Steel and Composite Structures, *Vol. 35, No. 6 (2020) 753-763*
DOI: https://doi.org/10.12989/scs.2020.35.6.753

Nonlinear deflection responses of layered composite structure using uncertain fuzzified elastic properties

B.K. Patle[1a], Chetan K. Hirwani[2b], Subrata Kumar Panda *[3], Pankaj V. Katariya[3c], Hukum Chand Dewangan[3d] and Nitin Sharma[4e]

[1]Department of Mechanical Engineering, CVR College of Engineering, Hyderabad, India
[2]Department of Mechanical Engineering, National Institute of Technology Patna, Bihar, India
[3]Department of Mechanical Engineering, National Institute of Technology Rourkela, Odisha, India
[4]School of Mechanical Engineering, KIIT Bhubaneswar, Odisha, India

(*Received September 26, 2019, Revised May 14, 2020, Accepted May 23, 2020*)

Abstract. In this article, the influence of fuzzified uncertain composite elastic properties on non-linear deformation behaviour of the composite structure is investigated under external mechanical loadings (uniform and sinusoidal distributed loading) including the different end boundaries. In this regard, the composite model has been derived considering the fuzzified elastic properties (through a triangular fuzzy function, α cut) and the large geometrical distortion (Green-Lagrange strain) in the framework of the higher-order mid-plane kinematics. The results are obtained using the fuzzified nonlinear finite element model via in-house developed computer code (MATLAB). Initially, the model accuracy has been established and explored later to show the dominating elastic parameter affect the deflection due to the inclusion of fuzzified properties by solving a set of new numerical examples.

Keywords: nonlinear bending; Green-Lagrange; laminated composite; fuzzy-FEM; Uncertain properties

1. Introduction

The composite structural components are gaining a significant role in different modern industries (aerospace, automobile, construction, etc.) because of their incomparable mechanical, physical and electrical (Mukhopadhyay 2009) properties and low cost to weight ratio in comparison to their metallic equivalents. In general, the fibre-reinforced composite elastic properties mostly depend on fibre orientation and individual properties. Moreover, the fibre distribution, the fractions of volume of individual constituent (fibre and matrix) and subsequent variation in geometry may also alter the properties from the expected line. Additionally, the composite components exposed to the service environment, may experience vibration (fatigue due to cyclic loading) and/or bending (excess geometrical distortion). These loadings lead to the final failure of the structural components. In this regard, a list published research items correlated to the modelling of a composite including their experimental and numerical characteristic under the influence of single or multiple loading of different structural configurations (plate, beam, shell etc.) are discussed. Also, the composite properties and geometry may vary during the process of manufacturing or due to the external environmental effect, which may induce uncertainty and affect the final performances too. To emphasize the requirement of the current analysis, a few earlier relevant research articles are discussed considering the deterministic as well as the uncertain properties in the subsequent upcoming lines.

The flexural and the free vibration frequencies of the layered structure (Akhras and Li 2005) are reported via the generic mathematical model considering Reddy's model (higher-order kinematic theory) in conjunction with spline finite strip method. Also, several article reported on the structural modelling and subsequent responses (dynamic, flexural, buckling/post-buckling, etc.) of the layered (Szerkrenyes and Jozsef 2007, Ghannadpour *et al.* 2014) /graded structure (Mehrparvar and Ghannadpour, 2018) considering the deterministic type of properties and von-Karman nonlinear strain (Ovesy and Ghannadpour 2006, 2007, Ovesy *et al*. 2006, Ghannadpour *et al*. 2006, Ovesy *et al.* 2015). Also, a few research reported considering the lower order kinematic models i.e., the classical plate theory (CPT) and the first-order shear deformation theory (FSDT).

As discussed previously, the uncertainty in composite properties may have demonstrative impact on the final structural characteristics and a few research articles are

*Corresponding author, Associate Professor
E-mail: pandask@nitrkl.ac.in
[a]Associate Professor
E-mail: balu_patle@rediffmail.com
[b]Assistant Professor
E-mail: chetank.me@nitp.ac.in
[c]Ph.D. Student
E-mail: pk.pankajkatariya@gmail.com
[d]Ph.D. Student
E-mail: hukumdewangan@gmail.com
[e]Associate Professor
E-mail: nits.iiit@gmail.com

http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

discussed in the subsequent lines. An analytical technique proposed by Cherki *et al*. (2000) to understand the structural sensitivities using the fuzzy numbers to include the uncertainties of the prescribed displacements. Noor *et al.* (2000) adopted the two-phase approach to analyse the effect of the material and geometrical parameters und uncertainty on nonlinear structural responses of the composite structure. Akpan (2000) formulated the uncertainties within the smart structure through the fuzzy set approach considering the influences of piezoelectric effect, mechanical loading, thermal environment and physical conditions. Akpan (2001) utilized finite element method (FEM) for the modelling of structural components considering the fuzzified material properties. Massa *et al*. (2004) demonstrated the competent approach for the calculation of fuzzified eigenvalues and the eigenvectors of the structure considering the imprecise fuzzy parameters through an α-cut material and geometrical parameters (fuzzy numbers within Taylor's expansion). Subsequently, the flexural and the modal frequency responses of the beam type structural components are reported using the deterministic type finite element (FE) theory as well as the fuzzified properties by Liu and Rao (2005). Gersem *et al*. (2007) presented an efficient non-probabilistic approach including fuzzy steps and mode synthesis technique for saving the time required to do calculation. Giannini and Hanss (2008) also presented the mathematical model (coupled standard transformation method and fuzzy arithmetic) which allow a relatively fast calculation to solve the problem of uncertainty management in the structural analysis. To predict the experimental structural characteristics, Massa *et al.* (2009) presented the experimental frequency and compared with the numerical responses (fuzzy eigenvalue analysis). The nonlinear flexural characteristics of the laminated composite are further by Dash and Singh (2010) via the higher-order type mathematical model. Similarly, the nonlinear flexural characteristics of the layered composite structure are also reported by the same authors (Dash and Singh 2010) using their customized higher-order FE model considering the large distortion. Subsequently, Luo *et al.* (2011) utilize the fuzzified FE model (FFEM) approach for the prediction of failure (serviceability) in a spatially random field. Pawar *et al*. (2012) introduced fuzzy arithmetic operators to evaluate the uncertainties of material properties of the thin-walled composite beams and investigated the stochastic behaviours of the same. Bui and his co-authors (2013, 2013a, 2014) evaluate the bending and buckling behaviour of composite structural components using the various techniques (meshfree Galerkin Kriging method and Improved knowledge-based neural network). Behera and Chakraverty (2013) proposed a method for the static deflections of the beam type structure using the fuzzified linear equilibrium equation considering system uncertainty. Adhikari and Khodaparast (2014) presented the uncertainty analysis of the composite structure using a spectral approach including the fuzzy variable. Xia and Friswell (2014) improved the fuzzy eigenvalue approach by reusing the interval analysis, demonstrated on a cantilever beam and obtained very accurate fuzzy eigenvalue solutions. The flexural characteristics of the functionally graded (FG) and sandwich structures analysed using the FEM (Taghizadeh *et al.* 2015) in the framework of Mindlin plate theory (Heydari *et al*. 2014), the higher-order theory (Bennai *et al*. 2015) and sinusoidal shear deformation theory (Kolahchi *et al*. 2015). In the recent past, the bending deflections of the single-layer graphene sheet (Shahsavari and Janghorban, 2017), advanced composite (Houari *et al*. 2018) and thermo-elastic analysis of FG-carbon nanotube-reinforced (Arefi *et al.* 2018) pressure vessels performed via various theories. Static deflection of FGM plates (Bouiadjra *et al*. 2018), carbon fibre reinforced plastic (CFRP) strengthened reinforced concrete (RC) slab (Razavi *et al.* 2015) and functionally graded carbon nanotube-reinforced composite (FG-CNTR) structure (Keleshteri *et al*. 2019) are analysed developing the mathematical model based on various theories.

From the above literature review, it can be understood easily that an ample amount of work has already been completed on the linear analysis for plate structures using different theories (CPT, FSDT, Reddy's higher-order theory). Moreover, a few research also dedicated to the thin-walled structure considering the fuzzified property variation. However, the bending behaviour of the composite structure considering large distortion and fuzzified properties has been not reported yet in published domain. In this regard, an effort has been made to derive a generic nonlinear fuzzified finite element model (FFEM) to investigate the influence of fuzzified elastic properties on the nonlinear deflection characteristics of the layered composite panels and geometrical configurations (single and double curvature).

2. Theoretical formulation

2.1 Fuzzy concept

Now, to introduce the uncertainity in the current analysis, the fuzzy set theory has been adopted to convert the deterministic input parameter of the laminated structure. The fuzzy set theory utilizes the membership function and the corresponding step wise details provided in the following lines. Let '$\mathfrak{R}$' be a crisp set of objects forming a *universe* (Patle *et al.* 2018) and element from the universe be denoted as 'ς'. Let Ψ be a subset (crisp) of $\mathfrak{R}$. Now, the corresponding member of Ψ is mapped to the valuation set [0, 1] from $\mathfrak{R}$ through χ (a characteristic function) as

$$\chi(\varsigma)=\begin{cases}1, & \text{for} \quad \varsigma\in\Psi, \\ 0, & \text{for} \quad \varsigma\notin\Psi\end{cases} \tag{1}$$

Now, for the given real valued and bounded valuation set, a fuzzy set 'Ψ' in '$\mathfrak{R}$' can be defined as

$$\Psi=\{[\varsigma,\chi(\varsigma)],\ \varsigma\in\mathfrak{R}\} \qquad \chi(\varsigma)\in[0,1] \tag{2}$$

where, $\chi(\varsigma)$ is the compatibility of the membership function and the corresponding element i.e. ς in 'Ψ'.

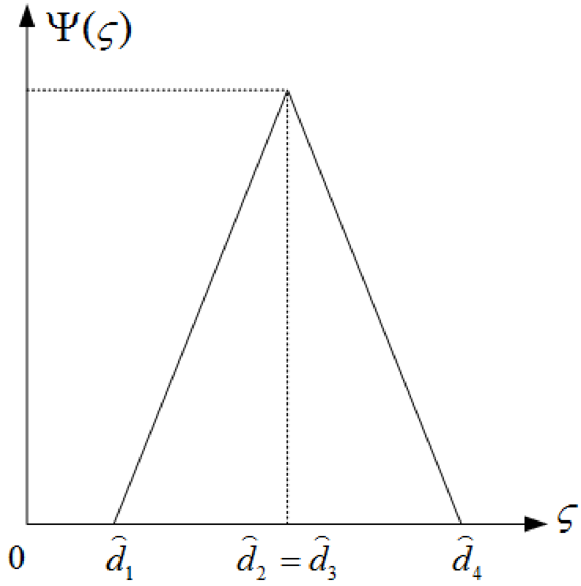

Fig. 1 Triangular fuzzy set representation

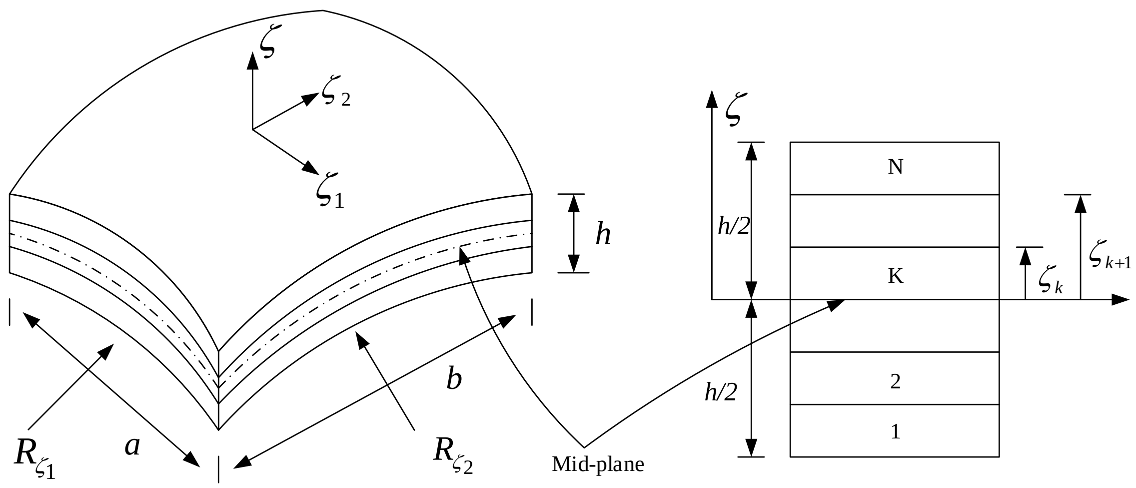

Fig. 2 Representation of shell panel type composite

Similarly, the membership function can be defined within the interval [0, 1].

Further, the fuzzification has been introduced using the α-cut function parameter i.e. an element $\varsigma \in \Re$. Fig. 1 denotes a triangular fuzzy set 'Ψ', such that $\Psi = \left\langle \hat{d}_1, \hat{d}_2, \hat{d}_4 \right\rangle$. The α-cut of 'Ψ' defined as 'Ψ_α' can be expressed as

$$\Psi_\alpha = \left[\hat{d}_1 + \alpha(\hat{d}_2 - \hat{d}_1), \hat{d}_4 - \alpha(\hat{d}_4 - \hat{d}_3) \right], \qquad \alpha \in [0,1] \quad (3)$$

2.2 Fuzzy kinematic model

The curved panel is defined using the spherical coordinate system ($0, \xi_1, \xi_2, \zeta$) and shown in Fig. 2. The panel's geometrical parameter is presented by 'a', 'b' and 'h', which denotes the length, the width and the thickness along ξ_1, ξ_2 and ζ direction, respectively. Similarly, R_{ξ_1} and R_{ξ_2} denotes the principal radii along ξ_1 and ξ_2, respectively. The composite panel made of 'N' numbers of the lamina having identical thickness and the lamina orientation is measured from ξ_1 direction.

Now the displacement kinematics (Reddy and Liu 1985) obtained from Taylor series expansion is used for the modelling purpose is presented in the following lines

$$
\begin{aligned}
X_{\alpha}(\xi_1,\xi_2,\zeta,t) &= X_{0_\alpha}(\xi_1,\xi_2)(+)\zeta(.)\phi_{\xi_1\alpha}(\xi_1,\xi_2)(+)\zeta^2(.)X^*_{0_\alpha}(\xi_1,\xi_2)(+)\zeta^3(.)\phi^*_{\xi_1\alpha}(\xi_1,\xi_2)\\
Y_{\alpha}(\xi_1,\xi_2,\zeta,t) &= Y_{0_\alpha}(\xi_1,\xi_2)(+)\zeta(.)\phi_{\xi_2\alpha}(\xi_1,\xi_2)(+)\zeta^2(.)Y^*_{0_\alpha}(\xi_1,\xi_2)(+)\zeta^3(.)\phi^*_{\xi_2\alpha}(\xi_1,\xi_2)\\
Z_{\alpha}(\xi_1,\xi_2,\zeta,t) &= Z_{0_\alpha}(\xi_1,\xi_2)
\end{aligned} \tag{4}
$$

where, X_α, Y_α and Z_α are the displacement of any arbitrary point along ξ_1, ξ_2 and ζ direction respectively. Similarly, X_{0_α}, Y_{0_α} and Z_{0_α} are mid-plane displacement and $\phi_{\xi_1\alpha}$ and $\phi_{\xi_2\alpha}$ are the rotation of the normal to the mid-plane about ξ_2 and ξ_1 directions, respectively. The remaining notations $\left(X^*_{0_\alpha}, Y^*_{0_\alpha}, \phi^*_{\xi_1\alpha} \text{ and } \phi^*_{\xi_2\alpha}\right)$ are the higher-order terms of the Taylor series expansion.

2.3 Strain-displacement relation

The strain field for the layered composite can be represented as (Singh 2015)

$$
\begin{Bmatrix} \varepsilon_{\xi_1\xi_{1\alpha}} \\ \varepsilon_{\xi_2\xi_{2\alpha}} \\ \varepsilon_{\xi_2\zeta_\alpha} \\ \varepsilon_{\xi_1\zeta_\alpha} \\ \varepsilon_{\xi_1\xi_{2\alpha}} \end{Bmatrix} = \begin{Bmatrix} X_{\alpha,\xi_1} \\ Y_{\alpha,\xi_2} \\ \left(Y_{\alpha,\zeta} + Z_{\alpha,\xi_2}\right) \\ \left(X_{\alpha,\zeta} + Z_{\alpha,\xi_1}\right) \\ \left(X_{\alpha,\xi_2} + Y_{\alpha,\xi_1}\right) \end{Bmatrix} + \begin{Bmatrix} \frac{1}{2}\left\{\left(X_{\alpha,\xi_1}\right)^2 + \left(Y_{\alpha,\xi_1}\right)^2 + \left(Z_{\alpha,\xi_1}\right)^2\right\} \\ \frac{1}{2}\left\{\left(X_{\alpha,\xi_1}\right)^2 + \left(Y_{\alpha,\xi_2}\right)^2 + \left(Z_{\alpha,\xi_2}\right)^2\right\} \\ \left\{X_{\alpha,\xi_1}X_{\alpha,\xi_2} + Y_{\alpha,\xi_1}Y_{\alpha,\xi_2} + Z_{\alpha,\xi_1}Z_{\alpha,\xi_2}\right\} \\ \left\{X_{\alpha,\xi_1}X_{\alpha,\zeta} + Y_{\alpha,\xi_1}Y_{\alpha,\zeta} + Z_{\alpha,\xi_1}Z_{\alpha,\zeta}\right\} \\ \left\{X_{\alpha,\xi_2}X_{\alpha,\zeta} + Y_{\alpha,\xi_2}Y_{\alpha,\zeta} + Z_{\alpha,\xi_2}Z_{\alpha,\zeta}\right\} \end{Bmatrix} \tag{5}
$$

or

$$
\left\{\varepsilon_\alpha\right\} = \left\{\varepsilon_\alpha\right\}^{Lin} (+) \left\{\varepsilon_\alpha\right\}^{NoL} \tag{6}
$$

The Eq. (6) can be rewritten as

$$
\left\{\varepsilon_\alpha\right\} = \left[T\right]^{Lin}(.)\left\{\bar{\varepsilon}_\alpha\right\}^{Lin} + \left[T\right]^{NoL}(.)\left\{\bar{\varepsilon}_\alpha\right\}^{NoL} \tag{7}
$$

where, $[T]^{Lin}$ and $[T]^{NoL}$ represents the corresponding linear and nonlinear thickness coordinate matrices, respectively. Similarly, the mid-plane linear and nonlinear strain matrices are defined as the terms $\left\{\bar{\varepsilon}_\alpha\right\}^{Lin}$ and $\left\{\bar{\varepsilon}_\alpha\right\}^{NoL}$, respectively. The detail expansion of terms used in the above Eq. (7) can be seen in the reference (Singh and Panda 2015).

Now, the lamina stress-strain relationship for any 'k^{th}' layer can be considered as (Liu and Rao 2003)

$$
\begin{Bmatrix} \sigma_{\xi_1\xi_{1\alpha}} \\ \sigma_{\xi_2\xi_{2\alpha}} \\ \sigma_{\xi_2\zeta_\alpha} \\ \sigma_{\varepsilon\xi_1\zeta_\alpha} \\ \sigma_{\varepsilon\xi_1\xi_{2\alpha}} \end{Bmatrix}^k = \begin{Bmatrix} \sigma_{1_\alpha} \\ \sigma_{2_\alpha} \\ \sigma_{4_\alpha} \\ \sigma_{5_\alpha} \\ \sigma_{6_\alpha} \end{Bmatrix}^k = \begin{bmatrix} Q_{11_\alpha} & Q_{12_\alpha} & 0 & 0 & 0 \\ Q_{12_\alpha} & Q_{22_\alpha} & 0 & 0 & 0 \\ 0 & 0 & Q_{45_\alpha} & 0 & 0 \\ 0 & 0 & 0 & Q_{55_\alpha} & 0 \\ 0 & 0 & 0 & 0 & Q_{66_\alpha} \end{bmatrix} (.) \left[\begin{Bmatrix} \varepsilon_{\xi_1\xi_{1\alpha}} \\ \varepsilon_{\xi_2\xi_{2\alpha}} \\ \varepsilon_{\xi_2\zeta_\alpha} \\ \varepsilon_{\xi_1\zeta_\alpha} \\ \varepsilon_{\xi_1\xi_{2\alpha}} \end{Bmatrix}\right]^k \tag{8}
$$

$$
\left\{\sigma_{ij_\alpha}\right\}^k = \left[\bar{Q}_{ij_\alpha}\right]^k (.) \left\{\varepsilon_{ij_\alpha}\right\}^k \tag{9}
$$

where, $\left\{\sigma_{ij_\alpha}\right\}^k$ and $\left\{\varepsilon_{ij_\alpha}\right\}^k$ is the stress and strain tensor, respectively, whereas $\left[\bar{Q}_{ij_\alpha}\right]^k$ is the stiffness matrix constituted by elastic constants.

2.4 Calculation of strain energy

For the current layered composite structure, the strain energy function can be expressed in the following form

$$
\hat{V} = \frac{1}{2}\iint\left[\int_{-h/2}^{+h/2}\left\{\varepsilon_{ij_\alpha}\right\}^T\left\{\sigma_{ij_\alpha}\right\}d\zeta\right]d\xi_1 d\xi_2 \tag{10}
$$

2.5 External work

The following Eq. (11) represents the external work done by the transverse force applied on the structure

$$
\hat{W} = \int_A \left\{\delta_\alpha\right\}^T \left\{F_\alpha\right\} dA \tag{11}
$$

2.6 Fuzzy finite element method

The isoparametric lagrangian element with eighty-one degrees of freedom is utilised for the discretization of the curved structure. Now, the elemental fuzzy displacement vector $\left\{\delta_\alpha\right\}$ can be given as (Cook *et al.* 2000)

$$
\left\{\delta_\alpha\right\} = \left[N_i\right](.)\left\{\delta_\alpha\right\}_i \tag{12}
$$

where, $\left\{\delta_\alpha\right\}$ is the elemental displacement vector, $\left[N_i\right]$ is the well-defined shape function of the discussed element whose detail of the same can be seen in (Cook *et al.* 2000) and $\left\{\delta_\alpha\right\}_i$ represents the nodal displacement vector.

Now, based on the isoparametric formulation, the mid-plane strain vector can be rewritten in terms of nodal displacement vectors and presented as

$$
\left\{\bar{\varepsilon}_\alpha\right\}^{Lin} = \left[B\right]^{Lin}\left\{\delta_\alpha\right\}_i \tag{13}
$$

$$
\left\{\bar{\varepsilon}_\alpha\right\}^{NoL} = \left[B\right]^{NoL}\left\{\delta_\alpha\right\}_i = \left[A_\alpha\right]\left[G_\alpha\right]\left\{\delta_\alpha\right\}_i \tag{14}
$$

where, the linear and nonlinear corresponding strain-displacement relation matrices are defined as $\left[B\right]^{Lin}$ and $\left[B\right]^{NoL}$, that can be further represented in the product form of the $\left[A_\alpha\right]$ and $\left[G_\alpha\right]$ whose detail can be seen in the reference (Singh and Panda 2015).

2.7 Governing equation for static analysis

The static analysis of the advanced fibre-reinforced laminated composite curved panel is performed by solving the governing equation derived by utilizing the variational principle and depicted as

$$
\partial\Pi = \partial\left(\hat{V} - \hat{W}\right) = 0 \tag{15}
$$

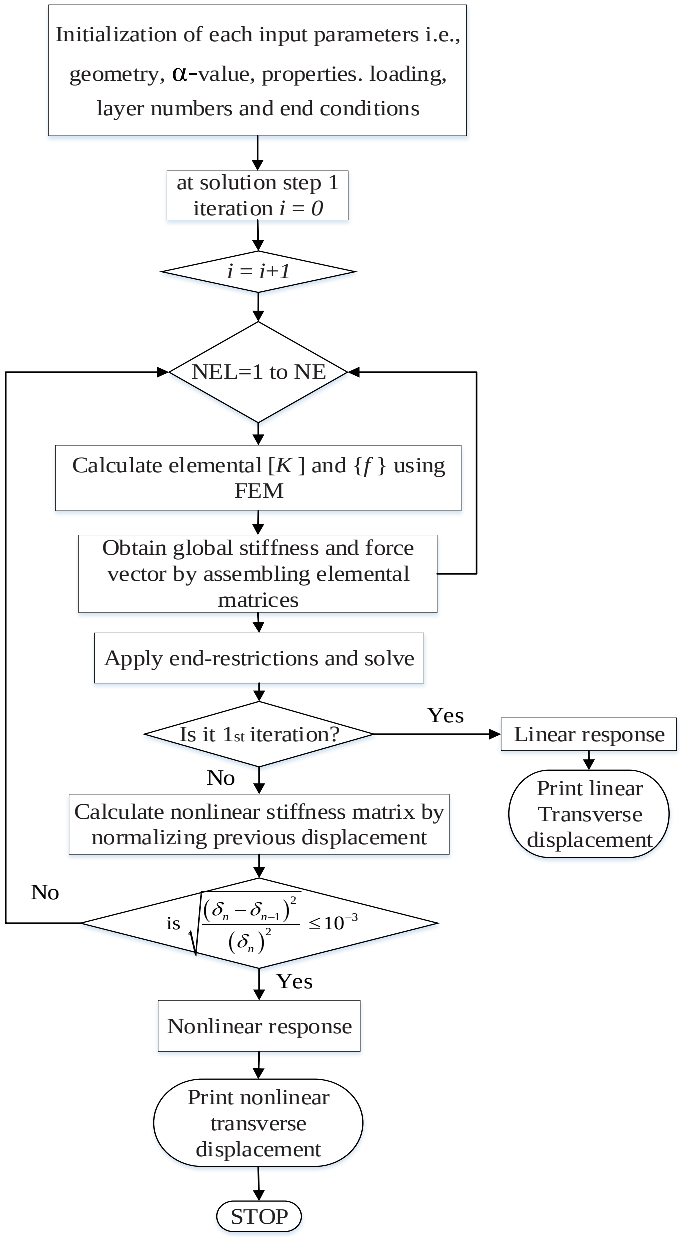

Fig. 3 Flowchart presentation for linear/nonlinear bending analysis

Eq. (15) is finally expressed as in Eq. (16) by using Eqs (10) and (11)

$$[K_\alpha]\{\delta_\alpha\}=\{F_\alpha\}\, or \left[[K_\alpha]^{Lin}+[K_\alpha]^{NoL}\right]\{\delta_\alpha\}=\{F_\alpha\} \quad (16)$$

2.8 End boundary conditions

The boundary conditions utilized hereafter are as follows:

Simply support

$$\left.\begin{array}{l} Y_{0_\alpha}=Z_{0_\alpha}=\phi_{\xi_{2\alpha}}=Y_{0_\alpha}^*=\phi_{\xi_{2\alpha}}^*=0 \text{at } \xi_1\text{=0and}a; \\ X_{0_\alpha}=Z_{0_\alpha}=\phi_{\xi_{1\alpha}}=X_{0_\alpha}^*=\phi_{\xi_{1\alpha}}^*=0 \text{at } \xi_2\text{=0and}b; \end{array}\right\} \quad (17)$$

Clamped

$$X_{0_\alpha}=Y_{0_\alpha}=Z_{0_\alpha}=\phi_{\xi_{1\alpha}}=\phi_{\xi_{2\alpha}}=X_{0_\alpha}^*=Y_{0_\alpha}^*= \phi_{\xi_{1\alpha}}^*=\phi_{\xi_{2\alpha}}^*=0 \text{at } \xi_1=0\text{and}a; \text{at } \xi_2=0\text{and}b \quad (18)$$

Free

$$X_{0_\alpha}\neq Y_{0_\alpha}\neq Z_{0_\alpha}\neq\phi_{\xi_{1\alpha}}\neq\phi_{\xi_{2\alpha}}\neq X_{0_\alpha}^*\neq Y_{0_\alpha}^*\neq \phi_{\xi_{1\alpha}}^*\neq\phi_{\xi_{2\alpha}}^*\neq 0 \text{at } \xi_1=0\text{and}a; \text{at } \xi_2=0\text{and}b \quad (19)$$

2.9 Solution procedure

The governing equation of equilibrium for the static analysis is derived and solved using the direct iterative method. The solution procedure is presented through a flow chart and presented in Fig. 3.

Table 1 Deterministic elastic properties of composite

Material properties	E_{ξ_1} (GPa)	E_{ξ_2} (GPa)	$G_{\xi_1\xi_2}$ (GPa)	$G_{\xi_1\zeta}$ (GPa)	$G_{\xi_2\zeta}$ (GPa)	$\upsilon_{\xi_1\xi_2}$	$\upsilon_{\xi_1\zeta}$	$\upsilon_{\xi_2\zeta}$
Values	25	1	0.5	0.5	0.2	0.25	0.25	0.25

Table 2 Fuzzified elastic properties of composite

E_{ξ_1}	E_{ξ_2}	$G_{\xi_1\xi_2} = G_{\xi_1\zeta}$	$G_{\xi_2\zeta}$	$\upsilon_{\xi_1\xi_2} = \upsilon_{\xi_1\zeta} = \upsilon_{\xi_2\zeta}$	α
30	1.2	0.6	0.24	0.3	0
29	1.16	0.58	0.232	0.29	0.2
28	1.12	0.56	0.224	0.28	0.4
27	1.08	0.54	0.216	0.27	0.6
26	1.04	0.52	0.208	0.26	0.8
25	1	0.5	0.2	0.25	1
24	0.96	0.48	0.192	0.24	0.8
23	0.92	0.46	0.184	0.23	0.6
22	0.88	0.44	0.176	0.22	0.4
21	0.84	0.42	0.168	0.21	0.2
20	0.8	0.4	0.16	0.2	0

3. Results and discussion

Now, a suitable computer code (MATLAB) is developed based on the derived model (FFEM). The material properties of the laminated composite are fuzzified. This computer code is utilized further to investigate the deflection parameter and discussed in this section. The deterministic (Table 1) and fuzzified (Table 2) material properties are varied within ±20% from the conventional values (Liu *et al.* 1986, Singh and coauthors 2012, 2014).

3.1 Convergence and validation study

The convergence and the comparison behaviour of the currently model have been verified by solving a simply supported flat-panel example. The panel is consisting of eight-layer cross-ply arrangement (0°/90°/90°/0°/0°/90°/90°/0°) and deterministic type composite properties (Table 1). The deflection results are plotted in Fig. 4 for varying mesh divisions and a (5×5) mesh selected for the evaluation of new results. Now, the responses computed using the higher-order FE model are compared with the published experimental (Zaghloul and Kennedy 1975) results. The results are computed for a clamped (0°/90°/90°/0°) square flat structure ($a = b$ =12 inch and h = 0.096 inches) under the uniformly distributed loading (UDL). The present and the reference results are presented in Table 3. The table values show that the current values are in close agreement with those of the theoretical, as well as the reference results. However, the deviation with experimental responses may be due to the inexact replication of the boundary condition used in the experimentation.

3.2 Non-dimensional non-linear deflection of a composite by variation of aspect ratio

The influence of the aspect ratio on the non-linear deflection is checked by considering a simply supported spherical shell geometry under the UDL loading for different amplitude (2 kPa, 4 kPa, 6 kPa, 8 kPa and 10 kPa). The responses are obtained using the fuzzified properties for each elastic constant α = 0.8 (refer to Table 2) including a/h = 60 and R/a = 50. The responses are plotted in Fig. 5, which indicates an incremental path for a few aspect ratios and slope become constant after a/b = 2.5. However, the deflection values are following increasing type of trend when load values increase for a particular aspect ratio.

3.3 Non-dimensional non-linear deflection of a composite by variation of curvature ratio

To demonstrate the effect of different curvature ratios (5, 10, 20, 30, 50, 80 and 100) on the non-linear deflection parameter of a cylindrical shell panel is checked in this problem under the influence of the sinusoidal loading (SDL) and fuzzified properties (α = 0.8). The responses are plotted Fig. 6 considering a clamped shell panel under different loading amplitude (2 kPa, 4 kPa, 6 kPa, 8 kPa and 10 kPa) and a/h =40. From the graph, it can be concluded that the non-dimensional non-linear deflections are increasing with load intensity and the slope become flatten after R/a = 20. The is because the structural panel becomes flat and the curvature has an insignificant effect on the structural deflection. Also, the shell panel has high stretching energy in comparison to bending for the small value of the curvature ratio.

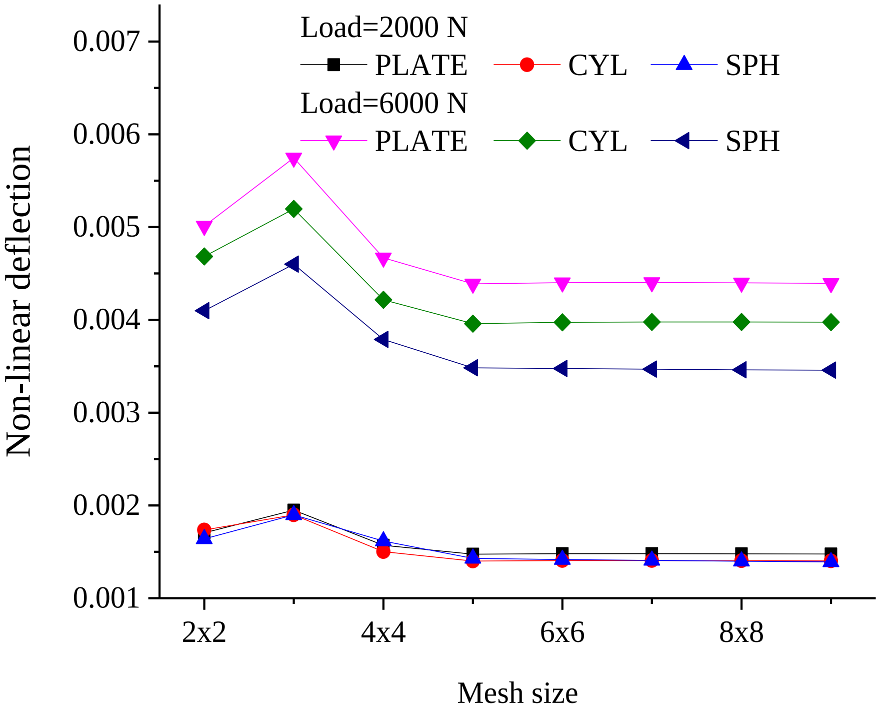

Fig. 4 Variation of non-dimensional non-linear deflection with the mesh size

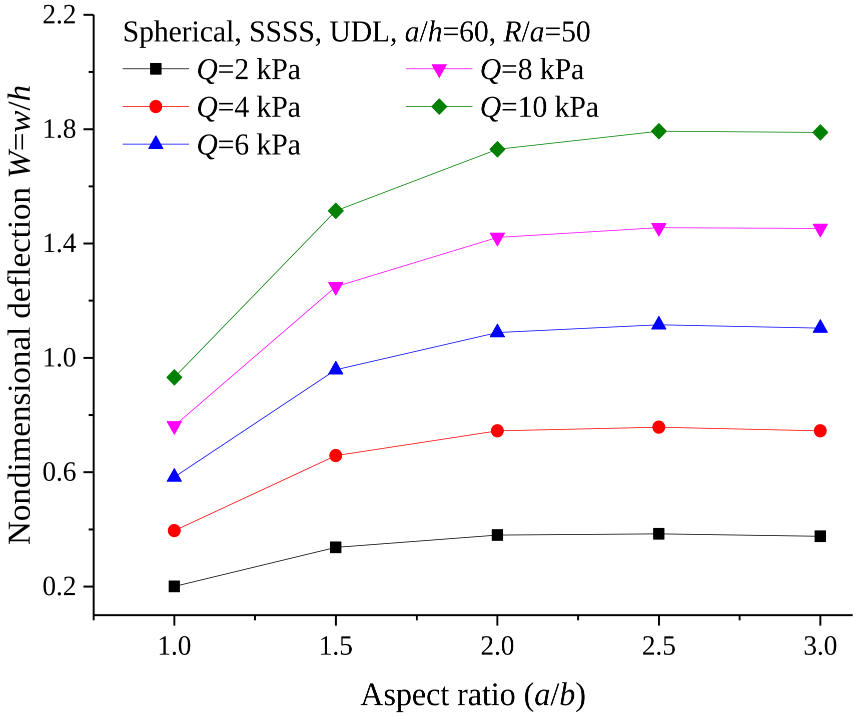

Fig. 5 Variation of non-dimensional non-linear deflection with aspect ratio (a/b)

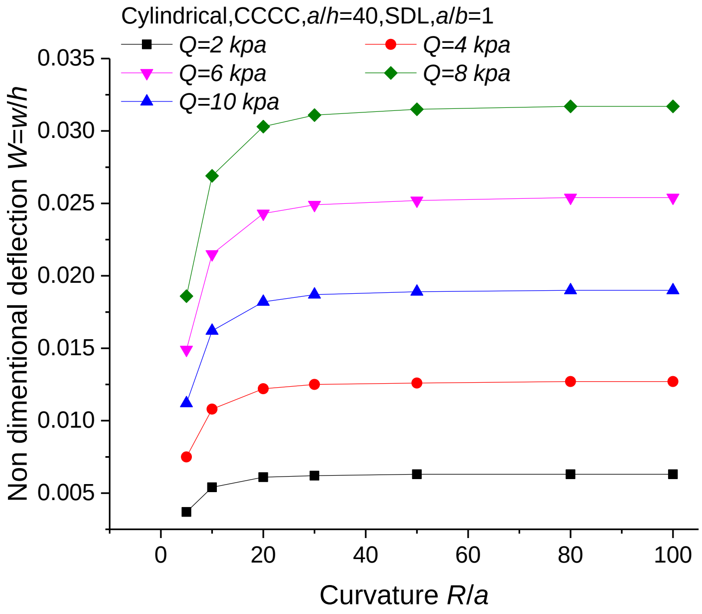

Fig. 6 Variation of non-dimensional non-linear deflection with curvature (R/a)

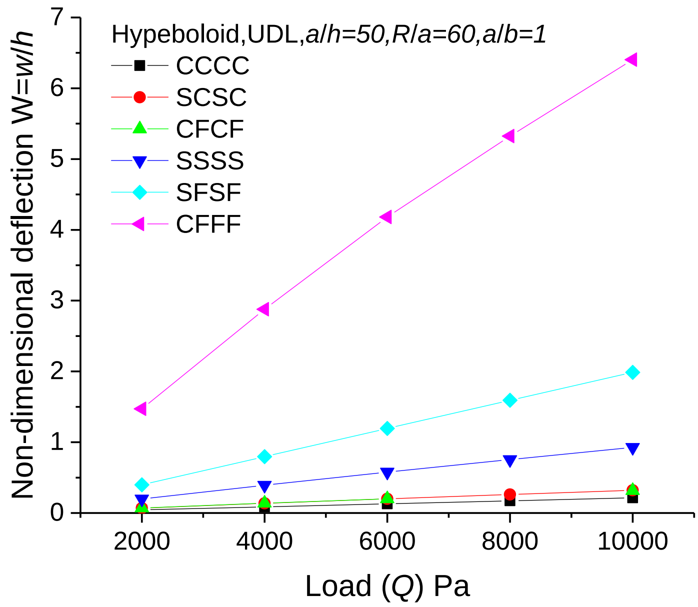

Fig. 7 Variation of non-dimensional non-linear deflection with load (Q)

3.4 Non-dimensional non-linear deflection of a composite by variation of loading and edge support conditions

Fig. 7 is showing the deflection responses of a thin (a/h = 50) hyperboloid composite panel (R/a = 60) under different loading amplitude (Q) and the end boundaries (CCCC, SCSC, CFCF, SSSS, SFSF and CFFF) considering fuzzified uncertain (α = 0.8) properties. The non-dimensional non-linear deflection values are following an incremental line while the load intensity increases for a particular end boundary. The maximum and the minimum deflections are showing correspondingly for the clamped and cantilever panel. The responses are following the anticipated line and the uncertainty has not much effect in comparison to the end boundaries.

Table 3 Present nonlinear deflections (inch) in comparison with the benchmark values

Load (psi)	Zaghloul and Kennedy (1975)		Present
	Experimental	Large deformation	
0.4	0.08079	0.09011	0.0766
1.2	0.15700	0.18079	0.1724
2	0.18930	0.23241	0.2299

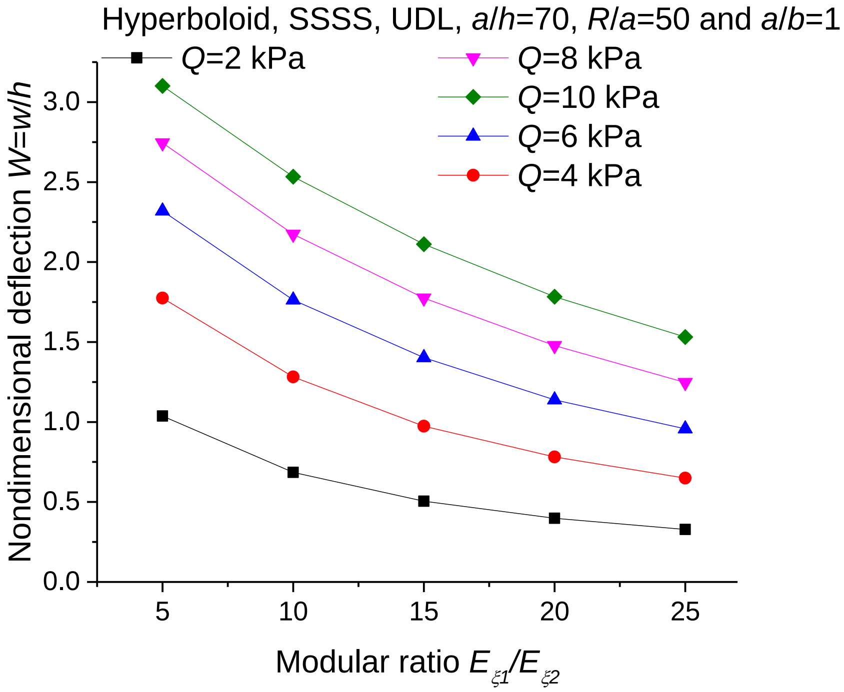

Fig. 8 Variation of non-dimensional non-linear deflection with modular ratio ($E_{\xi 1}/E_{\xi 2}$)

3.5 Non-dimensional non-linear deflection of a composite by variation of modular ratio

The influence of major and minor moduli ratio including the fuzzification (α = 0.8) on the deflection values have been computed in this example considering a simply supported hyperboloid shell structure. The results are obtained under the UDL of various loading amplitude (2 kPa, 4 kPa, 6 kPa, 8 kPa and 10 kPa) and input parameters i.e. a/h = 70 and R/a = 50. The computational test results are plotted in Fig.8. The graphical deflections are showing that values are decreasing when the modular ratio increases whereas follow a reverse line for the load intensity. This is because the structural stiffness becomes higher for the modular ratio and the subsequent deflection decreases.

3.6 Effect of fuzzified properties on non-linear deflection of elliptical panel

In this numerical example, the influence of fuzzified elastic properties on the non-linear deflection values are obtained considering a clamped elliptical (R/a = 60) shell panel. The results are plotted in Fig. 9 considering the fuzzification for each constant ($E_{\xi_1}, E_{\xi_2}, G_{\xi_1\xi_2}, G_{\xi_2\zeta}, \upsilon$) and thin panel type i.e. a/h = 60. The results are indicating large deviation of responses for the fuzzified longitudinal properties in comparison to each case.

4. Conclusions

This research mainly reported the implementation of higher-order nonlinear Fuzzy-FEM model for the computation of deflection parameters of the laminated curved/flat panel structure using the varied composite material properties via Fuzzy operator. The fuzzified uncertainty is included for the first time to compute the nonlinear deflections in association with Green-Lagrange type of geometrical nonlinearity. Initially, the model is established by considering the deterministic type properties by comparing the results with published numerical and experimental data. Moreover, the model is engaged to compute the deflection parameter considering the structure

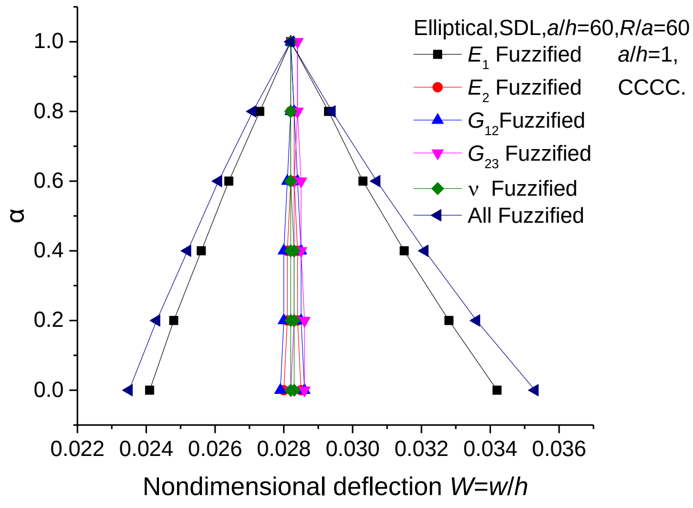

Fig. 9 Variation of non-dimensional nonlinear deflection with fuzzified properties

geometrical parameter with the fuzzified material properties. The responses are showing the consistent variation of deflection values for all kind of structural design parameters. However, the fuzzified properties are showing the interesting variation i.e., highest for the longitudinal property whereas insignificant for all other values. The study also reveals that the structural uncertainty can be examined with adequate accuracy similar to the earlier statistical methodologies with less computational effort.

References

Adhikari, S. and Khodaparast, H.H. (2014), "A spectral approach for fuzzy uncertainty propagation in finite element analysis", *Fuzzy Sets Syst.*, **243**, 1-24. https://doi.org/10.1016/j.fss.2013.10.005.

Akhras, G. and Li, W. (2005), "Static and free vibration analysis of composite plates using spline finite strips with higher-order shear deformation", *Compos. Part B*, **36**(6-7), 496-503. https://doi.org/10.1016/j.compositesb.2005.03.001.

Akpan, U.O., Koko, T.S., Orisamolu, I.R. and Gallant, B.K. (2001), "Fuzzy finite-element analysis of smart structures", *Smart Mater. Struct.*, **10**(2), 273. https://doi.org/10.1088/0964-1726/10/2/312.

Akpan, U.O., Koko, T.S., Orisamolu, I.R. and Gallant, B.K. (2001), "Practical fuzzy finite element analysis of structures", *Finite Elem. Anal. Des.*, **38**(2), 93-111. https://doi.org/10.1016/S0168-874X(01)00052-X.

Arefi, M., Mohammadi, M., Tabatabaeian, A., Dimitri, R. and Tornabene, F. (2018), "Two-dimensional thermo-elastic analysis of FG-CNTRC cylindrical pressure vessels", *Steel Compos. Struct.*, **27**(4), 525-536. https://doi.org/10.12989/scs.2018.27.4.525.

Behera, D. and Chakraverty, S. (2013), "Fuzzy finite element based solution of uncertain static problems of structural mechanics", *Int. J. Comput. Appl. Technol.*, **69**(15), 6-11. https://doi.org/10.5120/11916-8040.

Bennai, R., Atmane, H.A. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams", *Steel Compos. Struct.*, **19**(3), 521-546. https://doi.org/10.12989/scs.2015.19.3.521.

Bouiadjra, R.B., Mahmoudi, A., Benyoucef, S., Tounsi, A. and Bernard, F. (2018), "Analytical investigation of bending response of FGM plate using a new quasi 3D shear deformation theory: Effect of the micromechanical models", *Struct. Eng. Mech.*, **66**(3), 317-328. https://doi.org/10.12989/sem.2018.66.3.317.

Bui, T. Q., Tran, A.V. and Shah, A.A., (2014), Improved knowledge-based neural network (KBNN) model for predicting spring-back angles in metal sheet bending, *Int. J. Modeling, Simul, Scientific Computing*, **5**, 1350026. https://doi.org/10.1142/S1793962313500268

Bui, T.Q. and Nguyen, M.N. (2013), "Mesh-free galerkin kriging model for bending and buckling analysis of simply supported laminated composite plates", *Int. J. Comp. Meth.*, **10**(3), 1350011-26. https://doi.org/10.1142/S0219876213500114.

Cherki, A., Plessis, G., Lallemand, B., Tison, T. and Level, P. (2000), "Fuzzy behavior of mechanical systems with uncertain boundary conditions", *Comput Method Appl. M.*, **189**(3), 863-873. https://doi.org/10.1016/S0045-7825(99)00401-6.

Cook, R.D., Malkus, D.S., Plesha, M.E. and Witt, R.J. (2000), *Concepts and Applications of Finite Element Analysis*, 3rd Edition, John Willy and Sons (Asia) Pvt. Ltd., Singapore.

Dash, P. and Singh B.N. (2010), "Geometrically nonlinear bending analysis of laminated composite plate", *Commun. Nonlinear Sci. Numer. Simul.*, **15**(10), 3170-3181. https://doi.org/10.1016/j.cnsns.2009.11.017.

Dash, P. and Singh, B.N. (2012), "Geometrically nonlinear free vibration of laminated composite plate embedded with piezoelectric layers having uncertain material properties", *J. Vib. Acoust.*, **134**(6). https://doi.org/10.1115/1.4006757

De Gersem, H., Moens, D., Desmet, W. and Vandepitte, D. (2007), "Interval and fuzzy dynamic analysis of finite element models with superelements", *Comput. Struct.*, **85**(5-6), 304-319. https://doi.org/10.1016/j.compstruc.2006.10.011.

Ghannadpour, S.A.M., Ovesy, H.R. and Zia-Dehkordi E. (2014), "An exact finite strip for the calculation of initial post-buckling stiffness of shear-deformable composite laminated plates", *Compos. Struct.*, **108**, 504–513. https://doi.org/10.1016/j.compstruct.2013.09.049

Ghannadpour, S.A.M. and Barekati, M. (2006), "Initial imperfection effects on post-buckling response of laminated plates under end-shortening strain using Chebyshev techniques", *Compos. Struct.,* **75**(1-4), 106-113. https://doi.org/10.1016/j.compstruct.2006.04.006

Giannini, O. and Hanss, M. (2008), "The component mode transformation method: a fast implementation of fuzzy arithmetic for uncertainty management in structural dynamics", *J. Sound Vib.*, **311**(3-5), 1340-1357. https://doi.org/10.1016/j.jsv.2007.10.029.

Heydari, M.M., Kolahchi, R., Heydari, M. and Abbasi, A. (2014), "Exact solution for transverse bending analysis of embedded laminated Mindlin plate". *Struct. Eng. Mech.,* **49**(5), 661-672. https://doi.org/10.12989/sem.2014.49.5.661.

Houari, T., Bessaim, A., Houari, M.S.A., Benguedia, M. and Tounsi, A. (2018), "Bending analysis of advanced composite plates using a new quasi 3D plate theory", *Steel Compos. Struct.,* **26**(5), 557-572. https://doi.org/10.12989/scs.2018.26.5.557.

Jones, R.M. (1975), *Mechanics of Composite Materials*, Taylor and Francis, *Philadelphia.*

Keleshteri, M.M., Asadi, H. and Aghdam, M. (2019), "Nonlinear bending analysis of FG-CNTRC annular plates with variable thickness on elastic foundation", *Thin Wall. Struct.*, **135**, 453-462. https://doi.org/10.1016/j.tws.2018.11.020.

Kolahchi, R., Mohammad, A., Bidgoli, M. and Heydari, M.M. (2015), "Size-dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium", *Struct. Eng. Mech.*, **55**(5), 1001-1014. https://doi.org/10.12989/sem.2015.55.5.1001.

Liu, Q. and Rao, S.S. (2005), "Fuzzy finite element approach for analysis of fiber-reinforced laminated composite beams", *AIAA J.*, **43**(3), 651-661. https://doi.org/10.2514/1.940.

Liu, W.K., Belytschko, T. and Mani, A. (1986), "Random field finite elements", Int. J. Numer. Methods Eng., 23(10), 1831–1845. https://doi.org/10.1002/nme.1620231004

Luo, Z., Atamturktur, S., Juang, C.H., Huang, H. and Lin, P.S. (2011), "Probability of serviceability failure in a braced excavation in a spatially random field: Fuzzy finite element approach", *Comput. Geotech.*, **38**(8), 1031-1040. https://doi.org/10.1016/j.compgeo.2011.07.009.

Massa, F., Lallemand, B., Tison, T. and Level, P. (2004), "Fuzzy eigensolutions of mechanical structures", *Eng. Computation*, **21**(1), 66-77. https://doi.org/10.1108/02644400410511846.

Massa, F., Tison, T. and Lallemand, B. (2009), "Fuzzy modal analysis: Prediction of experimental behaviours", *J. Sound Vib.*, **322**(1-2), 135-154. https://doi.org/10.1016/j.jsv.2008.10.032.

Mehrparvar, M. and Ghannadpour, S.A.M. (2018), "Plate assembly technique for nonlinear analysis of relatively thick functionally graded plates containing rectangular holes subjected to inplane compressive load", *Compos. Struct.*, **202**, 867-880. https://doi.org/10.1016/j.compstruct.2018.04.053

Moens, D. and Vandepitte, D. (2005), "A fuzzy finite element procedure for the calculation of uncertain frequency-response functions of damped structures: Part 1—Procedure", *J. Sound Vib.*, **288**(3), 431-462. https://doi.org/10.1016/j.jsv.2005.07.001.

Mukhopadhyay, M. (2009), *Mechanics of Composite Materials and Structures*, Universities Press, Hyderabad, India.

Noor, A.K., Starnes Jr, J.H. and Peters, J.M. (2000), "Uncertainty analysis of composite structures", *Comput Method Appl. M.,* **185**(2-4), 413-432. https://doi.org/10.1016/S0045-7825(99)00269-8.

Ovesy, H.R. and Ghannadpour, S.A.M. (2006), "Geometric non-linear analysis of imperfect composite laminated plates, under end shortening and pressure loading, using finite strip method", *Compos. Struct.*, **75**, 100–105. https://doi.org/10.1016/j.compstruct.2006.04.005.

Ovesy, H.R. and Ghannadpour, S.A.M. (2007), "Large deflection finite strip analysis of functionally graded plates under pressure loads", *Int. J. Struct. Stab. Dyn.*, **7**(2), 193-211. https://doi.org/10.1142/S0219455407002241.

Ovesy, H.R., Ghannadpour, S.A.M. and Morada, G. (2006), "Post-buckling behavior of composite laminated plates under end shortening and pressure loading, using two versions of finite strip method", *Compos. Struct.*, **75**, 106-113. https://doi.org/10.1016/j.compstruct.2006.04.006.

Ovesy, H.R., Ghannadpour, S.A.M. and Nassirnia, M. (2015), "Post-buckling analysis of rectangular plates comprising Functionally Graded Strips in thermal environments", *Comput. Struct.*, **147**, 209-215. https://doi.org/10.1016/j.compstruc.2014.09.011.

Patle, B.K., Hirwani, C.K., Singh, R.P. and Panda, S.K. (2018), "Eigenfrequency and frequency analysis of layered structure using uncertain elastic properties–a fuzzy finite element approach", *Int. J. Approximate Reasoning*, **98**, 163-176. https://doi.org/10.1016/j.ijar.2018.04.013.

Pawar, P.M., Nam Jung, S. and Ronge, B.P. (2012), "Fuzzy approach for uncertainty analysis of thin walled composite beams", *Aircr. Eng. Aerosp. Tec.*, **84**(1), 13-22. https://doi.org/10.1108/00022661211194942.

Rao, S. and Sawyer, J.P. (1995), "Fuzzy finite element approach for analysis of imprecisely defined systems", *AIAA J.*, **33**(12), 2364-2370. https://doi.org/10.2514/3.12910.

Razavi, S.V., Jumaat, M.Z., EI-Shafie, E.H. and Ronagh, H.R. (2015) "Load-deflection analysis prediction of CFRP strengthened RC slab using RNN", *Adv. Concr. Constr.,* **3**(2). https://doi.org/10.12989/acc.2015.3.2.091.

Reddy, J.N. and Liu, C.F. (1985), "A higher-order shear deformation theory of laminated elastic shells", *Int. J. Eng. Sci.*, **23**(3), 319-330. https://doi.org/10.1016/0020-7225(85)90051-5.

Shahsavari, D. and Janghorban, M. (2017), "Bending and shearing responses for dynamic analysis of single-layer graphene sheets under moving load", *J. Braz. Soc. Mech. Sci. Eng.*, **39**(10), 3849-3861. https://doi.org/10.1007/s4043.

Singh, R.P. (2015), Vibration and bending behavior of laminated composite plate with uncertain material properties using fuzzy finite element method, (M.Tech. thesis).

Singh, V.K. and Panda, S.K. (2014), "Nonlinear free vibration analysis of single/doubly curved composite shallow shell panels", *Thin Wall. Struct.*, **85**, 341-349. https://doi.org/10.1016/j.tws.2014.09.003.

Szekrenyes, A. and Jozsef, U.J. (2007), "Over-leg Bending Test for Mixed-mode I/II Inter laminar Fracture in Composite Laminates". *Int. J. Damage Mech.*, **16**(1), 5-33. https://doi.org/10.1177/1056789507060774.

Taghizadeh, M., Ovesy, H.R. and Ghannadpour, S.A.M. (2015), "Nonlocal integral elasticity analysis of beam bending by using finite element method", *Struct. Eng. Mech.*, **54**(4), 755-769 https://doi.org/10.12989/sem.2015.54.4.755.

Talha, M. and Singh, B.N. (2014), "Stochastic perturbation-based finite element for buckling statistics of FGM plates with uncertain material properties in thermal environments", *Compos. Struct.*, **108**(1), 823-833. https://doi.org/10.1016/j.compstruct.2013.10.013.

Valizadeh, N., Natarajan, S., Gonzalez-Estrada, O. A., Rabczuk, T., Bui, T.Q. and Bordas, S.P.A. (2013a), "NURBS-based finite element analysis of functionally graded plates: Static bending, vibration, buckling and flutter", *Compos. Struct.*, **99**, 309-326. https://doi.org/10.1016/j.compstruct.2012.11.008

Xia, Y. and Friswell, M. (2014), "Efficient solution of the fuzzy eigenvalue problem in structural dynamics", *Eng. Computation*, **31**(5), 864-878. https://doi.org/10.1108/EC-02-2013-0052.

Zaghloul, S.A. and Kennedy, J.B. (1975), "Nonlinear behaviour of symmetrically laminated plates", *J. Appl. Mech.*, **42**, 234-236. https://doi.org/10.1115/1.3423532.

CC