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Wave propagation of FG polymer composite nanoplates reinforced with GNPs

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Abstract. This study examines the wave propagation of the functionally graded polymer composite (FG-PC) nanoplates reinforced with graphene nanoplatelets (GNPs) resting on elastic foundations in the framework of the nonlocal strain gradient theory incorporating both stiffness hardening and softening mechanisms of nanostructures. To this end, the material properties are based on the Halpin-Tsai model, and the expressions for the classical and higher-order stresses and strains are consistently derived employing the second-order shear deformation theory. The equations of motion are then consistently derived using Hamilton's principle of variation. These governing equations are solved with the help of Trial function method. Extensive numerical discussions are conducted for wave propagation of the nanoplates and the influences of different parameters, such as the nonlocal parameter, strain gradient parameter, weight fraction of GNPs, uniform and non-uniform distributions of GNPs, elastic foundation parameters as well as wave number.

Keywords: wave propagation; nanocomposites; graphene nanoplatelet; Kerr foundation; second-order shear deformation theory

1. Introduction

Due to its excellent mechanical and thermal properties. Graphene is regarded as a revolutionary material of the future with broad application prospects in the fields of micro-nano processing, energy and biomedicine. In recent years, the buckling/bending (Arefi *et al.* 2019, Jiao and Alavi 2018, Yang 2020) and vibration (Aditya 2019, Arefi 2018, Liu *et al.* 2020, Sahmani *et al.* 2018) of structures reinforced by graphene nanoplatelets (GNPs) and graphene platelets (GPLs) have been systematically analyzed by some researchers.

However, the researches on the wave propagation characteristics in macro, micro and nano structures reinforced by GPNs or GPLs are very limited. Through literature search, in the existing literature, using classical mechanical theory, Gao *et al.* (2020) studied the propagation characteristics of elastic waves in the square plate reinforced by porous GPLs, they considered different porosity and GPL distribution, and Halpin Tsai model is used to characterize the material properties, according to different plate theories, the wave equations are derived, and the dispersion relation of wave in plate is obtained, and the influences of GPL volume fraction and geometry on dispersion relation are analyzed. Li *et al.* (2020) proposed a semi analytical method to study the wave propagation characteristics in the GPLs reinforced plate, based on the Reissner Mindlin plate theory and Hamilton principle, the wave equations are derived, and the non-uniform rational B-spline method is used to transform the equation into the eigenvalue problem. Employing the Reddy's high-order shear deformation theory and classical mechanics theory, Ebrahimi *et al.* (2019) analyzed the wave propagation of porous GPLs reinforced composite shells with the help of the first-order shear deformation theory. It is worth mentioning that the above articles about the wave propagation of plates and shells are all based on the classical continuum mechanical theory.

In fact, the existing articles about wave propagation of structures reinforced by GPLs or GPNs based on continuum mechanics are very limited, and the articles about wave propagation of structures reinforced by GPLs or GPNs based on non-classical mechanics are even more limited. Among these, within the framework of nonlocal theory (Eringen 1998), Ebrahimi and Dabbagh (2018) discussed the characteristics of wave propagation in graphene plates (GPLs) reinforced nanoplates. It is worth mentioning that the nonlocal theory contains only one size parameter, which can capture the mechanical characteristics of some nanostructures. However, this theory can only characterize the stiffness softening effect, and it cannot account for the stiffness strengthening effect. Therefore, the mechanical characteristics of some nanostructures cannot be characterized. Based on the nonlocal strain gradient (NSG) theory (Lim *et al.* 2015). It is worth mentioning that NSG theory is totally different from traditional classical theory (Moradi Mansouri 2012, Hadji, *et al*. 2018) and non-classical theory, such as non-local theory (Eringen 1998, Barretta and de Sciarra, 2019, Barretta *et al*. 2019, Civalek and Demir 2016, Civalek *et al*. 2020, Eltaher and Mohamed 2020a, Emam *et al.* 2018, Eltaher *et al*. 2020b, Uzun and Civalek 2019, Taherifar *et al*. 2019, Numanoglu *et al.* 2018, Heydari 2018, Heydari and Shariati 2018, Malikan *et al.* 2020b), strain gradient theory (Akgöz and Civalek 2015), surface elasticity (Almitani *et al.* 2020, Eltaher *et al.*

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2020c), doublet mechanics (Eltaher *et al.* 2020d), nonlocal couple stress theory (Ebrahimi *et al*. 2020), modified couple stress theory (Akgoz and Civalek 2017a, b), NSG theory (Lim *et al.* 2015, She *et al.* 2020, Malikan *et al.* 2020a) can characterize both the stiffness strengthening effect and the stiffness softening effect at the same time (Apuzzo *et al.* 2018, Barretta and Marotti de Sciarra 2018, Faleh 2018, Ghayesh and Farajpour 2018, Ghayesh *et al.* 2019, Jalaei and Civalek 2019, Lu *et al.* 2018, Malikan *et al.* 2020a, Pinnola *et al.* 2020, Gao *et al*. 2019), and this theory has been verified by molecular dynamics and experiments, which has made an important contribution to the development of nanomechanics theory.

In this paper, for the first time, the nonlocal strain gradient theory is used to study the propagation characteristics of elastic waves in polymer composite nanoplates reinforced with GNPs. Besides, the small-scale effect and a new second-order displacement field are also considered. In particular, the non-classical wave equation is derived using Hamiltonian variational principle. The dispersion relation is obtained by using the trial function method. The effects of various parameters on wave propagation characteristics are studied, and the propagation characteristics of FG polymer composite nanoplates reinforced with GNPs are revealed.

2. Material properties

Fig. 1 illustrates an FG-PC nanoplate reinforced with GNPs resting on elastic foundation. The [length; width; thickness, the total layers] of the nanoplate are symbolized by [a; b; h; N_l] with $\Delta h=h/N_l$. Using the Halpin-Tsai micromechanical model, material properties at the k-layer can be written as (Arefi *et al.* 2019)

$$E_c^{(k)}=\frac{3E_M}{8}\left[1-\frac{1+\frac{2l_{GNP}}{h_{GNP}}\left(\frac{\frac{E_{GNP}}{E_M}-1}{\frac{E_{GNP}}{E_M}+\frac{2l_{GNP}}{h_{GNP}}}\right)\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right)}{\left(\frac{\frac{E_{GNP}}{E_M}-1}{\frac{E_{GNP}}{E_M}+\frac{2l_{GNP}}{h_{GNP}}}\right)\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right)}\right]+\frac{5E_M}{8}\left[\frac{1+\frac{2w_{GNP}}{h_{GNP}}\left(\frac{\frac{E_{GNP}}{E_M}-1}{\frac{E_{GNP}}{E_M}+\frac{2w_{GNP}}{h_{GNP}}}\right)\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right)}{1-\left(\frac{\frac{E_{GNP}}{E_M}-1}{\frac{E_{GNP}}{E_M}+\frac{2w_{GNP}}{h_{GNP}}}\right)\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right)}\right] \tag{1}$$

$$\rho_c^{(k)}=\rho_{GNP}\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right)+\rho_M\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right) \tag{2}$$

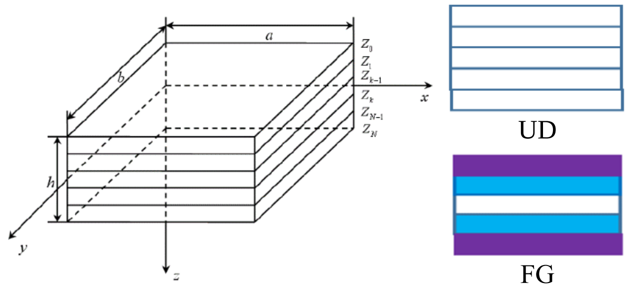

Fig. 1 A nanoplate reinforced with GNPs (modified from Fu *et al.* 2014)

$$\nu_c^{(k)}=\nu_{GNP}\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right)+\nu_M\left(\frac{g_{GNP}^{(k)}}{g_{GNP}^{(k)}+\left(\frac{\rho_{GNP}}{\rho_M}\right)(1-g_{GNP}^{(k)})}\right) \tag{3}$$

Here, Young' modulus of the polymeric-matrix and nanofillers are symbolized by E_M and E_{GNP}, mass density of the polymeric-matrix and nanofillers are symbolized by ρ_M and ρ_{GNP}, Poisson's ratio of the polymeric-matrix and nanofillers are symbolized by ν_M and ν_{GNP}, the average length, width and thickness of the GNPs are symbolized by l_{GNP}, w_{GNP}, and h_{GNP}, furthermore, the weight fraction of the GNP polymer nanocomposite for the k-layer is symbolized by $g_{GNP}^{(k)}$. Here, we consider two kinds of GNPs distributions with (Arefi *et al.* 2019):

For UD case

$$g_{GNP}^{(k)}=g_{GNP}^{*} \tag{4a}$$

For FG case

$$g_{GNP}^{(k)}=\frac{4g_{GNP}^{*}\left(\frac{N_L+1}{2}-\left|k-\frac{N_L+1}{2}\right|\right)}{(2+N_L)} \tag{4b}$$

Here, the weight fraction of GNPs is symbolized by g_{GNP}^{*}, in the later calculation, E_M=3GPa, ν_M=0.34, and ρ_M=1200 kg/m^3;E_{GNP}=1.01TPa,ν_{GNP}=0.186, ρ_{GPN}=1060 kg/m^3; h=20 nm, l_{GNP}=3 nm, h_{GNP}=0.7 nm, w_{GNP}=1.8 nm (Liu *et al.*2017).

3. Wave equations

The second-order shear deformation theory dictates a displacement field of (Khdeir and Reddy 1999)

$$\begin{aligned} u_1&=u+z\varphi_1+z^2\varphi_2\\ u_2&=v+z\psi_1+z^2\psi_2\\ u_3&=w \end{aligned} \tag{5}$$

with [u_1, u_2, u_3] being [axial; width; transverse] displacements generating the strain of (Khdeir and Reddy 1999)

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \varphi_1}{\partial x} \\ \frac{\partial \psi_1}{\partial y} \\ \frac{\partial \varphi_1}{\partial y} + \frac{\partial \psi_1}{\partial x} \end{Bmatrix} + z^2 \begin{Bmatrix} \frac{\partial \varphi_2}{\partial x} \\ \frac{\partial \psi_2}{\partial y} \\ \frac{\partial \varphi_2}{\partial y} + \frac{\partial \psi_2}{\partial x} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varphi_1 + \frac{\partial w}{\partial x} \\ \psi_1 + \frac{\partial w}{\partial y} \end{Bmatrix} + 2z \begin{Bmatrix} \varphi_2 \\ \psi_2 \end{Bmatrix} \tag{6}$$

The constitutive equation based on NSG theory is

$$\left(1-\mu^2\nabla^2\right)\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \left(1-l^2\nabla^2\right) \times \begin{bmatrix} Q_{11}^{(k)} & Q_{12}^{(k)} & 0 & 0 & 0 \\ Q_{12}^{(k)} & Q_{22}^{(k)} & 0 & 0 & 0 \\ 0 & 0 & Q_{44}^{(k)} & 0 & 0 \\ 0 & 0 & 0 & Q_{55}^{(k)} & 0 \\ 0 & 0 & 0 & 0 & Q_{66}^{(k)} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} \tag{7}$$

here

$$\begin{aligned} Q_{11}^{(k)} &= Q_{22}^{(k)} = \frac{E_c^{(k)}}{1-(\nu_c^{(k)})^2} \\ Q_{12}^{(k)} &= Q_{21}^{(k)} = \nu_c^{(k)} Q_{11}^{(k)} \\ Q_{44}^{(k)} &= Q_{55}^{(k)} = Q_{66}^{(k)} = \frac{E_c^{(k)}}{2(1+\nu_c^{(k)})} \end{aligned} \tag{8}$$

with [μ ; l] being [nonlocal; strain gradient] parameters, which are, respectively, used for describing the stiffness-softening and hardening mechanisms of nanostructures.

The Hamilton principle states that

$$\int_0^t \delta\Pi dt = \int_0^t \left(\delta\Pi_U + \delta\Pi_V - \delta\Pi_K\right) dt = 0 \tag{9}$$

with [Π ; Π_U ; Π_V ; Π_K] being [total energy; virtual strain energy; work done; kinetic energy], and

$$\begin{aligned} &\delta\Pi_U + \delta\Pi_V - \delta\Pi_K = \int_{\Omega_0} (N_x \frac{\partial \delta u}{\partial x} + M_x \frac{\partial \delta \varphi_1}{\partial x} \\ &+ L_x \frac{\partial \delta \varphi_2}{\partial x} + N_y \frac{\partial \delta v}{\partial x} + M_y \frac{\partial \delta \psi_1}{\partial y} + L_y \frac{\partial \delta \psi_2}{\partial y^2} \\ &+ N_{xy} (\frac{\partial \delta u}{\partial y} + \frac{\partial \delta v}{\partial x}) + M_{xy} (\frac{\partial \delta \varphi_1}{\partial y} + \frac{\partial \delta \psi_1}{\partial x}) \\ &+ L_{xy} (\frac{\partial \delta \varphi_2}{\partial y} + \frac{\partial \delta \psi_2}{\partial x}) - \phi_{xz} (\delta \varphi_1 + \frac{\partial \delta w}{\partial x}) \\ &- \phi_{yz} (\delta \psi_1 + \frac{\partial \delta w}{\partial y}) - 2R_{xz} \delta \varphi_2 - 2R_{yz} \delta \psi_2) dxdy \\ &+ (-\int_{\Omega_0} \left[(\frac{k_l k_u}{k_l + k_u}) w - (\frac{k_s k_u}{k_l + k_u}) \nabla^2 w \delta w \right] dxdy) \\ &+ \int_{\Omega_0} \Big[I_0 (\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \frac{\partial w}{\partial t}\frac{\delta\partial w}{\partial t}) + I_1 (\frac{\partial u}{\partial t}\frac{\delta\partial \varphi_1}{\partial t} \\ &+ \frac{\partial v}{\partial t}\frac{\delta\partial \psi_1}{\partial t} + \frac{\partial \varphi_1}{\partial t}\frac{\delta\partial u}{\partial t} + \frac{\partial \psi_1}{\partial t}\frac{\delta\partial v}{\partial t}) + I_2 (\frac{\partial u}{\partial t}\frac{\delta\partial \varphi_2}{\partial t} \\ &+ \frac{\partial v}{\partial t}\frac{\delta\partial \psi_2}{\partial t} + \frac{\partial \varphi_1}{\partial t}\frac{\delta\partial \varphi_1}{\partial t} + \frac{\partial \varphi_2}{\partial t}\frac{\delta\partial u}{\partial t} + \frac{\partial \psi_1}{\partial t}\frac{\delta\partial \psi_1}{\partial t} \\ &+ \frac{\partial \psi_2}{\partial t}\frac{\delta\partial v}{\partial t}) + I_3 (\frac{\partial \varphi_1}{\partial t}\frac{\delta\partial \varphi_2}{\partial t} + \frac{\partial \varphi_2}{\partial t}\frac{\delta\partial \varphi_1}{\partial t} + \frac{\partial \psi_1}{\partial t}\frac{\delta\partial \varphi_2}{\partial t} \\ &+ \frac{\partial \psi_2}{\partial t}\frac{\delta\partial \psi_1}{\partial t}) + I_4 (\frac{\partial \varphi_2}{\partial t}\frac{\delta\partial \varphi_2}{\partial t} + \frac{\partial \psi_2}{\partial t}\frac{\delta\partial \psi_2}{\partial t}) \Big] dxdy \end{aligned} \tag{10}$$

with Ω_0 being the domain of the nanoplate, [k_l; k_u; k_s] being [lower layer; upper layer; shear layer] of the elastic foundation (Rad 2015), and the stress resultants, the mass moment of inertias are defined as

$$\begin{aligned} &\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} \phi_{xz} \\ \phi_{yz} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz, \\ &\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} z \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} R_{xz} \\ R_{yz} \end{Bmatrix} = \int_{-h/2}^{h/2} z \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz, \\ &\begin{Bmatrix} L_x \\ L_y \\ L_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} z^2 \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz \end{aligned} \tag{11}$$

$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{Bmatrix} = \sum_{k=1}^{N_l} \int_{z(k)}^{z(k+z)} \begin{Bmatrix} 1 \\ z \\ z^2 \\ z^3 \\ z^4 \end{Bmatrix} \rho_c^{(k)} dz \tag{12}$$

Followed by Hamilton principle, the equations of motions are taken as below

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \varphi_1}{\partial t^2} + I_2 \frac{\partial^2 \varphi_2}{\partial t^2} \tag{13a}$$

$$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0 \frac{\partial^2 v}{\partial t^2} + I_1 \frac{\partial^2 \psi_1}{\partial t^2} + I_2 \frac{\partial^2 \psi_2}{\partial t^2} \tag{13b}$$

$$\frac{\partial \phi_{xz}}{\partial x} + \frac{\partial \phi_{yz}}{\partial y} - \left(\frac{k_l k_u}{k_l + k_u}\right) w + \left(\frac{k_l k_u}{k_l + k_u}\right) \nabla^2 w = I_0 \frac{\partial^2 w}{\partial t^2} \tag{13c}$$

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - \phi_{xz} = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \varphi_1}{\partial t^2} + I_3 \frac{\partial^2 \varphi_2}{\partial t^2} \tag{13d}$$

$$\frac{\partial L_{xx}}{\partial x} + \frac{\partial L_{xy}}{\partial y} - 2R_{xz} = I_2 \frac{\partial^2 u}{\partial t^2} + I_3 \frac{\partial^2 \varphi_1}{\partial t^2} + I_4 \frac{\partial^2 \varphi_2}{\partial t^2} \tag{13e}$$

$$\frac{\partial M_{yy}}{\partial y} + \frac{\partial M_{xy}}{\partial x} - \phi_{yz} = I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \psi_1}{\partial t^2} + I_3 \frac{\partial^2 \psi_2}{\partial t^2} \tag{13f}$$

$$\frac{\partial L_{yy}}{\partial y} + \frac{\partial L_{xy}}{\partial x} - 2R_{yz} = I_2 \frac{\partial^2 v}{\partial t^2} + I_3 \frac{\partial^2 \psi_1}{\partial t^2} + I_4 \frac{\partial^2 \psi_2}{\partial t^2} \tag{13g}$$

Using Eqs. (6), (7), (8), (11) and (12), Eq. (13) becomes

$$\begin{aligned} &\left(1-\ell_2^2\nabla^2\right)\Big(L_{11}(u) + L_{12}(v) + L_{13}(\varphi_1) + L_{14}(\varphi_2) \\ &+ L_{15}(\psi_1) + L_{16}(\psi_2)\Big) = \left(1-\ell_1^2\nabla^2\right) \\ &\times \left(I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \varphi_1}{\partial t^2} + I_2 \frac{\partial^2 \varphi_2}{\partial t^2}\right) \end{aligned} \tag{14}$$

$$\left(1-\ell_2^2\nabla^2\right)\left(L_{21}(u)+L_{22}(v)+L_{23}(\varphi_1)+L_{24}(\varphi_2)+L_{25}(\psi_1)+L_{26}(\psi_2)\right)=\left(1-\ell_1^2\nabla^2\right)\times\left(I_0\frac{\partial^2 v}{\partial t^2}+I_1\frac{\partial^2\psi_1}{\partial t^2}+I_2\frac{\partial^2\psi_2}{\partial t^2}\right) \tag{15}$$

$$\left(1-\ell_2^2\nabla^2\right)\left(L_{31}(w)+L_{32}(\varphi_1)+L_{33}(\varphi_2)+L_{34}(\psi_1)+L_{35}(\psi_2)\right)=\left(1-\ell_1^2\nabla^2\right)\left(I_0\frac{\partial^2 w}{\partial t^2}+\left(\frac{k_l k_u}{k_l+k_u}\right)w-\left(\frac{k_l k_u}{k_l+k_u}\right)\nabla^2 w\right) \tag{16}$$

$$\left(1-\ell_2^2\nabla^2\right)\left(L_{41}(u)+L_{42}(v)-L_{43}(w)+L_{44}(\varphi_1)+L_{45}(\varphi_2)+L_{46}(\psi_1)+L_{47}(\psi_2)\right)=\left(1-\ell_1^2\nabla^2\right)\times\left(I_1\frac{\partial^2 u}{\partial t^2}+I_2\frac{\partial^2\varphi_1}{\partial t^2}+I_3\frac{\partial^2\varphi_2}{\partial t^2}\right) \tag{17}$$

$$\left(1-\ell_2^2\nabla^2\right)\left(L_{51}(u)+L_{52}(v)-L_{53}(w)+L_{54}(\varphi_1)+L_{55}(\varphi_2)+L_{56}(\psi_1)+L_{57}(\psi_2)\right)=\left(1-\ell_1^2\nabla^2\right)\times\left(I_2\frac{\partial^2 u}{\partial t^2}+I_3\frac{\partial^2\varphi_1}{\partial t^2}+I_4\frac{\partial^2\varphi_2}{\partial t^2}\right) \tag{18}$$

$$\left(1-\ell_2^2\nabla^2\right)\left(L_{61}(u)+L_{62}(v)-L_{63}(w)+L_{64}(\varphi_1)+L_{65}(\varphi_2)+L_{66}(\psi_1)+L_{67}(\psi_2)\right)=\left(1-\ell_1^2\nabla^2\right)\times\left(I_1\frac{\partial^2 v}{\partial t^2}+I_2\frac{\partial^2\psi_1}{\partial t^2}+I_3\frac{\partial^2\psi_2}{\partial t^2}\right) \tag{19}$$

$$\left(1-\ell_2^2\nabla^2\right)\left(L_{71}(u)+L_{72}(v)-L_{73}(w)+L_{74}(\varphi_1)+L_{75}(\varphi_2)+L_{76}(\psi_1)+L_{77}(\psi_2)\right)=\left(1-\ell_1^2\nabla^2\right)\times\left(I_2\frac{\partial^2 v}{\partial t^2}+I_3\frac{\partial^2\psi_1}{\partial t^2}+I_4\frac{\partial^2\psi_2}{\partial t^2}\right) \tag{20}$$

with

$$\begin{Bmatrix}A_{ij}\\B_{ij}\\C_{ij}\\D_{ij}\\E_{ij}\end{Bmatrix}=\sum_{k=1}^{N_l}\int_{z(k)}^{z(k+z)}\begin{Bmatrix}1\\z\\z^2\\z^3\\z^4\end{Bmatrix}Q_{ij}^{(k)}dz,\quad\left(i,j=1,2,4,5,6\right) \tag{21}$$

and the linear operators are defined by

$$\begin{cases}L_{11}=A_{11}\frac{\partial^2}{\partial x^2}+A_{66}\frac{\partial^2}{\partial y^2}\\ L_{12}=\left(A_{12}+A_{66}\right)\frac{\partial^2}{\partial x\partial y}\\ L_{13}=B_{11}\frac{\partial^2}{\partial x^2}+B_{66}\frac{\partial^2}{\partial y^2}\\ L_{14}=D_{11}\frac{\partial^2}{\partial x^2}+D_{66}\frac{\partial^2}{\partial y^2}\\ L_{15}=\left(B_{12}+B_{66}\right)\frac{\partial^2}{\partial x\partial y}\\ L_{16}=\left(D_{12}+D_{66}\right)\frac{\partial^2}{\partial x\partial y}\end{cases}$$

$$\begin{cases}L_{21}=\left(A_{12}+A_{66}\right)\frac{\partial^2}{\partial x\partial y}\\ L_{22}=A_{22}\frac{\partial^2}{\partial y^2}+A_{66}\frac{\partial^2}{\partial x^2}\\ L_{23}=(B_{12}+B_{66})\frac{\partial^2}{\partial x\partial y}\\ L_{24}=(D_{12}+D_{66})\frac{\partial^2}{\partial x\partial y}\\ L_{25}=(B_{22}\frac{\partial^2}{\partial y^2}+B_{66}\frac{\partial^2}{\partial x^2})\\ L_{26}=(D_{22}\frac{\partial^2}{\partial y^2}+D_{66}\frac{\partial^2}{\partial x^2})\end{cases}$$

$$\begin{cases}L_{31}=(A_{44}\frac{\partial^2}{\partial y^2}+A_{55}\frac{\partial^2}{\partial x^2})\\ L_{32}=A_{55}\frac{\partial}{\partial x}\\ L_{33}=2B_{55}\frac{\partial}{\partial x}\\ L_{34}=A_{44}\frac{\partial}{\partial y}\\ L_{35}=2B_{55}\frac{\partial}{\partial y}\end{cases}$$

$$\begin{cases}L_{41}=B_{11}\frac{\partial^2}{\partial x^2}+B_{66}\frac{\partial^2}{\partial y^2}\\ L_{42}=(B_{12}+B_{66})\frac{\partial^2}{\partial x\partial y}\\ L_{43}=-A_{55}\frac{\partial}{\partial x}\\ L_{44}=D_{11}\frac{\partial^2}{\partial x^2}+D_{66}\frac{\partial^2}{\partial y^2}-A_{55}\\ L_{45}=E_{11}\frac{\partial^2}{\partial x^2}+E_{66}\frac{\partial^2}{\partial y^2}-2B_{55}\\ L_{46}=(D_{12}+D_{66})\frac{\partial^2}{\partial x\partial y}\\ L_{47}=(E_{12}+E_{66})\frac{\partial^2}{\partial x\partial y}\end{cases}$$

$$\begin{cases}L_{51}=D_{11}\frac{\partial^2}{\partial x^2}+D_{66}\frac{\partial^2}{\partial y^2}\\ L_{52}=(D_{12}+D_{66})\frac{\partial^2}{\partial x\partial y}\\ L_{53}=-2B_{55}\frac{\partial}{\partial x}\\ L_{54}=E_{11}\frac{\partial^2}{\partial x^2}+E_{66}\frac{\partial^2}{\partial y^2}-2B_{55}\\ L_{55}=F_{11}\frac{\partial^2}{\partial x^2}+F_{66}\frac{\partial^2}{\partial y^2}-4D_{55}\\ L_{56}=(E_{12}+E_{66})\frac{\partial^2}{\partial x\partial y}\\ L_{57}=(F_{12}+F_{66})\frac{\partial^2}{\partial x\partial y}\end{cases}$$

$$\begin{cases} L_{61} = (B_{12} + B_{66})\dfrac{\partial^2}{\partial x \partial y} \\ L_{62} = (B_{22}\dfrac{\partial^2}{\partial y^2} + B_{66}\dfrac{\partial^2}{\partial x^2}) \\ L_{63} = -A_{44}\dfrac{\partial}{\partial y} \\ L_{64} = (D_{12} + D_{66})\dfrac{\partial^2}{\partial x \partial y} \\ L_{65} = (E_{12} + E_{66})\dfrac{\partial^2}{\partial x \partial y} \\ L_{66} = (D_{22}\dfrac{\partial^2}{\partial y^2} + D_{66}\dfrac{\partial^2}{\partial x^2} - A_{44}) \\ L_{67} = (E_{22}\dfrac{\partial^2}{\partial y^2} + E_{66}\dfrac{\partial^2}{\partial x^2} - 2B_{44}) \end{cases}$$

$$\begin{cases} L_{71} = (D_{12} + D_{66})\dfrac{\partial^2}{\partial x \partial y} \\ L_{72} = (D_{22}\dfrac{\partial^2}{\partial y^2} + D_{66}\dfrac{\partial^2}{\partial x^2}) \\ L_{73} = -2B_{44}\dfrac{\partial}{\partial y} \\ L_{74} = (E_{12} + E_{66})\dfrac{\partial^2}{\partial x \partial y} \\ L_{75} = (F_{12} + F_{66})\dfrac{\partial^2}{\partial x \partial y} \\ L_{76} = (E_{22}\dfrac{\partial^2}{\partial y^2} + E_{66}\dfrac{\partial^2}{\partial x^2} - 2B_{44}) \\ L_{77} = (F_{22}\dfrac{\partial^2}{\partial y^2} + F_{66}\dfrac{\partial^2}{\partial x^2} - 4D_{44}) \end{cases} \tag{22}$$

4. Solution

To solve the wave propagation problem, we assume

$$\left[u, v, w, \varphi_1, \varphi_2, \psi_1, \psi_2\right] = \left[u^*, v^*, w^*, \varphi_1^*, \varphi_2^*, \psi_1^*, \psi_2^*\right] \times e^{i(xk_x + yk_y - \omega t)} \tag{23}$$

with $u^*, v^*, w^*, \varphi_1^*, \varphi_2^*, \psi_1^*, \psi_2^*$ being the wave amplitude, [k_x; k_y] being the wave number in [x-; y-] directions, ω being the circular frequency. Using Eq. (23), the eigenvalue problem can be derived by Eqs. (14)-(20), as

$$(\mathbf{K} - \omega^2 \mathbf{M})\left[u^*, v^*, w^*, \varphi_1^*, \varphi_2^*, \psi_1^*, \psi_2^*\right]^T = 0 \tag{24}$$

with $\mathbf{K}$ being the stiffness matrix, $\mathbf{M}$ being the mass matrix, defined as

$$\mathbf{K} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} & K_{17} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} & K_{27} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} & K_{37} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} & K_{47} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} & K_{57} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} & K_{67} \\ K_{71} & K_{72} & K_{73} & K_{74} & K_{75} & K_{76} & K_{77} \end{bmatrix} \tag{25}$$

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} & M_{27} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} & M_{37} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} & M_{47} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} & M_{57} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} & M_{67} \\ M_{71} & M_{72} & M_{73} & M_{74} & M_{75} & M_{76} & M_{77} \end{bmatrix} \tag{26}$$

The elements that appear in the matrix (Eqs. (25) and (26)) are defined as follows

$$\begin{aligned} K_{11} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](A_{11}k_x^2 + A_{66}k_y^2) \\ K_{12} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](A_{12} + A_{66})k_x k_y \\ K_{14} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](B_{11}k_x^2 + B_{66}k_y^2) \\ K_{15} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](D_{11}k_x^2 + D_{66}k_y^2) \\ K_{16} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](B_{12} + B_{66})k_x k_y \\ K_{17} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](D_{12} + D_{66})k_x k_y \end{aligned} \tag{27}$$

$$\begin{aligned} K_{21} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](A_{12} + A_{66})k_x k_y \\ K_{22} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](A_{22}k_y^2 + A_{66}k_x^2) \\ K_{24} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](B_{12} + B_{66})k_x k_y \\ K_{25} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](D_{12} + D_{66})k_x k_y \\ K_{26} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](B_{22}k_y^2 + B_{66}k_x^2) \\ K_{27} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](D_{22}k_y^2 + D_{66}k_x^2) \end{aligned} \tag{28}$$

$$\begin{aligned} K_{33} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](A_{44}k_y^2 + A_{55}k_x^2) \\ &+ \left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right]\left(\frac{k_l k_u}{k_l + k_u}\right)\left[1 + (k_x^2 + k_y^2)\right] \\ K_{34} &= \left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](iA_{55}k_x) \\ K_{35} &= \left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](2iB_{55}k_x) \\ K_{36} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](iA_{44}k_y) \\ K_{37} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](2iB_{44}k_y) \end{aligned} \tag{29}$$

$$\begin{aligned} K_{41} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](B_{11}k_x^2 + B_{66}k_y^2) \\ K_{42} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](B_{12} + B_{66})k_x k_y \\ K_{43} &= \left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](-iA_{55}\ k_x) \\ K_{44} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](D_{11}k_x^2 + D_{66}k_y^2 + A_{55}) \\ K_{45} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](E_{11}\ k_x^2 + E_{66}k_y^2 + 2B_{55}) \\ K_{46} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](D_{12} + D_{66})k_x k_y \\ K_{47} &= -\left[1 + \ell_2^2\left(k_x^2 + k_y^2\right)\right](E_{12} + E_{66})k_x k_y \end{aligned} \tag{30}$$

$$
\begin{aligned}
K_{51} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](D_{11}k_x^2+D_{66}k_y^2) \\
K_{52} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](D_{12}+D_{66})k_xk_y \\
K_{53} &= \left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](-2iB_{55}k_x) \\
K_{54} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](E_{11}k_x^2+E_{66}k_y^2+2B_{55}) \\
K_{55} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](F_{11}\,k_x^2+F_{66}k_y^2+4D_{55}) \\
K_{56} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](E_{12}+E_{66})k_xk_y \\
K_{57} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](F_{12}+F_{66})k_xk_y
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
K_{61} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](B_{12}+B_{66})k_xk_y \\
K_{62} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](B_{22}k_y^2+B_{66}\,k_x^2) \\
K_{63} &= \left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](-iA_{44}k_y) \\
K_{64} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](D_{12}+D_{66})k_xk_y \\
K_{65} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](E_{12}k_xk_y+E_{66}k_xk_y) \\
K_{66} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](D_{22}k_y^2+D_{66}k_x^2+A_{44}) \\
K_{67} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](E_{22}k_y^2+E_{66}\,k_x^2+2B_{44})
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
K_{71} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](D_{12}+D_{66})k_xk_y \\
K_{72} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](D_{22}k_y^2+D_{66}k_x^2) \\
K_{73} &= \left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](-2iB_{44}k_y) \\
K_{74} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](E_{12}+E_{66})k_xk_y \\
K_{75} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](F_{12}+F_{66})k_xk_y \\
K_{76} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](E_{22}\,k_y^2+E_{66}k_x^2+2B_{44}) \\
K_{77} &= -\left[1+\ell_2^2\left(k_x^2+k_y^2\right)\right](F_{22}k_y^2+F_{66}k_x^2+4D_{44})
\end{aligned} \tag{33}
$$

and

$$
\begin{aligned}
M_{11} = M_{22} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_0 \\
M_{14} = M_{26} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_1 \\
M_{15} = M_{27} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_2
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
M_{41} = M_{62} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_1 \\
M_{44} = M_{66} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_2 \\
M_{45} = M_{67} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_3
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
M_{51} = M_{71} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_2 \\
M_{54} = M_{76} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_3 \\
M_{55} = M_{77} &= \left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right]I_4
\end{aligned} \tag{36}
$$

$$
M_{33} = I_0\left[1+\ell_1^2\left(k_x^2+k_y^2\right)\right] \tag{37}
$$

$$
K_{13} = K_{23} = K_{31} = K_{32} = 0 \tag{38}
$$

$$
\begin{aligned}
M_{12} &= M_{13} = M_{21} = M_{23} = M_{24} = M_{25} \\
&= M_{31} = M_{32} = M_{34} = M_{35} = M_{36} \\
&= M_{37} = M_{42} = M_{43} = M_{46} = M_{47} \\
&= M_{52} = M_{53} = M_{56} = M_{57} = M_{61} \\
&= M_{63} = M_{64} = M_{65} = M_{72} = M_{73} \\
&= M_{74} = M_{75} = 0
\end{aligned} \tag{39}
$$

The phase velocity and group velocity can be given by the following formulas

$$
\begin{bmatrix} C_P \\ C_g \end{bmatrix} = \begin{bmatrix} \dfrac{\omega}{k} \\ \dfrac{d\omega}{dk} \end{bmatrix} \tag{40}
$$

with $k=k_x=k_y$.

5. Examples

Here, the verification studies are carried out, the present wave propagation is compared to the results of the general third-order shear deformation theory (GTSDT) by Karami *et al.* (2019), as well as the refined shear deformation theory (RSDT) by Karami *et al.* (2018), as seen, the agreement is excellent.

In Fig. 3, the small parameters is characterized by (μ,l)=(0, 0) for classical elasticity theory (CET), (μ,l)=(1 nm, 0) for nonlocal elasticity theory (NET), (μ,l)=(0, 0.2 nm) for strain gradient theory (SGT), (μ,l)=(1 nm, 0.2 nm) for nonlocal strain gradient theory (NSGT). As expected, the nonlocal parameter μ shows stiffness softening effect while the strain gradient parameter l shows the stiffness strengthening effect. Of course, the size effects only work when the wavenumber is larger enough.

Shown in Fig. 4 is the diagrams of the wave propagation for the nanoplates with different weight fraction g^*_{GNP} (=0, 0.5%, 1%). As seen, the phase velocity tends to increase with the increase of g^*_{GNP}.

In Fig.5, the nanoplates without any foundation ($k_l=k_u=k_s=0$), with Winkler-foundation ($k_l=k_u=0$, k_s=4 N/m), with Pasternak-foundation (k_l= 5×10[15] N/m[3], k_u=0, k_s=4 N/m), with Kerr-foundation ($k_l=k_u=5\times10^{15}$ N/m^3, k_s=4 N/m) are considered. As seen, the presence of the Winkler-foundation, Pasternak-foundation and Kerr-foundation all can increase the stiffness of the nanoplates, and correspondingly increase the phase velocity of the nanoplates.

Shown in Fig. 6 is the diagrams of the group velocity for the nanoplates with different theories (including NET, SGT, NSGT). As seen, different theories have great influence on the propagation characteristics of waves and the group velocity of the nanoplates.

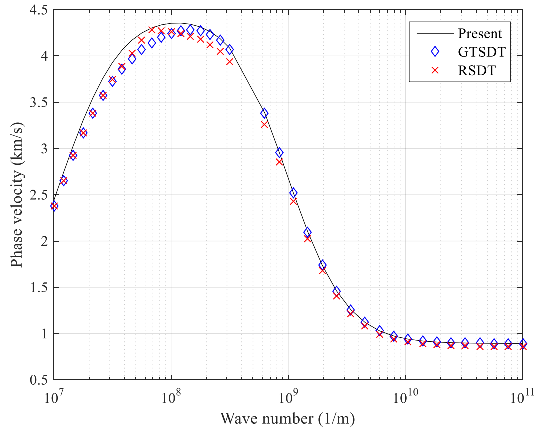

Fig. 2 Comparison of phase velocity in a rectangular nanoplate, (E=70GPa, ρ=2702 kg/m^3, ν=0.3, h=100 nm, μ=1 nm and l=0.2 nm)

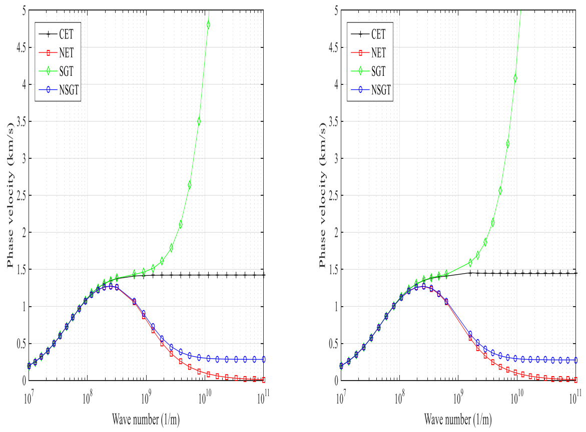

Fig. 3 Dispersion relation of the phase velocity for various theories at g^*_{GNP}=0.01

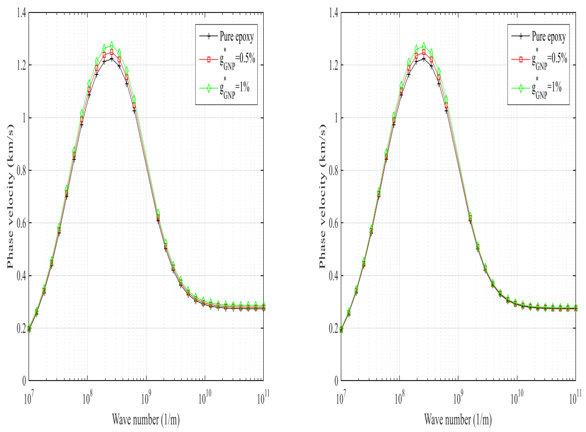

Fig. 4 Dispersion relation of the phase velocity for various weight fraction g^*_{GNP} at μ=1 nm and l=0.2 nm

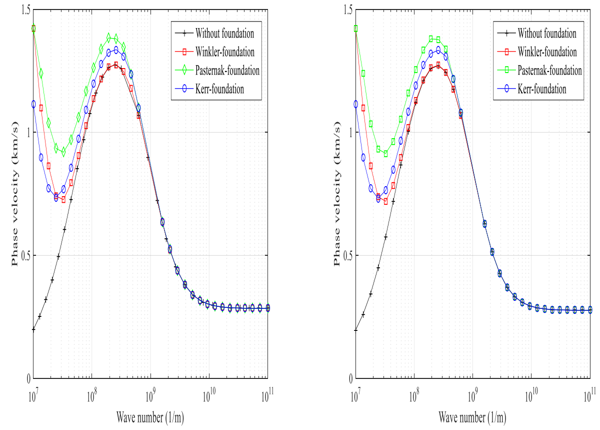

Fig. 5 Dispersion relation of the phase velocity for various elastic foundations at g^*_{GNP} =0.01, μ=1 nm and l=0.2 nm

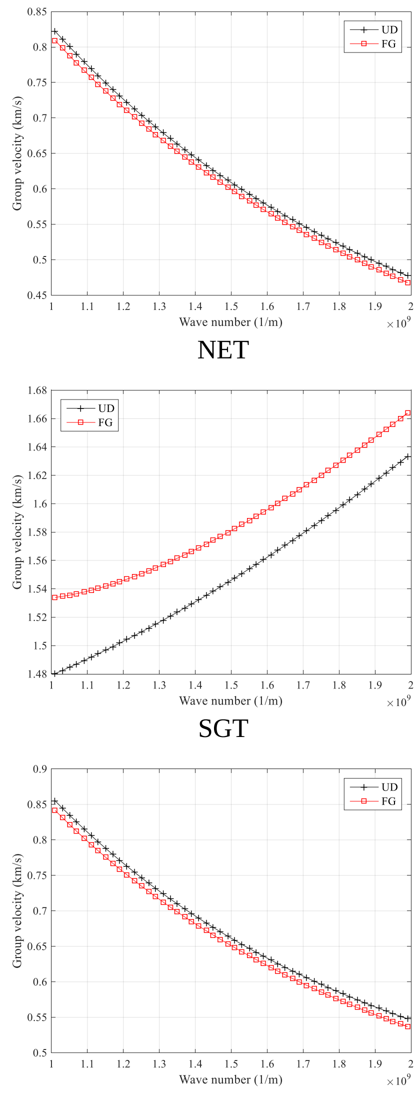

Fig. 6 Dispersion relation of the group velocity for various theories at g^*_{GNP} =0.01, k_l=k_u=5×10^{15} N/m^3, k_s=4 N/m, μ=1 nm and l=0.2 nm

6. Conclusions

The wave propagations of the FG-PC nanoplates reinforced with GNPs have been studied in this paper, the nonlocal strain gradient model for the nanoplates has been developed using the second-order shear deformation theory. The wave propagation analyses are solved using trial function. The correctness of this paper has been verified by comparing with the existing results. The numerical analyses shows that:

- The non-local parameters reduce the propagation velocity (including phase velocity and group velocity) of the wave, while the strain gradient parameter has the opposite effect.
- With the increase of weight fraction g^{*}_{GNP}, the propagation velocity of the wave shows an upward trend.
- The presence of the Winkler-foundation, Pasternak-foundation and Kerr-foundation all can increase the phase velocity of the nanoplates.

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