2,3),$$

$$H_{7m}=a_1b^2+a_{20}k_m^2+a_5(k_m^2-b^2)-a_4H_{1m},(\mathrm{m}=4,5),$$

$$H_{8m}=iba_{21}H_{1m},\ (\mathrm{m}=4,5),\ H_{8n}=iba_{22}H_{1n},\quad (n=1,2,3),$$

$$H_{9m}=-a_{21}k_mH_{1m},(\mathrm{m}=4,5),H_{9n}=-a_{22}k_nH_{1n},(n=1,2,3).$$