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Effect of pulsed laser heating on 3-D problem of thermoelastic medium with diffusion under Green-Lindsay theory

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Abstract. In this work, a novel three-dimensional model in the generalized thermoelasticity for a homogeneous an isotropic medium was investigated with diffusion, under the effect of thermal loading due to laser pulse in the context of Green-Lindsay theory was investigated. The normal mode analysis technique is used to solve the resulting non-dimensional equations of the problem. Numerical results for the displacement, the thermal stress, the strain, the temperature, the mass concentration, and the chemical potential distributions are represented graphically to display the effect of the thermal loading due to laser pulse and the relaxation time on the resulting quantities. Comparisons are made within the theory in the presence and absence of laser pulse.

Keywords: generalized thermoelasticity; 3-D modeling; laser pulse; diffusion; G-L theory

1. Introduction

The generalized theories of thermoelasticity, which admit the finite speed of the thermal signal, were the center of interest of active research during the last three decades. Biot (1956) introduced the theory of coupled thermo-elasticity to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity where it predicts two phenomena not compatible with physical observations. The generalized thermoelastic theories were introduced by Lord and Shulman (1967) and Green and Lindsay (1972) in the 1960s. The L-S theory postulated a wave-type heat conduction law to replace the classical Fourier's law. This law is the same as that suggested by Cattaneo (1958) and Vernote (1961). It contains the heat flux vector as well as its time derivative and also contains a new constant that acts as relaxation time. In the context of the Lord-Shulman (L-S) theory, the generalized thermoelastic problem with temperature dependent properties was studied by He *et al.* (2013). The (G-L) theory modified the energy equation and allows two relaxation times. Several authors have studied several problems of thermoelasticity Alimirzaei *et al*. (2019), Bhatti *et al*. (2020), Karami *et al.* (2019), Lata et al. (2016), Lata *et al.* (2019). Three new thermoelastic theories based on entropy equality rather than the usual entropy inequality introduced by Green and Naghdi (1991, 1992, 1993). The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories that they called thermoelasticity of type I, of type II and of type III. When the theory of type I is linearized, one can obtain the classical system of thermoelasticity. The theory of type II (a limiting case of type III) does not admit energy dissipation. The Green-Naghdi theory, has attracted a lot of attention in the recent years Othman and Atwa (2012), Sarkar *et al*. (2020), Othman *et al.* (2015). Ezzat and El-Bary (2017) studied fractional magneto-thermoelastic materials with phase-lag Green-Naghdi theories. Hosseini (2020 investigated a GN-based modified model for size-dependent coupled thermoelasticity analysis in nano-scale, considering non-locality in heat conduction and elasticity: An analytical solution for a nanobeam with energy dissipation. Lata and Singh (2019) explained the effect of nonlocal parameter on nonlocal thermoelastic solid due to inclined load. Kumar *et al.* (2016a) investigated the thermo-mechanical interactions in a transversely isotropic magneto-thermoelastic with and without energy dissipation with the combined effects of rotation, vacuum and two temperatures. Kumar *et al*. (2016b) studied the effects of Hall current in a transversely isotropic magneto-thermoelastic two temperature medium with rotation and with and without energy dissipation due to normal force. Kumar *et al.* (2017) discussed the effects of Hall current and two temperatures in transversely isotropic magneto-thermoelastic with and without energy dissipation due to Ramp type heat.

Diffusion can be defined as the movement of particles from an area of high concentration to an area of lower concentration until equilibrium is reached. It occurs as a result of the second law of thermodynamics, which states that the entropy or disorder of any system must always increase with time. Diffusion is important in many life processes. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In an integrated circuit fabrication, diffusion is used to introduce dopants in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in

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bipolar transistors, form integrated resistors, form the source/drain regions in MOS transistors and dope poly-silicon gates in MOS transistors. In most of these applications, the concentration is calculated using what is known as Fick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction. The phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermo-elastic diffusion for more efficient extraction of oil from oil deposits. The thermo-diffusion process also helps the investigation in the field associated with the advent of semiconductor devices and the advancement of micro-electronics.

Thermo-diffusion in the solids is one of the transport processes that have great practical importance. Most of the research associated with the presence of concentration and temperature gradients has been made with metals and alloys. Thermo-diffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. Heat and mass were exchanged with the environment during the process of thermo-diffusion in an elastic solid. The concept of thermo-diffusion is used to describe the processes of thermomechanical treatment of metals (carbonizing, nitriding steel, etc.) and these processes are thermally activated, their diffusing substances being, e.g., nitrogen, carbon, etc. They are accompanied by deformations of the solid. Othman *et al.* (2009) studied the effect of diffusion on the two-dimensional problem of generalized thermoelasticity with Green and Naghdi theory. Othman *et al.* (2013) discussed the effect of fractional parameter on plane waves of generalized thermoelastic diffusion with reference temperature-dependent elastic medium. Recently, Kumar and Kumar (2015) introduced 2-D deformation in a homogeneous micro-stretch thermo-elastic medium with mass diffusion due to mechanical forces. He *et al.* (2015) worked on the dynamic response of a 2-D generalized thermoelastic diffusion problem for a half-space is investigated in the context of the generalized thermoelastic diffusion theory.

The so-called ultra-short lasers are those with pulse duration ranging from nanoseconds to femtoseconds. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam have introduced situations where very large thermal gradients or an ultra-high heating rate may exist on the boundaries in Al-Qahtani and Datta (2008). Various research work related to the problem can be seen in the list of references (Sun *et al.* 2008, Ronghou *et al.* 2014, Marin 2010, Marin *et al.* 2015, Marin *et al.* 2017, Marin *et al.* 2019, Tounsi *et al.* 2020, Zarga *et al.* 2019).

The present investigation is devoted to the study of the effect of thermal loading due to laser pulse on the general three-dimensional model of the equations of the generalized thermoelasticity with diffusion for a homogeneous isotropic elastic half-space solid in the context of G-L theory without any body forces or heat sources. The problem has been solved numerically using a normal mode analysis. Numerical results for the displacement, thermal stress, strain, temperature, chemical potential, and mass concentration, with and without laser pulses, are represented graphically.

2. Governing equations and formulation of the problem

The governing equations of an isotropic and homogeneous elastic medium with generalized thermoelastic diffusion in the context of (G-L) theory in the absence of body forces are as Othman *et al.* (2009)

2.1 The constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e_{kk} - \gamma(1+\nu_0 \frac{\partial}{\partial t})T_{,i} - \beta_1(1+\nu_1 \frac{\partial}{\partial t})C\ ]\delta_{ij}, \quad (1)$$

$$P = -\beta_1\, e_{kk} - a_1(1+\nu_0 \frac{\partial}{\partial t})T + b_1(1+\nu_1 \frac{\partial}{\partial t})C. \quad (2)$$

2.2 The equation of motion

$$\rho \ddot{u}_i = (\lambda+\mu)u_{j,ij} + \mu u_{i,jj} - \gamma(1+\nu_0 \frac{\partial}{\partial t})T_{,i} - \beta_1(1+\nu_1 \frac{\partial}{\partial t})C_{,i}. \quad (3)$$

2.3 The equation of heat conduction

$$KT_{,ii} = \rho C_E (1+\tau_0 \frac{\partial}{\partial t})\dot{T} + \gamma T_0 \dot{e}_{kk} + a_1 T_0 (1+\tau_1 \frac{\partial}{\partial t})\dot{C} - \rho Q. \quad (4)$$

2.4 The equation of mass diffusion

$$d\,\beta_1 e_{kk,ii} + d\,a_1(1+\nu_0 \frac{\partial}{\partial t})T_{,ii} + \dot{C} - d b_1(1+\nu_1 \frac{\partial}{\partial t})C_{,ii} = 0. \quad (5)$$

2.5 The strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (6)$$

In the preceding equations, λ, μ are Lame' constants, ρ is the density, ν_0 and τ_0 are the thermal relaxation times such that $\nu_0 \geq \tau_0 > 0$, ν_1 and τ_1 are the diffusion relaxation times such that $\nu_1 \geq \tau_1 > 0$, a_1 is the measure of thermo-diffusion effect, b_1 is the measure of diffusive effect, d is the thermoelastic diffusion constant, C is the concentration of diffusive material in the elastic, P is the chemical potential, σ_{ij} are the components of the stress tensor, t is the time variable, γ is a material constant given by $\gamma = (3\lambda + 2\mu)\alpha_T$, where α_T is the coefficient of linear thermal expansion, $\beta_1 = (3\lambda + 2\mu)\alpha_c$, where α_c is the coefficient of linear diffusion expansion, K is the thermal conductivity, C_E is the specific heat at constant strain, T is the absolute temperature, and T_0 is the temperature of the medium in its natural state, assumed to be such that $\left|(T - T_0)/T_0\right| << 1$, Q is the heat input of the laser pulse.

We will consider that the plate surface is illuminated by a laser pulse given by the heat input as Al-Qahtani and Datta (2008) and Tang and Araki (1999)

$$Q = I_0 f(t) g(y) h(x), \tag{7}$$

where I_0 is the energy absorbed, the temporal profile $f(t)$ is represented as

$$f(t) = \frac{t}{t_0^2}\exp(-\frac{t}{t_0}), \tag{8}$$

where t_0 is the pulse rise time. The pulse is also

$$g(y) = \frac{1}{2\pi r^2}\exp(-\frac{y^2}{r^2}), \tag{9}$$

where r is the beam radius, and as a function of the depth x the heat deposition due to the laser pulse is assumed to decay exponentially within the solid

$$h(x) = \eta e^{-\eta x}. \tag{10}$$

From Eqs. (6)-(8) in Eq. (5) we get

$$Q = \frac{I_0 \eta t}{2\pi r^2 t_0^2}\exp(-\frac{y^2}{r^2} - \frac{t}{t_0})\exp(-\eta x). \tag{11}$$

We can rewrite the equation of motion as

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+\mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u - \gamma\nu_T\frac{\partial T}{\partial x} - \beta_1\nu_c\frac{\partial C}{\partial x}, \tag{12}$$

$$\rho\frac{\partial^2 v}{\partial t^2} = (\lambda+\mu)\frac{\partial e}{\partial y} + \mu\nabla^2 v - \gamma\nu_T\frac{\partial T}{\partial y} - \beta_1\nu_c\frac{\partial C}{\partial y}, \tag{13}$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\lambda+\mu)\frac{\partial e}{\partial z} + \mu\nabla^2 w - \gamma\nu_T\frac{\partial T}{\partial z} - \beta_1\nu_c\frac{\partial C}{\partial z}, \tag{14}$$

where $\nu_T = (1+\nu_0\frac{\partial}{\partial t})$ and $\nu_c = (1+\nu_1\frac{\partial}{\partial t})$, and the conduction equation takes the form

$$K\nabla^2 T = \rho C_E\tau_T\frac{\partial T}{\partial t} + \gamma T_0\frac{\partial e}{\partial t} + a_1 T_0\tau_c\frac{\partial C}{\partial t} - \rho Q, \tag{15}$$

where $\tau_T = (1+\tau_0\frac{\partial}{\partial t})$ and $\tau_c = (1+\tau_1\frac{\partial}{\partial t})$.

The constitutive equations can be written as

$$\sigma_{xx} = \lambda e + 2\mu\frac{\partial u}{\partial x} - \gamma\nu_T T - \beta_1\nu_c C, \tag{16}$$

$$\sigma_{yy} = \lambda e + 2\mu\frac{\partial v}{\partial y} - \gamma\nu_T T - \beta_1\nu_c C, \tag{17}$$

$$\sigma_{zz} = \lambda e + 2\mu\frac{\partial w}{\partial z} - \gamma\nu_T T - \beta_1\nu_c C, \tag{18}$$

$$\sigma_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}), \tag{19}$$

$$\sigma_{xz} = \mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}), \tag{20}$$

$$\sigma_{yz} = \mu(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}), \tag{21}$$

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}. \tag{22}$$

For simplifications we will use the following non-dimensional variables

$$\begin{gathered}(x',y',z') = \frac{\varpi}{C_1}(x,y,z),\ (u',v',w') = \frac{\rho C_1\varpi}{\gamma T_0}(u,v,w),\\ T' = \frac{T}{T_0},\ \sigma'_{ij} = \frac{\sigma_{ij}}{\gamma T_0},\ P' = \frac{P}{\beta_1},\ \varepsilon_T = \frac{\gamma^2 T_0}{\rho K\varpi},\\ \{t',\tau'_0,\nu'_0,\tau'_1,\nu'_1\} = \varpi\{t,\tau_0,\nu_0,\tau_1,\nu_1\},\ C' = \frac{\beta_1 C}{\gamma T_0},\\ Q' = \frac{Q}{\varpi T_0 C_E},\ C_1^2 = \frac{(\lambda+2\mu)}{\rho},\ \varpi = \frac{\rho C_E C_1^2}{K}.\end{gathered} \tag{23}$$

Eqs. (12)-(21) in the non-dimensional forms (after suppressing the primes) reduce to

$$\beta\nabla^2 u + (1-\beta)\frac{\partial e}{\partial x} - \nu_T\frac{\partial T}{\partial x} - \nu_c\frac{\partial C}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{24}$$

$$\beta\nabla^2 v + (1-\beta)\frac{\partial e}{\partial y} - \nu_T\frac{\partial T}{y} - \nu_c\frac{\partial C}{y} = \frac{\partial^2 v}{\partial t^2}, \tag{25}$$

$$\beta\nabla^2 w + (1-\beta)\frac{\partial e}{\partial z} - \nu_T\frac{\partial T}{\partial z} - \nu_c\frac{\partial C}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \tag{26}$$

$$\nabla^2 T = \tau_T\frac{\partial T}{\partial t} + \varepsilon_T\frac{\partial e}{\partial t} + \beta_2\tau_c\frac{\partial C}{\partial t} - Q, \tag{27}$$

$$\sigma_{xx} = 2\beta\frac{\partial u}{\partial x} + (1-2\beta)e - \nu_T T - \nu_c C, \tag{28}$$

$$\sigma_{yy} = 2\beta\frac{\partial v}{\partial y} + (1-2\beta)e - \nu_T T - \nu_c C, \tag{29}$$

$$\sigma_{zz} = 2\beta\frac{\partial w}{\partial z} + (1-2\beta)e - \nu_T T - \nu_c C, \tag{30}$$

$$\sigma_{xy} = \beta(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}), \tag{31}$$

$$\sigma_{xz} = \beta(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}), \tag{32}$$

$$\sigma_{yz} = \beta(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}), \tag{33}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},\ \beta = \frac{\mu}{(\lambda+2\mu)},\ \beta_2 = \frac{a_1\gamma T_0}{\beta_1\rho C_E}.$$

From Eqs. (28)-(30) by addition, we get

$$\sigma = \alpha e - \nu_T T - \nu_c C, \tag{34}$$

where $\sigma = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$, $\alpha = (3-4\beta)/3$.

From (23) into (2) and (5) we get

$$P = -e + \beta_3\nu_c C - \beta_4\nu_T T, \tag{35}$$

$$\nabla^2 e + \beta_5 \nu_T \nabla^2 T + \beta_6 \frac{\partial C}{\partial t} - \beta_7 \nu_c \nabla^2 C = 0, \tag{36}$$

where $\beta_3 = \frac{b_1 \gamma T_0}{\beta_1^2}$, $\beta_4 = \frac{a_1 T_0}{\beta_1}$, $\beta_5 = \frac{\rho C_1^2 a_1}{\gamma \beta_1}$,

$\beta_6 = \frac{K C_1^2}{d C_E \beta_1^2}$ and $\beta_7 = \frac{\rho C_1^2 b_1}{\beta_1^2}$.

From Eqs. (24)-(26) after using Eq. (22) we can get

$$\frac{\partial^2 e}{\partial t^2} = \nabla^2 e - \nu_T \nabla^2 T - \nu_c \nabla^2 C. \tag{37}$$

Eliminating e from Eqs. (27), (36) and (37) by using Eq. (34), we obtain

$$\nabla^2 T = (\tau_T + \frac{\nu_T \varepsilon_T}{\alpha})\frac{\partial T}{\partial t} + (\beta_2 \tau_c + \frac{\nu_c \varepsilon_T}{\alpha})\frac{\partial C}{\partial t} + \frac{\varepsilon_T}{\alpha}\frac{\partial \sigma}{\partial t} - Q, \tag{38}$$

$$\nabla^2 \sigma + (1-\alpha)(\nu_T \nabla^2 T + \nu_c \nabla^2 C) = \frac{\partial^2 \sigma}{\partial t^2} + \nu_T \frac{\partial^2 T}{\partial t^2} + \nu_c \frac{\partial^2 C}{\partial t^2}, \tag{39}$$

$$\nabla^2 \sigma + (1+\alpha\beta_5)\nu_T \nabla^2 T + (1-\alpha\beta_7)\nu_c \nabla^2 C + \alpha \beta_6 \frac{\partial C}{\partial t} = 0. \tag{40}$$

3. The solution of the problem

The solution of the considered physical variables can be written in terms of normal modes as in the following form

$$(u, v, w, e, T, \sigma_{ij}, C, P)(x, y, z, t) = (u^*, v^*, w^*, e^*, T^*, \sigma_{ij}^*, C^*, P^*)(x)\exp[\omega t - i(ay + bz)], \tag{41}$$

where $i = \sqrt{-1}$, ω is the angular frequency and *a, b* are the wave numbers in the *y* and *z*-directions respectively.
Using Eq. (41) into Eqs. (38)-(40), we can obtain the following equations

$$(\mathrm{D}^2 - A_2)T^* - A_3\sigma^* - A_4 C^* + Q_0 f(y,z,t)\exp(-\eta x) = 0, \tag{42}$$

$$(\mathrm{D}^2 - A_5)\sigma^* + (A_6\mathrm{D}^2 - A_7)T^* + (A_8\mathrm{D}^2 - A_9)C^* = 0, \tag{43}$$

$$(\mathrm{D}^2 - A_1)\sigma^* + A_{10}(\mathrm{D}^2 - A_1)T^* + A_{11}(\mathrm{D}^2 - A_{12})C^* = 0, \tag{44}$$

where

$\mathrm{D} = \frac{d}{dx}$, $A_1 = a^2 + b^2$, $A_2 = A_1 + [\tau_{T\omega} + \frac{\nu_{T\omega}\varepsilon_T}{\alpha}]\omega$, $A_3 = \frac{\varepsilon_T \omega}{\alpha}$,

$A_{12} = A_1 A_{11} - \alpha\beta_6\omega$, $A_{11} = (1-\alpha\beta_7)\nu_{c\omega}$, $A_{10} = (1+\alpha\beta_5)\nu_{T\omega}$,

$\nu_{T\omega} = (1+\nu_0\omega)$, $\tau_{T\omega} = (1+\tau_0\omega)$, $\nu_{c\omega} = (1+\nu_1\omega)$,

$\tau_{c\omega} = (1+\tau_1\omega)$, $Q_0 = (\frac{I_0\eta}{2\pi r^2 t_0^2})$,

$$f(y,z,t) = t\exp(-\frac{y^2}{r^2} - \omega t + i(ay + bz) - \frac{t}{t_0}).$$

Eliminating T^*, σ^* and C^* between Eqs. (42)-(44), we get the following two sixth order ordinary differential equations

$$(\mathrm{D}^6 - L_1\mathrm{D}^4 + L_2\mathrm{D}^2 - L_3)T^*(x) = B_1 Q_0 f(y,z,t)\exp(-\eta x), \tag{45}$$

$$(\mathrm{D}^6 - L_1\mathrm{D}^4 + L_2\mathrm{D}^2 - L_3)\sigma^*(x) = B_2 Q_0 f(y,z,t)\exp(-\eta x), \tag{46}$$

where $L_1 = \frac{A_8 A_{19} + A_{13}A_9 - A_{15}A_{18} - A_{11}A_{14}}{A_{18}A_8 - A_{11}A_{13}}$,

$L_2 = \frac{A_{16}A_7 - A_{15}A_{19} - A_{13}A_{21} + A_{14}A_{20}}{A_{18}A_8 - A_{11}A_{13}}$,

$L_3 = \frac{A_{16}A_{19} - A_{14}A_{21}}{A_{18}A_8 - A_{11}A_{13}}$,

$B_1 = \frac{(A_{13}A_{22} - A_{17}A_{18})\eta^2 - A_{14}A_{22} - A_{19}A_{17}}{A_{18}A_8 - A_{11}A_{13}}$,

$B_2 = \frac{(A_{11}A_{17} - A_8 A_{22})\eta^4 + (A_{17}A_{20} - A_{15}A_{22})\eta^2 + (A_{17}A_{21} - A_{16}A_{22})}{A_{18}A_8 - A_{11}A_{13}}$,

$A_{13} = A_4 - A_3 A_8$, $A_{14} = A_4 A_5 - A_3 A_9$, $A_{15} = A_4 A_6 - A_9 - A_2 A_8$,

$A_{16} = A_2 A_9 - A_4 A_7$, $A_{17} = A_8\eta^2 - A_9$, $A_{18} = A_4 - A_3 A_{11}$,

$A_{19} = A_1 A_4 - A_3 A_{12}$, $A_{20} = A_4 A_{10} - A_2 A_{11} - A_{12}$,

$A_{21} = A_2 A_{12} - A_1 A_4 A_{10}$, $A_{22} = (A_{11}\eta^2 - A_{12})$.

Eq. (38) can be factored as

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)T^*(x) = B_1 Q_0 f(y,z,t)\exp(-\eta x), \tag{47}$$

where k_i $(i = 1,2,3)$ are the roots of the characteristic equation of Eq. (45).
We can consider the general solution of Eqs. (45) and (46) bound at infinity in the form

$$T^*(x) = \sum_{i=1}^{2} M_i \exp(-k_i x) + AB_1 Q_0 f(y,z,t)\exp(-\eta x), \tag{48}$$

$$\sigma^*(x) = \sum_{i=1}^{2} H_i M_i \exp(-k_i x) + AB_2 Q_0 f(y,z,t)\exp(-\eta x), \tag{49}$$

where $A = \frac{1}{\eta^6 - L_1\eta^4 + L_2\eta^2 - L_3}$, $H_i = \frac{A_8 k_i^4 + A_{15}k_i^2 + A_{16}}{A_{14} - A_{13}k_i^2}$; $(i = 1,2,3)$.

From Eqs. (48), (49) into (42) we get

$$C^*(x) = \sum_{i=1}^{3} G_i M_i \exp(-k_i x) + B_3 Q_0 f(y,z,t)\exp(-\eta x), \tag{50}$$

where

$B_3 = \dfrac{1+(\eta^2 - A_2)AB_1 - A_3AB_2}{A_4}$, $G_i = \dfrac{k_i^2 - A_2 - A_3H_i}{A_4}$, $(i=1,2,3)$.

From Eqs. (48)-(50) into (34), (35) we obtain

$$P^*(x) = \sum_{i=1}^{3} R_i M_i \exp(-k_i x) + B_4 Q_0 f(y,z,t)\exp(-\eta x), \quad (51)$$

$$e^*(x) = \sum_{i=1}^{3} N_i M_i \exp(-k_i x) + B_5 Q_0 f(y,z,t)\exp(-\eta x), \quad (52)$$

where

$R_i = \dfrac{-H_i - A_{23}G_i - A_{22}}{\alpha}$, $N_i = \dfrac{H_i + \nu_{c\omega}G_i + \nu_{T\omega}}{\alpha}$, $(i=1,2,3)$,

$B_4 = \dfrac{-AB_2 - AB_1A_{22} - A_{23}B_3}{\alpha}$, $B_5 = \dfrac{AB_2 + AB_1\nu_{T\omega} + B_3\nu_{c\omega}}{\alpha}$,

$A_{22} = (1+\alpha\beta_4)\nu_{T\omega}$, $A_{23} = (1-\alpha\beta_3)\nu_{c\omega}$.

4. Applications

In order to complete the solution we have to know the parameters M_i, so we will consider the following boundary conditions at $x=0$:

4.1 Mechanical boundary condition that the bounding plane to the surface has no traction anywhere and has no variation of concentration, so we have

$$\sigma(0,y,z,t) = \sigma_{xx}(0,y,z,t) = \sigma_{yy}(0,y,z,t) = 0,$$
$$\sigma_{zz}(0,y,z,t) = 0, \qquad \frac{\partial C}{\partial x} = 0. \quad (53)$$

4.2 The thermal boundary condition is that the surface of the half-space is subjected to a thermal shock

$$T(0,y,z,t) = s(0,y,z,t) = s^* \exp[\omega t - i(ay+bz)]. \quad (54)$$

From the above boundary conditions together with Eqs. (48)-(50), we get

$$M_1 + M_2 + M_3 = s^*, \quad (55)$$

$$H_1M_1 + H_2M_2 + H_3M_3 = 0, \quad (56)$$

$$k_1G_1M_1 + k_2G_2M_2 + k_3G_3M_3 = 0. \quad (57)$$

Solving Eqs. (55)-(57), we obtain

$$M_1 = \frac{\Delta_1}{\Delta}, \quad M_2 = \frac{\Delta_2}{\Delta}, \quad M_3 = \frac{\Delta_3}{\Delta}, \quad (58)$$

where

$\Delta = H_2k_3G_3 - H_3k_2G_2 + H_3k_1G_1 - H_1k_3G_3 + H_1k_2G_2 - H_2k_1G_1$.

$\Delta_1 = s^*(H_2k_3G_3 - H_3k_2G_2)$, $\Delta_2 = s^*(H_3k_1G_1 - H_1k_3G_3)$,

$\Delta_3 = s^*(H_1k_2G_2 - H_2k_1G_1)$.

From Eqs. (48)-(50) into (24) after using (41) we get

$$\left(\frac{d^2}{dx^2} - \lambda_u^2\right)u^* = \sum_{i=1}^{3} \xi_i \exp(-k_i x) + \xi_4 \exp(-\eta x) \quad (59)$$

Where

$\lambda_u^2 = (a^2 + b^2 + \dfrac{\omega^2}{\beta})$ and $\xi_i = [\dfrac{(1-\beta)N_i - \nu_{T\omega} - \nu_{c\omega}G_i}{\beta}]k_i M_i$,

$\xi_4 = [\dfrac{(1-\beta)B_5 - \nu_{T\omega}AB_1 - \nu_{c\omega}B_3}{\beta}]\eta Q_0 f(y,z,t)$.

The solution of the ordinary differential equation (59) takes the form

$$u^*(x) = \sum_{i=1}^{3} r_i \exp(-k_i x) + r_4 \exp(-\eta x) + r_5 \exp(-\lambda_u x). \quad (60)$$

Where $r_i = \dfrac{\xi_i}{k_i^2 - \lambda_u^2}$, $r_4 = \dfrac{\xi_4}{\eta^2 - \lambda_u^2}$, and r_5 is a constant to be determined from the boundary conditions.

From Eqs. (34), (48), (50) and (60) after using the boundary condition (53) we get

$$r_5 = \frac{1}{\lambda_u}\left[\frac{(1-\alpha-2\beta)}{2\alpha\beta}\sum_{i=1}^{3}(\nu_{T\omega} + \nu_{c\omega}G_i)M_i + \sum_{i=1}^{3} k_i r_i + \eta r_4\right], \quad (61)$$

with the same manner, we can get the other components of the displacement (v^*, w^*).

Then, the final solutions for the dimensionless of the displacement *u*, stress *σ*, strain *e*, temperature *T*, mass concentration *C*, and chemical potential *P* can be deduced as follows

$$u(x,y,z,t) = [\sum_{i=1}^{2} r_i \exp(-k_i x) + r_4 \exp(-\eta x) + r_5 \exp(-\lambda_u x)]\exp[\omega t - i(ay+bz)], \quad (62)$$

$$\sigma(x,y,z,t) = [\sum_{i=1}^{2} H_i M_i \exp(-k_i x) + AB_2 Q_0 f(y,z,t)\exp(-\eta x)]\exp[\omega t - i(ay+bz)], \quad (63)$$

$$e(x,y,z,t) = [\sum_{i=1}^{3} N_i M_i \exp(-k_i x) + B_5 Q_0 f(y,z,t)\exp(-\eta x)]\exp[\omega t - i(ay+bz)], \quad (64)$$

$$T(x,y,z,t) = [\sum_{i=1}^{2} M_i \exp(-k_i x) + AB_1 Q_0 f(y,z,t)\exp(-\eta x)]\exp[\omega t - i(ay+bz)], \quad (65)$$

$$C(x,y,z,t) = [\sum_{i=1}^{3} G_i M_i \exp(-k_i x) + B_3 Q_0 f(y,z,t)\exp(-\eta x)]\exp[\omega t - i(ay+bz)], \quad (66)$$

$$P(x,y,z,t) = [\sum_{i=1}^{3} R_i M_i \exp(-k_i x) + B_4 Q_0 f(y,z,t)\exp(-\eta x)]\exp[\omega t - i(ay+bz)]. \quad (67)$$

5. Numerical calculation and discussion

In order to illustrate our theoretical results obtained in the preceding section, we now present some numerical results. In the calculation, we take the copper as the material subjected to mechanical thermal disturbances. Since ω is complex, we take $\omega = \omega_0 + i\zeta$, where i is the imaginary number. The numerical constants of the problem were taken at $T_0 = 293k$?as Othman *et al.* (2009):

$\lambda = 7.76 \times 10^{10} kg\ m^{-1} s^{-2}$, $\mu = 3.86 \times 10^{10} kg\ m^{-1} s^{-2}$, $\rho = 8.954 \times 10^{3} kg\ m^{-3}$,

$K = 386\, kg\, m\, k^{-1} s^{-3}$,? $\alpha_T = .78 \times 10^{-5} k^{-1}$,
$C_E = 0.3831 \times 10^{3} m^2 k^{-1} s^{-2}$, $a_1 = 1.2 \times 10^{4} m^2 k^{-1} s^{-2}$,
$b_1 = 0.9 \times 10^{6} m^5 kg^{-1} s^{-2}$, $\alpha_c = 1.98 \times 10^{-4}$?$m^3 kg^{-1}$ $d = 0.85 \times 10^{-8} kgm^{-3} s$,
$\tau_0 = 0.05 s$, ν_0?= $.05 s$, τ_1?= $.04\, s$, ν_1?= $.04\, s$, $\omega_0 = 2.5$,
$\zeta = 0.1$, $a = 0.2$,
$b = 1.2$, $s^* = 10$, $r = 100\ \mu m$, $t_0 = 8\ nan.s$, $\eta = 5\ m^{-1}$.

The numerical technique, outlined above, was used for the distribution of the real part of the displacement component u, stress σ, strain e, the temperature T, mass concentration C and chemical potential P for the problem. Here, all the variables are taken in non-dimensional form.

Figs. 1-6 represented 2D curves for the distributions of the physical quantities against the distance x at $y = z = 0.1$ and $t = 0.1$ in the case of the absence and presence of laser pulse effect $(I_0 = 0,\ 10^9,\ 10^{10})$. In these figures, the solid line, dashed line and dotted line correspond forrespectively, which is 10^{10} 10^{9}, $I_0 = 0$, furthermore precisely explained in each figure in the legend. Fig. 1 illustrates the variations of the displacement component u with a distance x. This figure shows that the displacement component u increases with the increase of the distance x and finally all curves terminate at the zero value at $x > 1.5$ approximately. It can be observed from this figure that the laser pulse value has a decreasing effect on the displacement component u.

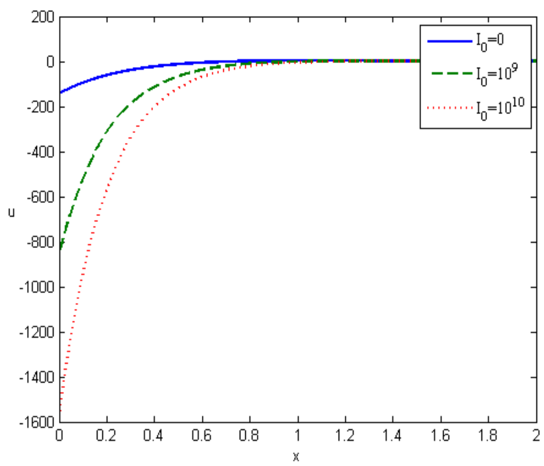

Fig. 1 Displacement distribution at $y = z = 0.1$, $t = 0.1$

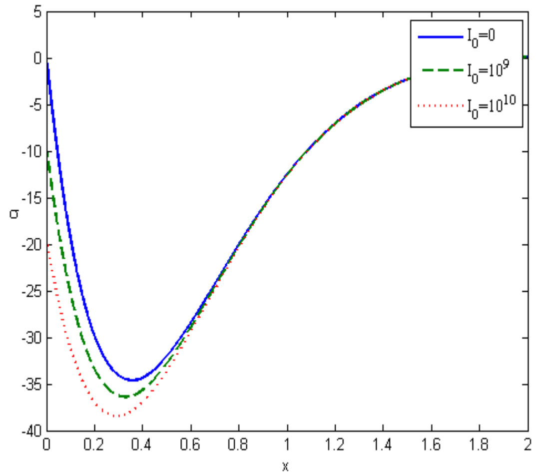

Fig. 2 Stress distribution at $y = z = 0.1$, $t = 0.1$

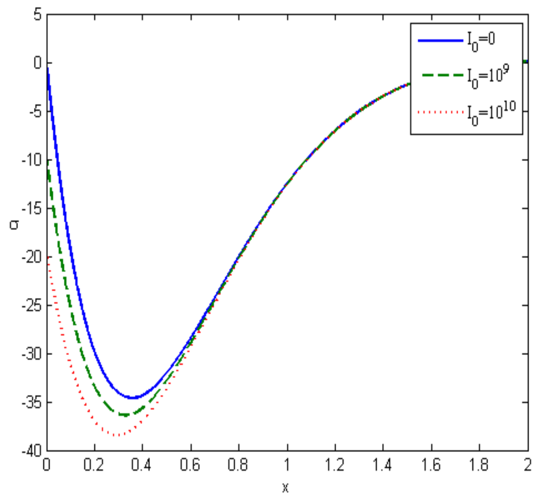

Fig. 3 Strain distribution at $y = z = 0.1$, $t = 0.1$

Fig. 2 describes the variations of the stress σ with a distance x. This figure shows that stress σ decreases with the increase of the distance x and has a minimum value at $x = 0.36$, $x = 0.34$ and $x = 0.30$ for $I_0 = 0$, 10^9, 10^{10} respectively, and then all curves increase tending to zero for $x > 2$. From this figure, it can be seen that the laser pulse value has a decreasing effect on the stress σ.

Fig. 3 exhibits the variations of the strain e, with distance x. This figure shows that the strain e, decreases with the increase of the distance x and finally all curves converge to zero for $x > 1.5$ approximately.

It can be observed from this figure that the laser pulse value has an increasing effect on the strain *e*. Fig. 4 depicts the variations of the temperature T with a distance x. We can see from this figure that the temperature T starts from a positive value in the case of the absence of laser pulse effect ($I_0 = 0$) and then decreases with the increase of the distance x, while it starts from negative values in the case of the presence of laser pulse effect ($I_0 = 10^9,\ 10^{10}$) and then increases with the increase of the distance x and finally all curves terminate at the zero value at $x > 1.5$ approximately. It is clearly observed that the laser pulse value has a decreasing effect on the temperature distribution. Fig. 5 displays the variations of the mass concentration C with a distance x and shows that the mass concentration C decreases with the increase of the distance x for all values of I_0 and finally all curves converge to zero for $x > 1.5$. It is noticed from this figure that the laser pulse value has an increasing effect on the mass concentration.

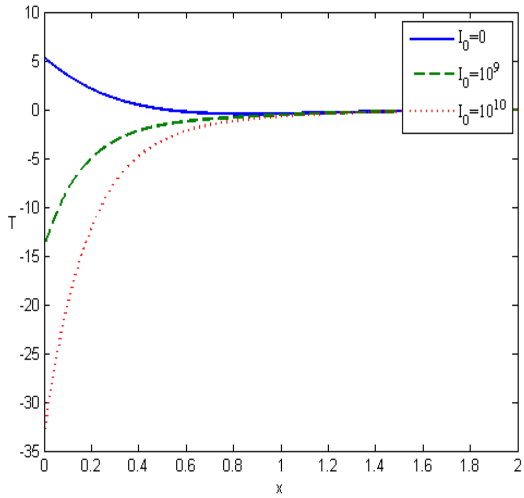

Fig. 4 Temperature distribution at $y = z = 0.1$, $t = 0.1$

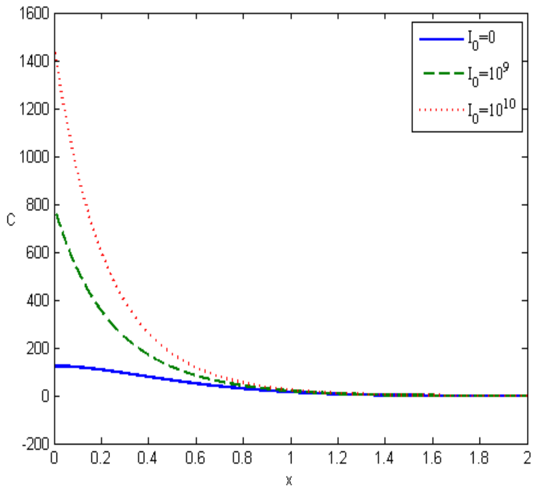

Fig. 5 Mass concentration distribution at $y = z = 0.1$, $t = 0.1$

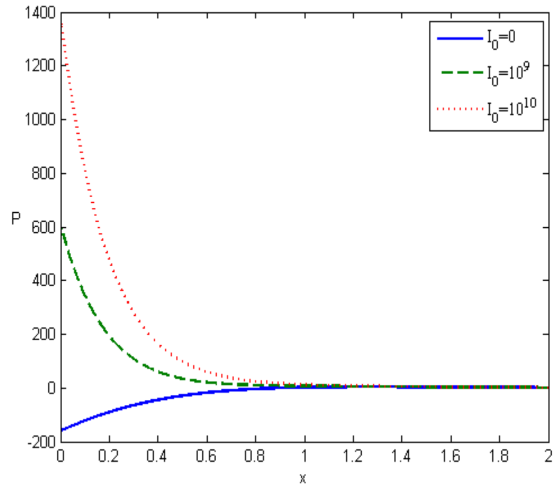

Fig. 6 Chemical potential distribution at $y = z = 0.1$, $t = 0.1$

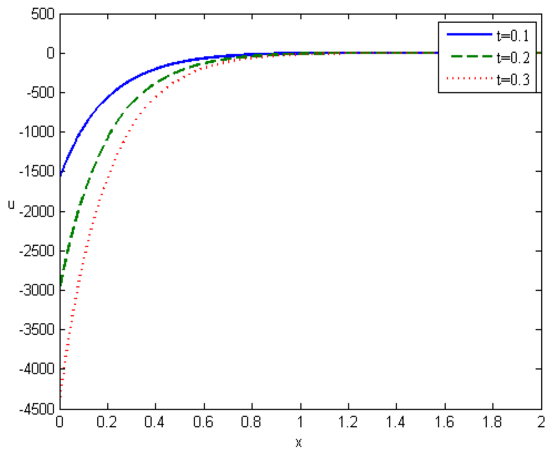

Fig. 7 Displacement distribution at $y = z = 0.1$, $I_0 = 10^{10}$.

Fig. 6 explains the variations of the chemical potential P with a distance x. We can see from this figure that the chemical potential starts from a negative value in the case of the absence of laser pulse effect ($I_0 = 0$) and then increases with the increase of the distance x, while it starts from positive values in the case of the presence of laser pulse effect ($I_0 = 10^9,\ 10^{10}$) and then decreases with the increase of the distance x and finally all curves vanish identically for $x > 1.5$ approximately. It is clearly observed that the laser pulse value has a decreasing effect on the temperature distribution.

Figs. 7-12 represent 2D curves for the distributions of the physical quantities against the distance x at $y = z = 0.1$ (with a fixed value of $I_0 = 10^{10}$) taking three values of the dimensionless time, namely $t = 0.1,\ 0.2,\ 0.3$.

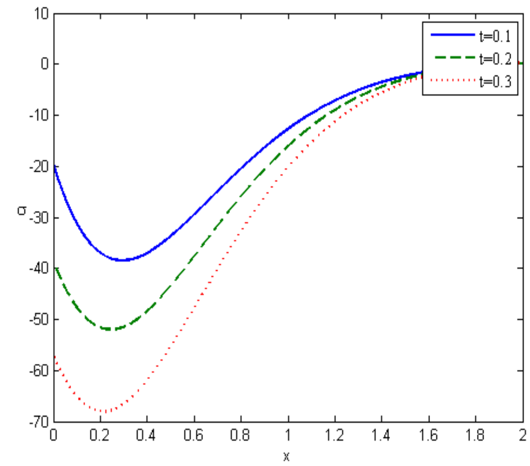

Fig. 8 Stress distribution at $y = z = 0.1$, $I_0 = 10^{10}$.

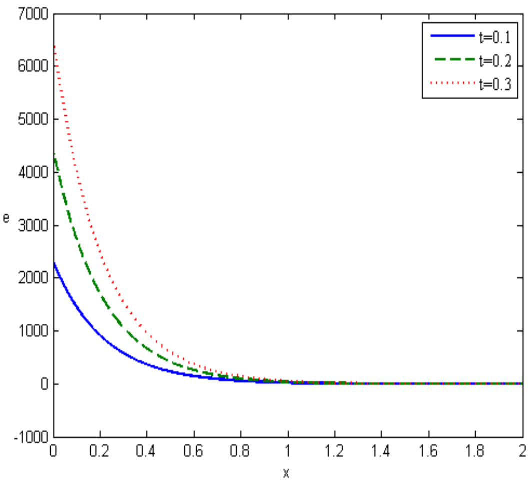

Fig. 9 Strain distribution at $y = z = 0.1$, $I_0 = 10^{10}$.

In these figures, the solid line, dashed line and dotted line correspond for $t = 0.1,\ 0.2,\ 0.3$ respectively, which is furthermore precisely explained in each figure in the legend. Figs. 7 and 10 illustrate the variations of the displacement component u and temperature e with a distance x. These figures show that the displacement component u and temperature T increase with the increase of the distance x and finally all curves terminate at the zero value at $x > 1.5$ approximately and it is observed from these figures that the dimensionless time t has a decreasing effect on the displacement component u and temperature T.

Fig. 8 describes the variations of the stress σ with a distance x. We can see from this figure that stress σ decreases with the increase of the distance x and has a minimum values at $x = 0.30$, $x = 0.25$ and $x = 0.22$ for $t = 0.1,\ 0.2,\ 0.3$ respectively and then all curves increase tending to zero for $x > 2$. From this figure, it can be seen that the dimensionless time t has a decreasing effect on the stress σ. Figs. 9, 11, 12 display the variations of the strain e, the mass concentration C and the chemical potential P with a distance x and it is clear that these figures show that the above mentioned physical quantities decrease with the increase of the distance x for all values of t and finally all curves converge to zero for $x > 1$. It is noticed from these figures that the dimensionless time t has an increasing effect on the strain e, the mass concentration C and the chemical potential P.

Figs. 13-18 represent 3D curves for the variations of the physical quantities against the distance x at $z = 0.1$, $t = 0.1$ and $I_0 = 10^{10}$.

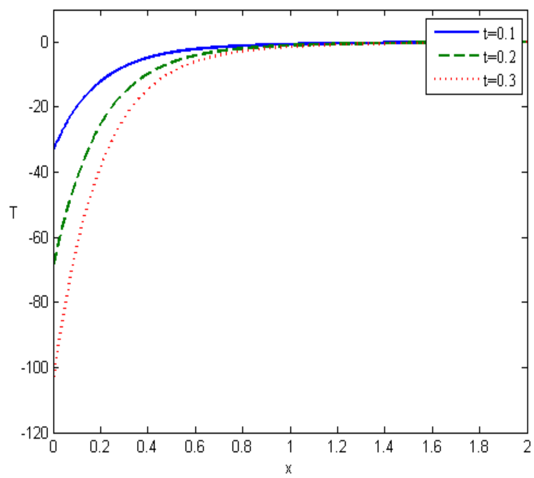

Fig. 10 Temperature distribution at $y = z = 0.1$, $I_0 = 10^{10}$.

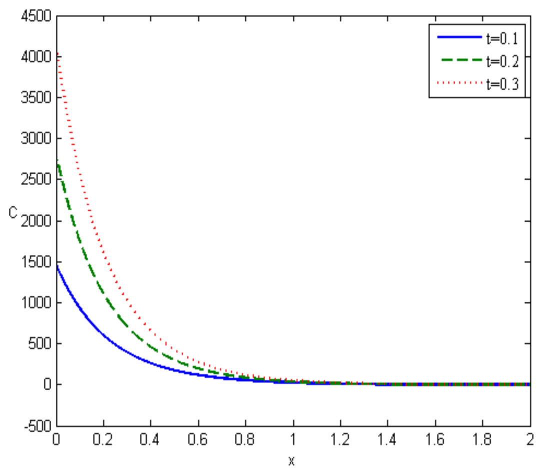

Fig. 11 Mass concentration distribution at $y = z = 0.1$, $I_0 = 10^{10}$.

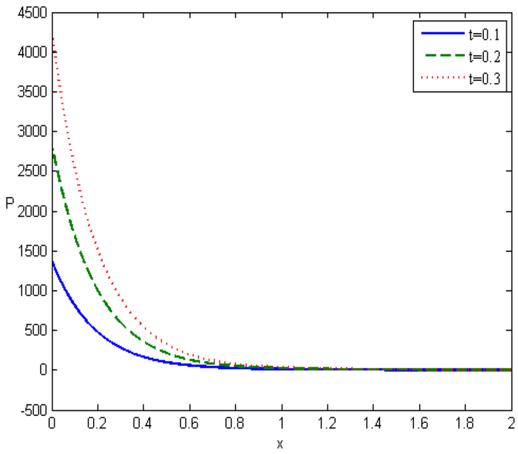

Fig. 12 Chemical potential distribution at $y = z = 0.1$, $I_0 = 10^{10}$.

These figures are very important to study the dependence of the physical quantities on both components of distance x, y. It can be clearly seen that the curves obtained are highly depending on both distance components and we can see that some quantities increase on the negative direction of the distance, while some on a positive direction.

6. Conclusions

A three-dimensional model of the generalized thermo-elasticity with diffusion under the influence of thermal loading due to laser pulse was established and according to the results the following conclusions can be obtained:

* The results indicate that the effect of the thermal loading due to laser pulse on the components of the displacement, stress, strain, temperature, mass concentration and chemical potential distributions is very pronounced.
* It was observed that the time has a significant effect on the distributions of all physical quantities.
* The normal mode analysis, used in this article to solve the problem, is applicable to a wide range of problems in thermodynamics and thermoelasticity. This method gives exact solutions without any assumed restrictions on either the temperature or stress distributions.
* The values of the distributions of all physical quantities converge to zero with increasing distance *x*. Using these results; it possible to investigate the disturbance caused by more general sources for practical applications.
* The physical applications are found in the mechanical engineering, geophysical, and industrial sectors.

Declaration of conflicting interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

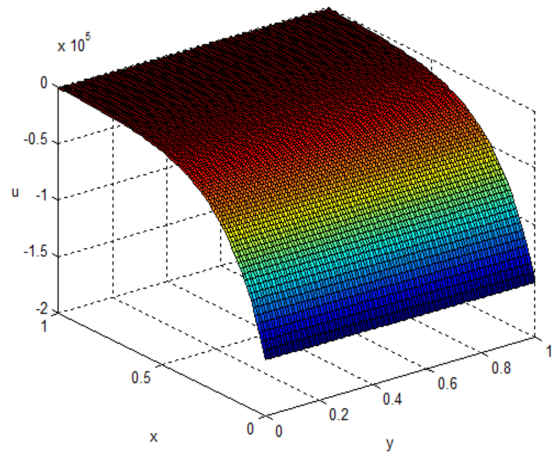

Fig. 13 Displacement distribution at $z = 0.1$, $t = 0.1$, $I_0 = 10^{10}$

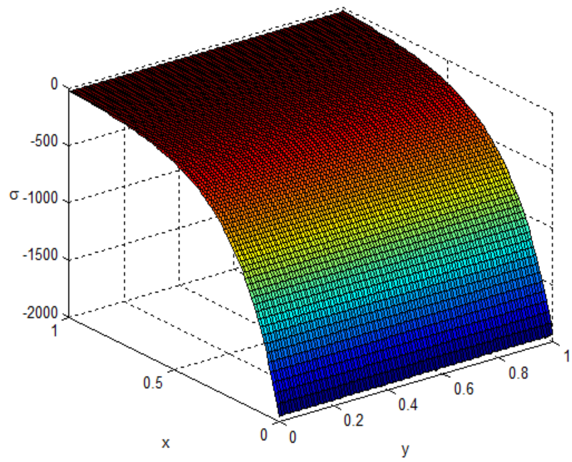

Fig. 14 Stress distribution at $z = 0.1$, $t = 0.1$, $I_0 = 10^{10}$

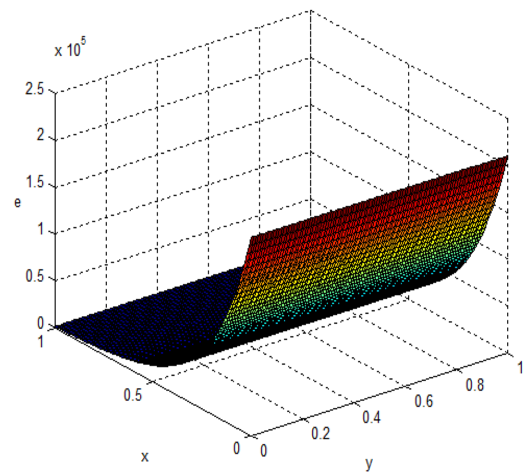

Fig. 15 Strain distribution at $z = 0.1$, $t = 0.1$, $I_0 = 10^{10}$

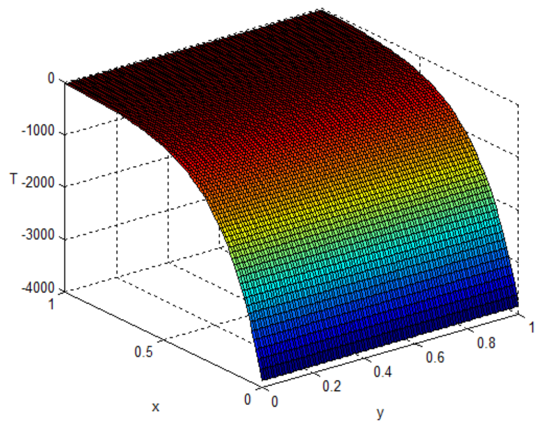

Fig. 16 Temperature distribution at $z = 0.1$, $t = 0.1$, $I_0 = 10^{10}$

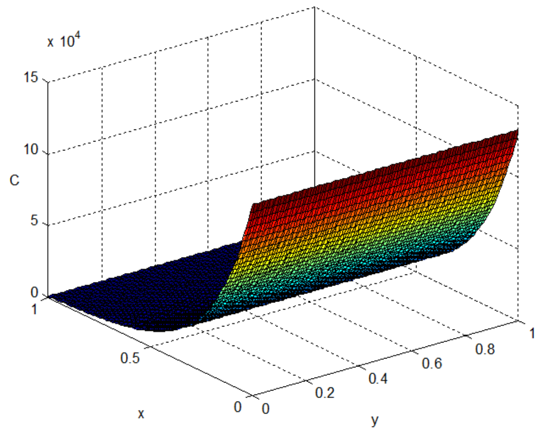

Fig. 17 Mass concentration distribution at $z = 0.1$, $t = 0.1$, $I_0 = 10^{10}$

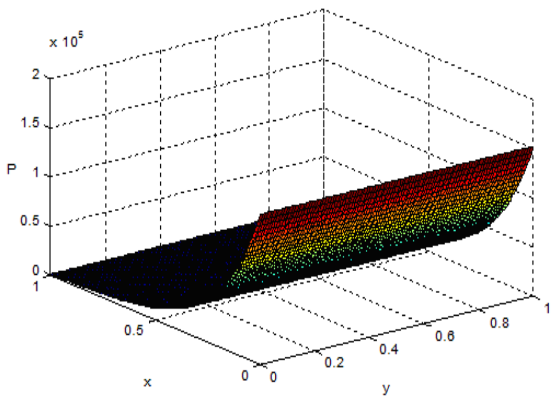

Fig. 18 Chemical potential distribution at $z = 0.1$, $t = 0.1$, $I_0 = 10^{10}$

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