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Strain gradient based static stability analysis of composite crystalline shell structures having porosities

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Abstract. This paper studies nonlinear stability behavior of a nanocrystalline silicon curved nanoshell considering strain gradient size-dependency. Nanocrystallines are composite materials with an interface phase and randomly distributed nano-size grains and pores. Imperfectness of the curved nanoshell has been defined based on an initial deflection. The formulation of nanocrystalline nanoshell has been established by thin shell theory and an analytical approach has been used in order to solve the buckling problem. For accurately describing the size effects related to nano-grains or nano-pores, their surface energies have been included. Nonlinear stability curves of the nanoshell are affected by the size of nano-grain, curvature radius and nano-pore volume fraction. It is found that increasing the nano-pore volume fraction results in lower buckling loads.

Keywords: nanocrystalline material; nonlinear stability; porous nanoshell; Strain gradient; Mori-Tanaka scheme

1. Introduction

Silicon is a basic material used in sensing systems and structures which may have macro, micron or nano dimensions. This material has not a perfect and ideal structure and it may possess small size pores. Pores or voids in material texture of silicon leads to the variation in material attributes. Also, grains are possible to be created within silicon and hence this type of material would be a crystalline material. Actually, the crystalline materials have grains or crystals of silicone together with voids and an interface zone between the grains and voids (Wang *et al.* 2003). The distribution of grains and voids within material structure would be random and it is not possible to place them in prescribed locations. In fact, the grains growth in possible positions during the fabrication of crystalline materials. Moreover, if the dimensions of grains are reduced to nano scales, the material would be a nanocrystalline material (Meyers *et al.* 2006). There are diverse approaches for describing material properties of nanocrystalline materials (Zhou *et al.* 2013) having grains and voids.

Shell structures have great application in mechanical devices and system form macro to micro/nano dimensions. Macro size shells are extensively researched via classic elasticity theory in the view of structural dynamic analysis. However, classic elasticity theory is not appropriate for nano dimension shells for which small scale impacts exist. Thus, another theories to carry out size-dependent dynamical analysis of nano dimension structural components are strain gradient and nonlocal elasticity theories (Aydogdu 2009, Thai *et al.* 2012, Ke *et al.* 2012, Eltaher *et al.* 2013, Barati 2017, Al-Maliki *et al.* 2019, Ahmed *et al.* 2019). Nonlocal elastic theory were used by various authors in order to incorporate small scale impacts in analysis of nanostructures based on a single scale factor (Lim 2010, Li 2014, Li *et al.* 2013, Zenkour and Abouelregal 2014, Ebrahimi and Barati 2016, 2017a, Barati and Shahverdi 2016, 2017a, Bounouara *et al.* 2016, Besseghier *et al.* 2017, Mokhtar *et al.* 2018). The scale factor defined by nonlocal elastic theory leads to structural rigidity reduction which highlights that nano size structures have different mechanical performance from macro scale counterparts. One another scale factor is defined by strain gradient theory leading to structural rigidity increment. The strain gradient theory express that the strains are not uniform within the material structures. Therefore, this theory would be useful for modeling of nanocrystalline materials and structures. For various types of materials and structures, the strain gradient theory has shown its efficacy (Lim *et al.* 2015, Li *et al.* 2016, Mehralian *et al.* 2017, Barati and Shahverdi 2017b).

Mechanical analysis of nanocrystalline structures has been carried out by few researches. Especially nanocrysalline nanoshells having nano-size grains and pores are not studied before. However, some papers are published on nanocrystalline nanoplates and nanobeams based on strain gradient theory taking into account the size of pores and grains (Ebrahimi and Barati 2017b, 2018, Barati and Shahverdi 2017c,d). For other types of materials rather than nanocrystalline materials, some researchers studied the mechanical properties of elastic nanoshells based on nonlocal and strain gradient theories and proved the efficacy of the theories (Zaera *et al.* 2013, Ke *et al.* 2014, Mehralian *et al.* 2016, Farajpour *et al.* 2017, Sun *et al.* 2016).

Within the context of strain gradient elasticity, the present article investigates static stability characteristics of a crystalline nano-sized shell having voids and nanograins. Hence, the nano-size shell has been constructed from a

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multi-phase porous composite for which each material property dependents on the scale of nanograins. Furthermore, for taking into account low size impacts more properly, the surface energies associated with grains and voids have been included. For taking into account all above-mentioned parameters, a micro-mechanic formulation has been incorporated for defining each material property of the shell. An analytical trend has been introduced for solving the governing equations. It would be stated that the static stability of the crystalline curved shell relies on pores size, grains size, pores percentages, initial deflection and strain gradient factor.

2. Model of nanocrystalline nanoshells

As shown in Fig. 1, the curved micro-size shell has been made of crystalline composites containing nano-pores and nano-grains. Elastic properties (Young's moduli and Poisson's ratio) for a crystalline composite might be introduced as functions of bulk and shear moduli (K_{NcM}, μ_{NcM}) based on below relations

$$E_{NcM} = \frac{9K_{NcM}\mu_{NcM}}{3K_{NcM} + \mu_{NcM}} \tag{1}$$

$$v_{NcM} = \frac{3K_{NcM} - 2\mu_{NcM}}{2(3K_{NcM} + \mu_{NcM})} \tag{2}$$

So that

$$K_{NcM} \cong k_{H_1} \times k_{H_2} \times \frac{1}{\eta k_g} \tag{3a}$$

$$\mu_{NcM} \cong \mu_{H_1} \times \mu_{H_2} \times \frac{1}{\eta \mu_g} \tag{3b}$$

So that $\eta = E_{in} / E_g$ and also

$$k_{H_1} = k_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta\mu_g, k_g, \mu_g, f_g, k_g^s, \mu_g^s, v_{in} = v_g, R_g) \tag{4a}$$

$$\mu_{H_1} = \mu_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta\mu_g, k_g, \mu_g, f_g, k_g^s, \mu_g^s, v_{in} = v_g, R_g) \tag{4b}$$

$$k_{H_2} = k_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta\mu_g, k_g = 0, \mu_g = 0, f_v, k_v^s, \mu_v^s, v_v, R_v) \tag{4c}$$

$$\mu_{H_2} = \mu_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta\mu_g, k_g = 0, \mu_g = 0, f_v, k_v^s, \mu_v^s, v_v, R_v) \tag{4d}$$

In the introduced equations, g defines the nano-grains material property. Moreover, v defines the void material properties. So, f_g and f_v define grain and void volume fraction introduced by

$$f_g = r(1 - f_v), \quad r = \frac{R_g^3}{(R_g + T_{in})^3} \tag{5}$$

Here, R_g, R_v and T_{in} respectively define the main radiuses of grains, void and interphase thickness. These relations have been utilized in order to define each material property including void effects. Without incorporating void effects, the material properties (Bulk and shear moduli) would be defined as (Ebrahimi and Barati 2017b)

$$k_{eff} = \frac{\begin{bmatrix} 3k_g\left(4f_g\mu_{in} + 3k_{in}\right) \\ +2\mu_{in}\left(4f_g\mu_{in}k_s^* + 3k_{in}\left(2 - 2f_g + k_s^*\right)\right)\end{bmatrix}}{3\left(3\left(1 - f_g\right)k_g + 3f_g k_{in} + 2\mu_{in}\left(2 + k_s^* - f_g k_s^*\right)\right)} \tag{6}$$

$$\mu_{eff} = \frac{\mu_{in}\left(5 - 8f_g\xi_3(7 - 5v_{in})\right)}{5 - f_g(5 - 84\xi_1 - 20\xi_2)} \tag{7}$$

So that

$$\xi_1 = \frac{15(1 - v_{in})(k_s^* + 2\mu_s^*)}{4H} \tag{8a}$$

$$\xi_2 = \frac{\begin{bmatrix} -15(1 - v_{in})((\frac{\mu_g}{\mu_{in}})(7 + 5v_g) \\ -8v_g(5 + 3k_s^* + \mu_s^*) + 7(4 + 3k_s^* + 2\mu_s^*))\end{bmatrix}}{4H} \tag{8b}$$

$$\begin{aligned}\xi_3 = \frac{5}{16H}&[2(\frac{\mu_g}{\mu_{in}})^2(7 + 5v_g) \\ &-4(7 - 10v_g)(2 + k_s^*)(1 - \mu_s^*) \\ &+(\frac{\mu_g}{\mu_{in}})(7(6 + 5k_s^* + 4\mu_s^*) - v_g(90 + 47k_s^* + 4\mu_s^*))]\end{aligned} \tag{8c}$$

and

$$\begin{aligned} H = &-2(\frac{\mu_g}{\mu_{in}})^2(7 + 5v_g)(4 - 5v_{in}) \\ &+7(\frac{\mu_g}{\mu_{in}})(-39 - 20k_s^* - 16\mu_s^* + 5v_g(9 + 5k_s^* + 4\mu_s^*)) \\ &+(\frac{\mu_g}{\mu_{in}})v_g(285 + 188k_s^* + 16\mu_s^* \\ &-5v_{in}(75 + 47k_s^* + 4\mu_s^*)) \\ &+4(7 - 10v_g)(-7 - 11\mu_s^* - k_s^*(5 + 4\mu_s^*) \\ &+v_{in}(5 + 13\mu_s^* + k_s^*(4 + 5\mu_s^*)))\end{aligned} \tag{8d}$$

so that $k_s^* = k_g^s / R_g\mu_{in}$ and $\mu_s^* = \mu_g^s / R_g\mu_{in}$ are surfaces bulks and shear moduli, respectively for which $k_g^s = 2(\mu_g^s + \lambda_g^s)$.

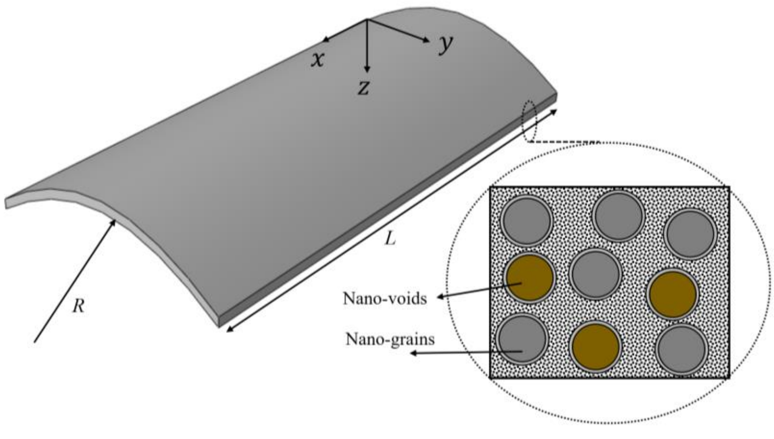

Fig. 1 A curved crystalline shell

3. Mathematical modeling for curved shell

Based on shell thickness *h,* Fig. 1 illustrates a curved shell having curvature radius of *R*. There are various plate/shell theories introduced in the literature (Abualnour *et al.* 2019, Addou *et al.* 2019, Alimirzaei *et al.* 2019, Balubaid *et al.* 2019, Batou *et al.* 2019, Bedia *et al.* 2019, Belbachir *et al.* 2019, Bellal *et al.* 2020, Berghouti *et al.* 2019, Bouamoud *et al.* 2019, Boulefrakh *et al.* 2019, Boukhlif *et al.* 2019, Bourada *et al.* 2019, Boussoula *et al.* 2020, Boutaleb *et al.* 2019, Chaabane *et al.* 2019, Draiche *et al.* 2019, Draoui *et al.* 2019, Hellal *et al.* 2019, Hussain *et al.* 2019, Kaddari *et al.* 2020, Khiloun *et al.* 2019, Khosravi *et al.* 2020, Mahmoudi *et al.* 2019, Medani *et al.* 2019, Meksi *et al.* 2019, Refrafi *et al.* 2020, Rahmani *et al.* 2020, Sahla *et al.* 2019, Younsi *et al.* 2018, Semmah *et al.* 2019, Soltani *et al.* 2019, Tlidji *et al.* 2019, Tounsi *et al.* 2020, Zarga *et al.* 2019, Zaoui *et al.* 2019). The curved micro-panel may be formulated employing thin shell model which represents the strain components as below forms

$$\begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} e_x^0 \\ e_y^0 \\ e_{xy}^0 \end{Bmatrix} - z \begin{Bmatrix} k_x \\ k_y \\ 2k_{xy} \end{Bmatrix} \tag{9}$$

in which

$$\begin{aligned} e_x^0 &= \frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2 + \frac{\partial w}{\partial x}\frac{\partial w^*}{\partial x}, \\ e_y^0 &= \frac{\partial v}{\partial y} + \frac{1}{2}(\frac{\partial w}{\partial y})^2 + \frac{\partial w}{\partial y}\frac{\partial w^*}{\partial y} - \frac{w}{R}, \\ e_{xy}^0 &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ &+ \frac{\partial w}{\partial x}\frac{\partial w^*}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial w^*}{\partial x}, \\ k_x &= \frac{\partial^2 w}{\partial x^2},\ k_y = \frac{\partial^2 w}{\partial y^2},\ k_{xy} = \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{10}$$

Defined strains incorporates deflection (w) and in-plane (u, v) displacements. Also, w^* defines the deflection due to imperfectness. In the case of micro-size structures, the relations for stresses σ_i ($i=x, y, xy$) may be defined in the context of strain gradient elasticity as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = (1 - l^2\nabla^2) \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} \tag{11}$$

in which Q_{ij} may be introduced as

$$\begin{aligned} Q_{11} &= Q_{22} = \frac{E_{NcM}}{1 - v_{NcM}^2}, \\ Q_{12} &= \frac{v_{NcM} E_{NcM}}{1 - v_{NcM}^2},\quad Q_{66} = \frac{E_{NcM}}{2(1 + v_{NcM})} \end{aligned} \tag{12}$$

Here l is called strain gradient coefficient. From integration of Eq. (11) over panel thickness, it may be possible to express the below relations of forces and moments

$$\begin{aligned} N_x &= (1 - l^2\nabla^2)[A_{11}e_x^0 + A_{12}e_y^0 \\ &-B_{11}k_x - B_{21}k_y] \end{aligned} \tag{13}$$

$$\begin{aligned} N_y &= (1 - l^2\nabla^2)[A_{12}e_x^0 + A_{22}e_y^0 \\ &-B_{12}k_x - B_{22}k_y] \end{aligned} \tag{14}$$

$$N_{xy} = (1 - l^2\nabla^2)[+A_{66}e_{xy}^0 - 2B_{66}k_{xy}] \tag{15}$$

$$\begin{aligned} M_x &= (1 - l^2\nabla^2)[B_{11}e_x^0 + B_{12}e_y^0 \\ &-D_{11}k_x + D_{12}k_y] \end{aligned} \tag{16}$$

$$M_y = (1-l^2\nabla^2)[B_{12}e_x^0 + B_{22}e_y^0 - D_{12}k_x - D_{22}k_y] \tag{17}$$

$$M_{xy} = (1-l^2\nabla^2)[+B_{66}e_{xy}^0 - 2D_{66}k_{xy}] \tag{18}$$

in which ∇^2 is Laplacian operator in Cartesian coordinates and

$$A_j = \int_{-h/2}^{h/2} Q_j dz,\ B_j = \int_{-h/2}^{h/2} Q_j z dz,\ D_j = \int_{-h/2}^{h/2} Q_j z^2 dz,\ j = \{11,12,22,66\} \tag{19}$$

A curved panel with curvature (R) owns three governing equations which may be calculated employing Hamilton's rule

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{20}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{21}$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{N_y}{R} + N_x\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w^*}{\partial x^2}\right) + 2N_{xy}\left(\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w^*}{\partial x \partial y}\right) + N_y\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w^*}{\partial y^2}\right) + P_x\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w^*}{\partial x^2}\right) = 0 \tag{22}$$

Note that P_x is applied mechanical load in axial direction. Based on the information that A_j, B_j and D_j are functions of x, the governing equations can be re-written by inserting Eqs. (13)-(18) in Eqs. (20) and (22) as

$$(1-l^2\nabla^2)[\frac{\partial}{\partial x}\{A_{11}e_x^0 + A_{12}e_y^0 - B_{11}k_x - B_{21}k_y\}] + (1-l^2\nabla^2)[\frac{\partial}{\partial y}\{+A_{66}e_{xy}^0 - 2B_{66}k_{xy}\}] = 0 \tag{23}$$

$$(1-l^2\nabla^2)[\frac{\partial}{\partial x}\{+A_{66}e_{xy}^0 - 2B_{66}k_{xy}\}] + (1-l^2\nabla^2)[\frac{\partial}{\partial y}\{A_{12}e_x^0 + A_{22}e_y^0 - B_{12}k_x - B_{22}k_y\}] = 0 \tag{24}$$

$$\begin{aligned}
&(1-l^2\nabla^2)[\frac{\partial^2}{\partial x^2}\{B_{11}e_x^0 + B_{12}e_y^0 - D_{11}k_x + D_{12}k_y\}] + 2(1-l^2\nabla^2)[\frac{\partial^2}{\partial x \partial y}\{+B_{26}e_y^0 + B_{66}e_{xy}^0 - 2D_{66}k_{xy}\}] \\
&+(1-l^2\nabla^2)[\frac{\partial^2}{\partial y^2}\{B_{12}e_x^0 + B_{22}e_y^0 - D_{12}k_x - D_{22}k_y\}] + \frac{1}{R}(1-l^2\nabla^2)[\{A_{12}e_x^0 + A_{22}e_y^0 - B_{12}k_x - B_{22}k_y\}] \\
&+ (1-l^2\nabla^2)[\{A_{11}e_x^0 + A_{12}e_y^0 - B_{11}k_x - B_{21}k_y\}](\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w^*}{\partial x^2}) \\
&+2(1-l^2\nabla^2)[\{+A_{66}e_{xy}^0 - 2B_{66}k_{xy}\}](\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w^*}{\partial x \partial y}) \\
&+(1-l^2\nabla^2)[\{A_{12}e_x^0 + A_{22}e_y^0 - B_{12}k_x - B_{22}k_y\}](\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w^*}{\partial y^2}) \\
&+P_x(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w^*}{\partial x^2}) = 0
\end{aligned} \tag{25}$$

4. Solution procedure

In order to solve Eqs. (23)-(25), it is crucial to express the displacements in reliable forms to satisfy the boundary conditions which in the case of simply-support edges become:

At x=0, L

$$v = w = 0,\quad \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^4 w}{\partial x^4} = \frac{\partial^4 w}{\partial y^4} = 0 \tag{26}$$

At y=0, b

$$u = w = 0,\quad \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^4 w}{\partial x^4} = \frac{\partial^4 w}{\partial y^4} = 0 \tag{27}$$

Then, according to above definitions one can use below approximations for displacement components based on their amplitudes (U, V, W) as (Mirjavadi *et al.* 2019a, b, c, d, e, Forsat 2020, Forsat *et al.* 2020)

$$u = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \frac{\partial f_m^x(x)}{\partial x} f_n^y(y) \tag{28}$$

$$v = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} f_m^x(x) \frac{\partial f_n^y}{\partial y} \tag{29}$$

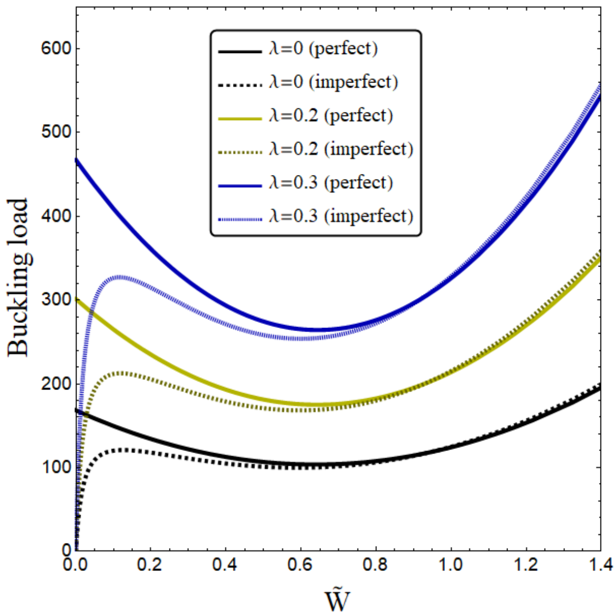

Fig. 2 Nonlinear stability curves of the curved shell based upon different values of the strain gradient coefficient (R =5L, f_v=0.1, R_g=20 nm)

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} f_m^x(x) f_n^y(y) \tag{30}$$

where the functions $f_m^x = \sin[m\pi x / L]$ and $f_n^y = \sin[n\pi y / b]$ are the applicable functions for satisfying the afore-mentioned conditions. Considering each governing equation as R_i (u, v, w)=0 with (i=1,2,3) and inserting displacement assumptions presented as Eqs. (28)-(30) into R_i yields below equations based upon Galerkin's technique

$$\int_0^b \int_0^L R_1 \frac{\partial f_m^x(x)}{\partial x} f_n^y(y) dxdy = 0 \tag{31}$$

$$\int_0^b \int_0^L R_2 f_m^x(x) \frac{\partial f_n^y(y)}{\partial y} dxdy = 0 \tag{32}$$

$$\int_0^b \int_0^L R_3 f_m^x(x) f_n^y(y) dxdy = 0 \tag{33}$$

The weighted residual methods such as Galerkin's method is well discussed in the works of Farrahi *et al.* (2009) and Faghidian *et al.* (2012). By substituting Eqs. (28)-(30) into Eqs. (23)-(25), and using the Galerkin's method, one obtains

$$S_{11}U + S_{21}V + S_{31}W + H_1 W^2 + Y_1 WW^* = 0 \tag{34}$$

$$S_{12}U + S_{22}V + S_{32}W + H_2 W^2 + Y_2 WW^* = 0 \tag{35}$$

$$\begin{aligned} &S_{13}U + S_{23}V + S_{33}W + H_3 W^2 + H_4 W^3 \\ &+H_5 UW + H_6 VW + Y_3 WW^* + Y_4 W^2 W^* \\ &+Y_5 W\left(W^*\right)^2 + Y_6 UW^* + Y_7 VW^* = 0 \end{aligned} \tag{36}$$

in which S_{ij} are linear stiffness matrix components; H_i denotes nonlinear stiffness components and Y_i are added stiffness due to geometric imperfection. With the aid of Eqs. (34) and (35) one can express that

$$\begin{aligned} U &= \frac{S_{21}S_{32} - S_{22}(S_{31} + Y_1 W^*)}{S_{11}S_{22} - S_{12}S_{21}} W + \frac{H_2 S_{21} - H_1 S_{22}}{S_{11}S_{22} - S_{12}S_{21}} W^2 \\ V &= \frac{S_{12}S_{31} - S_{11}(S_{32} + Y_2 W^*)}{S_{11}S_{22} - S_{12}S_{21}} W + \frac{H_1 S_{12} - H_2 S_{11}}{S_{11}S_{22} - S_{12}S_{21}} W^2 \end{aligned} \tag{37}$$

Then, applying Eq. (37) in Eq. (36) yields a single equation based on W and W* only. The solution of obtained for finding P_x (GPa) will give post-buckling curves.

5. Numerical results and discussions

Provided in the present section is post-buckling behavior of curved micro-panels made from crystalline composites containing interface as matrix and grains as inclusions. Material properties of the constituents are presented in Table 1. Geometric imperfectness of the micro-panel is also included. The elastic properties of crystalline materials were determined in the context of surface theory incorporating volume fraction of nano-pores. The dependency of post-buckling loads ($P=P_x/10^6h$)on the pore volume fraction (f_v), imperfectness (W*), normalized strain

Table 1 Material properties of nanocrystalline nanoshell (Shaat and Abdelkefi 2015)

Phase-1 (Interface)	E_{in}=45.56 GPa, v_{in}=0.064, ρ_{in}=2004.3 kg/m^3
Phase-2 (Si-nanograins)	E_g=169 GPa, v_g=0.064, ρ_g=2300 kg/m^3
Phase-3 (nanovoids)	E_v=0
Surface coefficients of grains and voids	λ_s =-4.488 N/m, μ_s =-2.774 N/m

Table 2 Comparison of post-buckling loads of perfect and imperfect panels for different normalized deflections

$\tilde{W}$	$W^*/h=0$		$W^*/h=0.1$	
	Chikh *et al.* (2016)	present	Chikh *et al.* (2016)	present
0	0.62411	0.62411	0	0
0.1	0.62627	0.62627	0.31853	0.31853
0.2	0.63274	0.63274	0.43334	0.43334
0.3	0.64354	0.64354	0.50047	0.50047

Table 3 Comparison of non-dimensional buckling load ($\tilde{N} = P_x L^2 / Eh^3$) of micro-panel for differentstrain gradient coefficients

l/h	Zhang *et al.* (2015)	present
0.1	297.304	297.306
0.2	88.3325	88.3328
0.5	29.7615	29.7617
1	21.3765	21.3768

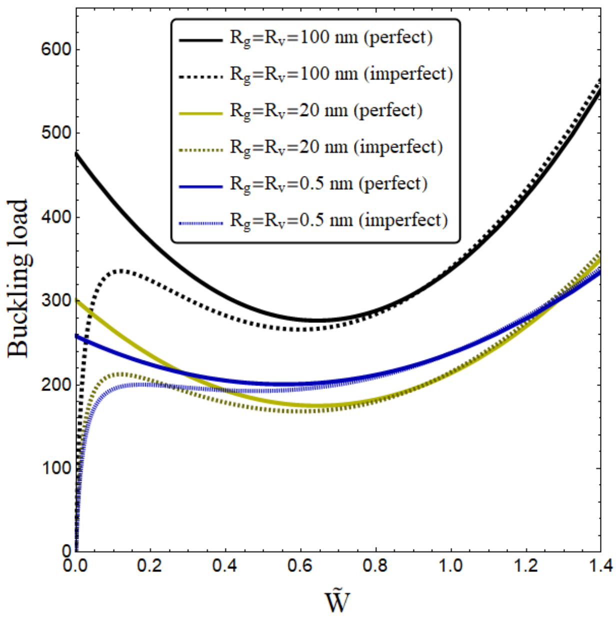

Fig. 3 Nonlinear stability curves of the curved shell based upon different size of the grains (R =5L, f_v=0.1, λ=0.2)

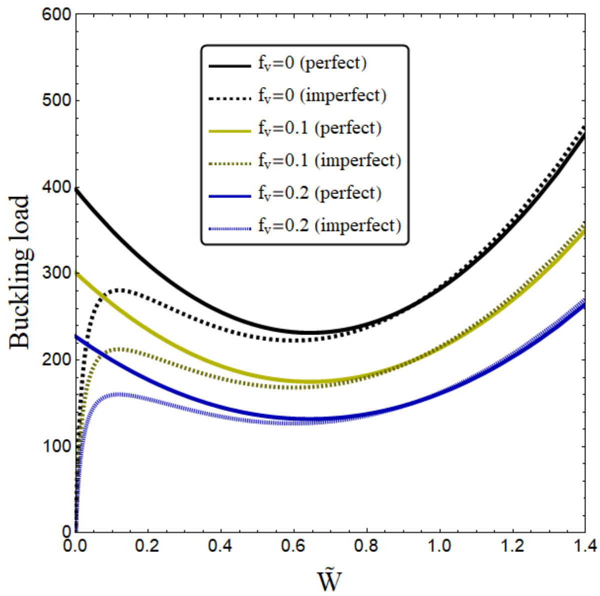

Fig. 4 Nonlinear stability curves of the curved shell based upon different volume of the pores (R_g=20 *nm*, λ=0.2)

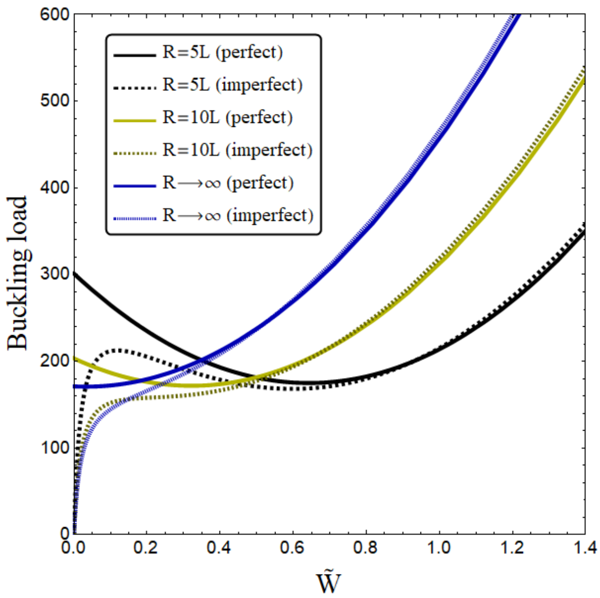

Fig. 5 Nonlinear stability curves of the curved shell based upon different values of curvature radius (R_g=20 *nm*, λ=0.2)

gradient coefficient (λ=l/L), and curvature radius (R) will be evaluated in detail.

As the first step, post-buckling loads of perfect and imperfect panels are validated in Table 2 with those reported by Chikh *et al*. (2016) considering the gradation of material properties. According to this table, buckling loads have been calculated for both perfect ($W^*/h=0$) and imperfect ($W^*/h=0.1$) panel taking into account various normalized deflection ($\tilde{W}=W/h$). Also, validation of the buckling load of a size-dependent micro-panel with the work of Zhang *et al*. (2015) is presented in Table 3 based on different strain gradient coefficients (l/h=0.1, 0.2, 0.5, 1). Obtained buckling loads are in good agreement with those obtained by Zhang *et al*. (2015).

Fig. 2 illustrates buckling load variation versus normalized deflection ($\tilde{W}$) of micro-scale curved panel for

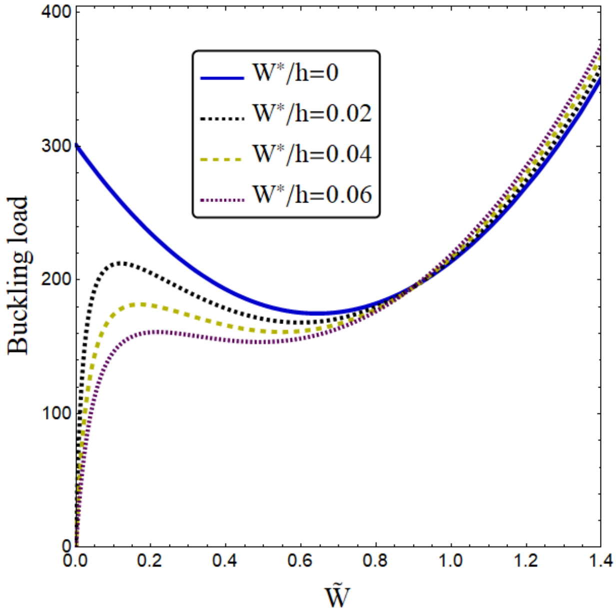

Fig. 6 Nonlinear stability curves of the curved shell based upon different values of imperfection (R_g=20 *nm*, λ=0.2, R=5L)

various strain gradient coefficient (λ). The panel radius is considered as constant, and the grain size is fixed at R_g=20 nm. For perfect panel it must be stated that W^*/h=0 and also for imperfect panel it is assumed that W^*h=0.02. The magnitude of pore volume fraction is selected as f_v=0.1 based on random pore distribution. It is found from the figure that higher values for strain gradient coefficient are corresponding to higher post-buckling curves. This is due to additional stiffness of the micro-panel when train gradient effects are included.

Nonlinear buckling load variation of nano-crystalline curved micro-panels with respect to normalized deflection according to different size of grains has been plotted in Fig. 3. It is assumed that the micro-panel has a curvature radius of R=5L. The present figure indicates that enlarging the magnitude of grain size may lead to greater post-buckling loads. The highest post-buckling curve has been obtained for the case of R_g=100 nm. The reason is additional stiffness of the micro-scale panel via increase of grains size.

Fig. 4 compares the post-buckling path of the nano-crystalline micro-panel between diverse volume fraction of pores at a fixed value of grain size R_g=20 nm. For an imperfect micro-panel it is assumed that W^*/h=0.02. This graph indicates that higher portion of the pores is corresponding to lower post-buckling load values. This is owing to the reason that pore content is increasing in thickness direction of the micro-panel by considering higher pore volume. So, a micro-panel based on lower pore content has extra stiffness compared to highly porous counterpart.

Fig. 5 illustrates the effects of curvature radius (R) on post-buckling curves of nano-crystalline curved micro-size panel when λ=0.2. It must be noted that post-buckling curve of flat panels will be derived by considering curvature radius as infinity (R→∞). Indeed, curvature radius specifies the structural behaviors of curved panel. As instance, by increment of curvature radius, the buckling behaviors of curved micro-panels become closer to flat panels. Thus, based on the graph, it can be seen that at smaller values for R, buckling loads of perfect micro-panels first decrease with the enlargement of normalized deflection because of remarkable effects of panel curvature. However, at larger values of R, buckling load decrement becomes less appreciable.

Depicted in Fig. 6 is the variation of buckling load of micro-size curved panel versus normalized deflection ($\tilde{W} = W / h$) by taking into account different values for geometric imperfection amplitude (W^*/h=0, 0.02, 0.04, 0.06). Strain gradient coefficient is fixed at λ=0.2. It must be clarified that $\tilde{W} = 0$ yields a bifurcation point called critical buckling load in the case of perfect micro-panel. However, there is no bifurcation point for an imperfect micro-panel. Therefore, for an imperfect micro-panel the post-buckling load starts from zero and reaches to post-buckling path of the perfect micro-panel at higher values of normalized deflection. Since micro-panels are not always ideal and they may have initial configuration, it is crucial to incorporate their imperfectness effects.

6. Conclusions

The presented research investigated post-buckling behaviors of geometrically imperfect curved micro-panels made of nano-crystalline composite. Micro-scale effects on the panel structure were included based on strain gradient elasticity. Post-buckling curves were determined based on both perfect and imperfect micro-panel assumptions. There are some interesting findings which can be classified as:

- For an imperfect micro-panel the post-buckling load starts from zero and reaches to post-buckling path of the perfect micro-panel at higher values of normalized deflection.
- Higher values for strain gradient coefficient are corresponding to higher post-buckling curves.
- Enlarging the magnitude of pore volume fraction yields lower post-buckling loads.
- At smaller values for micro-panel curvature, buckling loads of perfect micro-panels first decrease with the enlargement of normalized deflection.
- Post-buckling loads of the curved shell depend on the value of grain size.

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