The effect of embedding a porous core on the free vibration behavior of laminated composite plates

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Abstract. This paper proposes the use of a porous core between layers of laminated composite plates to examine its effect on the natural frequencies of the resulted porous laminated composite sandwich plate (PLCSP) resting on a two-parameter elastic foundation. Moreover, it has been suggested that the dispersion of porosity has two different functionally graded (FG) patterns which are compared with a uniformly dispersed (UD) profile to find their best vibrational efficiency in the proposed PLCSPs. In FG patterns, two types of dispersions, including symmetric (FG-S) and asymmetric (FG-A) patterns have been considered. To derive the governing Eigen value equation of such structures, the first order shear deformation theory (FSDT) of plates has been employed. Accordingly, a finite element method (FEM) is developed to solve the derived Eigen value equation. Using the mentioned theory and method, the effects of porosity parameters, fiber orientation of laminated composite, geometrical dimensions, boundary conditions and elastic foundation on the natural frequencies of the proposed PLCSPs have been studied. It is observed that embedding porosity in core layer leads to a significant improvement in the natural frequencies of PLCSPs. Moreover, the natural frequencies of PLCSPs with FG porous core are higher than those with UD porous core.

Keywords: porous core; laminated composite; functionally graded; sandwich plate; free vibrations

1. Introduction

Nowadays, porous materials are widely being employed in lightweight structures for decreasing structural weight, damping vibrations, filtering, carrying catalysts, absorbing impact energies and managing thermal responses because of their specific structures and high stiffness to weight ratio (Sciamanna et al. 2015). Depends on the demand concerns, different preparation methods have been introduced such as freeze casting (Deville 2008), salt-leaching (Yan et al. 2012), synthesis of bio-template (Wu et al. 2014), etc. According to the topology of such materials, they can be classified as close or open porous materials. Closed porous materials usually have higher thermal and sonic insulation effects as well as higher stiffness but less permeability (Studart et al. 2006). Laminated composite is material another widely used in different industries. This material is extensively utilized in the fabrications of engineering structures such as automotive, civil aircraft and civil infrastructures because laminated composites can address most of the structural concerns related to strength to weight ratio, design flexibility, corrosion resistance and fatigue (Vinson 2001, Zargar et al. 2017, Ghanati and Safaei 2019, Xu et al. 2019). Besides, sandwich structures basically include a thick low strength core layer to stabilize the structure and two thin stiff face sheets to support the applied loads. Also, the core layer significantly affects the thermomechanical behavior and the strength-to-weight ratio of the structure (Safaei *et al.* 2018a, Safaei *et al.* 2019a). Accordingly, it could be expected that the use of porous materials between the plies of laminated composites intensifies the advantages of the traditional laminated composites structures in terms of weight reduction, stiffness improvement, damping vibrations and thermal management (Chen *et al.* 2016, Cong *et al.* 2018). However, the precise design of such porous laminated composite or sandwich structures requires different mechanical analyses.

Due to the significant impacts of composite materials in engineering applications, many researchers have been attracted to research on the mechanical behaviors of structures made of different classes of composites (Safaei et al. 2016, Babaei et al. 2017, Qin et al. 2018, Safaei et al. 2019b, Yang et al. 2020, Zhang et al. 2020). For plates made of laminated composites, Mantari et al. (2012) developed an analytical model based on Navier solution and trigonometric theory of plates to study the deflections and stresses of the plates under static loads. For the same plates, Thai et al. (2013) established an isogeometric FEM to study vibrations and static responses. To eliminate shear correction factors, they considered FSDT for each layer. Malekzadeh et al. (2009) conducted a dynamic study for laminated composite plates subjected to moving loads using layerwise theory. Safaei et al. (2018b) investigated an FEM model based on the Galerkin method for analyzing elastic stress field in a platelet reinforced composite subjected to axial load. Setoodeh et al. (2009) utilized the theory of 3D elasticity and FEM to evaluate the transient response of laminated composite plates under low velocity impact loads. Tornabene et al. (2013) presented a higher order theory to study the vibrational frequencies of doubly-curved panels made of laminated composite materials. Wang

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(2014) studied the nonlinear wave vibrations of cylindrical shells made of laminated composite materials under radial loads using Galerkin's method. To accomplish this purpose, To eliminate shear locking effect, Yu et al. (2016) developed an FSDT based isogeometric analysis on the buckling and vibrational behaviors of laminated composite plates with a random hole. Considering zig-zag effect and utilizing generalized differential quadrature (DQ) method, the static responses of doubly-curved panels made of laminated composite materials located on a nonlinear elastic foundation were presented by Tornabene et al. (2017). By developing a Navier's method based on a refined theory of plates, the impact of plate dimensions on the mechanical deflections of laminate composite plates was studied in by Xiaohui et al. (2018). Other refined (Sehoul et al. 2017) and a higher order plate (Javed et al. 2018) theories were also proposed to study the natural frequencies of plates made of laminated composites. The effective material properties of bi-directional functionally graded material were extracted based on Mori-Tanaka, and Hashin-Shtrikman homogenization models proposed numerical analysis for the size-dependent nonlinear bending and postbuckling properties via different homogenization schemes (Sahmani and Safaei 2020). By using the Gurtin-Murdoch theory Li et al. (2020) studied nonlinear free vibration behavior of FG composite nanoshells incorporating the influence of modal vibration interaction. In the other work Yi et al. (2020) proposed the nonlinear large-amplitude free vibration response of FG porous materials nanoshells by taking into account higher symmetric vibration mode interactions and surface free energy effects. Lateral vibration analysis of simply-supported microbeam under thermal stress is investigated based on both classical and modified couple stress theories (Babaei and Rahmani 2018). Wave propagation was studied in a rotary laminated composite shell with an attached piezoelectric layer using FSDT and considering transverse shear effect in ((Bisheh and Wu, 2018)). Sharma et al. (2018) conducted vibroacoustic analysis of laminated composite plates by developing a finite boundary element method. The effect of arbitrary boundary conditions on the natural frequency of laminate composite plates was numerically investigated in by Benhenni et al. (2019). Yuan et al. (2020) proposed shear buckling behavior of a skew nanoplate made of the FG composite material by using nonlocal theory of elasticity.

Polymers reinforced with different types of nanofillers such as graphene and carbon nanotubes (CNTs) are another popular (nano)composite materials which are being used in engineering structures. Static (Moradi-Dastjerdi et al. 2018), vibrations and dynamic behaviors of polymer/CNT (Moradi-Dastjerdi and Payganeh 2017a, 2018, Moradi-Payganeh et al. 2017) as Dastjerdi well as polymer/graphene (Moradi-Dastjerdi and Behdinan 2019) cylinders, and polymer/nanoclay plates (Moradi-Dastjerdi et al. 2019) were investigated using an axisymmetric meshless method. A unified solution was proposed to study the effect of general boundary conditions on the vibrations and wave propagation responses of polymer/graphene shells in (Qin et al. 2019, 2020). The free vibrational responses of plates made of amorphous polyethylene/CNT were examined using DQ method based on different plate theories by Safaei et al. (2019c). The impact of CNT agglomeration formation on the nonlinear modal analysis and vibrations of sandwich beams with polymer/CNT was investigated in by Pourasghar and Chen (2019d). The effect of this factor on the thermoelastic dynamic responses of sandwich plates with polymer/CNT faces was also studied (Moradi-Dastjerdi et al. 2017; Safaei et al. 2019d). Due to the astonishing thermal conductivity of CNTs, the heat transfer (Moradi-Dastjerdi and Payganeh 2017b, Pourasghar and Chen 2019a, Behdinan et al. 2020), thermoelastic (Pourasghar and Chen 2019c) and thermoelastic vibrations (Pourasghar et al. 2018, Pourasghar and Chen 2019b) of polymer/CNT cylinders, panels and plates have also intensively been analyzed. In addition, the above-mentioned works showed that the use of the concept of ceramic/metal FG materials (Jalali et al. 2018, Jalali et al. 2018, Fattahi et al. 2019a) in the dispersion of nanofillers resulted in significant changes on the thermo-mechanical responses of nanocomposite structures.

Due to the weight advantage of porous materials, their application as single layer structures or as the middle layer of sandwich structures have become an interesting idea to propose lightweight engineering structures. Wang et al. (2016) successfully modeled a 3D porosity embedded in volume through an extended FEM simulation. The mechanical responses of metal/ceramic FG plates with FG patterns of embedded porosity dispersions were investigated using a developed polygonal FEM (Nguyen et al. 2018). Yang et al. (2018) conducted an FSDT based Chebyshev-Ritz solution for the buckling and free vibration Eigenvalue equations of a single FG porous graphene/polymer. For rotary cylindrical shells made of the same porous nanocomposite materials, the backward and forward frequencies of moving waves were calculated by FSDT in (Dong et al. 2018). In addition, the static deflection and buckling responses of curved beams made of FG porous metal/graphene mixture were studied using Navier's technique by Polit et al. (2019). Zargar et al. (2019) employed homotopy approach to study temperature contour in an axisymmetric porous fin. They verified the obtained temperature contour with some numerical results. Xue et al. (2019) presented the mode shapes and natural frequencies of circular and rectangular plates with FG embedded voids along the thickness using an expanded isogeometric FEM. The same vibrational analysis was performed for FG porous axisymmetric panels using modified Fourier series (Zhao et al. 2019b). Moreover, the vibrations of circular sector plates and annular disks with FG profiles of embedded pores were presented by applying FSDT in by Zhao et al. (2019a). Xie et al. (2020) investigated the nonlinear resonance behavior of a silicon nanobeam with FG profiles of embedded pore dispersions using the surface theory of elasticity. Nonlinear free vibrations of plates made of an FG mixture of two different piezoceramics with symmetric and asymmetric patterns of porosity dispersions were investigated using an analytical method based on the refined theory of plates (Barati and Zenkour 2018).

Regarding sandwich structures with porous layer(s), Jabbari et al. (2013) proposed a circular porous plate

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located between piezoelectric layers and studied buckling responses of the proposed plate. Askari et al. (2018) also presented free vibrational analysis of sandwich plates with the same layer arrangement by applying Mindlin plate theory. Mohammadi et al. (2019) attached two piezoelectric faces as sensor and actuator, around FG porous metal cylinders and studied stress and displacement profiles under electro-mechanical loads. Nguyen et al. (2019) considered two piezoceramic layers attached onto the faces of an FG porous graphene/polymer with FG patterns of embedded porosity dispersions as sensor and actuator, and developed an FEM model to control the vibrational behavior of the obtained sandwich plates. Also, five-layer sandwich structures with an FG porous core, nanocomposite middle layers and piezoelectric faces have been proposed in literatures. In this regards, the vibrations and buckling loads of such sandwich plates with FG graphene/polymer plates as nanocomposite layers were presented by developing a meshless method in (Moradi-Dastjerdi and Behdinan, 2020b, 2020a). Also, by similar method based on a modified Halpin-Tsai' equation and Reddy's third order theory (TSDT) they proposed the structural damping behavior for piezoelectric sandwich plate (Moradi-Dastjerdi et al. 2020c). Setoodeh et al. (2019) utilized FG CNT/polymer curved shells as nanocomposite layers and presented natural frequencies of such five-layer curved sandwich shells. Moreover, the buckling responses of polymeric plates located between FG CNT/polymer faces with uniform dispersion of pores embedded along the thickness were presented by applying meshless technique by Safaei et al. (2019e). In addition, the thermoelastic static performance of FG porous plates located between FG CNT/polymer faces was studied using a third order of plate theory by Safaei et al. (2019f). For rectangular sandwich plates with metal faces and an FG porous graphene/polymer core, a nonlinear natural frequency analysis was conducted in (Li et al. 2018).

This paper examines the effect of embedding a porous core between the layers of laminated composite plates on the natural frequencies of the resulted PLCSP. A twoparameter elastic foundation and FG profiles of porosity dispersion have been considered for PLCSP. To study the natural frequency of such structures, Hamilton's principal and FSDT are employed to obtain the governing Eigen value equation. Based on the utilized FSDT, a finite element method is developed to solve the obtained Eigen value equation. The effects of porosity parameters, fiber orientation of laminated composite, geometrical dimension, boundary conditions and elastic foundation on the natural frequencies of the proposed PLCSPs are investigated.

2. Modeling of PLCSP

As shown in Fig. 1, the proposed PLCSP consists of a laminated composite plate which symmetrically integrated a porous core as its middle layer. The fiber orientations α_i (i=1, 2, ..., *nl* where *nl* is the number of laminated layers) of composite layers are labeled as $[\alpha_1, \alpha_2, ..., \alpha_{nl}]$ starting from the inner to outer layers of each face. In the proposed

PLCSP, the length, width, core thickness, faces thickness and total thickness are shown with a, b, h_c , h_f and h, respectively. In addition, the normal and shear parameters of elastic foundation are sown with k_1 and k_2 , respectively.

Fig. 2 also illustrates the variations of porosity volume fraction V_p along the thickness of core layer for the three considered porosity patterns. The elasticity modulus E^c , density ρ^c , and Poisson's ratio υ^c of porous layer can be estimated as (Zhao *et al.* 2019b)

FG-S:
$$E^{p}(z) = \left(1 - p_{0}\cos\left(\frac{\pi z}{h_{c}}\right)\right)E^{m}$$
, (1)
 $\rho^{p}(z) = \left(1 - p_{m}\cos\left(\frac{\pi z}{h_{c}}\right)\right)\rho^{m}$ (2)
FG-A: $E^{p}(z) = \left(1 - p_{0}\cos\left(\frac{\pi z}{4h_{c}} + \frac{\pi}{4}\right)\right)E^{m}$, (2)
 $\rho^{p}(z) = \left(1 - p_{m}\cos\left(\frac{\pi z}{4h_{c}} + \frac{\pi}{4}\right)\right)\rho^{m}$ (2)
UD: $E^{p} = \left(\frac{2}{\pi}\sqrt{1 - p_{0}} - \frac{2}{\pi} + 1\right)^{2}E^{m}$, (3)

In all the profiles:

$$\upsilon^{p}(z) = 0.221\beta + \upsilon^{m} \left(0.342\beta^{2} - 1.21\beta + 1 \right)$$
(4)

where p_0 is porosity parameter, $p_m = 1 - \sqrt{1 - p_0}$, $\beta = 1 - \rho^p / \rho^m$ and superscript *m* shows the material properties of perfect ($p_0 = 0$) core.



Fig. 1 The proposed PLCSP with a porous core integrated between two laminated composite faces $[\alpha_1, \alpha_2, \alpha_3]$ and rested on an elastic foundation



Fig. 2 The distributions of the volume of porosities along the thickness of porous core

3. Governing equations

3.1 Basic equations

For the proposed PLCSP, the equation of energy function U, which includes strain energy, the energy of the two parameter foundation and kinematic energy are determined as follows (Moradi-Dastjerdi *et al.* 2016)

$$U = \frac{1}{2} \int_{V} \left[\varepsilon_{b}^{T} \boldsymbol{\sigma} + \boldsymbol{\gamma}^{T} \boldsymbol{\tau} \right] - \rho(z) (\dot{u}^{2} + \dot{v}^{2} + \dot{w}^{2}) dV$$

$$\frac{1}{2} \int_{A} \left[k_{1} w^{2} + k_{2} \left[\left(\frac{\partial w}{\partial x} \right)^{2} + \left(\frac{\partial w}{\partial y} \right)^{2} \right] \right] dA$$
(5)

where *A* and *V* are the toper (or downer) face area and the total volume of sandwich plate, σ and τ are out-ofplane and in-plane stress vectors, ε_b and γ are out-ofplane and in-plane strain vectors, and *u*, *v* and *w* are the components of displacement field along *x*, *y* and *z* directions, respectively.

As mentioned before, a first order shear deformation theory is utilized to determine displacement field in the proposed PLCSP. This theory has five unknown parameters including u_0 , v_0 , w_0 , φ_x and φ_y which are the displacement components and normal rotations of mid line in sandwich plate. According to the utilized FSDT, the components of displacement field can be defined as (Reddy 2004, Mohammadsalehi *et al.* 2017, Fattahi *et al.* 2019b; Liu *et al.* 2020)

$$u = u_0(x, y) + z\varphi_x(x, y)$$

$$v = v_0(x, y) + z\varphi_y(x, y)$$

$$w = w_0(x, y)$$
(6)

Based on the defined displacement field, the relation between displacement and strain can be given as (Reddy 2004)

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$
(7)

By dividing strain components to in-plane and out-ofplane terms, the strain vectors can be written as

$$\boldsymbol{\varepsilon}_{b} = \begin{cases} \frac{\partial u_{0}}{\partial x} \\ \frac{\partial v_{0}}{\partial y} \\ \frac{\partial u_{0}}{\partial y} + \frac{\partial v_{0}}{\partial x} \end{cases} + z \begin{cases} \frac{\partial \varphi_{x}}{\partial x} \\ \frac{\partial \varphi_{y}}{\partial y} \\ \frac{\partial \varphi_{y}}{\partial y} + \frac{\partial \varphi_{y}}{\partial y} \end{cases}$$

$$\boldsymbol{\gamma} = \begin{cases} \varphi_{x} + \frac{\partial w_{0}}{\partial x} \\ \varphi_{y} + \frac{\partial w_{0}}{\partial y} \end{cases}$$
(8)

These vectors can be rewritten as

$$\boldsymbol{\varepsilon}_{b} = \left\{ \boldsymbol{\varepsilon}_{xx} \quad \boldsymbol{\varepsilon}_{yy} \quad \boldsymbol{\gamma}_{xy} \right\}^{T} = \boldsymbol{\varepsilon}_{0} + \boldsymbol{z}\boldsymbol{\varepsilon}_{1} \quad , \boldsymbol{\gamma} = \left\{ \boldsymbol{\gamma}_{xz} \quad \boldsymbol{\gamma}_{yz} \right\}^{T} \qquad (9)$$

In addition, the constitutive equation, which relates the stress vectors (i.e., in-plane σ and out-of-plane τ stress vectors) to strain vectors, in the proposed PLCSP is defined as (Reddy 2004)

$$\begin{cases} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{cases} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{26} & D_{26} & D_{66} \end{bmatrix} \begin{cases} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{cases}$$

$$\begin{cases} \tau_{xz} \\ \tau_{yz} \end{cases} = \frac{5}{6} \begin{bmatrix} D_{55} & D_{45} \\ D_{45} & D_{44} \end{bmatrix} \begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

$$\sigma = \mathbf{D}_{\mathbf{b}} \varepsilon_b , \ \tau = \frac{5}{6} \mathbf{D}_s \gamma , \ \mathbf{D} = \begin{bmatrix} \mathbf{D}_{\mathbf{b}} & 0 \\ 0 & \mathbf{D}_s \end{bmatrix}$$
(11)

where **D** is elastic constant matrix, and the value 5/6 in Eqs. (10) and (11) is came from the use of FSDT as shear correction factor.

The utilized porous core is an isotropic material. In the isotropic materials, D_{ii} are defined as

$$D_{11} = D_{22} = \frac{E^{p}}{1 - (v^{p})^{2}}, D_{12} = v^{p} D_{11},$$

$$D_{44} = D_{55} = D_{66} = \frac{E^{p}}{2(1 + v^{p})},$$

$$D_{16} = D_{26} = D_{45} = 0$$
(12)

In each layer of laminated composite faces, the matrix of elastic constant is determined by reducing the components related to z direction as

$$D_{ij} = \begin{cases} \bar{Q}_{ij} - (\bar{Q}_{i3} \cdot \bar{Q}_{ij}) / Q_{33} & i, j = 1, 2, 6\\ \bar{Q}_{ij} & i, j = 4, 5 \end{cases}$$
(13)

where

$$Q_{11} = E_1 \frac{1 - v_{23} v_{32}}{\Delta} , \quad Q_{22} = E_2 \frac{1 - v_{13} v_{31}}{\Delta}$$

$$Q_{12} = E_1 \frac{v_{21} + v_{31} v_{32}}{\Delta}, \quad Q_{13} = E_3 \frac{v_{13} + v_{12} v_{23}}{\Delta}$$

$$Q_{23} = E_3 \frac{v_{23} + v_{13} v_{21}}{\Delta}, \quad Q_{33} = E_3 \frac{1 - v_{12} v_{21}}{\Delta}$$

$$Q_{44} = G_{23} , \quad Q_{55} = G_{13} , \quad Q_{66} = G_{12}$$

$$\Delta = \frac{1 - v_{32} v_{23} - v_{21} v_{12} - v_{13} v_{31} - 2 v_{32} v_{21} v_{13}}{E_1 E_2 E_3}$$
(14)

However, Eq. (14) is valid only for laminated composites with $\alpha = 90^{\circ}$. For other fiber orientations the components of elastic matrix \overline{Q}_{ij} are determined by the values of $m = \cos \alpha$ and $n = \sin \alpha$ as (Reddy 2004)

$$\begin{aligned} Q_{11} &= m^4 Q_{11} + 2m^2 n^2 \left(Q_{12} + 2Q_{66} \right) + n^4 Q_{22} ,\\ \bar{Q}_{22} &= n^4 Q_{11} + 2m^2 n^2 \left(Q_{12} + 2Q_{66} \right) + m^4 Q_{22} ,\\ \bar{Q}_{44} &= m^2 Q_{44} + n^2 Q_{55} , \ \bar{Q}_{55} &= m^2 Q_{55} + n^2 Q_{44} ,\\ \bar{Q}_{66} &= -m^2 n^2 \left(Q_{11} + Q_{22} - 2Q_{12} \right) + \left(m^4 - n^4 \right) Q_{66} ,\\ \bar{Q}_{12} &= m^2 n^2 \left(Q_{11} + 4Q_{22} Q_{66} \right) + \left(m^4 + n^4 \right) Q_{12} ,\\ \bar{Q}_{13} &= m^2 Q_{13} + n^2 Q_{23} , \ \bar{Q}_{23} &= n^2 Q_{13} + m^2 Q_{23} ,\\ \bar{Q}_{36} &= \left(Q_{32} - Q_{31} \right) mn , \ \bar{Q}_{45} &= \left(Q_{45} - Q_{55} \right) mn ,\\ \bar{Q}_{16} &= -mn \Big[n^2 Q_{11} - m^2 Q_{22} - \left(m^2 - n^2 \right) \left(Q_{12} + 2Q_{66} \right) \Big] \\ \bar{Q}_{26} &= -mn \Big[m^2 Q_{11} - n^2 Q_{22} - \left(m^2 - n^2 \right) \left(Q_{12} + 2Q_{66} \right) \Big] \end{aligned}$$

3.2 FEM formulations

After generating elements, finite element methods approximate the real values of domain at some specific points called nodes. In this work, the desired domain is displacement field, and rectangular elements with only four nodes have been employed. The relation between the displacement field \mathbf{u} and that approximated by FEM u_i is determined as

$$\mathbf{u} = \sum_{i=1}^{n} N_i u_i \tag{16}$$

where *n* is the total number of nodes and N_i is bilinear shape functions. The use of FSDT dictates five unknowns or degrees of freedom to each node. So, the vector of approximated nodal values is determined as (Moradi-Dastjerdi *et al.* 2016)

$$u_{i} = \begin{bmatrix} u_{0i}, v_{0i}, w_{0i}, \varphi_{xi}, \varphi_{yi} \end{bmatrix}^{T}$$
(17)

Introducing Eq. (16) in Eq. (7) results in the FEM forms of strain vectors

$$\boldsymbol{\varepsilon}_{b} = \left\{ \mathbf{B}_{m} + z \, \mathbf{B}_{b} \right\} \mathbf{u} \quad , \; \boldsymbol{\gamma} = \mathbf{B}_{s} \, \mathbf{u} \tag{18}$$

where

$$\mathbf{B}_{s} = \begin{bmatrix} 0 & 0 & N_{i,x} & N_{i} & 0\\ 0 & 0 & N_{i,y} & 0 & N_{i} \end{bmatrix}$$
$$\mathbf{B}_{m} = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0\\ 0 & N_{i,y} & 0 & 0 & 0\\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \end{bmatrix}$$
(19)
$$\mathbf{B}_{b} = \begin{bmatrix} 0 & 0 & 0 & N_{i,x} & 0\\ 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} \end{bmatrix}$$

Introducing the FEM definitions of displacement, strain and stress vectors in the total energy function (Eq. (15)) results in

$$U = \frac{1}{2} \int_{\Omega} \mathbf{u}^{T} \left\{ \mathbf{B}_{m}^{T} \mathbf{A} \mathbf{B}_{m} + \mathbf{B}_{m}^{T} \overline{\mathbf{B}} \mathbf{B}_{b} + \mathbf{B}_{b}^{T} \overline{\mathbf{B}} \mathbf{B}_{m} \right\} \mathbf{u} \, dA$$

+
$$\frac{1}{2} \int_{\Omega} \mathbf{u}^{T} \left\{ \mathbf{B}_{n}^{T} k_{1} \mathbf{B}_{n} + \mathbf{B}_{s}^{T} k_{s} \mathbf{B}_{s} \right\} \mathbf{u} \, dA \qquad (20)$$

-
$$\frac{1}{2} \int_{A} \ddot{\mathbf{u}}^{T} \int_{z} \left[\mathbf{G}_{i}^{T} \overline{\mathbf{M}} \mathbf{G}_{j} \right] \ddot{\mathbf{u}} \, dA$$

where

$$(\mathbf{A}, \overline{\mathbf{B}}, \mathbf{D}) = \int_{-h/2}^{h/2} \mathbf{D}_b(1, z, z^2) dz, \mathbf{A}_s = \frac{5}{6} \int_{-h/2}^{h/2} \mathbf{D}_s dz \qquad (21)$$

$$\mathbf{B}_{n} = \begin{bmatrix} 0 & 0 & N_{i} & 0 & 0 \end{bmatrix}, \\ \mathbf{B}_{s2} = \begin{bmatrix} 0 & 0 & N_{i,x} & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 0 \end{bmatrix}$$
(22)

$$\mathbf{G}_{i} = \begin{bmatrix} \varphi_{i} & 0 & 0 & 0 & 0 \\ 0 & \varphi_{i} & 0 & 0 & 0 \\ 0 & 0 & \varphi_{i} & 0 & 0 \\ 0 & 0 & 0 & \varphi_{i} & 0 \\ 0 & 0 & 0 & 0 & \varphi_{i} \end{bmatrix}$$
(23)
$$\mathbf{\overline{M}} = \begin{bmatrix} I_{0} & 0 & 0 & I_{1} & 0 \\ 0 & I_{0} & 0 & 0 & I_{1} \\ 0 & 0 & I_{0} & 0 & 0 \\ I_{1} & 0 & 0 & I_{2} & 0 \\ 0 & I_{1} & 0 & 0 & I_{2} \end{bmatrix}$$
(24)

In addition, I_0 , I_1 and I_2 are the coefficient of inertia which are defined as

$$(I_0, I_1, I_2) = \int_{-h/2}^{h/2} \rho(z)(1, z, z^2) dz$$
(25)

The use of Hamilton's principle in Eq. (20) results in the following Eigen value governing equation for the proposed PLCSP

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = 0 \tag{26}$$

where **M** and **K** are the global stiffness and mass matrices which are determined as

$$\mathbf{M} = \int_{A} \mathbf{G}_{i}^{T} \overline{\mathbf{M}} \mathbf{G}_{j} dA$$
(27)

$$\mathbf{K} = \int_{\Omega} \left\{ \mathbf{B}_{m}^{T} \mathbf{A} \mathbf{B}_{m} + \mathbf{B}_{m}^{T} \overline{\mathbf{B}} \mathbf{B}_{b} + \mathbf{B}_{b}^{T} \overline{\mathbf{B}} \mathbf{B}_{m} \\ + \mathbf{B}_{b}^{T} \mathbf{D} \mathbf{B}_{b} + \mathbf{B}_{s}^{T} \mathbf{A}_{s} \mathbf{B}_{s} \\ + \int_{\Omega} \left\{ \mathbf{B}_{n}^{T} k_{1} \mathbf{B}_{n} + \mathbf{B}_{s2}^{T} k_{2} \mathbf{B}_{s2} \right\} d\Omega$$
(28)

4. Results and discussions

In this section, first, the reliability of the developed FEM is established and then, the free vibration behavior of the proposed PLCSP is studied. It is assumed that the laminate composite faces and the porous core of PLCSPs are made of graphite–epoxy (Gr/Ep) and neat epoxy, respectively. Moreover, in the following simulations, clamped (CCCC) square PLCSPs with four-layer laminated composite faces [0, 90, 0, 90], $h_c/a = 0.01$, $h_f/h_c = 0.2$, and $K_l = K_2 = 0$ (without elastic foundation) have been considered, unless otherwise clearly stated. In addition, the following material properties have been considered (Talebitooti *et al.* 2016):

Epoxy: *ρ*=1150 Kg/m³, *E*=4.5 GPa, *v*=0.4

Gr/Ep: ρ =1578 Kg/m³, E_{11} =132.38 GPa, E_{12} = E_{13} =10.756 GPa, G_{23} =3.606e9 GPa, G_{12} = G_{13} =5.6537 GPa, v_{23} =0.49, v_{12} = v_{13} =0.24

The following normalized parameters are also utilized for natural frequency Ω and foundation coefficients K_1 , K_2 in this section

$$\Omega = \omega H_0 \sqrt{\rho^p / E^p} \tag{29}$$

$$K_1 = k_1 a^4 / D$$
, $K_2 = k_2 a^4 / D$

where

$$D = E^{p} h^{3} / 12 \left(1 - \left(v^{p} \right)^{2} \right)$$
 and $H_{0} = 0.1 m$ (30)

In this paper, sandwich plates with different types of boundary conditions on their edges including clamped (C), simply supported (S) and free (F) edges have been considered. To implement such boundary conditions, the following descriptions have been utilized:

C edges: " $u = v = w = \varphi_x = \varphi_y = 0$ "

S edges: " $u = w = \varphi_x = 0$ at y = 0 or a" and " $v = w = \varphi_y = 0$ at x=0 or b"

4.1 Validation of models

In order to examine the accuracy of the developed method, square simply supported isotropic plates with E=380 GPa, ρ =3800 Kg/m³, v=0.3 and different thicknesses have been considered and their natural frequencies obtained from our developed FEM are compared with those available in literatures as summarized in Table 1. The comparison of the nondimensional fundamental frequency of plates shows that the developed FEM has a very good agreement with the reported results, especially for thin and moderately thick plates. This is because our formulation is based on FSDT formulation; however, higher order theories have been employed in by Baferani et al. (2011). This comparison also reveals that the formulation developed in this paper is computationally efficient because it is based on FSDT with only five unknowns and employs bilinear rectangular element which do not impose huge computational costs.

In addition, the convergence of the developed FEM in the calculation of the fundamental natural frequency of the proposed PLCSP with a porous core is also examined. Fig. 3 shows the variation of Ω as a function of node number along each direction N (x or y) when the porosity state is FG-S and $p_0=0.9$. This figure shows that the addition of node number after the use of 15×15 nodes does not significantly change the value of Ω which shows the convergence of the developed FEM.



Fig. 3 Nondimensional fundamental natural frequency of PLCSP as a function of node number

Table1 The nondimensional fundamental natural frequency of square simply supported

h/a	Method	Ω
0.05	Baferani et al. (2011)	0.0291
	Thai and Choi (2011)	0.0291
	Moradi-Dastjerdi et al. (2016)	0.0291
	Present	0.0292
0.1	Baferani et al. (2011)	0.1134
	Thai and Choi (2011)	0.1135
	Moradi-Dastjerdi et al. (2016)	0.1135
	Present	0.1136
0.2	Baferani et al. (2011)	0.4154
	Thai and Choi (2011)	0.4154
	Moradi-Dastjerdi et al. (2016)	0.4167
	Present	0.4159

4.2 Free vibration of PLCSPs

One of the main parameters in structures made of laminated composite materials is their fiber orientation or stacking sequence. For the proposed PLCSP with the porosity state of FG-S and $p_0=0.9$, Table 2 shows the effect of stacking sequence in laminate composite faces on the first four natural frequencies. Among the four considered stacking sequence, it is observed that [0, 90, 0, 90] offers the highest values of natural frequencies while, PLCSP with [45, -45, 45, -45] faces has the lowest values of natural frequencies.

Fig. 4 illustrates the first four mode shapes of vibrations for PLCSPs with [0, 90, 0, 90] and [45, -45, 45, -45]. This figure confirms that stacking sequence has a significant effect on the mode shapes of vibrations.

Another parameter in such structures is the number of layers nl. The effect of this parameter on the fundamental natural frequency of PLCSPs with porous (FG-S and $p_0=0.2$) and perfect cores is shown in Fig. 5. This figure shows that the increase of nl has an insignificant effect on natural frequency such that PLCSPs with nl > 2 have almost the same Ω_1 . However, embedding porosity in core layer results in higher natural frequencies.



Fig. 4 First four mode shapes of vibration in PLCSP with (a) [0, 90, 0, 90] (b) [45, -45, 45, -45] faces



Fig. 5 The variation of fundamental Ω versus the number of laminated layers in PLCSP with porous (FG-S and $p_0=0.2$) and perfect core

Table 2 The first four Ω for PLCSPs with FG-S, $p_0=0.9$ and different stacking sequences

[45, -45, 45, -45] 0.4613 0.9284 0.9441 1.4574 [0, 30, 60, 90] 0.4648 0.8936 1.0270 1.3308 [0, 45, 90, -45] 0.4702 0.9416 0.9922 1.4180	[α1, α2, α3, α4]	Ω_1	Ω_2	Ω_3	Ω_4
[0, 30, 60, 90] 0.4648 0.8936 1.0270 1.3308 [0, 45, 90, -45] 0.4702 0.9416 0.9922 1.4180	[45, -45, 45, -45]	0.4613	0.9284	0.9441	1.4574
[0, 45, 90, -45] 0.4702 0.9416 0.9922 1.4180	[0, 30, 60, 90]	0.4648	0.8936	1.0270	1.3308
	[0, 45, 90, -45]	0.4702	0.9416	0.9922	1.4180
[0, 90, 0, 90] 0.4784 0.9734 1.0203 1.3609	[0, 90, 0, 90]	0.4784	0.9734	1.0203	1.3609

The effect of porosity state, including the volume of porosity and the profile of their dispersion, is also investigated in Fig. 6. This figure shows the Ω_1 of such PLCSPs with two different values of core thickness; i.e.,



Fig. 6 The variation of fundamental Ω versus porosity parameter in PLCSPs with (a) $h_c/a = 0.01$ (b) $h_c/a = 0.02$

 $h_c/a = 0.01$ and 0.02. The figures show that the increase of p_0 , which implies the increase of porosity volume, leads to the enhancement of natural frequency. The reason is the structural weight is remarkably reduced by embedding higher voids in the thickest layer of PLCSP. In addition, it is observed that the dispersion pattern of voids does not have a significant impact on the natural frequency of PLCSP although FG-S type sandwich plates have higher Ω_1 . The comparison between Figs. 6(a) and 6(b) shows that the increase of core thickness significantly enhances the natural frequency of the PLCSP.

The effects of aspect ratio (b/a) and the length of sides (a/h_c) on the natural frequency of PLCSPs are shown in Figs. 7 and 8, respectively. These figures show fundamental Ω_1 for PLCSPs with porous and perfect cores and two values of $h_f/h_c=0.1$ and 0.2. Fig. 7 shows that, in all cases, a small increase of the aspect ratio of plates from b/a=1 dramatically reduces the natural frequencies of PLCSPs. The rate of this reduction becomes insignificant for b/a>4. Fig. 8 also shows that the increase of the length of sides continuously reduces the natural frequency of PLCSPs.

Moreover, the significant effect of the thickness of composite faces are evident in both perfect and porous sandwich plates such that the use of thicker composite faces offers much higher natural frequencies. The comparison between the results of PLCSPs with porous and perfect cores (Figs. 7(a) and 7(b) and Figs. 8(a) and 8(b)) shows that the use of porous core results in higher PLCSPs with higher natural frequencies.

Finally, the effect the remaining parameters of the PLCSP, i.e., boundary conditions (F, C and S imply free, clamped and simply supported edges, respectively) and the coefficients of elastic foundation, on the natural frequencies of the proposed PLCSPs are studied. Table 3 shows Ω_1 for PLCSP with FG-S type core. As expected, PLCSPs with four clamped edges have the highest values of natural frequency and by changing clamped edges to simply supported and then free ones, the natural frequency of the plate is decreased due to losing some constrains in each edge such that the lowest natural frequencies are observed in sandwich SFSF plates. Moreover, the enhancement of natural frequency is observed due to the use of elastic foundation such that the shear effect of foundation K_2 has stronger impact on the natural frequency of PLCSPs than the normal one K_l .



Fig. 7 The variation of fundamental Ω versus the aspect ratio of plate in PLCSPs with (a) perfect and (b) porous cores



Fig. 8 The variation of fundamental Ω versus a/h_c of for PLCSPs with (a) perfect and (b) porous cores

Table 3 Fundamental Ω for PLCSPs with FG-S type core

B.C.	K_1, K_2	$p_0 = 0$	$p_0=0.5$	$p_0=0.9$
CCCC	(0,0)	0.4106	0.4350	0.4784
	(500,0)	0.4144	0.4391	0.4830
	(0,50)	0.4199	0.4449	0.4895
CSCS	(0.0)	0.3273	0.3468	0.3815
	(500.0)	0.3322	0.3520	0.3872
	(0,50)	0.3379	0.3581	0.3941
CFCF	(0,0)	0.2905	0.3081	0.3392
	(500,0)	0.2959	0.3139	0.3459
	(0,50)	0.2972	0.3152	0.3471
SSSS	(0.0)	0.1940	0.2054	0.2259
	(500.0)	0.2020	0.2140	0.2355
	(0,50)	0.2096	0.2222	0.2446
SFSF	(0,0)	0.1279	0.1358	0.1498
	(500,0)	0.1398	0.1486	0.1639
	(0,50)	0.1398	0.1485	0.1638

5. Conclusions

This paper examined the effect of embedding a porous core between the layers of laminated composite plates on the natural frequencies of a new PLCSP resting on elastic foundations. The governing Eigen value equation of such structures was obtained using Hamilton's principal and FSDT. To solve the obtained equation, a finite element method was developed for this particular structure. The significant results of this study are as follows:

- The use of laminated faces with stacking sequence of [0, 90, 0, 90] results in PLCSPs with the highest natural frequencies.
- The addition of layers in laminated composite faces after *nl* >2 has an insignificant effect on the natural frequency of PLCSPs.
- Embedding a core between the layers of laminated composite plates enhances the natural frequencies of the resulted structures.
- In addition, embedding porosity in core layer leads to a significant improvement in the natural frequencies of PLCSPs.
- The natural frequencies of PLCSPs with FG porous cores are higher than those with UD porous cores.
- A small increase in the aspect ratio of plates from *b*/*a*=1 dramatically reduces the natural frequencies of PLCSPs.
- The effect of face thickness on the natural frequency of PLCSPs are stronger than core thickness.

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