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Modeling of memory-dependent derivative in a rotating magneto-thermoelastic diffusive medium with variable thermal conductivity

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Abstract. The purpose of this paper is to depict the effect of rotation and initial stress on a magneto-thermoelastic medium with diffusion. The problem discussed within memory-dependent derivative in the context of the three-phase-lag model (3PHL), Green-Naghdi theory of type III (G-N III) and Lord and Shulman theory (L-S). Analytical expressions of the considered variables are obtained by using Laplace-Fourier transforms technique. Numerical results for the field quantities given in the physical domain and illustrated graphically in the absence and presence of a magnetic field, initial stress as well as the rotation. The differences in variable thermal conductivity are also presented at different parameter of thermal conductivity. The numerical results of the field variables are presented graphically to discuss the effect of various parameters of interest. Some special cases are also deduced from the present investigation.

Keywords: diffusion; rotation; initial stress; variable thermal conductivity; memory-dependent derivative

1. Introduction

In recent years, inspired by the successful applications of fractional calculus in different areas of physics and engineering, generalized thermoelasticity (GTE) models have been further extended into temporal fractional ones (Povstenko 2004, Othman *et al*. 2013, Bhatti 2019, Riaz *et al*. 2019, Hendy *et al*. 2020) to express memory-dependence in a heat conductive sense. The memory-dependent derivative is defined in an integral form of a common derivative with a kernel function on a slip in the interval. So this kind of definition is better than the fractional one for reflecting the memory effect (instantaneous change rate depends on the past state). Its definition is more intuitionists for understanding the physical meaning and the corresponding memory-dependent differential equation has more expressive force. Wang and Li (2011) introduced a memory-dependent derivative (MDD). Recently, an interesting application of memory-dependent derivative is given by Yu *et al.* (2014). Al-Jamel *et al.* (2016) studied a memory-dependent derivative model with respect to displacement is proposed to describe damping in various oscillatory systems of complex dissipation mechanisms where memory effects could not be ignored. The two-dimensional problem of two-temperature generalized thermoelasticity using memory-dependent heat transfer: an integral transform approach was discussed by Sarkar and Mondal (2020). Othman and Mondal (2020) introduced the phase-lag models (Lord-Shulman, dual-phase-lag and the three-phase-lag) to study the effect of memory-dependent derivative and the influence of thermal loading due to laser pulse on the wave propagation of generalized micropolar thermoelasticity. Mondal *et al*. (2019) discussed transient response in a piezoelastic medium due to the influence of magnetic field with memory-dependent derivative.

Thermal conductivity is an important parameter of a material which is typically considered constant. However, several experimental and theoretical studies have indicated that the thermal conductivity is closely related to temperature change (Singh 2014, Dogonchi and Ganji 2016, Zarga *et al*. 2019, Medani 2019). The effect of gravity and hydrostatic initial stress with variable thermal conductivity on a magneto-fiber-reinforced was studied by Said and Othman (2020). Othman *et al*. (2019) applied the dual-phase-lag theory to study the two-dimensional problem of generalized thermoelasticity for a fiber-reinforced thick plate under initial stress and variable thermal conductivity. Abd-Elaziz *et al*. (2019) explained the effect of Thomson and initial stress in a thermo-porous elastic solid under the GN electromagnetic theory. Marin *et al*. (2016), Hosseinzadeh *et al.* (2019a), Hosseinzadeh *et al.* (2019b), Hosseinzadeh *et al*. (2020a), Hosseinzadeh *et al.* (2020b), Hosseinzadeh *et al.* (2020c), Rostami *et al*. (2020); Gholinia *et al.* (2020); Salehi *et al*. (2020) have done pioneer works on this subject.

The theory of magneto-thermoelasticity is concerned with the effect of magnetic field on elastic and thermo-elastic deformations of a solid body and has received the attention of many researchers due to its extensive use in various fields like optics, geophysics, and acoustics. The

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problem of generalized electro-magneto-thermoelastic plane waves by thermal shock problem in a finite conductivity half-space with one relaxation time was studied by Othman (2005). The effect of rotation on the two-dimensional problem of a fibre-reinforced thermoelastic with one relaxation time was investigated by Othman and Said (2012). Othman and Song (2009) used the normal mode method to study the effect of rotation on 2-D thermal shock problems for a generalized magneto-thermoelasticity half-space under three theories. Deswal *et al.* (2014) studied the magneto-thermoelastic interactions in an initially stressed, isotropic, homogeneous half-space. The 2-D problem of magneto–thermoelasticity fiber-reinforced medium under temperature-dependent properties with three-phase-lag theory was explored by Othman and Said (2014).

Since the large bodies like the earth, the moon, and other planets have an angular velocity, it appears more realistic to study the thermoelastic problems in a rotating medium. Some results in thermoelastic rotating medium are due to Roy Choudhuri and Debnath (1983) and Othman and Said (2013). Othman and Abd-Elaziz (2017) used the normal mode technique to study the effect of rotation on a micropolar magneto-thermoelastic medium with dual-phase-lag model under gravitational field.

The present paper concerned with the investigations related to the effect of rotation, initial stress, variable thermal conductivity and magnetic field in the context of the 3PHL model of thermoelasticity with diffusion. The combined Laplace–Fourier transforms are applied to solve the non-dimensional governing equations to find the solutions for the temperatures, displacement components, concentration, chemical potential and stresses in the transform domain. An application is considered to enable us to get complete solutions. The variations of the considered variables with the horizontal distance illustrated graphically. Comparisons made between the three models of the theories of thermoelasticity in the absence and presence of the rotation, initial stress, variable thermal conductivity as well as the magnetic field.

2. Basic equations

The problem of a thermo-diffusion medium is permeated into a uniform magnetic field with a constant intensity $\boldsymbol{H}=(0,H_0,0)$ and rotates with angular velocity $\boldsymbol{\Omega}=(0,\Omega,0)$.

We are interested in a plane strain in the xz-plane with displacement vector $\boldsymbol{u}=(u,0,w)$.

The basic equations in the absence of the body force are

1) The equation of motion as Othman and Said (2013), Schoenberg and Censor (1973)

$$\rho[\ddot{u}_i+\{\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})\}_i+2(\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})]=(\lambda+\mu+\frac{p_1}{2})u_{j,ij}$$
$$+(\mu-\frac{p_1}{2})u_{i,jj}-(\beta_1\theta_{,i}+\beta_2 C_{,i})+\mu_0(\boldsymbol{J}\times\boldsymbol{H})_i\,. \tag{1}$$

2)The heat conduction equation as Roy Choudhuri (2007)

$$K^*(1+\tau_v \mathrm{D}_{w_3})\nabla^2\theta+K(1+\tau_T \mathrm{D}_{w_2})\nabla^2\theta_{,t}=(1+\tau_q \mathrm{D}_{w_1}+\frac{1}{2}\tau_q^2 \mathrm{D}_{w_1}^2)[\rho C_E(n_0\theta_{,tt}+n_1\theta_{,t})$$
$$+\beta_1 T_0(n_0 e_{,tt}+n_1 e_{,t})+aT_0(n_0 C_{,tt}+n_1 C_{,t})]. \tag{2}$$

3) The generalized diffusion equation as Othman *et al.* (2013)

$$d\,\beta_2 e_{kk,ii}+da\theta_{,ii}+(n_1+n_0\tau_q\frac{\partial}{\partial t}+\frac{1}{2}n_0\tau_q^2\frac{\partial^2}{\partial t^2})\,C-dbC_{,ii}=0. \tag{3}$$

4) The constitutive equation

$$\sigma_{ij}=\lambda e_{kk}\delta_{ij}+2\mu e_{ij}-(\beta_1\theta+\beta_2 C)\delta_{ij}-P_1(\omega_{ij}+\delta_{ij}), \tag{4}$$

$$P=-\beta_2 e_{kk}-a\theta+bC, \tag{5}$$

where $\mu_0(\boldsymbol{J}\times\boldsymbol{H})$ is the Lorentz force given as Othman and Said (2013).

D_{w_i} is the memory-dependent derivative operator is defined as Yu *et al.* (2014)

$$\mathrm{D}_{w_i} f(t)=\frac{1}{w_i}\int_{t-w_i}^{t} L(t-\beta)f'(\beta)d\beta. \tag{6}$$

The parameter w_i is the time-delay and $L(t-\beta)$ is the kernel function in which they can be chosen freely, see Caputo and Mainardi (1971 a,b,c) for more explanations.

$$L(t-\beta)=1-\frac{2a_0}{\varpi}(t-\beta)+b_0^2\frac{(t-\beta)^2}{\varpi^2}. \tag{7}$$

In the present paper, we take $L(t-\beta)=q+n(t-\beta)$, where a_0, b_0 are constants.

The field Eqs. (1) and (4) become

$$\rho\,(\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t})=A_1\frac{\partial^2 u}{\partial x^2}+A_4\frac{\partial^2 w}{\partial x\partial z}+A_3\frac{\partial^2 u}{\partial z^2}-\beta_1\frac{\partial\theta}{\partial x}$$
$$-\beta_2\frac{\partial C}{\partial x}-\mu_0 H_0\frac{\partial h}{\partial x}-\varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$\rho\,(\frac{\partial^2 \omega}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t})=A_3\frac{\partial^2 w}{\partial x^2}+A_4\frac{\partial^2 u}{\partial x\partial z}+A_1\frac{\partial^2 w}{\partial z^2}-\beta_1\frac{\partial\theta}{\partial z}$$
$$-\beta_2\frac{\partial C}{\partial z}-\mu_0 H_0\frac{\partial h}{\partial z}-\varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 \omega}{\partial t^2}, \tag{9}$$

where $A_1=\lambda+2\mu$, $A_2=\mu+\frac{P_1}{2}$, $A_3=\mu-\frac{P_1}{2}$, $A_4=\lambda+A_2$.

Introducing the following non-dimension quantities

$$(x',z',u',w')=c_0\eta\,(x,z,u,w),\ (t',\tau_q',\tau_v',\tau_T')=c_0^2\eta\,(t,\tau_q,\tau_v,\tau_T),$$
$$h'=\frac{h}{H_0},\quad \sigma_{ij}'=\frac{\sigma_{ij}}{\mu},\quad \theta'=\frac{\beta_1\theta}{(\lambda+2\mu)},\ C'=\frac{C}{\rho},\ P'=\frac{P}{\beta_2},\quad \Omega'=\frac{\Omega}{c_0^2\eta}, \tag{10}$$
$$P_1'=\frac{1}{\mu}P_1,\qquad i,j=1,2.$$

Where $\eta=\frac{\rho C_E}{K^*}$, $c_0^2=\frac{(\lambda+2\mu)}{\rho}$.

Using Eq. (10), thus we have

$$A_5\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t}=A_6\frac{\partial^2 u}{\partial x^2}+A_8\frac{\partial^2 w}{\partial x\partial z}+A_7\frac{\partial^2 u}{\partial z^2}-\frac{\partial\theta}{\partial x}-A_9\frac{\partial C}{\partial x}, \tag{11}$$

$$A_5\frac{\partial^2 \omega}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}=A_7\frac{\partial^2 w}{\partial x^2}+A_8\frac{\partial^2 u}{\partial x\partial z}+A_6\frac{\partial^2 w}{\partial z^2}-\frac{\partial\theta}{\partial z}-A_9\frac{\partial C}{\partial z}, \tag{12}$$

$$e_{kk,ii}+A_{10}\theta_{,ii}+(n_1+n_0\tau_q\frac{\partial}{\partial t}+\frac{1}{2}n_0\tau_q^2\frac{\partial^2}{\partial t^2})A_{11}C-A_{12}C_{,ii}=0, \tag{13}$$

$$(1+\tau_v\frac{\partial}{\partial t})\nabla^2\theta+A_{13}(1+\tau_T\frac{\partial}{\partial t})\nabla^2\theta_{,t}=(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})[(A_{14}\theta_{,tt}+A_{17}\theta_{,t})$$
$$+(A_{15}e_{,tt}+A_{18}e_{,t})+(A_{16}C_{,tt}+A_{19}C_{,t})]. \tag{14}$$

Where, $A_5=1+\frac{\varepsilon_0\mu_0^2H_0^2}{\rho}$, $A_6=\frac{A_1+\mu_0H_0^3}{\rho c_0^2}$, $A_7=\frac{A_3}{\rho c_0^2}$, $A_8=\frac{A_4+\mu_0H_0^3}{\rho c_0^2}$,

$A_9=\frac{\beta_2}{c_0^2}$, $A_{10}=\frac{a(\lambda+2\mu)}{\beta_1\beta_2}$, $A_{11}=\frac{\rho}{d\,\beta_2c_0^2\eta^2}$, $A_{12}=\frac{b\rho}{\beta_2}$, $A_{13}=\frac{Kc_0^2\eta}{K^*}$,

$A_{14}=\frac{\rho C_E n_0c_0^2}{K^*}$, $A_{15}=\frac{\beta_1^2T_0n_0c_0^2}{K^*(\lambda+2\mu)}$, $A_{16}=\frac{aT_0n_0\rho\beta_1c_0^2}{K^*(\lambda+2\mu)}$, $A_{17}=\frac{\rho C_E n_1}{\eta K^*}$,

$A_{18}=\frac{\beta_1^2T_0n_1}{\eta K^*(\lambda+2\mu)}$, $A_{19}=\frac{aT_0n_1\rho\beta_1}{\eta K^*(\lambda+2\mu)}$.

We will consider the thermal conductivity as a linear function of thermodynamically temperature as follows

$$K=K(\theta)=K_0(1+K_1\theta). \tag{15}$$

Where K_0 is a constant which is equal to the thermal conductivity of the material when it does not depend on the thermo-dynamical temperature (θ) and K_1 is a non-positive small parameter.

$$\psi=\frac{1}{K_0}\int_0^{\theta}K(\theta')d\theta'. \tag{16}$$

The above equations with the aid of Eq. (15) give

$$\psi=\theta(1+\frac{K_1}{2}\theta) \tag{17}$$

For linearity, then the above equation will be reduced to

$$\frac{\partial\theta}{\partial x_i}=\frac{\partial\psi}{\partial x_i},\qquad \frac{\partial\theta}{\partial t}=\frac{\partial\psi}{\partial t}. \tag{18}$$

3. Solution of the problem

In this part, the governing equations derived in the previous section are going to be solved analytically. Applying the Laplace and Fourier transform defined by

$$\bar f(x,z,p)=\int_0^{\infty}f(x,z,t)e^{-pt}dt, \tag{19}$$

$$f^*(\zeta,z,p)=\int_{-\infty}^{\infty}\bar f(x,z,p)e^{i\zeta x}dx. \tag{20}$$

Using Eqs. (19), (20) in Eqs. (11) - (14), we obtain

$$(A_7\mathrm{D}^2-N_1)u^*+(i\zeta A_8\mathrm{D}-2\Omega p)w^*-i\zeta\psi^*-i\zeta A_9C^*=0, \tag{21}$$

$$(i\zeta A_8\mathrm{D}+2\Omega p)u^*+(A_6\mathrm{D}^2-N_2)w^*-\mathrm{D}\psi^*-A_9\mathrm{D}C^*=0, \tag{22}$$

$$i\zeta(\mathrm{D}^2-\zeta^2)u^*+\mathrm{D}(\mathrm{D}^2-\zeta^2)w^*+A_{10}(\mathrm{D}^2-\zeta^2)\psi^*-(A_{12}\mathrm{D}^2-N_3)C^*=0, \tag{23}$$

$$i\zeta N_4u^*+N_4\mathrm{D}w^*-(N_5\mathrm{D}^2-N_6)\psi^*+N_7C^*=0, \tag{24}$$

where, $N_1=A_5p^2-\Omega^2+A_6\zeta^2$, $N_2=A_7\zeta^2-\Omega^2+A_5p^2$,

$N_3=A_{12}\zeta^2+(n_1+n_0G_1+n_0G_2)A_{11}$, $N_4=(A_{15}p^2+A_{18}p)(1+G_1+G_2)$,

$N_5=1+G_3+A_{13}p(1+G_4)$, $N_6=N_5\zeta^2+(1+G_1+G_2)(A_{14}p^2+A_{17}p)$,

$N_7=(1+G_1+G_2)(A_{16}p^2+A_{19}p)$,

$G_1=\frac{\tau_q}{w_1}[\frac{qp+n}{p}(1-e^{-pw_1})-nw_1e^{-pw_1}]$,

$G_2=\frac{p\,\tau_q^2}{2w_1}[\frac{qp+n}{p}(1-e^{-pw_1})-nw_1e^{-pw_1}]$,

$G_3=\frac{\tau_T}{w_2}[\frac{qp+n}{p}(1-e^{-pw_2})-nw_2e^{-pw_2}]$,

$G_4=\frac{\tau_\nu}{w_3}[\frac{qp+n}{p}(1-e^{-pw_3})-nw_3e^{-pw_3}]$, $\mathrm{D}=\frac{\mathrm{d}}{\mathrm{d}z}$,

w_1, w_2, w_3 are the time delay for three-phase-heat equation.

Solving Eqs. (21)- (24), (by using the Matlab program) we have

$$(\mathrm{D}^8-L_1\mathrm{D}^6+L_2\mathrm{D}^4-L_3\mathrm{D}^2+L_4)(u^*,w^*,\psi^*,C^*)(z)=0. \tag{25}$$

Where $L_0=N_5N_7A_7(N_{11}A_{12}-N_{12}A_9)$,

$$L_1=\frac{1}{L_0}\{N_2N_5N_7^2A_7A_{12}+2N_{12}N_5N_9A_9\zeta^2+N_{11}N_5N_8A_{12}+N_7N_{11}N_{14}A_7-N_{11}N_{12}N_5A_9\zeta^2+N_{10}N_{12}N_7A_7-N_5N_7N_{13}A_9A_7-N_5N_8N_{12}A_9-N_5N_9^2A_{12}\zeta^2\},$$

$$L_2=\frac{1}{L_0}\{N_2N_5N_7N_8A_{12}+N_2N_{14}N_7^2A_7-N_{12}N_5N_2N_7A_9\zeta^2-2N_9N_{10}N_{12}\zeta^2+2N_5N_9N_{13}A_9\zeta^2+N_{11}N_{15}N_7A_7+N_8N_{14}N_{11}+N_{10}N_{11}N_{12}\zeta^2-N_{13}N_{11}N_5A_9\zeta^2+N_{10}N_{13}N_7A_7+N_8N_{12}N_{10}-N_5N_{13}N_8A_9-N_{14}N_9^2\zeta^2+4\Omega^2p^2N_5N_7^2A_{12}\},$$

$$L_3=\frac{1}{L_0}\{N_2N_{15}N_7^2A_7+N_2N_7N_8N_{14}+N_2N_7N_{10}N_{12}\zeta^2-N_2N_7N_5N_{13}A_9\zeta^2-2N_9N_{10}N_{13}\zeta^2+N_8N_{11}N_{15}+N_{10}N_{11}N_{13}\zeta^2+N_8N_{10}N_{13}-N_9^2N_{15}\zeta^2+4\Omega^2p^2N_7^2N_{14}\},$$

$$L_4=\frac{1}{L_0}\{N_2N_7N_8N_{15}+N_2N_7N_{10}N_{13}\zeta^2+4\Omega^2p^2N_7^2N_{15}\},$$

$N_8=N_1N_7+N_4A_9\zeta^2$, $N_9=A_8N_7+N_4A_9$, $N_{10}=N_7-N_6A_9$,

$N_{11}=A_6N_7+N_4A_9$, $N_{12}=N_7+N_4A_{12}$, $N_{13}=N_3N_4+N_7\zeta^2$,

$N_{14}=A_{10}N_7+N_3N_5+N_6A_{12}$, $N_{15}=N_3N_6+N_7A_{10}\zeta^2$.

Solution of Eq. (25), which bound as $z\to\infty$, is given by

$$u^*(z)=\sum_{n=1}^{4}M_n\exp(-k_nz), \tag{26}$$

where $k_n^2(n=1,2,3,4)$ are the roots of the following characteristic equation: $k^8-L_1k^6+L_2k^4-L_3k^2-L_4=0$

In a similar manner, we get that

$$w^*(z)=\sum_{n=1}^{4}H_{1n}M_n\exp(-k_nz), \tag{27}$$

$$\psi^*(z)=\sum_{n=1}^{4}H_{2n}M_n\exp(-k_nz), \tag{28}$$

$$C^*(z)=\sum_{n=1}^{4}H_{3n}M_n\exp(-k_nz), \tag{29}$$

where, $H_{1n}=\frac{A_7N_7k_n^3+(N_9\zeta^2-N_8)k_n+2\mathrm{i}\zeta\Omega pN_7}{\mathrm{i}\zeta(N_9-N_{11})k_n^2+2\Omega pN_7k_n+\mathrm{i}\zeta N_2N_7}$,

$H_{2n}=\frac{A_7N_7k_n^2-N_8-(\mathrm{i}\zeta N_9k_n+2\Omega pN_7)H_{1n}}{\mathrm{i}\zeta(A_9N_5k_n^2+N_{10})}$,

$H_{3n}=\frac{-\mathrm{i}\zeta N_4+N_4k_nH_{1n}+(N_5k_n^2-N_6)H_{2n}}{N_7}$.

Using the above relations, we get

$$\sigma^*_{zz} = \sum_{n=1}^{4} H_{4n} M_n \exp(-k_n z), \tag{30}$$

$$\sigma^*_{xz} = \sum_{n=1}^{4} H_{5n} M_n \exp(-k_n z), \tag{31}$$

$$P^* = \sum_{n=1}^{4} H_{6n} M_n \exp(-k_n z), \tag{32}$$

where $H_{4n} = \frac{1}{\mu}\left(-(\lambda+2\mu)(k_n H_{1n} + H_{2n}) + \mathrm{i}\zeta\lambda - \beta_2 \rho H_{3n}\right)$,

$H_{5n} = \frac{1}{\mu}\left(-A_2 k_n + \mathrm{i}\zeta A_3 H_{1n}\right)$, $H_{6n} = -\mathrm{i}\zeta + k_n H_{1n} - \frac{a(\lambda+2\mu)}{\beta_1} H_{2n} + b\rho H_{3n}$.

By solving the Eq. (17), with the aid of Eqs. (19), (20) and (28), the temperature is

$$\theta^* = \frac{\sqrt{1+2K_1\psi^*} - 1}{K_1}. \tag{33}$$

4. Boundary conditions

The boundary surface of half-space is subjected to mechanical strip load; the boundary conditions on the surface at $z=0$ are as follows

$$\psi = \sigma_{xz} = 0, \quad \frac{\partial C}{\partial z} = 0, \quad \sigma_{zz} = -F_0\,\delta(x)\,\delta(t). \tag{34}$$

Using the expressions of the variables considered into the above boundary conditions (Eqs. (34)), we can obtain the following equations satisfied with the parameters

$$\sum_{n=1}^{4} H_{2n} M_n = 0, \tag{35}$$

$$\sum_{n=1}^{4} k_n H_{3n} M_n = 0, \tag{36}$$

$$\sum_{n=1}^{4} H_{4n} M_n = -\frac{F_0}{p}, \tag{37}$$

$$\sum_{n=1}^{4} H_{5n} M_n = 0. \tag{38}$$

Invoking Eqs. (35)-(38), we obtain a system of four equations. After applying the inverse of the matrix method, we have the values of the four constants M_n $(n = 1,2,3,4)$.

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{21} & H_{22} & H_{23} & H_{24} \\ k_1 H_{31} & k_2 H_{32} & k_3 H_{33} & k_4 H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \\ H_{51} & H_{52} & H_{53} & H_{54} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ -\frac{F_0}{p} \\ 0 \end{pmatrix}. \tag{39}$$

5. Inversion of the transforms

The transformed displacements, thermodynamic temperature the conductive temperature, the stress components and the tangential couple stress are the functions of z and the parameters p and ζ of Laplace and Fourier transforms respectively and hence are of the form $f(z,p,\zeta)$. To obtain the solution of the problem in the physical domain, we invert the Laplace and Fourier transforms by using the method described by Kumar and Rani (2005).

6. Particular cases

a) Equations of the 3PHL model when, $n_0 = 1$, $n_1 = 0$, $K, \tau_T, \tau_q, \tau_\nu > 0$ and the solutions are always (exponentially) stable if $\frac{2K\tau_T}{\tau_q} > \tau^*_\nu > K^*\tau_q$ as in Quintanilla and Racke (2008).

b) Equations of the GN-II theory when, $n_0 = 1$, $n_1 = 0$, $K = \tau_T = \tau_q = \tau_\nu = 0$.

c) Equations of the GN-III theory when, $n_0 = 1$, $n_1 = 0$, $\tau_T = \tau_q = \tau_\nu = 0$.

d) Equations of the L-S theory when, $n_0 = 1$, $n_1 = 0$, $\tau_T = \tau_\nu = 0$, $K, \tau_q \neq 0$.

e) Equations of the CD theory when, $n_0 = 0$, $n_1 = 1$, $K = \tau_T = \tau_q = \tau_\nu = 0$.

7. Numerical results and discussion

In this section, the goal to illustrate numerical results of the analytical expressions obtained in the above section and elucidates the influence of initial stress, rotation, magnetic parameter and variable of thermal conductivity on the behavior of the field quantities. For the computation, Matlab Software package is used as a tool. The physical constants are taken as Thomas (1980)

$\lambda = 7.76\times10^9\,\mathrm{N/m^2}$, $\mu = 3.83\times10^{10}\,\mathrm{N/m^2}$, $T_0 = 298\mathrm{K}$,

$\rho = 8954\,\mathrm{kg/m^3}$, $K^* = 386\,\mathrm{K\times N/s}$, $\alpha_t = 1.98\times10^{-5}\,\mathrm{K^{-1}}$,

$\varepsilon_0 = 0.3\,\mathrm{F\times m^{-1}}$, $\alpha_c = 3.38\times10^{-4}\,\mathrm{Kg^{-1}m^3}$, $C_E = 1383.1\,\mathrm{K.J/kg}$,

$\mu_0 = 1.7\,\mathrm{N\times A^{-2}}$.

The diffusion parameters are

$a = 1.2\times10^4\,\mathrm{m^2/Ks^2}$, $b = 0.9\times10^6\,\mathrm{m^5Kg^{-1}s^{-2}}$,

$d = 0.85\times10^{-8}\,\mathrm{Kg/m^3}$.

In calculation, the other constants of the problem are chosen as

$q = 0.3$, $n = 0.5$, $K_0 = 150\,\mathrm{K\times N/s}$, $\omega_1 = 0.04\,\mathrm{s}$, $\omega_2 = 0.06\,\mathrm{s}$,

$\omega_3 = 0.07\,\mathrm{s}$, $\tau_q = 0.7$, $\tau_T = 0.5$, $\tau_\nu = 0.2$, $\beta = 0.07$,

$F_0 = 100\,\mathrm{N}$, $x = 0.2\mathrm{m}$, $t = 0.03\,\mathrm{s}$, $0 \le z \le 14$.

The comparisons have been made in the context of the three theories, namely; three-phase-lag theory (3PHL), Green and Naghdi theory of type III (G-N III) and Lord and Shulman theory (L-S), in four situations:

(i) With and without rotation ($\Omega = 0.2$ and $\Omega = 0$).

(ii) With and without magnetic field $H_0 = 120$ and $H_0 = 0$.

(iii) With and without initial stress ($p_1 = 30$ and $p_1 = 0$).

(iv) Effect of thermal conductivity ($K_1 = -0.5$ and $K_1 = -1$).

In Figs. 1-7, we established the effect of the rotation parameter, ($\Omega = 0.2$ and $\Omega = 0$), on the behavior of the displacement components u, w, the temperature θ, the mass concentration C, the stress components σ_{xz}, σ_{zz} and chemical potential per unit mass P with respect to z. This study is considered in the presence of the magnetic field ($H_0 = 120$), initial stress ($p_1 = 30$) and the thermal conductivity with ($K_1 = -0.5$). The graphs show six curves predicted by three different theories of thermoelasticity. In these figures, the solid lines represent the solution in the 3PHL model, the dot lines represent the solution in L-S theory and the dashed lines represent the solution derived using the G-N III theory. Here all the variables are taken in non-dimensional forms. We noticed that for different values of the rotation parameter ($\Omega = 0.2$ and $\Omega = 0$) have a significant effect on all fields. Fig. 1 depicts the variation of the displacement component u against the distance z for the rotation parameter ($\Omega = 0.2$ and $\Omega = 0$). It is observed that: in the context of the three theories, the values of the displacement component u for $\Omega = 0$ are small compared to those for $\Omega = 0.2$ in the range $0 \le z \le 7$, while the values are the same for three theories at $z \ge 7$. Fig. 2 shows the variation of the displacement component w against the distance z. It is observed that: the values of the displacement component w for $\Omega = 0$ are large compared to those for $\Omega = 0.2$ in the range $0 \le z \le 5$, while the values are the same for three theories at $z \ge 5$. Fig. 3 investigated the variation of temperature distribution θ, with distance z. in this figure, the presence of rotation causes increases in the magnitude of temperature distribution.

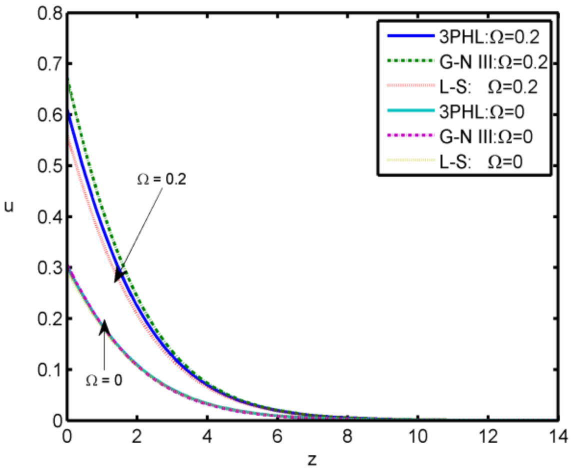

Fig. 1 Variation of displacement component u with distance z.

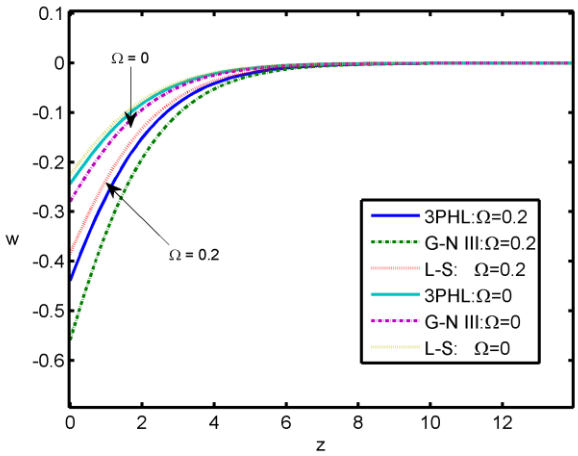

Fig. 2 Variation of displacement component w with distance z.

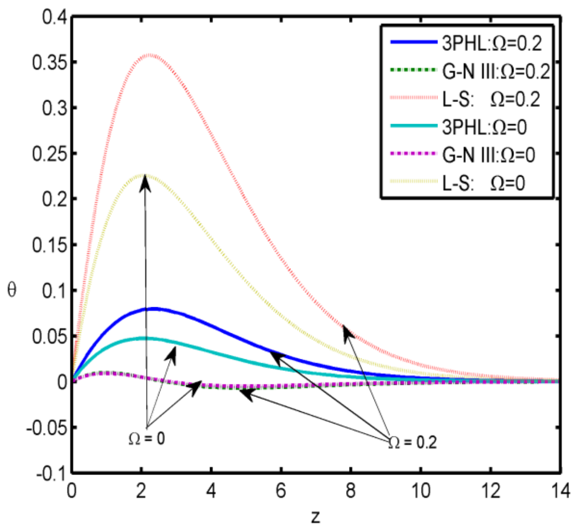

Fig. 3 Variation of temperature distribution θ with distance z.

Figs. 4 and 5 display the variation of the stress components σ_{zz}, σ_{xz} against the distance z when the rotation parameter has two values $\Omega (\Omega = 0,\ 0.2)$. We noticed that the stress components have been affected by the rotation, where the presence of rotation parameter causes decreasing in value of σ_{xz} and decreasing in the value of σ_{zz}. These figures also show that the boundary conditions are identically satisfied by the stress components. Figs. 6 and 7 illustrate the variation of the concentration C and the chemical potential P versus z. It observes from these figures that, the rotation acts to decrease the values of the concentration and the chemical potential.

Figs. 8-14 show the variations of the non-dimensional displacements, temperature, stresses, concentration and chemical potential, respectively, which demonstrate the effects of the magnetic field parameter H_0 on the variations of the considered variables. As a representation of the effect of H_0, two typical values of H_0, $H_0 = 0$ and $H_0 = 120$ in the context of three theories of thermo-elasticity, namely: 3PHL theory, G-N III theory and L-S theory, are considered. These figures evidence that, the magnetic field increases and decreases the values of the non-dimensional considered variables. Figs. 8, 9, 10, 13 and 14 exhibit the variation of the displacement components u, w, the temperature θ, the concentration C and the chemical potential P against the distance z. We notice from these figures that, the presence of the magnetic field increases the values of these variables. In Figs. 11 and 12 show the variation of the dimensionless normal stress component σ_{zz} and the tangential stress component σ_{xz} according to the different magnetic field parameter ($H_0 = 120$ and $H_0 = 0$). In the two figures, the presence of a magnetic field decreases the magnitude of the stress components.

Figs. 15-21 show the behavior of the physical quantities against distance z in 2D with and without initial stress effect ($p_1 = 0$ and $p_1 = 30$). The initial stress parameter has a significant role in the distribution of all physical quantities in the problem. Figs. 15 and 16 indicate the distribution of the displacement components u, w. The values of the displacement components u, w for $p_1 = 30$ are small compared to those for $p_1 = 0$, i.e. the initial stress acts to decrease the displacement field. Fig. 17 shows the distribution of the temperature θ versus the distance z. The values of the temperature θ for presence initial stress are small compared to those for absence initial stress in the range $0 \le z \le 11$, while the same values at $z \ge 11$.

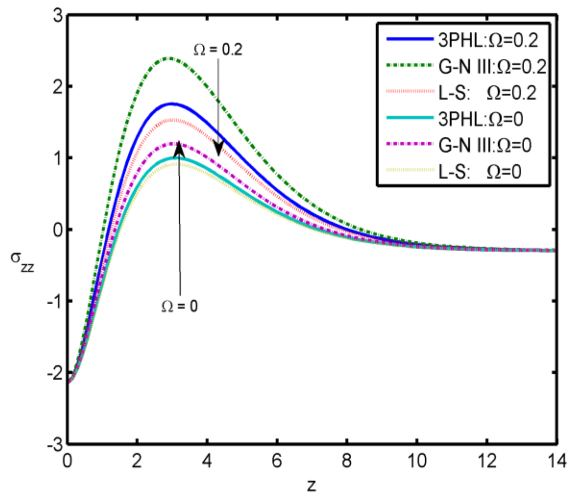

Fig. 4 Variation of the normal force stress σ_{zz} with distance z.

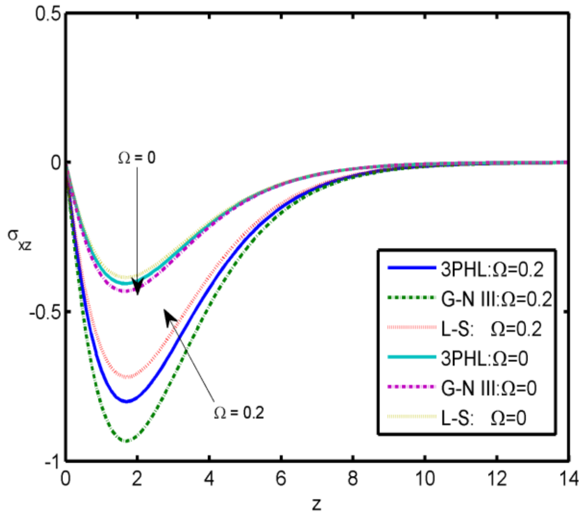

Fig. 5 Variation of shearing force stress σ_{xz} with distance z.

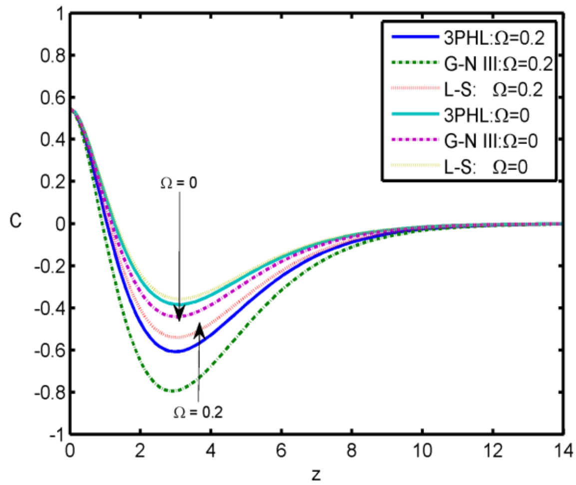

Fig. 6 Variation of the concentration C with distance z.

Figs. 18 clarifies the distribution of the normal stress σ_{zz} versus the distance z. It is clear that, the magnitude of the stress componentincreases with the increase of σ_{zz} the parameter of initial stress. Fig. 19 shows the distribution of the normal stress component σ_{zz} versus the distance z. It can be seen that the magnitude of the shear stress component σ_{xz} is found to be small for the presence initial stress in the range $0 \le z \le 7$, while the values are the same for two cases at $z \ge 7$. Figs. 20 and 21 show the variation of the concentration C and the chemical potential P versus z. It observes from this figure that the initial stress parameter shows a decreasing effect on the values of the concentration and the chemical potential. It can be also seen that C and P have qualitatively similar behaviors, but

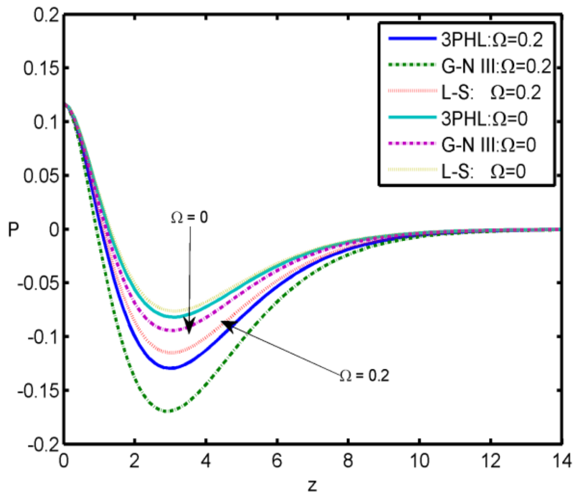

Fig. 7 Variation of the chemical potential P versus z.

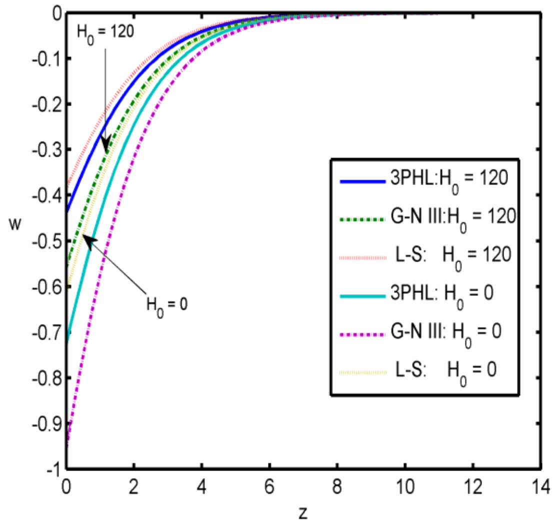

Fig. 9 Variation of the displacement component w with distance z.

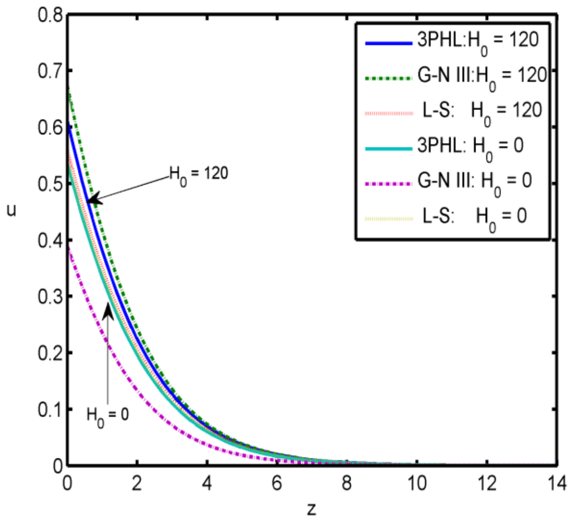

Fig. 8 Variation of normal displacement u with distance z.

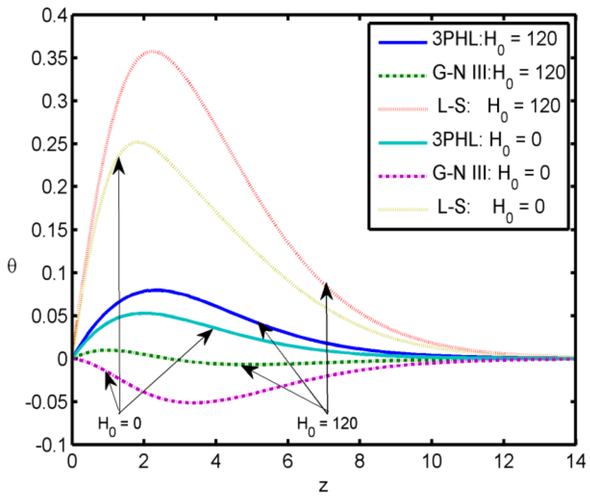

Fig. 10 Variation of temperature distribution θ with distance

different magnitudes in the presence and absence of the initial stress.

Figs. 22-28 display the variation of the non-dimensional displacements, temperature, stresses, concentration and chemical potential, respectively, for different values of the thermal conductivity ($K_1 = -0.5$ and $K_1 = -1$), i.e. this study considers that, the thermal conductivity depends on the temperature**.** Figs. 22, 23, 24, 27 and 28 explain the variation of the displacement components u, w, the temperature θ, the concentration C and the chemical potential P, respectively, against the distance z. We observation from these figures that, the values of variables at $K_1 = -0.5$ are larger than the values of variables at $K_1 = -1$. Figs. 25 and 26 demonstrate the variation of the dimensionless stress components σ_{zz} and σ_{xz}, the values of variables σ_{zz}, σ_{xz} at $K_1 = -0.5$ are smaller than the values of variables at $K_1 = -1$. It is observed that any small changes in the thermal conductivity lead to a considerable change in the propagation of wave behavior. Moreover, the distinguishing curves of the physical quantities start from its initial values and then begin to coincide, final approach zero value for large distance z.

8. Conclusions

In this study, we discussed the effect of rotation, initial stress, magnetic field and variable of the thermal conductivity on a thermoelastic medium with diffusion. We depicted the problem of memory-dependent derivative into the context of 3PHL, G-N III and L-S theories of thermo-elasticity. We concluded that:

* The variable thermal conductivity and rotation play a great role in our study, which has a significant effect on all fields. The magnetic field and initial stress have a good effect on all the physical fields. It is observed that the kernel functions and time-delay of memory dependent derivative can be arbitrarily chosen freely according to the necessity of applications.

* All figures display that the variations of all field quantities in the context of the 3PHL, G-N III and L-S theories of thermoelasticity follow similar trends.

* The method used in the present article, Laplace-Fourier transforms technique, is applicable to a wide range of problems in thermodynamics**.**

* The use of diffusion phase-lags in the equation of mass diffusion allows a more actual model of thermoelastic diffusion media as it leads to a delayed response between the relative mass flux vector and the potential gradient.

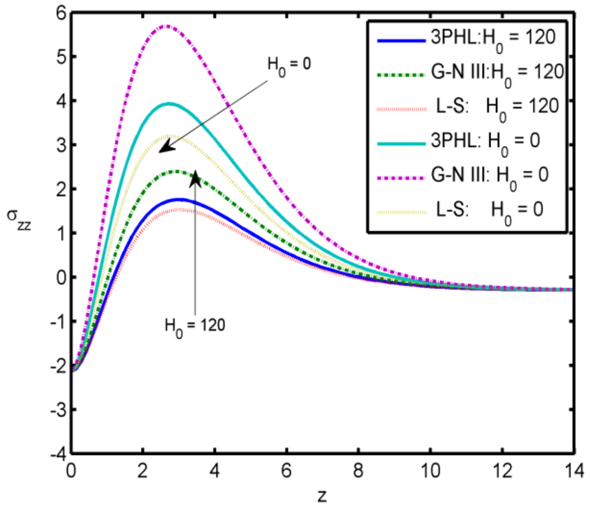

Fig. 11 Variation of normal force stress σ_{zz} with distance z.

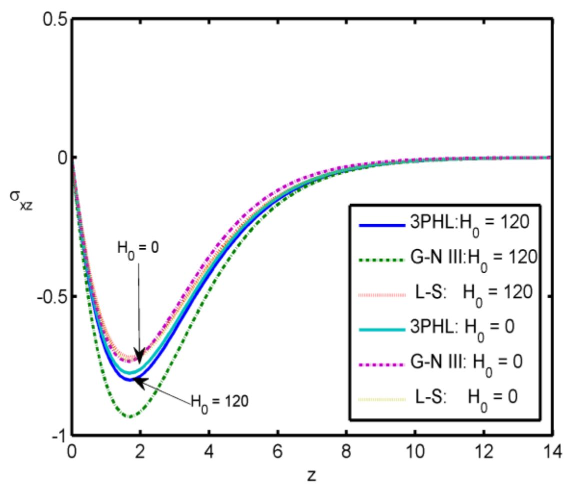

Fig. 12 Variation of shearing force stress σ_{xz} with distance

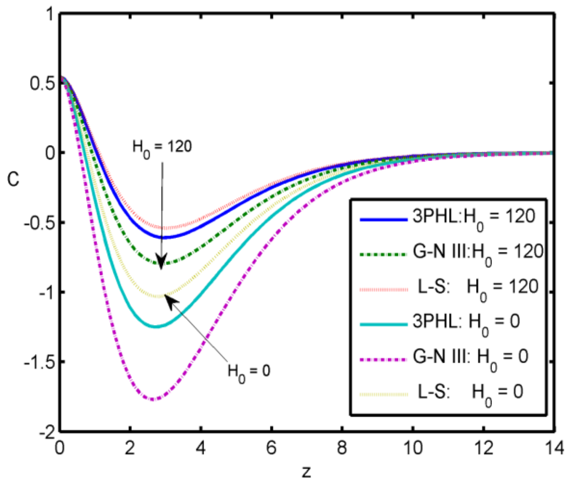

Fig. 13 Variation of the concentration field C with distance z.

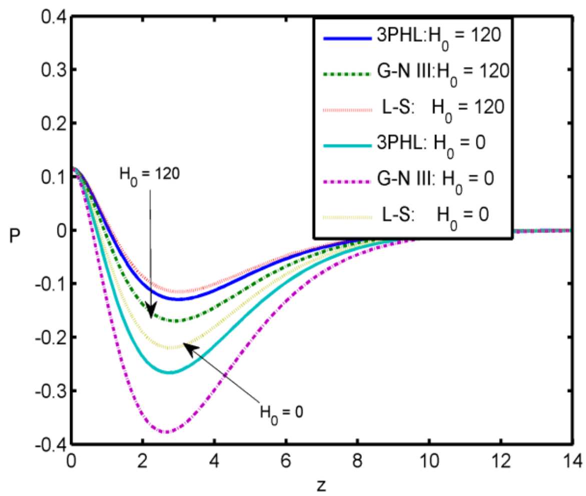

Fig. 14 Variation of the chemical potential P versus z.

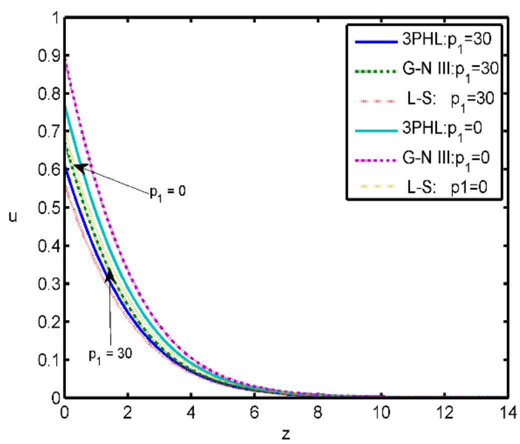

Fig. 15 Variation of displacement component u with distance z.

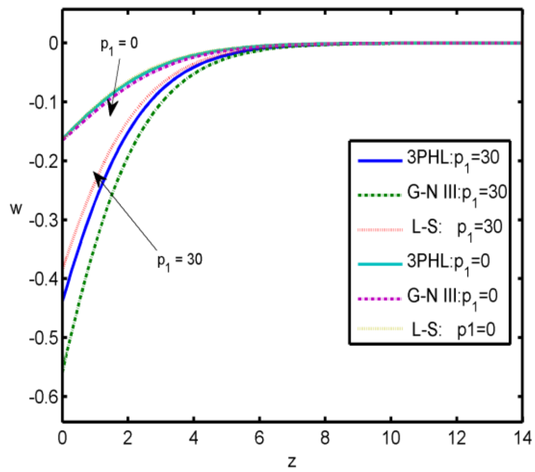

Fig. 16 Variation of displacement component w with distance z.

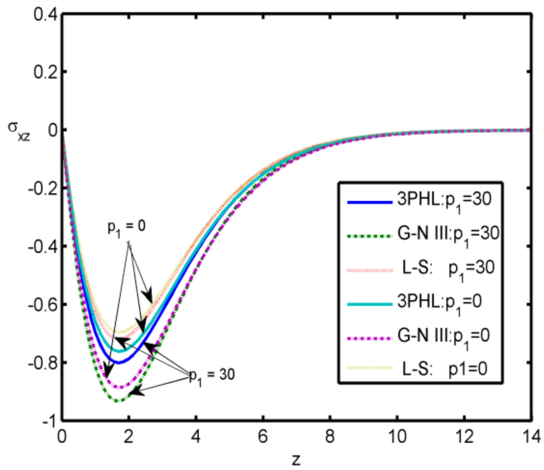

Fig. 19 Variation of shearing force stress σ_{xz} with distance

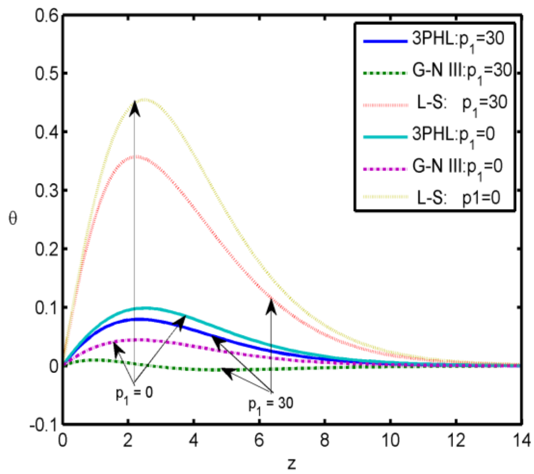

Fig. 17 Variation of temperature distribution θ with distance z.

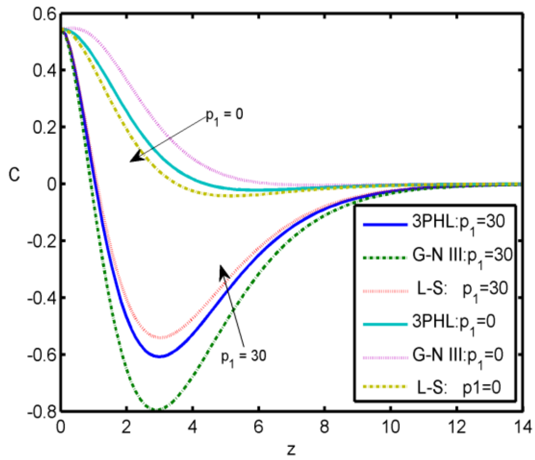

Fig. 20 Variation of the concentration C with distance z.

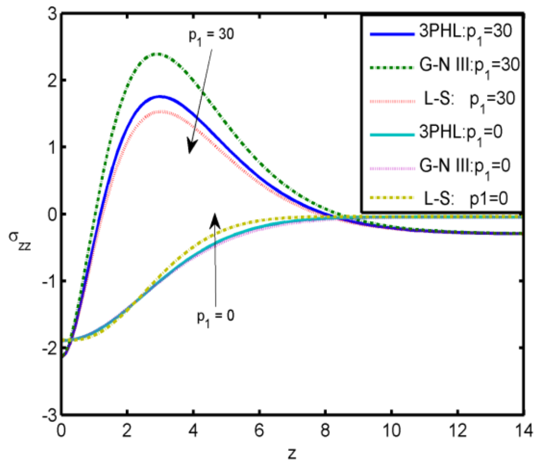

Fig. 18 Variation of the normal force stress σ_{zz} with distance z.

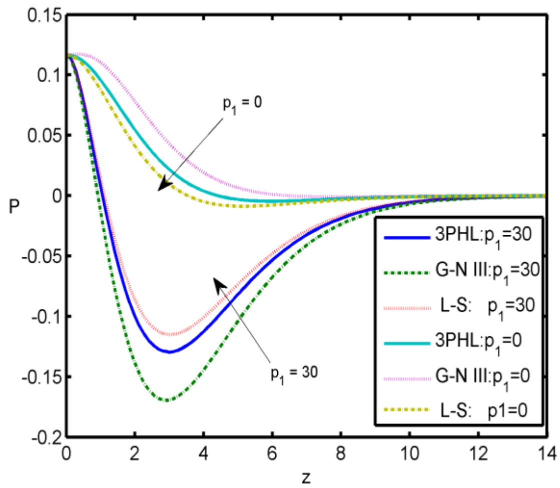

Fig. 21 Variation of the chemical potential P versus z.

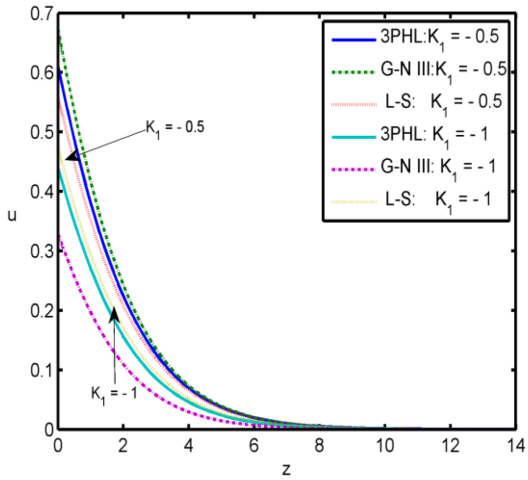

Fig. 22 Variation of displacement component u with distance z.

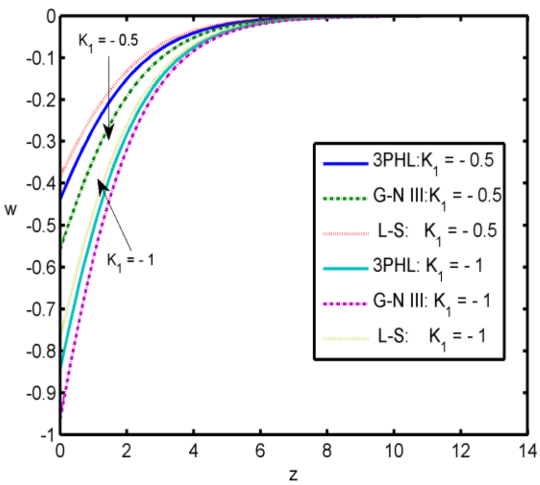

Fig. 23 Variation of displacement component w with distance z.

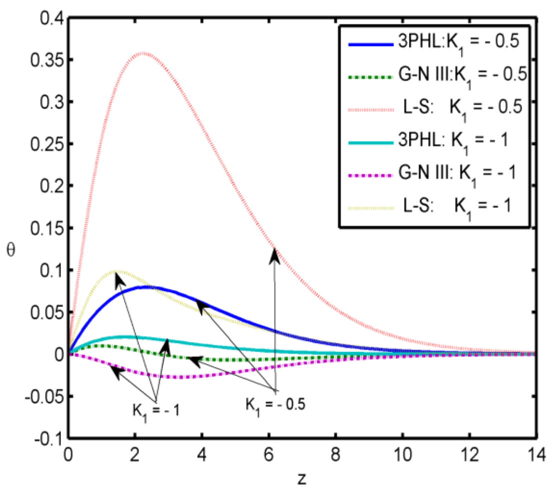

Fig. 24 Variation of temperature distribution θ with distance z.

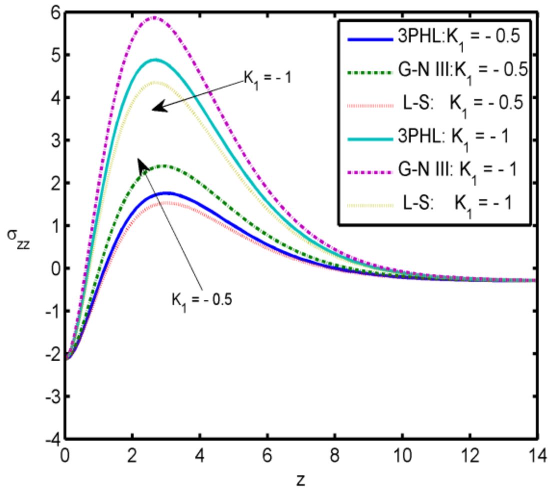

Fig. 25 Variation of the normal force stress σ_{zz} with distance z.

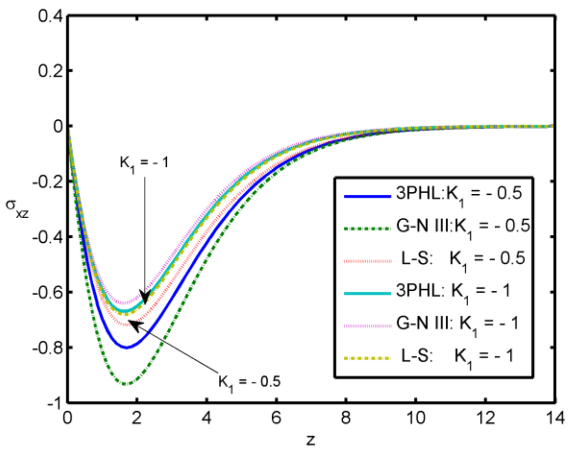

Fig. 26 Variation of shearing force stress σ_{xz} with distance z.

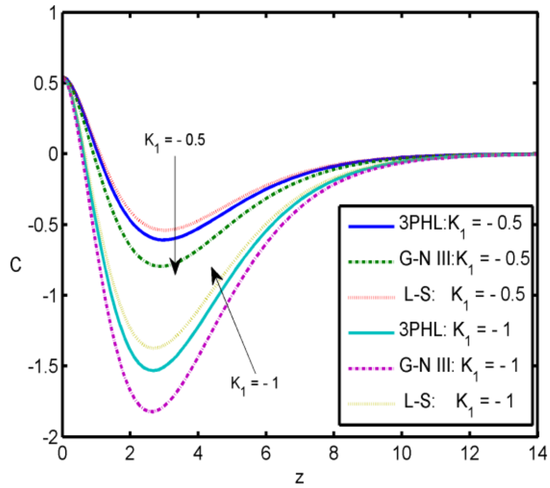

Fig. 27 Variation of the concentration C with distance z.

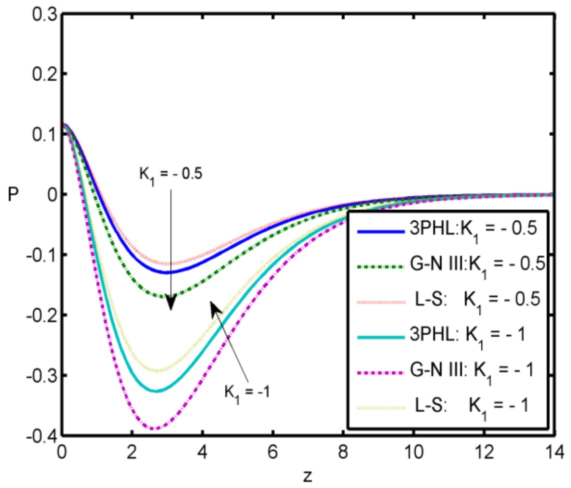

Fig. 28 Variation of the chemical potential P versus z.

Declaration of conflicting interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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CC

Nomenclature

σ_{ij} are the components of stress

e_{ij} are the components of strain

e_{kk} is the dilatation

λ, μ are the elastic constants

T is the temperature above the reference temperature T_0

δ_{ij} is the Kronecker delta

K is the coefficient of thermal conductivity

K^* is the additional material constant

ρ is the mass density

C_E is the specific heat at constant strain

P is chemical potential per unit mass

C is mass concentration

d thermo-diffusion constant

a measure of thermo-diffusion effect

b measure of diffusive effect

P_1 is the initial stress

$\boldsymbol{J}$ is the current density vector

τ_T is the phase-lag of temperature gradient

τ_q is the phase-lag of heat flux

τ_v is the phase-lag of thermal displacement gradient.

$\mu_0(\boldsymbol{J} \times \boldsymbol{H})$ is the Lorentz force

$\beta_1 = (3\lambda + 2\mu)\alpha_t$, α_t is the linear thermal expansion coefficient

$\beta_2 = (3\lambda + 2\mu)\alpha_c$, α_c is the linear diffusion expansion coefficient

$\theta = T - T_0$,

μ_0 is the magnetic permeability

ε_0 is the electric permeability

F_0 is a constant

$\delta(x)$ is the Dirac-delta.