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# Transversely isotropic thin circular plate with multi-dual-phase lag heat transfer

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**Abstract.** The present research deals with the multi-dual-phase-lags thermoelasticity theory for thermoelastic behavior of transversely isotropic thermoelastic thin circular plate The Laplace and Hankel transform techniques have been used to find the solution of the problem. The displacement components, stress components, and conductive temperature distribution are computed in the transformed domain with the radial distance and further determined in the physical domain using numerical inversion techniques. The effect of rotation and two temperature are depicted graphically on the resulting quantities.

**Keywords:** transversely isotropic thermoelastic; thin plate; laplace and hankel transform; multi-dual-phase lag heat transfer; rotation effect

## 1. Introduction

A lot of research and interest has been given to deformation and heat flow in a continuum using thermoelasticity theories in past few decades. Ventsel and Krauthammer (2001), Zhao (2008) categorized the plates into three classes: membranes, thick plates, and thin plates subject to the ratio of $a/h$ (i.e., aspect ratio), where $a$ is diameter and $h$ is the thickness of plate. Tikhe and Deshmukh (2005, 2006) considered a thin finite circular plate with integral transform technique and heating temperatures in the form of Bessel functions and with integral techniques. Kanoria *et al.* (2011) studied the axisymmetric thermoelastic loading response of fiber reinforced thin circular disc with three phase lag(TPL). Gaikwad and Deshmukh (2005) discussed the inverse thermoelastic problem for thermal deflection in a thin isotropic circular plate. Gaikwad *et al.* (2012) studied the inverse thermoelastic problem of circular plate, whereas, Gaikwad (2016) considered the circular plate for known interior temperature under Steady-state field due to uniform internal energy generation. Gaikwad (2019) discussed the thin circular plate under an instable temperature field due to internal heat generation using Fourier and Hankel transform techniques. Elsheikh *et al.* (2019) investigated thermal effects on the deflection and stresses in a thin-circular plates with an axisymmetric input where the perimetric edge of thin circular plate is fixed and insulated, whereas upper and lower sides of the plate are exposed to heat source. Varghese *et al*. (2018) studied, induced transverse vibration of a thin elliptic annulus plate using integral operational methods. Despite of this several researchers worked on different theory of thermoelasticity as Marin (2010), Abbas and Youssef (2009, 2012), Mohamed *et al.* (2009), Abbas *et al.* (2009), Abd-Alla and Mahmoud (2011), Bouderba *et al.* (2013), Marin and Florea (2014) , Mahmoud *et al.* (2011, 2015), Atmane *et al.* (2015), Meradjah *et al.* (2015), Bousahla *et al.* (2016), Yang *et al.* (2016), Menasria *et al.* (2017), Marin *et al.* (2013, 2016), Bijarnia and Singh (2016), Marin *et al.* (2017a), Shahani and Torki (2018), Eftekhari (2018), Altunsaray (2018), Banh *et al*. (2018), Zenkour (2018), Bhatti *et al.* (2019), Bhatti and Lu (2019b), Kaur and Lata (2019a,b,c), Lata and Kaur (2019a,b,c).

The present research deals with the deformation in transversely isotropic thermoelastic (TIT) thin circular plate with the rotation effect. The Laplace and Hankel transform techniques have been used to find the solution to the problem. The displacement components, conductive temperature distribution and stress components with the radial distance are computed in the transformed domain and further calculated in the physical domain using numerical inversion techniques. The effects of rotation and two temperature are represented graphically.

## 2. Basic equations

Following Kumar *et al.* (2016), equation of motion for a uniformly rotating medium with an angular velocity $\mathbf{\Omega} = \Omega\mathbf{n}$, where $\mathbf{n}$ is vector of unit magnitude directed towards the axis where rotation takes place, in the absence of body forces and heat sources, is given by

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$$\boldsymbol{t}_{ij,j} = \rho\{\ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{u})) + (2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})\}_i, \tag{1}$$

The constitutive relations for an anisotropic thermoelastic medium

$$t_{ij} = c_{ijkl}e_{kl} - \beta_{ij}T, \tag{2}$$

Following Zenkour (2018) heat conduction equation with multi dual phase lag heat transfer is

$$K_{ij}\mathcal{L}_\theta\varphi_{,ij} = \mathcal{L}_q\frac{\partial}{\partial t}\left(\beta_{ij}T_0u_{i,j} + \rho C_E T\right) \tag{3}$$

Where $C_E$ denotes specific heat at uniform strain and $K_{ij}$denote themal conductivity coefficients. Here we will propose two differential parameters $\mathcal{L}_\theta$and $\mathcal{L}_q$ in the form

$$\mathcal{L}_\theta = 1 + \sum_{i=1}^{R_1}\frac{\tau_\theta^i\,\partial^i}{i!\,\partial t^i}, \quad \text{and } \mathcal{L}_q = \left(\varrho + \tau_0\frac{\partial}{\partial t} + \sum_{i=2}^{R_2}\frac{\tau_q^i\,\partial^i}{i!\,\partial t^i}\right) \tag{4}$$

The thermal relaxation parameters $\tau_\theta, \tau_q$and $\tau_0$ are the thermal memories in which $\tau_q$is the phase lag of heat flux, $(0 \le \tau_\theta < \tau_q)$, while $\tau_\theta$is the phase lag of the temperature gradient. For example, L-S theory will be appearing when $\tau_\theta = \tau_q = 0$ and $\varrho = 1$. Generally the value of $R_1 = R_2 = R$ may reach 5 or more according to refined multi-dual-phase-lag theory required while $\varrho$ is a non-dimensional parameter (=0 or 1 according to the thermoelasticity theory). Also we have

$$T = \varphi - a_{ij}\varphi_{,ij},$$
$$\beta_{ij} = C_{ijkl}\alpha_{ij}, \tag{5}$$
$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \quad i = 1,2,3$$

## 3. Formulation of the problem

We consider a transversely isotropic thin circular plate of thickness 2b occupying the space D defined by $0 \le r \le \infty, -b \le z \le b$ in the context of the multi-dual-phase-lag model. We assume that the medium, is transversely isotropic in such a way that the planes of isotropy are perpendicular to the z axis. Thin plates are usually characterized by the ratio a / b (the ratio between the length of a side, a, and the thickness of the material, b, falling between the values of 8 and 80 as mentioned by Ventsel *et. al.* (2001). Let the plate be subjected to axisymmetric heat supply into its boundary having an initially undisturbed state at a uniform temperature $T_0$. We use plane cylindrical coordinates (*r, θ, z*) with the center of the plate as the origin. Applying the transformation:

$$x' = x\cos\phi + y\sin\phi, y' = -x\sin\phi + y\cos\phi, z' = z.$$

where $\phi$ is angle of rotation in x-y plane, on the set of Eqs. (1)-(3) to derive the equations for TIT solid with two temperatures, to obtain Equation of motion for the transversely isotropic medium in cylindrical polar coordinates are

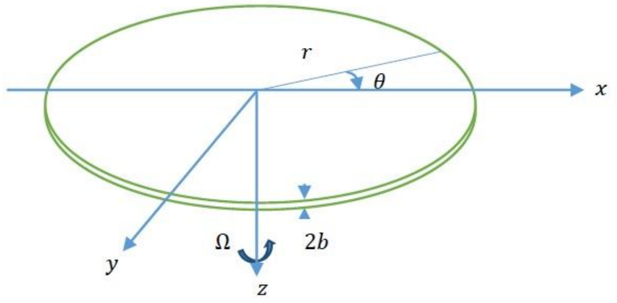


Fig. 1 Geometry of the problem

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r^2}u\right) + c_{12}\left(\frac{1}{r}\frac{\partial^2 v}{\partial r\partial\theta}\right) + c_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + c_{44}\frac{\partial^2 u}{\partial z^2} + c_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + c_{66}\left(\frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} + \frac{1}{r}\frac{\partial^2 v}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial v}{\partial\theta}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right), \tag{6}$$

$$c_{66}\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial u}{\partial\theta} + \frac{1}{r}\frac{\partial^2 u}{\partial r\partial\theta} - \frac{v}{r^2}\right) + c_{12} + c_{11} + c_{13}\left(\frac{1}{r}\frac{\partial^2 w}{\partial z\partial\theta}\right) + c_{44}\frac{\partial^2 v}{\partial z^2} + c_{44}\left(\frac{1}{r}\frac{\partial^2 w}{\partial z\partial\theta}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{7}$$

$$(c_{13} + c_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial^2 v}{\partial z\partial\theta}\right) + c_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) + c_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right) \tag{8}$$

and Heat conduction Eq. (2) becomes

$$K_1\left(1 + \sum_{i=1}^{R_1}\frac{\tau_\theta^i\,\partial^i}{i!\,\partial t^i}\right)\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}\right) + K_3\left(1 + \sum_{i=1}^{R_1}\frac{\tau_\theta^i\,\partial^i}{i!\,\partial t^i}\right)\frac{\partial^2\varphi}{\partial z^2} = \left(\varrho + \tau_0\frac{\partial}{\partial t} + \sum_{i=2}^{R_2}\frac{\tau_q^i\,\partial^i}{i!\,\partial t^i}\right)\left\{T_0\frac{\partial}{\partial t}\left(\beta_1\frac{\partial u}{\partial r} + \beta_2\frac{\partial v}{\partial\theta} + \beta_3\frac{\partial w}{\partial z}\right) + \rho C_E\frac{\partial}{\partial t}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\}\right\} \tag{9}$$

In above equations, following contracting subscript notations are used (11→ 1, 22→ 2, 33→ 3, 23→ 5, 13→ 4, 12→ 6) to relate $c_{ijkl}$ to $c_{mn}$ . As the problem considered is plane axisymmetric, $u, v, w,$and $\varphi$ are independent of $\theta$. We restrict our analysis to two-dimension problem with $\vec{u} = (u, 0, w)$. Thus Eqs. (6)-(9) becomes

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r^2}u\right) + c_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + c_{44}\frac{\partial^2 u}{\partial z^2} + c_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right), \tag{10}$$

$$(c_{11} + c_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + c_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \tag{11}$$

$$C_{33}\frac{\partial^2 w}{\partial z^2}-\beta_3\frac{\partial}{\partial z}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}=\rho\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}\right),$$

$$K_1\left(1+\sum_{i=1}^{R_1}\frac{\tau_\theta^i}{i!}\frac{\partial^i}{\partial t^i}\right)\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)+K_3\left(1+\sum_{i=1}^{R_1}\frac{\tau_\theta^i}{i!}\frac{\partial^i}{\partial t^i}\right)\frac{\partial^2\varphi}{\partial z^2}=\left(\varrho+\tau_0\frac{\partial}{\partial t}+\sum_{i=2}^{R_2}\frac{\tau_q^i}{i!}\frac{\partial^i}{\partial t^i}\right)\left[T_0\left(\beta_1\frac{\partial\dot{u}}{\partial r}+\beta_3\frac{\partial\dot{w}}{\partial z}\right)+\rho C_E\left\{\dot{\varphi}-a_1\left(\frac{\partial^2\dot{\varphi}}{\partial r^2}+\frac{1}{r}\frac{\partial\dot{\varphi}}{\partial r}\right)-a_3\frac{\partial^2\dot{\varphi}}{\partial z^2}\right\}\right]. \tag{12}$$

where $a_1$and $a_3$ are two temperature parameters.

To facilitate the solution, the dimensionless quantities defined by

$$r'=\frac{r}{L},\quad z'=\frac{z}{L},\quad t'=\frac{c_1}{L}t,\quad u'=\frac{\rho c_1^2}{L\beta_1 T_0}u,\quad w'=\frac{\rho c_1^2}{L\beta_1 T_0}w, T'=\frac{T}{T_0},\ t'_{zr}=\frac{t_{zr}}{\beta_1 T_0},\ t'_{zz}=\frac{t_{zz}}{\beta_1 T_0},\ t'_{rr}=\frac{t_{rr}}{\beta_1 T_0},\quad \varphi'=\frac{\varphi}{T_0},\ a'_1=\frac{a_1}{L^2},\ a'_3=\frac{a_3}{L^2},\rho c_1^2=c_{11}\ ,(\tau'_0,\tau'_\theta,\tau'_q,t')=\frac{C_1}{L}(\tau_0,\tau_\theta,\tau_q,t),\Omega'=\frac{L}{C_1}\Omega\ . \tag{13}$$

are introduced. Using these dimensionless quantities in Eqs. (10)-(12) and suppressing the primes gives

$$\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{1}{r^2}u\right)+\frac{c_{13}}{c_{11}}\left(\frac{\partial^2 w}{\partial r\partial z}\right)+\frac{c_{44}}{c_{11}}\frac{\partial^2 u}{\partial z^2}+\frac{c_{44}}{c_{11}}\left(\frac{\partial^2 w}{\partial r\partial z}\right)-\frac{\partial}{\partial r}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}=\left(\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t}\right) \tag{14}$$

$$\frac{(c_{13}+c_{44})}{c_{11}}\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial z}\right)+\frac{c_{44}}{c_{11}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\frac{c_{33}}{c_{11}}\frac{\partial^2 w}{\partial z^2}-\frac{\beta_3}{\beta_1}\frac{\partial}{\partial z}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}=\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}\ , \tag{15}$$

$$K_1\left(1+\sum_{i=1}^{R_1}\frac{\tau_\theta^i}{i!}\frac{\partial^i}{\partial t^i}\right)\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)+K_3\left(1+\sum_{i=1}^{R_1}\frac{\tau_\theta^i}{i!}\frac{\partial^i}{\partial t^i}\right)\frac{\partial^2\varphi}{\partial z^2}=\left(\varrho+\tau_0\frac{\partial}{\partial t}+\sum_{i=2}^{R_2}\frac{\tau_q^i}{i!}\frac{\partial^i}{\partial t^i}\right)\left[\frac{\beta_1 T_0 L}{\rho C_1}\frac{\partial}{\partial t}\left(\beta_1\frac{\partial u}{\partial r}+\beta_3\frac{\partial w}{\partial z}\right)+\rho C_E C_1 L\frac{\partial}{\partial t}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}\right]\ . \tag{16}$$

The Laplace transform of a function $f$ with respect to time variable $t$ , with $s$ as a Laplace Transform variable is defined as

$$f^*(r,z,s)=\int_0^\infty f(r,z,t)e^{-st}dt, \tag{17}$$

Hankel transforms defined by

$$\tilde{f}(\xi,z,s)=\int_0^\infty f^*(r,z,s)rJ_n(r\xi)dr. \tag{18}$$

applying the Laplace and Hankel transforms defined by (17)-(18), on the Eqs. (14)-(16), we obtain

$$(-\xi^2-s^2+\Omega^2+\delta_2 D^2)\tilde{u}+[\xi\delta_1 D-2\Omega s]\tilde{w}+\left(\xi(1-a_3D^2)+a_1\xi^3)\right)\tilde{\varphi}=0, \tag{19}$$

$$(\delta_1\xi D+2\Omega s)\tilde{u}+(\delta_3 D^2-\delta_2\xi^2-s^2+\Omega^2)\tilde{w}-\left(\frac{\beta_3}{\beta_1}D[(1-a_3D^2)+\xi^2 a_1]\right)\tilde{\varphi}=0, \tag{20}$$

$$\delta_{11}\delta_6 s\xi\tilde{u}+\delta_{11}\delta_5 sD\tilde{w}+\left(\delta_{11}\delta_7 s(1+\xi^2 a_1)+K_1\delta_{10}\xi^2-D^2(K_3\delta_{10}+a_3\delta_7 s\delta_{11})\right)\tilde{\varphi}=0, \tag{21}$$

Where

$\delta_1=\frac{c_{13}+c_{44}}{c_{11}}$, $\delta_2=\frac{c_{44}}{c_{11}}$, $\delta_3=\frac{c_{33}}{c_{11}}$, $\delta_6=\frac{\beta_1^2 T_0}{\rho C_1}$, $\delta_5=\frac{\beta_1\beta_3 T_0}{\rho C_1}$,

$\delta_7=\rho C_E C_1 L$, $\delta_8=\frac{C_{13}}{C_{11}}$, $\delta_9=\frac{C_{12}}{C_{11}}$,

$\delta_{10}=1+\sum_{i=1}^{R_1}\frac{\tau_\theta^i s^i}{i!}$, $\delta_{11}=\varrho+\tau_0 s+\sum_{i=2}^{R_2}\frac{\tau_q^i s^i}{i!}$ and $D\equiv\frac{d}{dz}$ .

The stress relations after application of non-dimensional quantities defined by (13) and after suppressing primes becomes

$$\widetilde{t_{zz}}=\delta_8\xi\tilde{u}+\delta_3 D\tilde{w}-\frac{\beta_3}{\beta_1}(1+a_1\xi^2-a_3D^2)\tilde{\varphi}, \tag{22}$$

$$\widetilde{t_{rz}}=\delta_2 D\tilde{u}-\xi\delta_2\tilde{w}, \tag{23}$$

$$\widetilde{t_{rr}}=-\xi\tilde{u}+\delta_9\widetilde{\xi u}+\delta_8 D\tilde{w}-(1+a_1\xi^2-a_3D^2)\tilde{\varphi}. \tag{24}$$

The non-trivial solution of (19)-(21) exists if the determinant of the coefficient $\tilde{u}$, $\tilde{w}$, and $\tilde{\varphi}$ vanishes, which yields to the following characteristic equation

$$AD^6+BD^4+CD^2+E=0, \tag{25}$$

where

$$A=\delta_2\delta_3\zeta_{12}-\delta_2\zeta_{10}\zeta_8,$$

$$B=\zeta_1\zeta_{12}\delta_3-\zeta_1\zeta_{10}\zeta_8+\delta_2\delta_3\zeta_{11}+\delta_2\zeta_{12}\zeta_6-\delta_2\zeta_{10}\zeta_7-\zeta_2^2\zeta_{12}-\zeta_2\zeta_9\zeta_8+\zeta_5\zeta_{10}\zeta_2-\zeta_5\zeta_9\delta_3,$$

$$C=\delta_3\zeta_1\zeta_{11}+\delta_2\zeta_6\zeta_{11}+\zeta_1\zeta_6\zeta_{12}-\zeta_1\zeta_{10}\zeta_7-\zeta_2^2\zeta_{11}+\zeta_2\zeta_7\zeta_9-\zeta_5\zeta_6\zeta_9+\zeta_4\zeta_2\zeta_{10}-\delta_3\zeta_4\zeta_9+\zeta_3^2\zeta_{12},$$

$$E=\zeta_6\zeta_1\zeta_{11}-\zeta_4\zeta_6\zeta_9.$$

$$\zeta_1=-\xi^2-s^2+\Omega^2,$$

$$\zeta_2=\delta_1\xi$$

$$\zeta_3=2\Omega s,$$

$$\zeta_4=\xi(1+a_1\xi^2),$$

$$\zeta_5=a_3\xi,$$

$$\zeta_6=-\delta_2\xi^2-s^2+\Omega^2,$$

$$\zeta_7=-\frac{\beta_3}{\beta_1}(1+a_1\xi^2),$$

$$\zeta_8=\frac{\beta_3}{\beta_1}a_3$$

$$\zeta_9=\delta_{11}\delta_6 s\xi,$$

$$\zeta_{10}=\delta_{11}\delta_5 s,$$

$$\zeta_{11}=\delta_{11}\delta_7 s(1+\xi^2 a_1)+K_1\delta_{10}\xi^2,$$

$$\zeta_{12}=-(K_3\delta_{10}+a_3\delta_7 s\delta_{11}).$$

The solutions of the Eq. (25) can be written in the form

$$\tilde{u}=\sum A_i(\xi,s)cosh(q_i z), \tag{26}$$

$$\tilde{w} = \sum d_i A_i(\xi, s) cosh(q_i z), \tag{27}$$

$$\tilde{\varphi} = \sum l_i A_i(\xi, s) cosh(q_i z), \tag{28}$$

where $A_{i,} i = 1, 2, 3$ being arbitrary constants, $\pm q_i (i = 1,2,3)$ are the roots of the equation (25) and $d_i$ and $l_i$ are given by

$$d_i = \frac{(\zeta_2\zeta_{12} - \zeta_8\zeta_9)q_i^3 + \zeta_3\zeta_{12}q_i^2 + (\zeta_2\zeta_{11} - \zeta_7\zeta_9)q_i + \zeta_3\zeta_{11}}{(-\zeta_8\zeta_{10} + \delta_3\zeta_{12})q_i^4 + (\delta_3\zeta_{11} + \zeta_6\zeta_{12} - \zeta_7\zeta_{10})q_i^2 + \zeta_6\zeta_{11}}$$

$$l_i = \frac{(-\zeta_9\delta_3 + \zeta_1\zeta_{10})q_i^2 + \zeta_3\zeta_{10}q_i - \zeta_8\zeta_9}{(-\zeta_8\zeta_{10} + \delta_3\zeta_{12})q_i^4 + (\delta_3\zeta_{11} + \zeta_6\zeta_{12} - \zeta_7\zeta_{10})q_i^2 + \zeta_6\zeta_{11}}$$

Also, using (26)-(28) in Eqs. (22)-(24) we have

$$\widetilde{t_{zz}} = \sum A_i(\xi, s)\eta_i cosh(q_i z) + \sum \mu_i A_i(\xi, s) \sinh(q_i z), \tag{29}$$

$$\widetilde{t_{rz}} = \sum A_i(\xi, s) M_i cosh(q_i z) + \sum N_i A_i(\xi, s) \sinh(q_i z), \tag{30}$$

$$t_{rr} = \sum A_i(\xi, s) R_i cosh(q_i z) + \sum S_i A_i(\xi, s) \sinh(q_i z). \tag{31}$$

Where

$$\eta_i = \delta_8\xi - \frac{\beta_3}{\beta_1} l_i (1 + a_1\xi^2 - a_3 q_i^2),$$
$$R_i = -\xi + \delta_9\xi - (1 + a_1\xi^2 - a_3 q_i^2),$$
$$S_i = \delta_8 d_i q_i,$$
$$\mu_i = \delta_3 d_i q_i,$$
$$M_i = \delta_2 d_i \xi,$$
$$N_i = \delta_2 q_i \ , i = 1, 2, 3.$$

## 4. Boundary conditions

We consider a cubical thermal source and vertical force of unit magnitude along with vanishing tangential stress components at the stress-free surface z = ±b. Mathematically, these can be written as

$$\frac{\partial\varphi}{\partial z} = \pm g_o F(r, z), \tag{32}$$

$$t_{zz}(r, z, t) = f(r, t), \tag{33}$$

$$t_{rz}(r, z, t) = 0. \tag{34}$$

Using dimensionless quantities defined by (13) on Eqs. (32)-(34) and after suppressing primes and then by taking Hankel and Laplace transform defined by (17)-(18), of resulting equations and using (29)-(30) and (28) yields

$$\sum A_i l_i q_i \sinh(q_i b) = \pm g_o \tilde{F}(\xi, b), \tag{35}$$

$$\sum A_i(\xi, s)\eta_i cosh(q_i b) + \sum \mu_i A_i(\xi, s) \sinh(q_i b) = \tilde{f}(\xi, s), \tag{36}$$

$$\sum A_i(\xi, s) M_i cosh(q_i b) + \sum N_i A_i(\xi, s) \sinh(q_i b), = 0. \tag{37}$$

Using Cramer's rule for solving Eqs. (35)-(37) to get value of $A_i(\xi, s)$ and substituting these values in (26)-(28) and (29)-(31), we obtain

$$\tilde{u} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1\vartheta_1 + \chi_2\vartheta_2 - \chi_3\vartheta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\vartheta_1 - \chi_5\vartheta_2 + \chi_6\vartheta_3\}, \tag{38}$$

$$\tilde{w} = \frac{\tilde{f}(\xi, s)}{\Delta}\{-\chi_1 d_1 \vartheta_1 + \chi_2 d_2 \vartheta_2 - \chi_3 d_3 \vartheta_3\} + \frac{g_o\tilde{F}(\xi, z)}{\Delta}\{\chi_4 d_1 \vartheta_1 - \chi_5 d_2\vartheta_2 + \chi_6 d_3 \vartheta_3\}, \tag{39}$$

$$\tilde{\varphi} = \frac{\tilde{f}(\xi, s)}{\Delta}\{-\chi_1 l_1 \vartheta_1 + \chi_2 l_2 \vartheta_2 - \chi_3 l_3 \vartheta_3\} + \frac{g_o\tilde{F}(\xi, z)}{\Delta}\{\chi_4 l_1 \vartheta_1 - \chi_5 l_2\vartheta_2 + \chi_6 l_3 \vartheta_3\}, \tag{40}$$

$$\tilde{t}_{zz} = \frac{\tilde{f}(\xi, s)}{\Delta}\{-\chi_1 G_4 + \chi_2 G_5 - \chi_3 G_6\} + \frac{g_o\tilde{F}(\xi, z)}{\Delta}\{\chi_4 G_4 - \chi_5 G_5 + \chi_6 G_6\}, \tag{41}$$

$$\tilde{t}_{zr} = \frac{\tilde{f}(\xi, s)}{\Delta}\{-\chi_1 G_7 + \chi_2 G_8 - \chi_3 G_9\} + \frac{g_o\tilde{F}(\xi, z)}{\Delta}\{\chi_4 G_7 - \chi_5 G_8 + \chi_6 G_9\}, \tag{42}$$

$$\tilde{t}_{rr} = \frac{\tilde{f}(\xi,s)}{\Delta}\{-\chi_1 G_{10} + \chi_2 G_{11} - \chi_3 G_{12}\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 G_{10} - \chi_5 G_{11} + \chi_6 G_{12}\}, \tag{43}$$

where

$$G_i = l_i q_i \phi_i,$$
$$G_{i+3} = \eta_i \psi_i + \mu_i \phi_i,$$
$$G_{i+6} = N_i \phi_i + M_i \psi_i,$$
$$G_{i+9} = S_i \phi_i + R_i \vartheta_i, i = 1,2,3.$$
$$\Delta = G_1\chi_4 - G_2\chi_5 + G_3\chi_6,$$
$$\Delta_1 = -\tilde{f}(\xi, s)\chi_1 + g_o\tilde{F}(\xi, z)\chi_4,$$
$$\Delta_2 = \tilde{f}(\xi, s)\chi_2 - g_o\tilde{F}(\xi, z)\chi_5,$$
$$\Delta_3 = -\tilde{f}(\xi, s)\chi_3 + g_o\tilde{F}(\xi, z)\chi_6,$$
$$\chi_1 = [G_2G_9 - G_8G_3],$$
$$\chi_2 = [G_1G_9 - G_7G_3],$$
$$\chi_3 = [G_1G_8 - G_2G_7],$$
$$\chi_4 = [G_5G_9 - G_8G_6],$$
$$\chi_5 = [G_4G_9 - G_6G_7],$$
$$\chi_6 = [G_4G_8 - G_5G_7],$$
$$\cosh(q_i b) = \phi_i, \qquad \sinh(q_i b) = \psi_i, i = 1,2,3.$$
$$\vartheta_i = \cosh(q_i z), \theta_i = \sinh(q_i z), i = 1,2,3$$

## 5. Applications

As application of the problem, we take the source function a

$$F(\mathrm{r}, \mathrm{z}) = \frac{1}{\sqrt{r^2 + z^2}}, \tag{44}$$

Consider the instantaneous distributed load defined by

$$f(r,t) = \delta(t)H(\alpha - r), \tag{45}$$

Where $\delta(\ )$ is the dirac delta function, H( ) denotes Heaviside stepsize function. Applying Laplace and Hankel Transform, on Eqs. (44) and (45), gives

$$\tilde{F}(\xi,z) = \frac{e^{-\xi|z|}}{\xi} \tag{46}$$

$$\bar{f}(\xi,s) = \frac{\alpha J_1(\alpha\xi)}{\xi}, \tag{47}$$

## 6. Inversion of the transforms

To find the solution of the problem in the physical domain, we must invert the transforms in Eqs. (38)-(43). These equations are functions of $\xi$ and z, hence are of the form $\tilde{f}(\xi,z,s)$. To get the function $f(r,z,t)$in the physical domain, first, we invert the Hankel transform using

$$f^*(r,z,s) = \int_0^\infty \xi\tilde{f}(\xi,z,s)J_n(\xi r)d\xi \tag{48}$$

Following Honig and Hirdes (1984), the Laplace transform function $\tilde{f}(x,z,s)$ can be inverted to f(x, z, t). The last step is to calculate the integral in Eq. (48). The method for evaluating this integral by using Romberg's integration with adaptive step size is described in Press *et al.* (1986).

## 7. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of rotation for the first problem and the effect of frequency of time harmonic sources for the second problem, we now present some numerical results. Cobalt material is chosen for the purpose of numerical calculation, which is transversely isotropic. The physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal *et al.* (1980)is given by

$$c_{11} = 3.07 \times 10^{11} Nm^{-2},$$

$$c_{12} = 1.650 \times 10^{11} Nm^{-2},$$

$$c_{13} = 1.027 \times 10^{10} Nm^{-2},$$

$$c_{33} = 3.581 \times 10^{11} Nm^{-2}$$

$$c_{44} = 1.510 \times 10^{11} Nm^{-2},$$

$$C_E = 4.27 \times 10^2 JKg^{-1}deg^{-1},$$

$$\beta_1 = 7.04 \times 10^6 Nm^{-2}deg^{-1}, \rho = 8.836 \times 10^3 Kgm^{-3}$$

$$\beta_3 = 6.90 \times 10^6 Nm^{-2}deg^{-1},$$

$$K_1 = 0.690 \times 10^2 Wm^{-1}Kdeg^{-1},\ K_3 = 0.690 \times 10^2 Wm^{-1}K^{-1},$$

$$K_1^* = 0.02 \times 10^2 NSec^{-2}deg^{-1},$$

$$K_3^* = 0.04 \times 10^2 NSec^{-2}deg^{-1}.$$

$$L = 1, b = 0.01m$$

The values of radial displacement *u,* axial displacement w, shear stress $t_{zr}$, radial stress $t_{rr}$ and conductive temperature $\varphi$ for a TIT solid with two temperature is illustrated graphically to show the effect of rotation.

- The solid line with centre symbol square corresponds to rotation $\Omega = 0.0$.
- The dash line with centre symbol circle corresponds to rotation $\Omega = 0.25$,
- The dotted line with centre symbol triangle corresponds to rotation $\Omega = 0.5$,

The dash dotted line with centre symbol diamond corresponds rotation $\Omega = 0.75$.

Fig. 2 shows the variations of radial displacement u with radius r. In the initial range of radius r, there is a sharp decrease in the value of displacement component for all the cases. Moreover, away from source applied, it follows oscillatory behavior. We can see that the rotation have a significant effect on the displacement component in all the cases as there are more variations in u in case of rotation as compared to when rotation is zero.

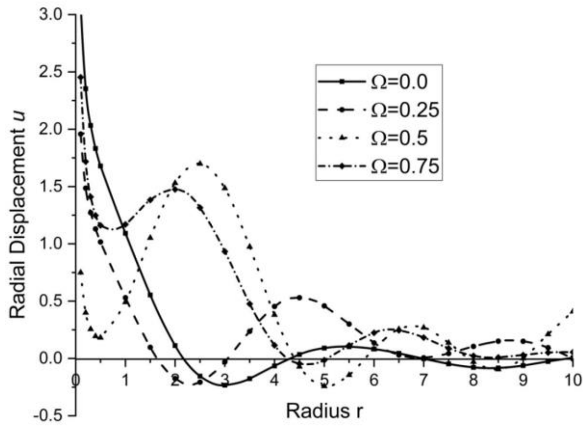


Fig. 2 variations of radial displacement u with radius r

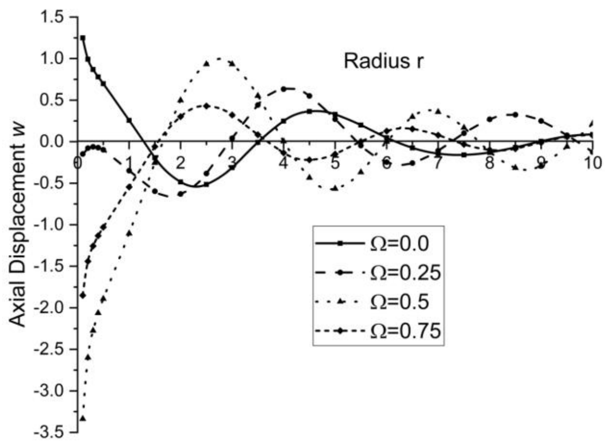


Fig. 3 variations of displacement component *w* with radius r

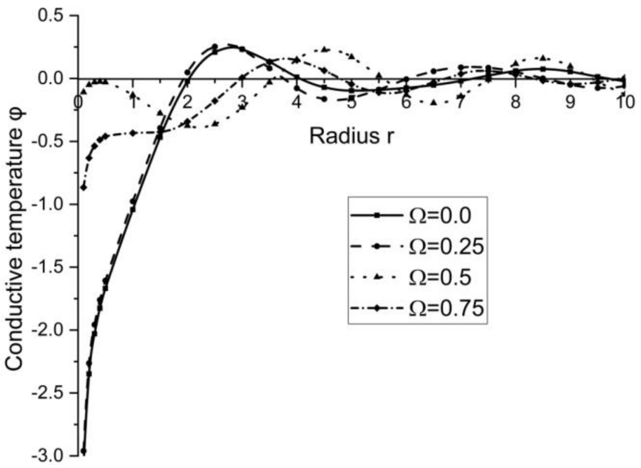


Fig. 4 variations of Conductive temperature $\varphi$ with radius r

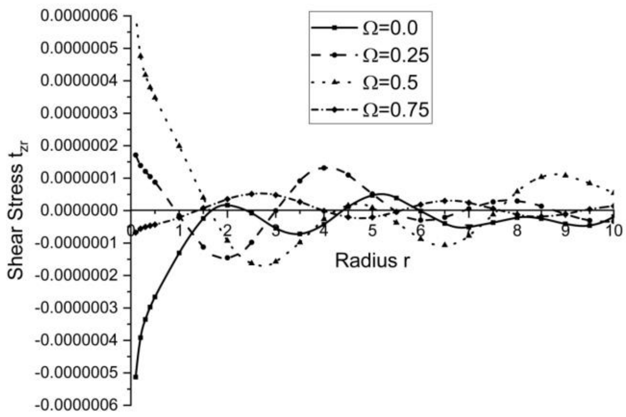


Fig. 5 variations of shear stress $t_{zr}$ with radius r

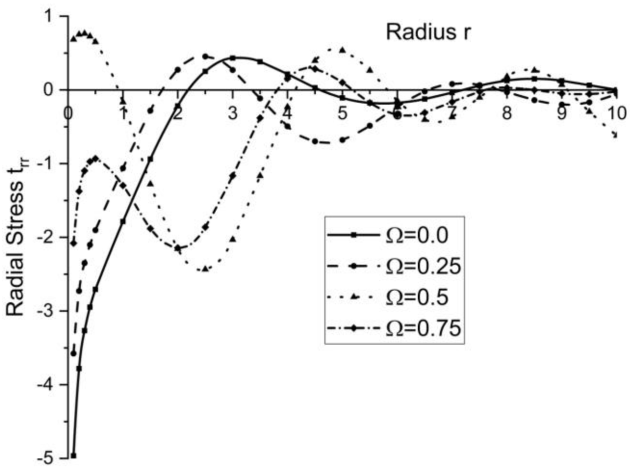


Fig. 6 variations of radial stress with radius r

Fig. 3 illustrates the variations of displacement component $w$ with radius r. In the initial range of radius r, there is a decrease in the value of displacement component for $\Omega = 0.0$ and then follows oscillatory behaviour. We can see that the rotation have a major effect on the displacement component $w$ as with increase in value of rotation $\Omega$, the amplitude of displacement increases which reduces with increase in radius r.

Fig. 4 illustrates the variations of conductive temperature $\varphi$ with radius r. In the initial range of radius r, there is a sharp increase in the value of $\varphi$ for all the cases. Moreover, away from source applied, it follows opposite oscillatory behavior nearby the zero value.

Fig. 5 illustrates the variations of shear stress $t_{zr}$ with radius r. In the initial range of radius r, there is a small oscillation in the value of stress component $t_{zr}$ for all the cases. Moreover, away from source applied, it follows opposite oscillatory behavior nearby the zero value with increase in rotation. Fig. 6 illustrates the variations of radial stress with radius r. In the initial range of radius r, there is a large oscillation in the value of radial stress for all the cases.

## 8. Conclusions

In this paper, we have discussed the thermoelastic problem for a transversely isotropic thin circular plate with rotation, two temperature and with multi-dual-phase lag heat transfer. The finite Hankel transform technique is used to obtain numerical results.

In the present research article, conductive temperature, displacement, and stresses along with rotation, two temperature, have been outlined. Since the thickness of plate is very small, the series solution given here will be definitely convergent. The temperature, displacement and thermal stresses that are obtained can be applied to the design of pressure sensors, microphones, gas flow meters, optical telescopes, radar antennae and many other devices structures or machines in engineering applications.

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## Nomenclature

| | |
|---|---|
| $\delta_{ij}$ | Kronecker delta, |
| $C_{ijkl}$ | Elastic parameters, |
| $\beta_{ij}$ | Thermal elastic coupling tensor, |
| $T$ | Absolute temperature, |
| $T_0$ | Reference temperature, |
| $\varphi$ | conductive temperature, |
| $t_{ij}$ | Stress tensors, |
| $e_{ij}$ | Strain tensors, |
| $u_i$ | Components of displacement, |
| $\rho$ | Medium density, |
| $C_E$ | Specific heat, |
| $a_{ij}$ | Two temperature parameters, |
| $\alpha_{ij}$ | Linear thermal expansion coefficient, |
| $K_{ij}$ | Materialistic constant, |
| $K_{ij}^*$ | Thermal conductivity, |
| $\omega$ | Frequency |
| $\tau_0$ | Relaxation Time |
| $\mathbf{\Omega}$ | Angular Velocity of the Solid |
| $\vec{j}$ | Current Density Vector |
| $\vec{u}$ | Displacement Vector |
| $\delta(t)$ | Dirac's delta function |