Steel and Composite Structures, *Vol. 36, No. 2 (2020) 179-186*
DOI: https://doi.org/10.12989/scs.2020.36.2.179

On resonance behavior of porous FG curved nanobeams

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(*Received May 14, 2020, Revised June 17, 2020, Accepted June 24, 2020*)

Abstract. In this paper, the forced resonance vibration of porous functionally graded (FG) curved nanobeam is examined. In order to capture the hardening and softening mechanisms of nanostructure, the nonlocal strain gradient theory is employed to build the size-dependent model. Using the Timoshenko beam theory together with the Hamilton principle, the equations of motion for the curved nanobeam are derived. Then, Navier series are used in order to obtain the dynamical deflections of the porous FG curved nanobeam with simply-supported ends. It is found that the resonance position of the nanobeam is very sensitive to the nonlocal and strain gradient parameters, material variation, porosity coefficient, as well as geometrical conditions. The results indicate that the resonance position is postponed by increasing the strain gradient parameter, while the nonlocal parameter has the opposite effect on the results. Furthermore, increasing the opening angle or length-to-thickness ratio will result in resonance position moves to lower-load frequency.

Keywords: resonance phenomena; porous materials; curved nano-beams; nonlocal strain gradient theory

1. Introduction

When the load frequency of the physical system is equal to the natural frequency of the system, the amplitude of the physical system will reach its maximum and resonance phenomenon will occur. The phenomena of resonance exist widely in nature and many engineering fields, which involve in the acoustic resonance of musical instruments, bridges and circuits. Scientists exploit or try to avoid resonance. In engineering, in most cases, resonance can be very destructive to the system. Machine tool, for example, when the moving parts of the machine tool are running, when the external excitation frequency is equal to the natural frequency of the machine tool, resonance will occur, which will affect the machining accuracy and aggravate the damage to the machine tool. In addition, the collapse of Bridges and the crash of airplanes are sometimes caused by resonance phenomena. Therefore, the study of resonance phenomena not only has important engineering background but also has important theoretical value. In addition, as common structures in engineering, the resonance characteristics of beams, plates and shells have attracted the attention of many scholars. For example, Du and Li studied the resonance behavior of FG cylindrical shells under pure bending loadings (Du and Li 2013) and combined bending and thermal loadings (Du and Li 2014) based on multiple scale method. Ahmadi *et al*. (2019) investigated the effect of damping and nonlinear elastic foundations on the resonance of imperfect spiral stiffened FG cylindrical shells. Sebastián *et al*. (2013) performed the resonance analysis of the FG beams under combined harmonic transverse and thermal loadings. However, the resonance vibration of micro/nano structures is a new topic and has not yet been study enough.

Nanomaterials have better mechanical properties than traditional materials (Marami *et al.* 2016) and have been widely used in nano electromechanical systems (NEMS). Recently, the dynamic characteristics of micro and nano structures have become a hot topic (e.g., Akgöz and Civalek 2014, 2015, 2016, 2017, Amar *et al*. 2018, Apuzzo *et al.* 2019, Attia and Rahman 2018, Barretta, Faghidian and Marotti de Sciarra, 2019, Barretta *et al.* 2020, Barretta and de Sciarra 2018, 2019, Civalek and Demir 2011, 2016, Civalek, Uzun, Yaylı, and Akgöz 2020, Demir and Civalek 2013, Eltaher, Fouda, El-Midany, and Sadoun 2018, Fattahi, Safaei, and Moaddab 2019, Malikan *et al.* 2020, Fourn *et al*. 2018, Faleh *et al.* 2018, Gürses, Akgöz, and Civalek 2012, Heydari 2018, Karami, Shahsavari, and Janghorban 2019c, Khaniki 2018, Jandaghian and Rahmani 2017, Moradi-Dastjerdi and Behdinan 2019, Numanoğlu, Akgöz, and Civalek 2018, Zenkour 2018, Zenkour and Radwan 2019).

The resonance phenomenon of nanostructures is an important factor in the design of NEMS. Some scholars have carried out researches in this field. For example, based on Kirchhoff plate theory, Nami and Janghorban (2014) presented the resonance vibration of FG rectangular plate in micro and nano scale with simply-supported ends. According to nonlocal strain gradient (NSG) theory, Tang *et al*. (2018) analyzed the resonance behaviors of FG nanobeams surrounded by nonlinear elastic foundations. Based on modified couple stress theory (Farokhi *et al.* 2015, Farokhi and Ghayesh 2015, Ghayesh *et al*. 2017), Farokhi and Ghayesh (2015) examined the nonlinear

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http://www.techno-press.org/?journal=scs&subpage=8
ISSN: 1229-9367 (Print), 1598-6233 (Online)

resonance responses of Timoshenko microbeams, this work was then extended to the resonance responses of geometrically imperfect FG extensible microbeams by the same authors (Ghayesh *et al.* 2017). Farokhi *et al.* (2015) analyzed the effects of axial load and geometric imperfection on the resonance of Timoshenko mircobeams. Employing the third-order shear deformable plate theory, Ansari and Gholami (2016) predicted the resonance behaviors of the FG plates reinforced by carbon nanotubes. Karami *et al.* (2019a) discussed the resonance characteristic of FG nanoplates, in this work, the Kirchhoff plate theory are adopted. According to the nonlocal stress theory, Karami *et al*. (2019b) studied the resonance of FG polymer composite nanoplates reinforced with graphene nanoplatelets, their work shows that the reinforcement patterns can determine the resonance position.

Curved beams are the basic structural elements of NEMS, and their design requires appropriate scale effect models (Barretta *et al*. 2019). At present, some scholars have studied the mechanical behavior of curved beams at micro/nano scale. For example, employing the stress-driven nonlocal model, Barretta *et al*. (2019) applied the nonlocal integral methodology to study the static bending of nanobeams. According to the strain gradient theory, Qi *et al*. (2018) investigated the static bending and free vibration characteristics of the flexoelectric curved micro beams. Employing the nonlocal stress theory and sinusoidal shear deformation theory, Arefi and Zenkour presented the thermal stress, deformation analyses (Arefi and Zenkour 2018a) and vibrations (Arefi and Zenkour 2018b) of FG curved nanobeams. Based on NSG theory, Ebrahimi and Barati (2017) investigated the buckling characteristics of FG curved nanobeams with the help of various beam theories. Taking into the thickness stretching effect, Ganapathi and Polit (2017) performed the buckling and bending analyses of the curved nanobeams. Employing the nonlocal theory, Polit and his partners discussed the elastic stability (Polit, Merzouki, and Ganapathi 2018) and vibration (Ganapathi, Merzouki, and Polit 2018) analyses of curved nanobeam via finite element approach. Using the experimental method, Medina *et al*. (2014) presented the symmetric and asymmetric buckling of curved microbeam under axial and electrostatic force. Based on NSG theory, She and his partners performed the snap-buckling analyses of FG curved nanobeams subjected to combined bending and thermal loadings (She *et al.* 2019).

From the literature survey, it is indicated that all of the existing valuable articles about curved micro/nano beams are limited to the buckling/bending problems or the free vibration of curved nanobeams, and the existing literature about forced resonance vibration are limited to flat micro/nano beams or shells. In addition, there is no works investigating the forced resonance vibration of the curved beams. Inspired this fact, this paper aims at predicting the forced resonance vibration of porous FG curved nanobeams. To this end, based on NSG theory, the governing equations with simply-supported ends are derived and solved by Navier's series. According to the numerical analysis, it is found that the material composition, nonlocal and strain gradient parameters, porosity, opening angle and length-to-thickness ratio have a crucial role to play in the frequency-deflection response of the porous FG curved nanobeam.

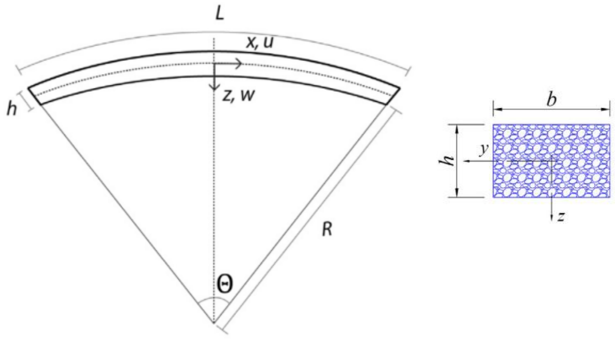

Fig. 1 Configuration of FG curved nanobeam (From Anirudh *et al*. 2020 and Lei *et al*. 2020)

2. Governing equations

A curved nanobeam made from a mixture of ceramic (denoted by *c*) and metal (denoted by *m*) is considered in Fig. 1. The length and thickness of the nanobeam are denoted by *L* and *h*, respectively. Herein, the effective material properties (including mass density ρ(z), Poisson ratio v(z), and Young's modulus E(z)) of the nanobeams are approximated by a modified power-law rule as follows (She *et al.* 2019, Jalaei and Civalek 2019)

$$P(z) = -\frac{\xi}{2}\left(P_c + P_m\right) + P_m + (P_c - P_m)(\frac{z}{h} + \frac{1}{2})^n \quad \text{for } 0 \le n \le \infty \text{ and } \xi << 1 \tag{1}$$

Here, ξ refers to the porosity volume fraction, n stands for the power law index.

Here, we assume that the displacement field at any point of the nanobeam according to Timoshenko beam theory can be expressed as (Hosseini and Rahmani 2016)

$$\begin{aligned} u(x,z,t) &= u(x,t) - z\,\varphi(x,t) \\ w(x,z,t) &= w(x,t) \end{aligned} \tag{2}$$

Here, *u* and *w* are tangential and radial displacement of a point and φ represents the rotation. The non-zero strains are

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} - z\frac{\partial \varphi}{\partial x} + \frac{w}{R} \\ \gamma_{xz} &= \frac{\partial w}{\partial x} - \varphi - \frac{u}{R} \end{aligned} \tag{3}$$

To obtain the governing equations, the extended Hamilton principle is utilized

$$\delta\int_{t_1}^{t_2}(T - U + W)dt = 0 \tag{4}$$

The strain energy's variation *U* can be written as

$$\delta U=\int_V(\sigma_{ij}\delta\varepsilon_{ij})dV$$
$$=\int_0^L(N_{xx}(\frac{\partial\delta u}{\partial x}+\frac{\delta w}{R})-M_{xx}\frac{\partial\delta\varphi}{\partial x}$$
$$+Q_{xz}(\frac{\partial\delta w}{\partial x}-\frac{\delta u}{R}-\delta\varphi))dx \tag{5}$$

where

$$N_{xx}=\int_{-h/2}^{h/2}\sigma_{xx}dz$$
$$M_{xx}=\int_{-h/2}^{h/2}z\,\sigma_{xx}dz \tag{6}$$
$$Q_{xz}=\kappa\int_{-h/2}^{h/2}\tau_{xz}dz$$

K=5/6 denotes the shear correction factor (Romano *et al*. 2012, Faghidian 2017).

Kinetic energy's variation *T* is

$$\delta T=\int_V(\dot u\,\delta\dot u+\dot w\,\delta\dot w)dV$$
$$=\int_0^L\{I_0(\dot u\,\delta\dot u+\dot w\,\delta\dot w)-I_1(\dot u\,\delta\dot\varphi+\dot\varphi\delta\dot u)+I_2\dot\varphi\delta\dot\varphi\}dx \tag{7}$$

in which

$$I_0=\int_{-h/2}^{h/2}\left[-\frac{\xi}{2}(\rho_c+\rho_m)+\rho_m+(\rho_c-\rho_m)(\frac{z}{h}+\frac{1}{2})^n\right]dz$$
$$I_1=\int_{-h/2}^{h/2}z\left[-\frac{\xi}{2}(\rho_c+\rho_m)+\rho_m+(\rho_c-\rho_m)(\frac{z}{h}+\frac{1}{2})^n\right]dz \tag{8}$$
$$I_2=\int_{-h/2}^{h/2}z^2\left[-\frac{\xi}{2}(\rho_c+\rho_m)+\rho_m+(\rho_c-\rho_m)(\frac{z}{h}+\frac{1}{2})^n\right]dz$$

The variation of the virtual work done *W* by the transverse load q can be obtained as

$$\delta W=\int_0^L q_{\text{dynamic}}\delta w dx \tag{9}$$

Substituting Eqs. (5), (7) and (9) into Eq. (4), and integrating through the thickness, the following governing equations can be arrived at

$$\delta u:\frac{\partial N_{xx}}{\partial x}+\frac{Q_{xz}}{R}=I_0\ddot u-I_1\ddot\varphi \tag{10}$$

$$\delta w:\frac{\partial Q_{xz}}{\partial x}-\frac{N_{xx}}{R}-q_{\text{dynamic}}=I_0\ddot w \tag{11}$$

$$\delta\varphi:-\frac{\partial M_{xx}}{\partial x}+Q_{xz}=-I_1\ddot u+I_2\ddot\varphi_x \tag{12}$$

It is noted that this Timoshenko beam theory can be appropriately disused from the classical elasticity formulation of the Saint-Venant flexure problem Faghidian (2016). Since classical continuum theories cannot predict the behavior of nanostructures, the non-classical theories have been put forward to overcome the shortcoming of classical theories. In the present work, based on the NSG theory (Lim *et al.* 2015, Ghayesh and Farajpour 2018, Lu *et al*. 2019), the stress-strain relation can be written as

$$\sigma_{xx}-(ea)^2\nabla^2\sigma_{xx}$$
$$=\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]\varepsilon_{xx}$$
$$-\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]\ell^2\nabla^2\varepsilon_{xx} \tag{13}$$

$$\tau_{xz}-(ea)^2\nabla^2\tau_{xz}=(1-\ell^2\nabla^2)$$
$$\times\left\{\frac{\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]\kappa\gamma_{xz}}{2\left(1+\left[-\frac{\xi}{2}(\upsilon_c+\upsilon_m)+\upsilon_m+(\upsilon_c-\upsilon_m)(\frac{z}{h}+\frac{1}{2})^n\right]\right)}\right\} \tag{14}$$

Here, (ea) is the nonlocal parameter, and ℓ denotes the strain gradient parameter. According to the stress resultants (Eq. (6)), then, we have

$$N_{xx}-(ea)^2\nabla^2N_{xx}=A_{11}(\frac{\partial u}{\partial x}+\frac{w}{R})-B_{11}\frac{\partial\varphi}{\partial x}$$
$$-\ell^2\nabla^2(A_{11}(\frac{\partial u}{\partial x}+\frac{w}{R})-B_{11}\frac{\partial\varphi}{\partial x}) \tag{15}$$

$$M_{xx}-(ea)^2\nabla^2M_{xx}=B_{11}(\frac{\partial u}{\partial x}+\frac{w}{R})-C_{11}\frac{\partial\varphi}{\partial x}$$
$$-\ell^2\nabla^2(B_{11}(\frac{\partial u}{\partial x}+\frac{w}{R})-C_{11}\frac{\partial\varphi}{\partial x}) \tag{16}$$

$$Q_{xz}-(ea)^2\nabla^2Q_{xz}=A_{13}(\frac{\partial w}{\partial x}-\varphi-\frac{u}{R})$$
$$-\ell^2\nabla^2(A_{13}(\frac{\partial w}{\partial x}-\varphi-\frac{u}{R})) \tag{17}$$

where

$$A_{11}=\int_{-h/2}^{h/2}\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]dz$$
$$B_{11}=\int_{-h/2}^{h/2}\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]zdz$$
$$C_{11}=\int_{-h/2}^{h/2}\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]z^2dz \tag{18}$$
$$A_{13}=\kappa\int_{-h/2}^{h/2}\frac{\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]}{2\left(1+\left[-\frac{\xi}{2}(E_c+E_m)+E_m+(E_c-E_m)(\frac{z}{h}+\frac{1}{2})^n\right]\right)}dz$$

By inserting the rewritten stress resultants (Eqs. (15)-(17)) into the equilibrium equations (Eqs. (10)-(12)), we have the following governing equations

$$A_{11}(1-\ell^2\frac{\partial^2}{\partial x^2})(\frac{\partial^2u}{\partial x^2}+\frac{\partial w}{R\partial x})-B_{11}(1-\ell^2\frac{\partial^2}{\partial x^2})\frac{\partial^2\varphi}{\partial x^2}$$
$$+\frac{1}{R}\left\{A_{13}(1-\ell^2\frac{\partial^2}{\partial x^2})(\frac{\partial w}{\partial x}-\varphi-\frac{u}{R})\right\}=I_0\frac{\partial^2u}{\partial t^2}$$
$$-I_1\frac{\partial^2\varphi}{\partial t^2}-(ea)^2(I_0\frac{\partial^4u}{\partial x^2\partial t^2}-I_1\frac{\partial^4\varphi}{\partial x^2\partial t^2}) \tag{19}$$

$$A_{13}(1-\ell^2\frac{\partial^2}{\partial x^2})(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \varphi}{\partial x}-\frac{\partial u}{R\partial x})$$
$$-\frac{1}{R}\left\{A_{11}(1-\ell^2\frac{\partial^2}{\partial x^2})(\frac{\partial u}{\partial x}+\frac{w}{R})-B_{11}\frac{\partial \varphi}{\partial x}\right\} \tag{20}$$
$$-(1-(ea)^2\frac{\partial^2}{\partial x^2})q_{\mathrm{dynamic}}=I_0\frac{\partial^2 w}{\partial t^2}-(ea)^2(I_0\frac{\partial^4 w}{\partial x^2\partial t^2})$$

$$-B_{11}(1-\ell^2\frac{\partial^2}{\partial x^2})(\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{R\partial x})+C_{11}\frac{\partial^2\varphi}{\partial x^2}$$
$$+A_{13}(1-\ell^2\frac{\partial^2}{\partial x^2})(\frac{\partial w}{\partial x}-\varphi-\frac{u}{R})=I_2\frac{\partial^2\varphi}{\partial t^2} \tag{21}$$
$$-I_1\frac{\partial^2 u}{\partial t^2}-(ea)^2(I_2\frac{\partial^4\varphi}{\partial x^2\partial t^2}-I_1\frac{\partial^4 u}{\partial x^2\partial t^2})$$

4. Solution method

According to the suitable admissible functions, the forced resonance vibration of the curved nanobeams with simply-supported ends can be determined with the help of the Navier's series. To this end, the displacements of Timoshenko nanobeam are selected as

$$\begin{Bmatrix} u \\ w \\ \varphi \end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix} U_m \cos\alpha x \sin\Omega t \\ W_m \sin\alpha x \sin\Omega t \\ \Phi_m \cos\alpha x \sin\Omega t \end{Bmatrix} \tag{22}$$

where $\alpha=m\pi/L$; Ω denotes excitation frequency; U_m, W_m, Φ_m represent undetermined displacement amplitudes. Furthermore, for the simply supported ends,

$$u(x,t)=0, w(x,t)=0, M(x,t)=0 \text{ at } x=0,L \tag{23}$$

Substituting Eq. (22) into Eqs. (19)-(21), and collecting the coefficients of displacement amplitudes, we can obtain the following matrix

$$(\Omega^2[M]+[K])\begin{Bmatrix} U_m \\ W_m \\ \Phi_m \end{Bmatrix}=\begin{Bmatrix} 0 \\ q_{\mathrm{dynamic}} \\ 0 \end{Bmatrix} \tag{24}$$

in which $[M]$, and $[K]$ are, respectively, mass matrix, and the total stiffness matrix. In the current work, we assume that the applied load is distributed on the surface of the nanobeam, and the specific load frequency can be expressed as

$$q_{\mathrm{dynamic}}=\sum_{m=1}^{\infty}Q_m \sin\alpha x \sin\Omega t \tag{25}$$

Herein, Q_m is the load amplitude.

5. Numerical results

This work deals with the forced resonance vibrations of curved Al_3O_3/SUS304 nanobeam. The adopted material properties are (Ebrahimi and Barati 2017): ρ_c=2370 kg/m^3, v_c=0.24, E_c=348.43 GPa; ρ_m=8166 kg/m^3, v_m=0.3262, E_m=201.04 GPa. To ensure the accuracy of present model, the free vibration of Steel/Alumina nanobeam is compared to the results of Ebrahimi and Barati (2017) and tabulated in Table 1, from which good agreement can be seen. It is worth noting that the curved nanobeam behave like a straight one when $R=+\infty$. It is important to note that, in the following analysis, $\mu=(ea)^2$ and $\lambda=\ell^2$. In addition, we define the following dimensionless notation (Karami *et al.* 2019a,b):

$$\text{Deflection raio}=\frac{\text{Deflection using nonloacl/strain gradient theory}}{\text{Deflection using local theory}}$$

Figs. 2 and 3 depict the resonance phenomena of the curved nanobeams with different strain gradient and nonlocal parameters, respectively. From Fig. 2 it is observable that the resonance position is postponed by increasing the strain gradient parameter due to the fact that the totally stiffness of the curved nanobeams increases as the strain gradient parameter rises. Furthermore, as seen in Fig. 3, the resonance position will move to the lower load frequency which is due to the reduction in total stiffness of the nanobeams. Clearly, the resonance position of nanostructures can be changed by the small-scale parameters.

Fig. 4 shows the effect of opening angle on the resonance position of the curved porous nanobeams. Furthermore, the resonance position of straight beam is studied. It is found that increasing opening angle results in resonance position moves to lower-load frequencies and it's because of decreasing the radius of curvature ($R=L/\alpha$).

The effect of length-to-thickness ratio L/h is illustrated in Fig. 5. It is observed that increasing the length-to-thickness ratio leads to resonance position moves to lower-load frequencies.

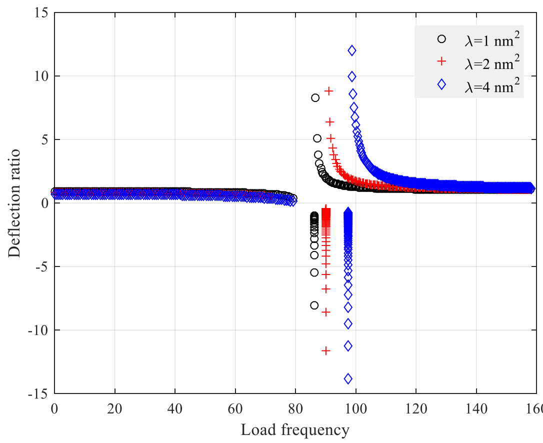

Fig. 2 The effect of the strain gradient parameter on the resonance position of the curved nanobeam, (L=10nm, α=π/3, L/h=20, n=1, ξ=0.1, μ=0)

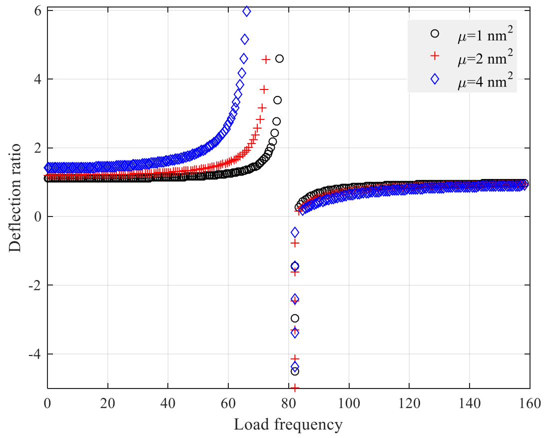

Fig. 3 The effect of the nonlocal parameter on the resonance position of the curved nanobeams, (L=10 nm, $\alpha=\pi/3$, L/h=20, n=1, ξ=0.1, λ=0)

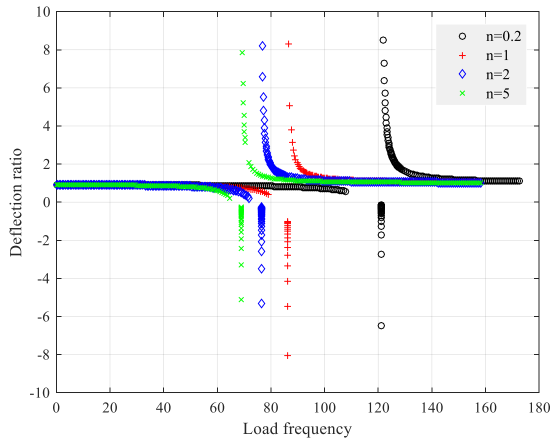

Fig. 6 The effect of the power-law index on the resonance position of the curved nanobeam, (L=10 nm, $\alpha=\pi/3$, L/h=20, ξ=0.1, μ=0, λ=1 nm^2)

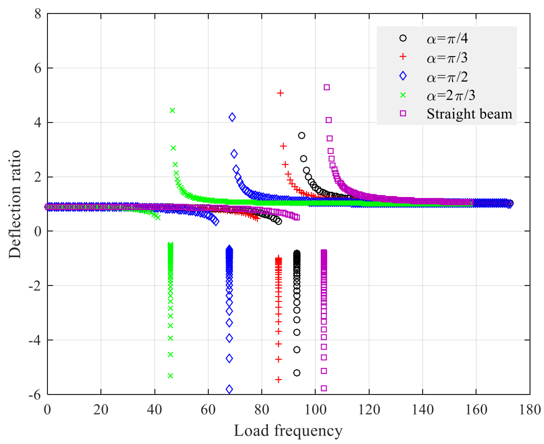

Fig. 4 The effect of the opening angle on the resonance position of the curved nanobeam, (L=10 nm, L/h=20, n=1, ξ=0.1, μ=0, λ=1 nm^2)

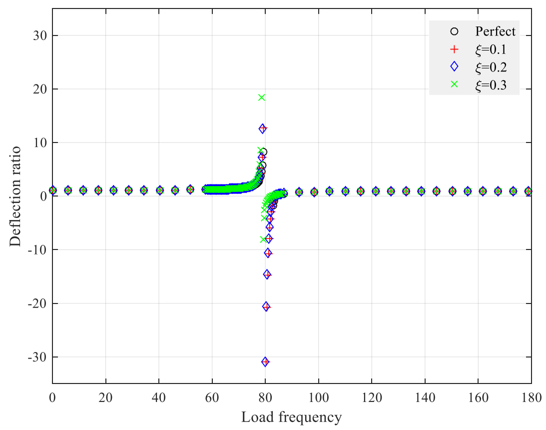

Fig. 7 The effect of the porosity coefficient on the resonance position of the curved nanobeam, (L=10 nm, $\alpha=\pi/3$, L/h=20, n=1, μ=1 nm^2, λ=0)

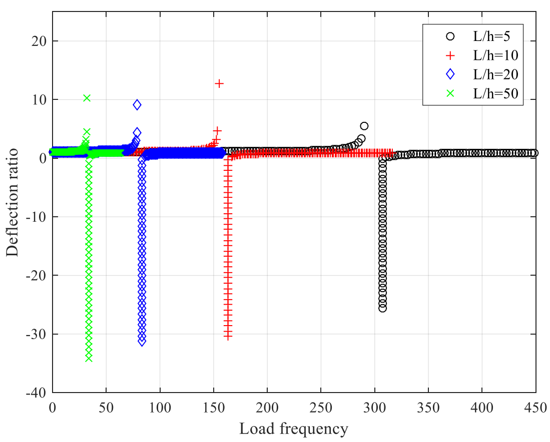

Fig. 5 The effect of the length-to-thickness ratio on the resonance position of the curved nanobeam, (L=10 nm, $\alpha=\pi/3$, n=1, ξ=0.1, μ=1 nm^2, λ=0)

In other words, the resonance position is postponed by increasing the thickness of the nanobeams. From Fig. 5, it can be concluded that the vibration of nanostructures can be controlled by geometrical parameters.

The effect of material composition considering power-law index on the dynamical deflection ratio of the nanobeams is illustrated in Fig. 6. It is brightly shown that the rise of power-law indices yields to move resonance position to lower-load frequencies. It is because of increasing the metal phase which has lower value of Young's modulus compared to ceramic one that leads to decrease the stiffness of the nanobeams.

Porosity affected resonance poison of the nanobeam is depicted in Fig. 7. As seen, by increasing the porosity, the resonance position will move to lower-load frequencies. Furthermore, it worth mentioning that this coefficient has no valuable impact on the resonance phenomenon, but maybe propounding this factor leads to provide a better response compared to the nature of FGMs.

Table 1 Comparisons of non-dimensional frequency $\Omega = \omega R^2 \sqrt{\rho_c A / E_c I}$ of curved Steel/Alumina nanobeams (L/h=10, μ=1 nm^2, λ=0.5 nm^2, ξ=0)

	n=0.2			n=1			n=5		
(n,α)	$\alpha=\pi/3$	$\alpha=\pi/2$	$\alpha=2\pi/3$	$\alpha=\pi/3$	$\alpha=\pi/2$	$\alpha=2\pi/3$	$\alpha=\pi/3$	$\alpha=\pi/2$	$\alpha=2\pi/3$
Ebrahimi and Barati (2017)	6.38035	2.25189	0.87262	5.10491	1.79836	0.69638	4.36455	1.5386	0.59575
Present	6.37976	2.25017	0.87198	5.10376	1.79767	0.69567	4.36369	1.5307	0.59540

6. Conclusions

This work aims at investigating the forced resonance phenomenon of a curved nano-size beam whose was made of FGMs. The influence of porosities was also considered. Timoshenko beam model and NSG theory were used to obtain the motion governing equations. Then, an analytical technique based Navier series was adopted to solve the dynamic problem and get the resonance position. Through the numerical examples, the conclusions can be summarized as below:

- It was revealed that the resonance position will move to higher load frequencies by increasing the strain gradient size dependency. In other word, body stiffness's growth has been observable for the nanobeam with the rise of gradient length scale parameter.
- It was revealed that the resonance position will move to lower load frequencies by increasing the nonlocality. In other word, body stiffness's reduction has been observable for the nanobeam with the rise of nonlocal parameter.
- The resonance position will move to lower-load frequencies with increasing the opening angle.
- The resonance position was postponed by increasing the thickness of the nanobeams.
- The resonance position of the porous FG curved nanobeam will move to lower-load frequency by increasing the power-law indices.
- It was revealed that the porosity doesn't play an important role in the resonance phenomenon of the nanobeam.

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