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Dynamic responses of laminated beams under a moving load in thermal environment

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Abstract. The goal of this study is to investigate dynamic responses of laminated composite beams under a moving load with thermal effects. The governing equations of problem are derived by using the Lagrange procedure. The transverse-shear strain and rotary inertia are considered within the Timoshenko beam theory. The material properties of laminas are considered as the temperature dependent physical property. The differential equations of the problem are solved by the Ritz method. The solution step of dynamic problem, the Newmark average acceleration method is used in the time history. A compassion study is performed for accuracy of used formulations and method. In the numerical results, the effects of velocity of moving load, temperature values, the fiber orientation angles and the stacking sequence of laminas on the dynamic responses of the composite laminated beam are investigated.

Keywords: laminated composites; moving load problems; temperature effect; ritz method

1. Introduction

Laminated composite structures have been used in a lot of engineering fields such as civil, mechanical and aerospace engineering projects because of higher strength and low density properties. With the development of production technology, the using of laminated composite have been increasing in engineering applications. In the laminated composite structures, the moving-load dynamic problems are very important topic as such in other structures elements. As is known, the mechanical results of dynamic moving loads are bigger than those of static loadings. Also, instantaneous failures and local cracks can be occurred affected by dynamically moving loads in contrast with other load types. Another important problem in the laminated structures is temperature effects. In the higher temperature rising, the interlaminar cracking can be frequently occurred in the laminated structures. So, the temperature and moving load effects are very important problems in the laminated structures.

In recent decades, many researchers investigated vibration and thermal analysis of laminated beams. In the literature, the studies about thermal problems in the laminated and composite beams in brief as follows: Karnaukhov and Kirichok (2005) presented an investigation about the vibration of viscoelastic beams in dissipative heating, moving load. Mazur-Śniady *et al.* (2009) investigated the moving load problems of micro-periodic composite rods by using the perturbation method. Akbaş and Kocatürk (2012, 2013), Kocatürk and Akbaş (2011, 2012), Akbaş (2017b) investigated post-buckling analysis of homogeneous and non-homogeneous beams with thermal effects by using finite element method. Bahmyari *et al*. (2014) analyzed the dynamics of laminated beams under distributed moving masses by using finite element method and first shear deformation theory. Hadji *et al*. (2014) studied static and free vibration of FGM beams based on higher order beam theory. Akbaş (2014, 2015, 2017a) presented dynamic analysis of functionally graded beams with thermal effects by using finite element method. Bourada *et al.* (2016) studied stability of ortotropic and isotropic plates by using higher order plate theory. Ebrahimi and Barati (2016, 2018) investigated effects of thermal loading on the bucking and dynamics of smart piezoelectrically actuated nanobeams. Malekzadeh and Monajjemzadeh (2016) investigated effects of temperature rising on the dynamic responses of functionally graded beams under moving load. They used the first shear deformation theory and finite element method. Tao *et al*. (2016) studied nonlinear vibration responses of fiber metal laminated beams under both moving load and temperature effects by using the Galerkin method. Wang and Wu (2016) investigated forced vibration results of axially functionally graded beams under both moving load and temperature rising with temperature dependent material property. Chen *et al*. (2017) examined nonlinear vibration responses of laminated beams embedded tensionless elastic medium under moving load. Vosoughi and Anjabin (2017) investigated dynamic analysis identification of laminated beams under moving load based on first shear deformation theory by using hybrid finite element, time marching differential quadrature and genetic algorithms methods. Akbaş (2018a, 2018b, 2019a, 2019b) investigated thermal and hygro-thermal effects of laminated beams for nonlinear static and post-buckling responses. Yüksel and Akbaş (2018) analzed free vibration of laminated composite plates

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by using Navier method with first-order shear deformation plate theory. Ghayesh (2018a, 2018b, 2019) presented nonlinear vibration analysis of multilayered and functionally garaded beams. Abualnour *et al*. (2019) analyzed thermomechanical behavior of laminated reinforced composite plates with a higher order plate theory. Draiche *et al.* (2019) investigated static analysis of lamineted plates by using Navier method with first-order shear deformation plate theory. Akbaş (2019c, 2019d) presented effects of hygro-thermal loads on the nonlinear and post-buckling behaviour of functionally graded beams. Abdelrahman *et al.* (2019) analyzed the vibration of perforated beams. Li *et al*. (2018) investigated nonlinear dynamics of laminated beams under both blast and thermal loads.

It is seen from literature survey, an investigation about the effects of temperature on the moving load problems of the laminated beams has not been presented. It is aimed to fill this gap for laminated beams in this study. In this study, the effects of velocity of moving load, temperature values, the fiber orientation angles and the stacking sequence of laminas on the dynamic results of the laminated beam are presented and discussed. In considered problem, Timoshenko beam theory is used and the temperature dependent physical property is considered. In order to validate accuracy of used formulations and method, a compassion study is studied. In the solution of the problem, the Ritz method is used and the solution step of dynamic problem is solved by using the Newmark average acceleration method in the time history.

2. Theory and formulation

A simply supported laminated beam subjected to a moving load under uniform temperature rising (ΔT) is shown in figure 1 with *X*,*Y*,*Z* Cartesian coordinate system. Three identical laminas are considered in the laminated beam. The length, height and width of beam indicate *L*, *h* and *b*, respectively. The magnitude and the constant velocity of load indicate *F* and v_f, respectively.

The equivalent Young's modulus of *i*th layer for *X* direction (E_x^i) is used considered as following model with temperature function (Vinson and Sierakowski 2006)

$$\frac{1}{E_x^i(T)} = \frac{cos^4(\theta_i)}{E_{11}(T)} + \left(\frac{1}{G_{12}(T)} - \frac{2v_{12}}{E_{11}(T)}\right) cos^2(\theta_i)\ sin^2(\theta_i) + \frac{sin^4(\theta_i)}{E_{22}(T)} \quad (1)$$

where, E_{11} and E_{22} are the Young's modulus in X and Y directions, respectively. G_{12}, G_{13}, G_{23} indicate the shear modulus. The material properties of laminas are considered as temperature-dependent as following functions (Shen 2001 and Li and Qiao 2015)

$$E_{11}(T) = E_{01}(1 - 0.5\ \ 10^{-3}T) \quad \text{GPa} \quad (2a)$$

$$E_{22}(T) = E_{02}(1 - 0.2\ \ 10^{-3}T) \quad \text{GPa} \quad (2b)$$

$$G_{12}(T) = G_{13} = G_{012}(1 - 0.2\ \ 10^{-3}T) \quad \text{GPa} \quad (2c)$$

$$G_{23}(T) = G_{023}(1 - 0.2\ \ 10^{-3}T) \quad \text{GPa} \quad (2d)$$

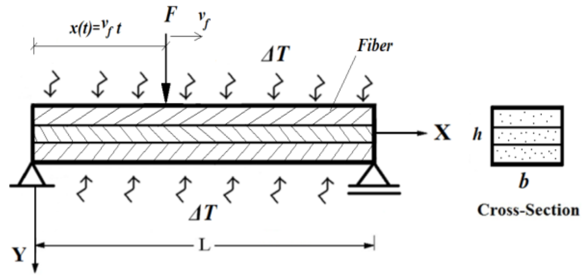


Fig. 1 A simply supported laminated beam subjected a moving load under uniform temperature rising and cross-section

$$\alpha_{11}(T) = \alpha_{011}(1 + 0.5\ \ 10^{-3}T) \quad 1/\,^0C \quad (2e)$$

$$\alpha_{22}(T) = \alpha_{022}(1 + 0.5\ \ 10^{-3}T) \quad 1/\,^0C \quad (2f)$$

where, α_{11} and α_{22} indicate the thermal expansion coefficients in X and Y directions, respectively. θ is the fiber orientation angle, *m=cos* θ and *n= sin* θ. E_{01}, E_{02}, G_{012}, G_{023}, α_{011}, α_{022} indicate the material values at initial temperature value. $T= T_0 +\Delta T$, T_0 is initial temperature and ΔT is the temperature rising.

On the Timoshenko beam theory, the axial strain (ε_{xx}) and shear strain (γ_{xy}) are presented as follows

$$\varepsilon_{xx} = \frac{\partial u(X,t)}{\partial X} - Y\frac{\partial \emptyset(x,t)}{\partial X} \quad (3a)$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} - \emptyset(x,t) \quad (3b)$$

where, *u, v* and $\emptyset$ are axial displacement, vertical displacement and rotation, respectively. *t* indicated the time. The constitutive equations are presented as follows

$$\sigma_{xx} = E_x^i(T)\left(\frac{\partial u(X,t)}{\partial X} - Y\frac{\partial \emptyset(x,t)}{\partial X} - \alpha_{11}^i(T)\Delta T\right) \quad (4a)$$

$$\tau_{xy} = k_s\ \ G_{12}{}_x^i(T)\left(\frac{\partial v}{\partial x} - \emptyset(x,t)\right) \quad (4b)$$

where, σ_{xx} and τ_{xy} are the normal and shear stresses, respectively. k_s is the shear correction factor. The strain energy (U), the kinetic energy (K) and potential energy of the external loads (W) are presented as follows

$$U = \frac{1}{2}\int_0^L \left(A_0\left(\frac{\partial u}{\partial x}\right)^2 - 2B_0\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial \emptyset}{\partial x}\right) + D_0\left(\frac{\partial \emptyset}{\partial x}\right)^2 + A_5\left(\frac{\partial v}{\partial x} - \emptyset(x,t)\right)^2 - A_{XT}\left(\frac{\partial u}{\partial x}\right) + A_{YT}\left(\frac{\partial \emptyset}{\partial x}\right)\right)dX \quad (5a)$$

$$K = \frac{1}{2}\int_0^L \left(I_0\left(\frac{\partial u}{\partial t}\right)^2 - 2I_1\left(\frac{\partial u}{\partial t}\right)\left(\frac{\partial \emptyset}{\partial t}\right) + I_2\left(\frac{\partial \emptyset}{\partial t}\right)^2 + I_0\left(\frac{\partial v}{\partial t}\right)^2\right)dX \quad (5b)$$

$$W = \int_0^L \left[F(t)\ \ \delta(x - v_f t)v(x,t)\right]dX \quad (5c)$$

where, δ is the Dirac delta operator, A_0, B_0, D_0, A_5, A_{XT}, A_{YT}, I_0, I_1, I_2 are given as follows

$$A_0 = \sum_{i=1}^n bE_x^i(y_{i+1} - y_i\ ) \quad (6a)$$

$$B_0 = \frac{1}{2}\sum_{i=1}^n bE_x^i(y_{i+1}^2 - y_i^2) \quad (6b)$$

$$D_0 = \frac{1}{3}\sum_{i=1}^{n} bE_x^i (y_{i+1}^3 - y_i^3) \tag{6c}$$

$$A_{55} = \frac{5}{4}\sum_{i=1}^{n} bQ_{55}^i (z_{i+1} - z_i \; - \frac{4}{3h^2}(y_{i+1}^3 - y_i^3)) \tag{6d}$$

$$A_{XT} = \sum_{i=1}^{n} bE_x^i \alpha_{11}^i \Delta T (y_{i+1} - y_i \;) \tag{6e}$$

$$A_{YT} = \frac{1}{2}\sum_{i=1}^{n} bE_x^i \alpha_{11}^i \Delta T (y_{i+1}^2 - y_i^2) \tag{6f}$$

$$I_0 = \sum_{i=1}^{n} b\rho^i (y_{i+1} - y_i \;) \tag{6g}$$

$$I_1 = \frac{1}{2}\sum_{i=1}^{n} b\rho^i (y_{i+1}^2 - y_i^2) \tag{6h}$$

$$I_2 = \frac{1}{3}\sum_{i=1}^{n} b\rho^i (y_{i+1}^3 - y_i^3) \tag{6i}$$

where *n* is number of layers, ρ^i is the mass density of *i*th layer and Q_{55}^k is given below

$$Q_{55}^i = G_{13}(T)\cos^2(\theta_i) + G_{23}(T)\sin^2(\theta_i) \tag{7}$$

The Lagrangian functional of the problem is presented as follows

$$I = K - (U + W) \tag{8}$$

The Ritz method is used in the solution of the problem. According to the Ritz method, approximate solution of the axial, vertical displacements and rotation functions are presented with series of *m* terms as follows

$$u(x,t) = \sum_{m=1}^{\infty} a_m \;(t)\alpha_m(x) \tag{9a}$$

$$v(x,t) = \sum_{m=1}^{\infty} b_m \;(t)\beta_m(x) \tag{9b}$$

$$\emptyset(x,t) = \sum_{m=1}^{\infty} c_m \;(t)\gamma_m(x) \tag{9c}$$

where a_m, b_m and c_m are the unknown coefficients, $\alpha_m\;(x,t)$, $\beta_m\;(x,t)$, $\gamma_m\;(x,t)$ are the coordinate functions depend on the boundary conditions over the interval [*0*,*L*]. For the simply supported beam, the coordinate functions are given as algebraic polynomials

$$\alpha_m(x) = \; x^{(m)} \tag{10a}$$

$$\beta_m(x) = \; (L-x) \;\; x^{(m)} \tag{10b}$$

$$\gamma_m(x) = \; x^{(m-1)} \tag{10c}$$

After substituting Eq. (9) into energy Eqs. (5), and then using the Lagrange's equation gives the following equation

$$\frac{\partial I}{\partial q_m} - \frac{\partial}{\partial t}\frac{\partial I}{\partial \dot{q}_m} = 0 \tag{11}$$

where q_m is the unknown coefficients which are a_m, b_m and c_m. After implementing the Lagrange procedure, the motion equation of the problem is obtained as follows

$$[K]\{q(t)\} + [M]\{\ddot{q}(t)\} = \{F(t)\} + \{F\}_T \tag{12}$$

where [K], [M], {F(t)} and $\{F\}_T$ are the stiffness matrix and mass matrix load vector due to dynamic external loads and load vector due to temperature effect, respectively. Details of these matrixes are presented as follows

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \tag{13}$$

where

$$\begin{aligned}
&K_{11} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L A_0 \frac{\partial\alpha_i}{\partial x}\frac{\partial\alpha_j}{\partial x}dx \;,\; K_{12} = 0, \\
&K_{13} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L B_0 \frac{\partial\alpha_i}{\partial x}\frac{\partial\gamma_j}{\partial x}dx, \quad K_{21} = 0, \\
&K_{22} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L A_{55} \frac{\partial\beta_i}{\partial x}\frac{\partial\beta_j}{\partial x}dx, \\
&K_{23} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L A_{55} \frac{\partial\beta_i}{\partial x}\gamma_j dx \\
&K_{31} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L B_0 \frac{\partial\gamma_i}{\partial x}\frac{\partial\alpha_j}{\partial x}dx \\
&K_{32} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L A_{55}\gamma_i \frac{\partial\beta_j}{\partial x}dx \\
&K_{33} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L D_0 \frac{\partial\gamma_i}{\partial x}\frac{\partial\gamma_j}{\partial x}dx
\end{aligned} \tag{14}$$

$$[M] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \tag{15}$$

where

$$\begin{aligned}
&M_{11} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_0 \;\alpha_i\alpha dx \;,\; M_{12} = 0, \\
&M_{13} = -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_1 \;\alpha_i\gamma_j dx, \quad M_{21} = 0, \\
&M_{22} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_0 \;\beta_i\beta_j dx, \\
&M_{23} = M_{32} = 0 \\
&M_{31} == -\sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_1 \;\gamma_i\alpha_j dx \\
&M_{33} = \sum_{i=1}^{m}\sum_{j=1}^{m}\int_0^L I_2 \;\gamma_i\gamma_j dx
\end{aligned} \tag{16}$$

$$\{F(t)\} = F\beta_j(v_p t) \qquad 0 \le t \le L/v_p \tag{17a}$$

$$\{F\}_T = \begin{Bmatrix} A_{XT}\dfrac{\partial\alpha_j}{\partial x} \\ 0 \\ -A_{YT}\dfrac{\partial\gamma_j}{\partial x} \end{Bmatrix} \tag{17b}$$

The motion equation of eq. (12) is solved within the time domain by using Newmark average acceleration method. The dimensionless quantities are presented as follows

$$\bar{v} = \frac{v}{L}, \qquad t^* = \frac{t}{L}v_f \tag{18}$$

3. Findings and discussion

In the section, the effects of temperature rising, fiber orientation angles, the stacking sequence of laminates and velocity of load on the dynamic responses of the laminated beam are presented and discussed. The material of each laminas are selected as Graphite/Epoxy and its material properties are temperature-dependent according to equation 2. The material constants of Graphite/Epoxy are E_{01}=150 GPa, E_{02}=9 GPa, G_{012}=7,1 GPa, G_{023}=2,5 GPa, α_{011}=1,1x10^{-6} $1/{}^0C$, α_{022}= 25,2x10^{-6} $1/{}^0C$, ν=0.3 at

30 ^0C. The geometry values are selected as b=0.2 m, h=0.2 m and L=6 m. The initial temperature is considered as T_0=30 ^0C. The magnitude of load is selected as F=100 kN. In the numerical results, number of the series term is taken as 10.

In order to validate using method, a comparison study is performed. In the comparison study, the dynamic displacements at midspan (v_m) of a homogeneous-isotropic simply-supported beam are obtained and compared in figure 2 with the analytical formulation of homogeneous-isotropic beams under moving load without thermal effect for E=210 GPa, ρ=7800 kg/m^3, v=0.3, $v_f = 30\ m/s$. The vertical displacement function of simply-supported beams under moving load by using analytical solution is presented as follows Tao *et al.* (2016)

$$v(x,t) = \sum_{i=1}^{m} \frac{2}{I_0 L} \frac{F}{\left((i\pi v_p/L)^2 - \omega_i^2\right)} \left(sin\left(\frac{i\pi v_p t}{L}\right) - \frac{i\pi v_p}{L\omega_i} sin(\omega_i t)\right) sin\left(\frac{i\pi x}{L}\right) \qquad 0 \le t \le L/v_f \tag{19}$$

where

$$\omega_i = \left(\frac{i\pi}{L}\right)^2 \sqrt{\frac{D_0}{I_0}} \qquad i=1,2,\ldots m \tag{20}$$

It is seen from Fig. 2, that results of this study are approximately identical with results of analytical solution.

Figs. 3 and 4 present the time response of the mid-span displacements of laminated with different values of temperature rising, fiber orientation angles in [$0/\theta/0$] and [$\theta/0/\theta$] stacking sequences, respectively. It is noted that the dimensionless displacements ($\bar{v}$) and time (t^*) quantities which defines Eq. (18) are used in the time history graphs. In the figures 3 and 4, the velocity of load is taken as $v_p = 30\ m/s$.

As shown, by increasing the temperature, the displacement responses of the laminated beam increase considerably. Increasing the temperature accompanying by reduction in strength of materials, tends to increase displacements. With increasing the temperature, the Young's Modulus and other strength parameters of the material decrease according to temperature-dependent relations which given Eq. 2. So, the stiffness of the beam reduces and displacements increase naturally. Especially, in higher temperature values, the displacements of laminated beam more increase.

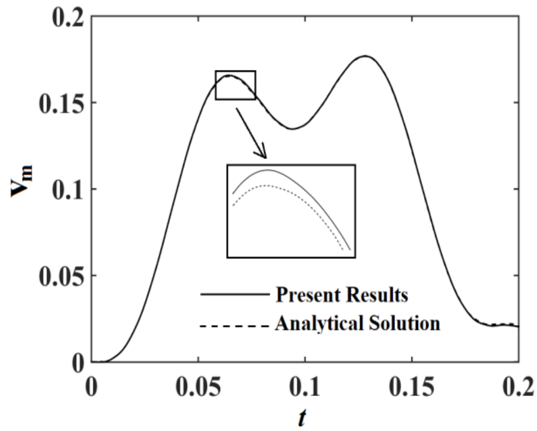


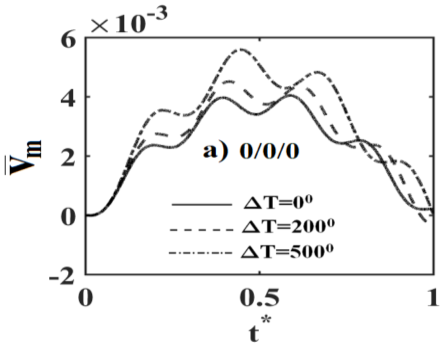
Fig. 2 Comparison study: Dynamic vertical deflections at midspan of a homogeneous and isotropic beam under moving load

As seen from figures 3 and 4, increasing the fiber orientation angles (θ) yields to an increase in the displacements in both [$0/\theta/0$] and [$\theta/0/\theta$] stacking sequences. By increasing the fiber orientation angles, the stiffness of the beam decrease according to the equation (1). By comparing Fig. 3 by Fig. 4, it is noted that, the stacking sequence of [$\theta/0/\theta$] is more effect on the dynamical responses rather that the stacking sequence of [$0/\theta/0$]. The dynamical deflections in [$\theta/0/\theta$] are greater than those of [$0/\theta/0$].

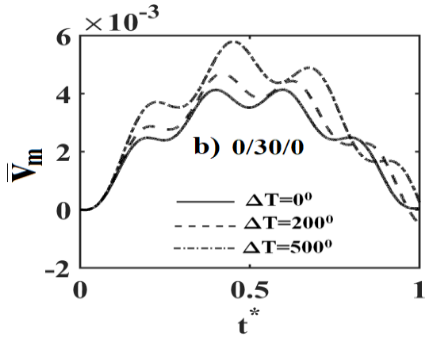
Another result from Figs. 3 and 4, the dynamical responses due to thermal effects vary with different stacking sequences. In the stacking sequence of [$\theta/0/\theta$], the dynamical results of temperature values change dramatically by increasing the fiber orientation angles. However, this situation is not occurred in [$0/\theta/0$] like the results of [$\theta/0/\theta$]. This is because; the maximum stress and strain occurred at the bottom and top surfaces of the beam and the strength of the these surfaces only changes in case of [$\theta/0/\theta$] in contrast with [$0/\theta/0$]. Hence, the laminated composite beam is more sensitive in the stacking sequence of [$\theta/0/\theta$].

In Figs. 5-7, the effects velocity of moving load on the dynamical deflection of the laminated composite beam investigated with different fiber orientation angles for for ΔT=0 ^0C, ΔT=200 ^0C and for ΔT=500 ^0C in time histories. It is seen from figures 5,6 and 7, the fiber orientation angles and stacking sequences of laminas are very effective to behavior of the moving load. For different fiber orientation angles and stacking sequences, the effects of load velocity on the dynamical displacements of the laminated beam change considerably. By comparing with the dimensionless displacements at midspan for different velocity of moving load are $\bar{v}_{10\ m/s} > \bar{v}_{30\ m/s} > \bar{v}_{50\ m/s}$ for t^*=0.5 for stacking sequence of [0/30/0] in all values of temperature rising. However, this sorting is very different from the [30/0/30]. The dimensionless displacements are $\bar{v}_{50\ m/s} > \bar{v}_{30\ m/s} > \bar{v}_{10\ m/s}$ for t^*=0.5 for stacking sequence of [30/0/30] in all temperature rising. Another same comparison is given for fiber orientation angles θ=60 as follows; the dimensionless displacements are $\bar{v}_{10\ m/s} > \bar{v}_{30\ m/s} > \bar{v}_{50\ m/s}$ for stacking sequence of [0/60/0] and $\bar{v}_{30\ m/s} > \bar{v}_{50\ m/s} > \bar{v}_{10\ m/s}$ for stacking sequence of [60/0/60] for t^*=0.5 in all values of temperature rising. As shown, the stacking sequence and fiber orientation angle play important role on the effects of moving load on the dynamic responses of the laminated composite beams.

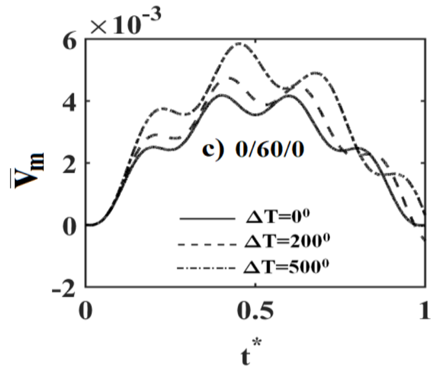
In order to learn the effects of temperature values on the velocity of moving load, the following finding are obtained from Figs. 5-7. As seen from Figs. 5-7, the differences among of the displacements due to different load velocities increase considerably for 0/30/0, 0/60/0, 0/90/0, namely [$0/\theta/0$], by increasing the temperature. However, this



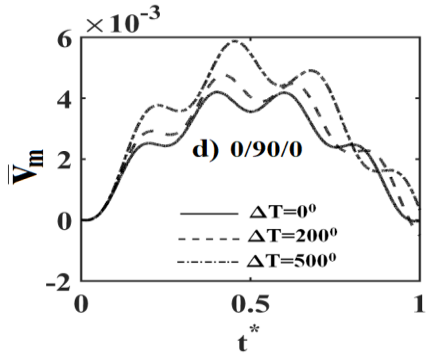
(a) for the stacking sequence [*0/0/0*]



(b) for the stacking sequence [*0/30/0*]

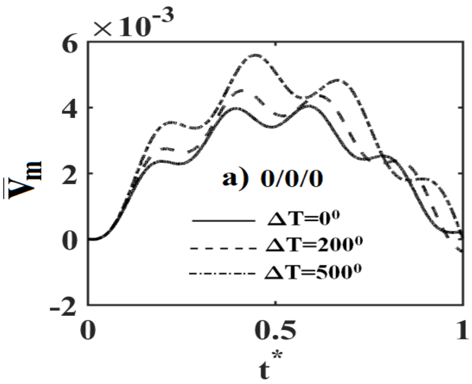


(c) for the stacking sequence [*0/60/0*]

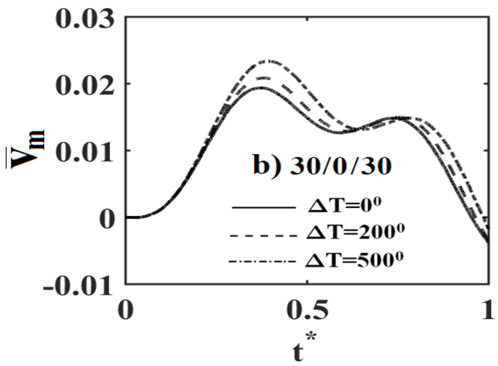


(d) for the stacking sequence [*0/90/0*]

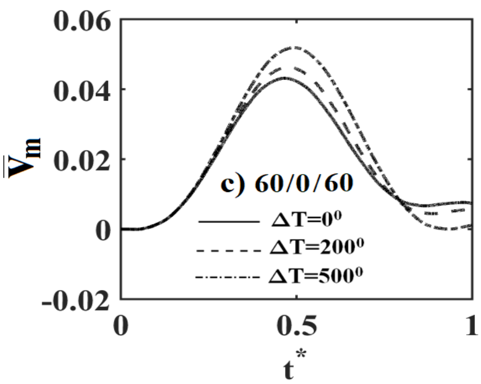
Fig. 3 Time history of dimensionless dynamic displacements at midspan of *0/θ/0* laminated beam under moving load for different temperature values



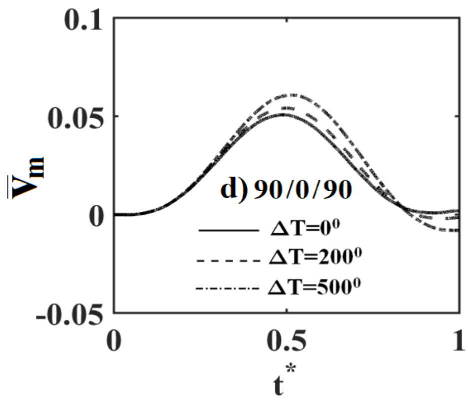
(a) for the stacking sequence [*0/0/0*]



(b) for the stacking sequence [3*0/0/30*]

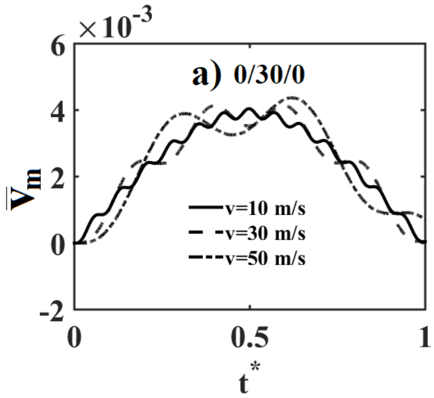


(c) for the stacking sequence [6*0/0/60*]

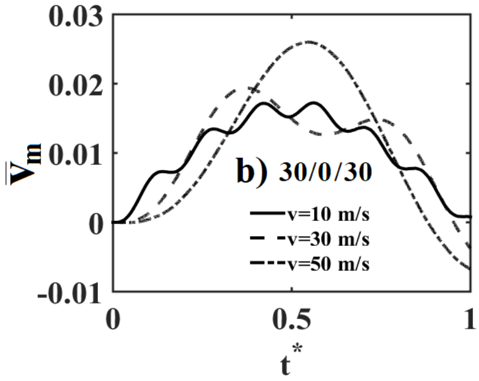


(d) for the stacking sequence [9*0/0/90*]

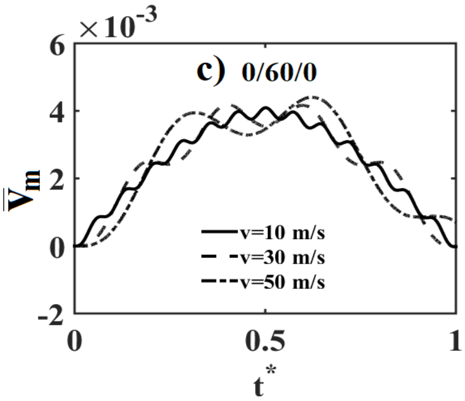
Fig. 4 Time history of dimensionless dynamic displacements at midspan of *θ/0/θ* laminated beam under moving load for different temperature values



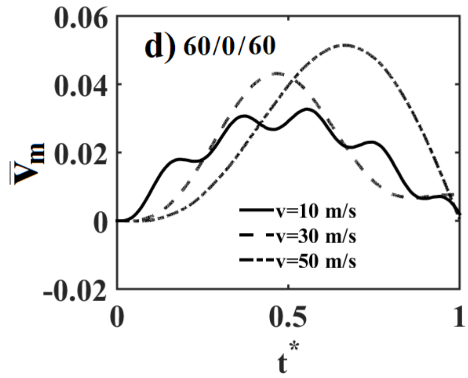
(a) for the stacking sequence [*0/30/0*]



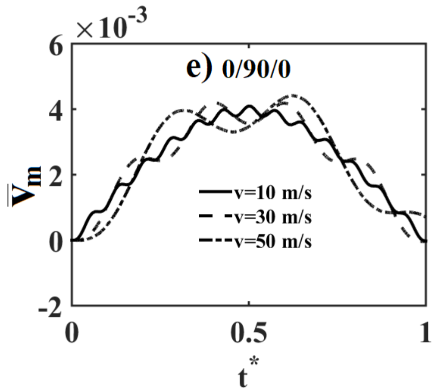
(b) for the stacking sequence [3*0/0/30*]



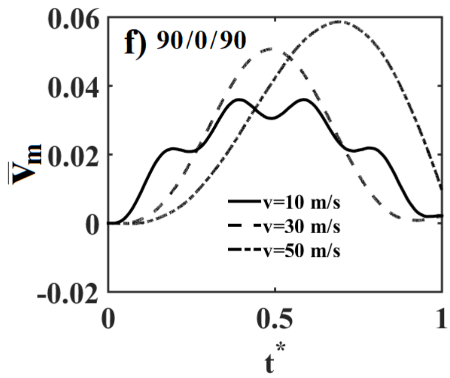
(c) for the stacking sequence [6*0/0/60*]



(d) for the stacking sequence [*0/60/0*]



(e) for the stacking sequence [*0/90/0*]



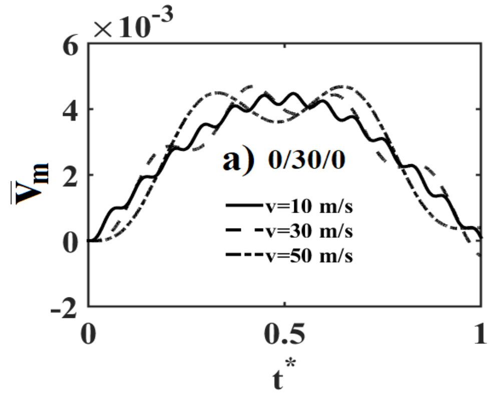
(f) for the stacking sequence [9*0/0/90*]

Fig. 5 Time history of dimensionless dynamic displacements at midspan for different stacking sequences and the velocity of load for ΔT=0 ^0C

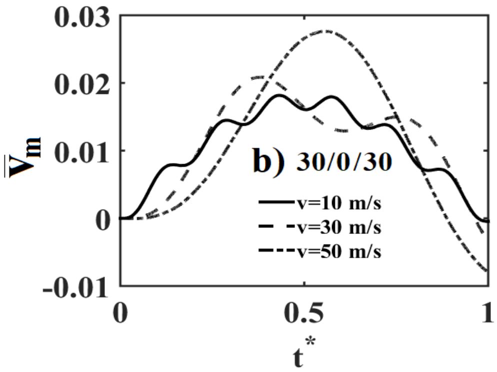
situation is very different for 30/0/30, 60/0/60, 90/0/90, namely [*θ/0/θ*]. In the stacking sequence of [*θ/0/θ*], the differences among of results for load velocities does not increase as [*0/θ/0*]. By comparing the stacking sequence of [*θ/0/θ*] by [*0/θ/0*], the stacking sequence of laminas is more significant on the time response of moving load and temperature effects.

4. Conclusions

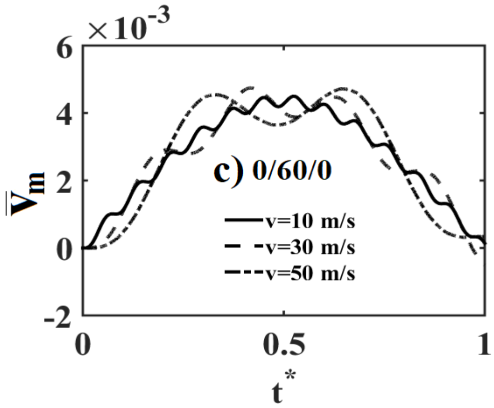
Dynamic responses of a simply supported laminated composite beam are analysed under moving load and temperature rising by using the Ritz method within the Timoshenko beam theory. The material properties of laminas are considered as the temperature dependent physical property. The solution step of dynamic problem, the Newmark average acceleration method is used in the time history. The effects of velocity of moving load, temperature values, the fiber orientation angles and the stacking sequence of laminas on the dynamic responses of the laminated beam are investigated. The comparison study shows the accuracy of proposed model. It is obtained from the results; major conclusions are presented as follows:



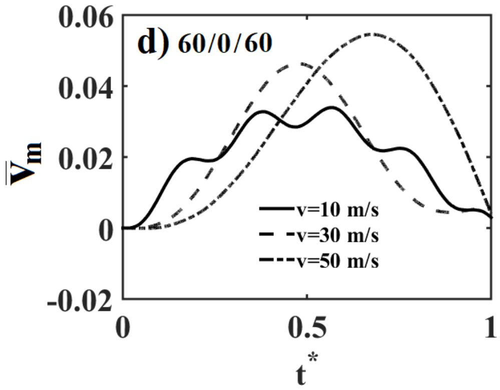
(a) for the stacking sequence [*0/30/0*]



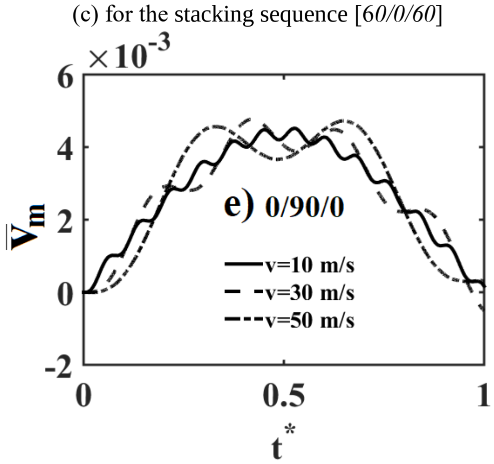
(b) for the stacking sequence [3*0/0/30*]



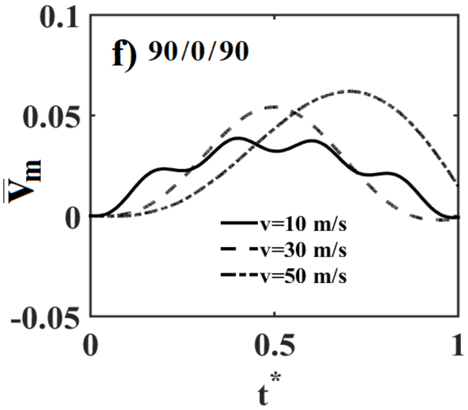
(c) for the stacking sequence [6*0/0/60*]



(d) for the stacking sequence [*0/60/0*]



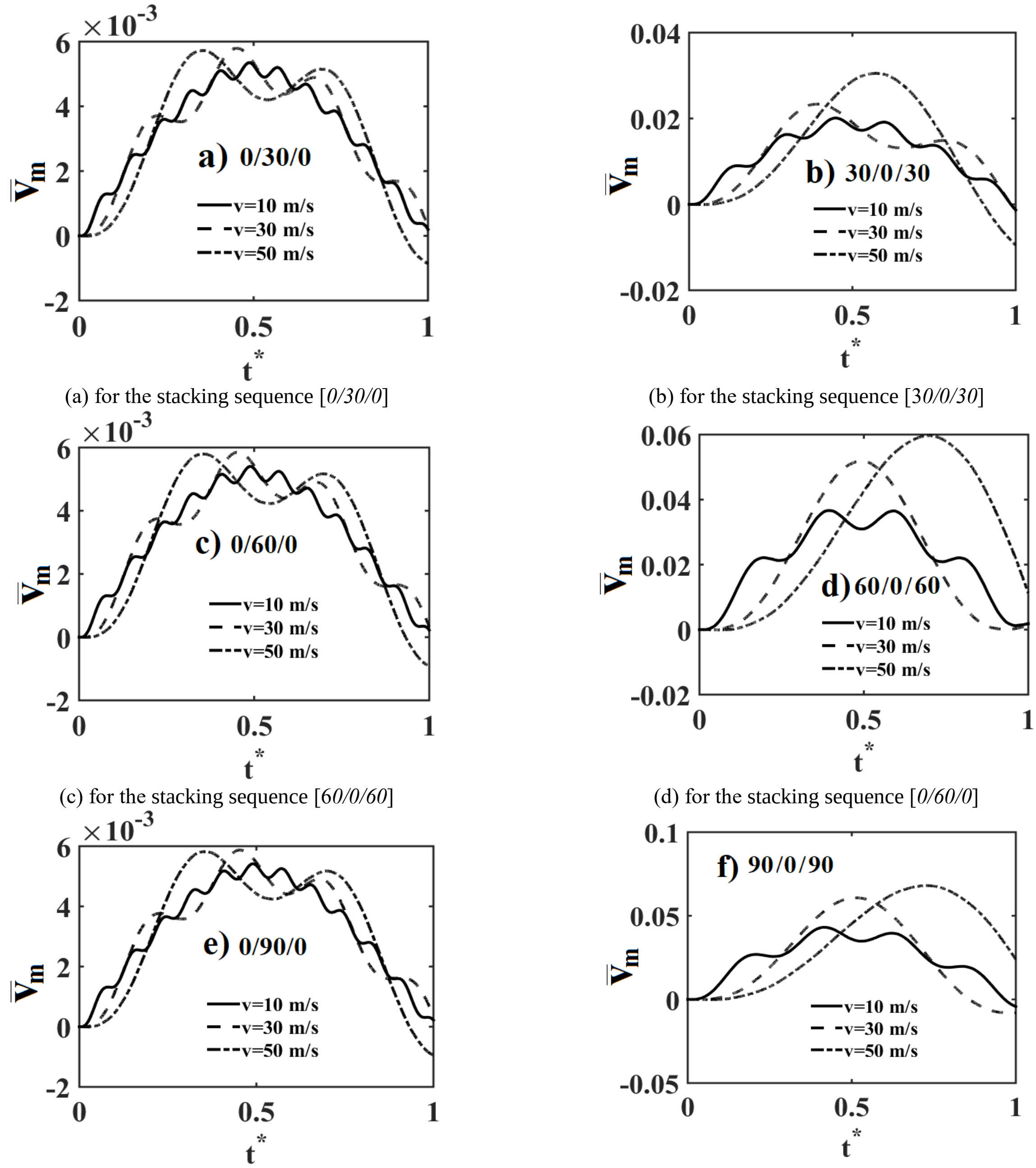
(e) for the stacking sequence [*0/90/0*]



(f) for the stacking sequence [9*0/0/90*]

Fig. 6 Time history of dimensionless dynamic displacements at midspan for different stacking sequences and the velocity of load for ΔT=200 ^0C

- By increasing the temperature, the dynamic responses of the laminated beam under moving load increase dramatically.
- The stacking sequence of [*θ/0/θ*] is more influence of the dynamical responses rather that the stacking sequence of [*0/θ/0*].
- The dynamical responses due to thermal effects vary significantly with different stacking sequences.
- The stacking sequence of laminas and fiber orientation angle play important role on the effects of moving load on the dynamic responses of the laminated composite beams.
- By changing in temperature and stacking sequence of laminas, the effects of load velocity on the dynamical responses change considerably.



(a) for the stacking sequence [*0/30/0*] (b) for the stacking sequence [3*0/0/30*]

(c) for the stacking sequence [6*0/0/60*] (d) for the stacking sequence [*0/60/0*]

(e) for the stacking sequence [*0/90/0*] (f) for the stacking sequence [9*0/0/90*]

Fig. 7 Time history of dimensionless dynamic displacements at midspan for different stacking sequences and the velocity of load for ΔT=500 ^0C

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