

Optimization of cables size and prestressing force for a single pylon cable-stayed bridge with Jaya algorithm

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Abstract. In recent years, due to the many advantages cable-stayed bridges have often constructed in medium and long span. These advantages can be listed as an aesthetically pleasing appearance, economic and easy construction, etc. The main structural elements of cable-stayed bridges are listed as deck, pylon, cables and foundation. Perhaps one of the most vital and expensive of these structural elements is stay-cables. Stay-cables ensure the allowable displacement and distribution of bending moments along the bridge deck with prestressing force. Therefore the optimum design of the stay-cables and prestressing force are very important in achieving the performance expected from the cable-stayed bridges. This paper aims to obtain the stay-cables size and prestressing force optimization of the cable-stayed bridge. For this purpose, single pylon and fan type cable configuration Manavgat Cable-Stayed Bridge was selected as an example. The three dimensional (3D) finite element model (FEM) of the bridge was created with SAP2000. Analysis of the 3D FEM of the bridge was conducted under the different combined effects of the self-weight of the structural element, prestressing force of stay-cable and live load. Stay-cable stress and deck displacement were taken into account as constraints for the optimization problem. To optimize this existing bridge a metaheuristic algorithm named Jaya was used in the optimization process. 3D FEM of the selected bridge was repeatedly analyzed by using Open Applicable Programming Interface (OAPI) properties of SAP2000. To carry out the optimization process the developed program which integrates the Jaya algorithm and the required codes for calling SAP2000 is coded in MATLAB. At the end of the study, the total weight of the stay-cables was reduced more than 40% according to existing stay cables under loads taken into account.

Keywords: SAP2000-OAPI; size optimization; prestressing force; cable-stayed bridge; Jaya algorithm

1. Introduction

Mankind has built different structures for different needs apart from the construction of the structures used as a shelter for centuries. One of the most important needs of human beings is transportation. Because many needs of communities depend on transportation. To overcome obstacles such as rivers and deep valleys, human beings have discovered the bridge. Many bridges have been built since the first bridge and each bridge created knowledge for the next one. This situation has resulted in the generation of new bridge types. In these bridge types, cable-stayed bridges (CSBs) are the most preferred types for the passing of medium and long spans in recent years (Hewson 2003, Leonhardt 1987). Russky Bridge is the world's longest CSB with 1104 m main span. The main structural components of the CSBs are foundation, pylon, deck, and cables similar to suspension bridge. Unlike the suspended bridge, main cable does not exist and deck is directly connected to the pylon with cable. One of the main tasks of stay-cable is to support the bridge deck by distributing the dead and live load of effects through bridge pylon. Also, the deflection of the bridge deck is adjusted with stay-cables and applied

prestressing force to them. Under the dead load of structural and non-structural elements of the bridge the vertical deflection of the deck must be well-nigh to zero (Hassan *et al.* 2013). To obtain prestressing force applied the stay-cables that will give zero vertical deck displacement by using the trial-and-error procedure is time-consuming, expensive and unable to find optimum solution generally. For this reason determination of the initial prestressing force of stay-cable is considered one of the common difficulties at the design stage of CSBs (Freire *et al.* 2006, Xiao *et al.* 2001, Lee *et al.* 2008). Moreover to find the minimum cross-sectional area of stay-cable and calculate the correct prestressing force to meet design requirements together may be near impossible. In this study, it is aims to overcome these difficulties at the design stage of CSBs with the help of a developed program which integrates the Jaya algorithm, Open Applicable Programming Interface (OAPI) properties of SAP2000 and MATLAB. For example, Artar (2016a) and Artar (2016b) used this property of SAP2000 for the optimization of frames and truss structures, respectively. Venkata Rao (2015) showed the performance of Jaya and compared this algorithm with Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Differential Evolution (DE), Artificial Bee Colony (ABC) and TLBO algorithms. He mentioned that the Jaya algorithm is a newly proposed algorithm and to have strong

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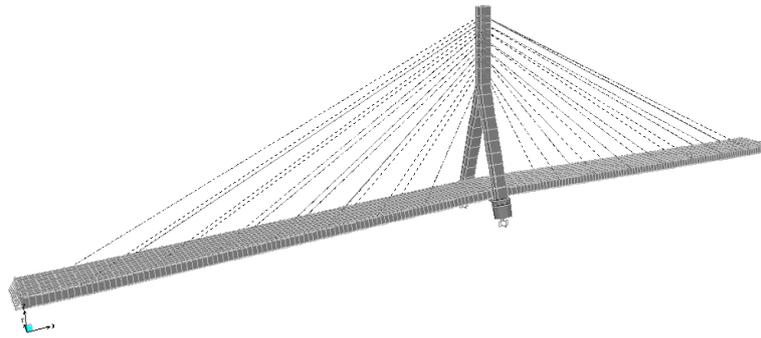


Fig. 2 FEM of selected bridge

formed by placing 25 cm thick concrete and 10 cm thick asphalt on I steel profiles used in longitudinal and transverse directions. Longitudinal steel I profiles are supported the deck at the outer edge. The pylon is located in the middle of the longitudinal direction in this manner bridge has symmetrically two 101 m lateral spans. The bridge has a total of 28 steel stay cables arranging modified fan type. 14 of them are supported to the deck at right side of the pylon and the others are supported deck at left side of the pylon (Atmaca and Ateş 2012). The distance between the stay-cables is 12 m. Stay-cables are formed using some tendons together. The number of tendon used to generate the stay-cables, diameters of the strands, total area of the stay-cables and initial strain value of existing bridge are shown in Table 1. Structural material properties used in stay-cable, deck and pylon of the bridge are shown Table 2.

3. Finite element model of the bridge

3D FEM was created with SAP2000 (2016) in order to obtain the structural behavior of selected bridge (Fig. 2). The bridge model consist of 28 truss elements (for stay-cables), 1102 beam elements (for the steel I profile of deck and pylon) and 1980 shell elements (for concrete part of deck).

Table 1 Properties of stay-cable

Stay cable number	Number of strand	Diameter of strand (mm)	Cross-sectional area of stay cable (mm ²)	Strain of stay-cable
A ₁ -B ₁	15	15.2	2100	-0.0034
A ₂ -B ₂	16	15.2	2240	-0.0010
A ₃ -B ₃	19	15.2	2660	-0.0039
A ₄ -B ₄	19	15.2	2660	-0.0010
A ₅ -B ₅	22	15.2	3080	-0.0034
A ₆ -B ₆	19	15.2	2660	-0.0021
A ₇ -B ₇	24	15.2	3360	-0.0030

Table 2 Material properties of the bridge

Material Properties			Unit
Strand	Type	Grade 270	-
	Ultimate strength	1862	MPa
	Weight per unit volume	78.5	kN/m ³
	Modulus of Elasticity	197000	MPa
	Poisson's Ratio	0.3	-
Structural steel	Type	S355	-
	Weight per unit volume	78.5	kN/m ³
	Modulus of Elasticity	200000	MPa
	Poisson's Ratio	0.3	-
Deck concrete	Type	C40/50	-
	Modulus of Elasticity	34000	MPa
	Weight per unit volume	25	kN/m ³
	Poisson's Ratio	0.2	-

For the purpose of prevent the transmission of moment between stay-cables connection at pylon and deck beam, the start and end point rotations of truss elements were released. As a boundary condition, both of rotations and translations are fixed at the pylon base, the left and right hand supports are selected as pinned and roller, respectively. In this study, static analysis performed under two different combined effect of self-weight of structural and non-structural element, prestressing force of stay-cable and live load. Combination 1 (Combo-1) consist of dead and prestressing force. Combination 2 (Combo-2) consist of dead, prestressing force and live load. Dead loads of structural and non-structural element of the bridge calculated with SAP2000 automatically. The magnitude of live load live acting on the road surface along the span length was taken into account as 4 kN/m² (Baldomir *et al.* 2010). Materials of structural elements of the bridge were assumed to behave linear elastic (Baldomir *et al.* 2010).

4. Size optimization of bridge with Jaya

The design variables for the optimization problem can be continuous or discrete. In the case of the continuous optimization, the number of potential solutions is unlimited.

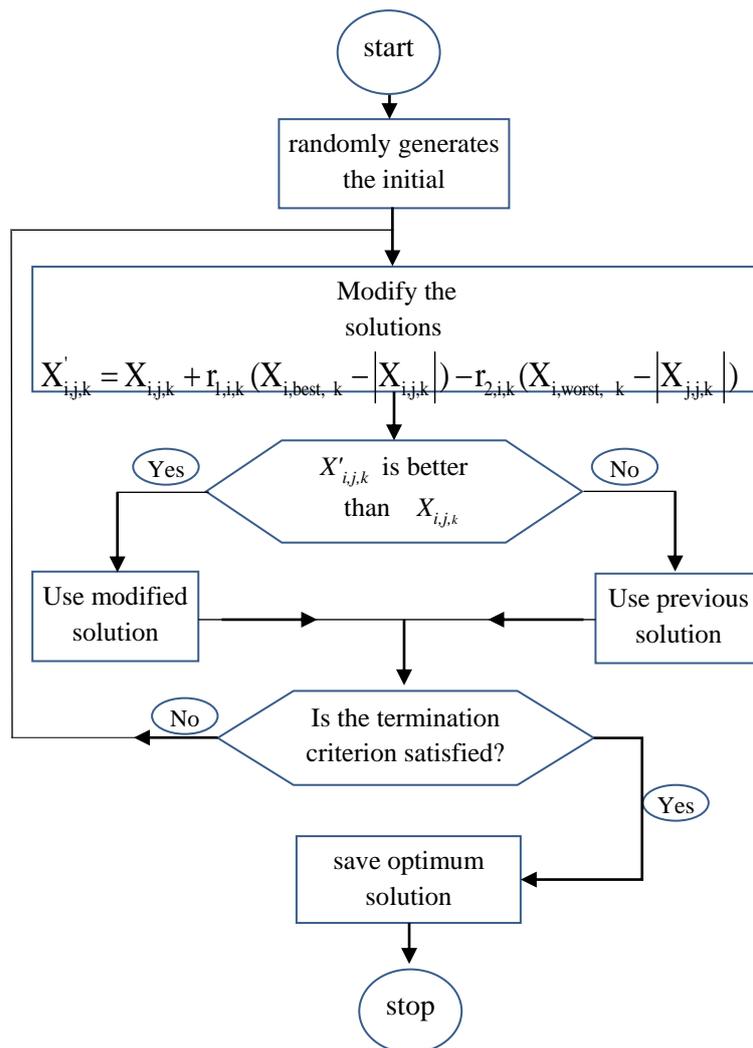


Fig. 3 Flow chart for Jaya algorithm

Thus, to find the global optimal solution will take so much computational times, many transaction volume and computer memory. Also, the design results must be made with special production or the designer should be used the nearest solution among the discrete solution. In the real life, the structural elements are the allowable-section such as the steel profile list. In the case of the discrete optimization the design result is a vector which including the members from the available list. So, the volume of the solution space is smaller when it compared the continuous case.

There are three main type of optimization for the structures. These are size, shape and the topology optimization. For the size optimization, the design variables are generally the cross-sectional areas of the structural members. As an example, the designer finds the optimal cross-sectional areas for the reinforced concrete beam or column. In the case of the shape optimization, the nodal connections of the structural members are changeable.

In this study design variables for optimization of cable-stayed bridge, the cross-sectional area of stay-cables which consist of a number of strand and prestressing force were

taken as discrete and continuous, respectively. The objective function of this study is to minimize the total weight of stay cables of the bridge. For this aim, the general objective function is written as given in the Eq. (1).

$$W_{\min} = \sum_{i=1}^{nm} \rho(L.A)_i \tag{1}$$

Where, ρ is the density of the strand, L is the length of the stay cable, A is the cross-sectional area of the stay cable, nm is total number of stay-cable and the W is the total weight of the cables. Structural constraints should not be violated while reducing the total weight of the stay-cable. In this study, stress of stay-cables and displacement of deck were taken into account as constraints.

$$g_i(x) = \frac{\delta_i}{\delta_u} - 1 \leq 0 \quad i = 1, \dots, n \tag{2}$$

$$g_j(x) = \frac{\sigma_i}{\sigma_u} - 1 \leq 0 \quad j = 1, \dots, m \tag{3}$$

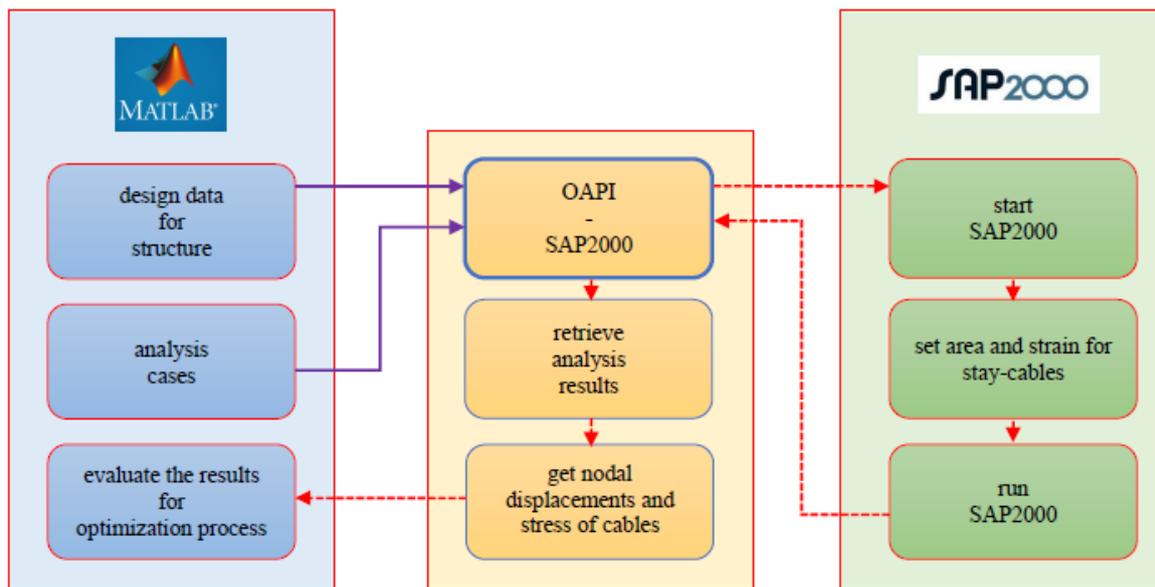


Fig. 4 The cooperation between SAP2000 and MATLAB via OAPI-SAP2000

$$\begin{aligned} \text{if } g(x) > 0 \quad c &= g(x) \\ \text{if } g(x) \leq 0 \quad c_m &= 0 \end{aligned} \quad (4)$$

Where, n is the number of nodal point on the deck, δ_i is calculated displacement, and δ_u is allowable displacement. m is the number of stay-cable, σ_i is calculated stress, and σ_u is allowable stress.

The specified displacement and stress constraints are determined according to the initial FEM analysis of the selected bridge. The summation of constraints (“ C ”) in the optimization problem was taken into account by following equation.

$$C = \sum_{k=1}^{n+m} c_k \quad (5)$$

In the calculation of constraints, there are two different Load Cases as Combo-1 (dead and prestressing force) and Combo 2 (dead, prestressing force and live load). For the case of Combo-1, the maximum vertical displacement at the point connecting stay-cables to the deck is 5 mm while this constraint was taken $L/550$ (L =lateral-span length) for all non-supporting point of deck for Combo-2. The allowable stay-cable stress was taken 1395 MPa ($0.75\sigma_u$) for both load combinations (AASHTO, 2012).

The objective function is written in terms of the constraint to take into the constraints. Thus, the penalized objective function φ is given in Eq. (5).

$$\varphi = W(x) \cdot [1 + P \cdot C] \quad (6)$$

Where, the P is a constant value which is determined according to the problem. At the end of the optimization process, the penalized objective function must be equal to the objective function.

The Jaya optimization algorithm used in this study was originally presented by Rao (2016). This metaheuristic algorithm is used by many researchers in the optimization of the civil engineering structures (Dede 2018, Ho-Huu *et al.* 2018, Grzywiński *et al.* 2019). As an optimization algorithm, Jaya can be used in constrained or unconstrained problems. Like the other population based algorithm this algorithm uses a randomly created initial population. The general steps used for this algorithm are given in the Fig. 3.

The main principle of the Jaya is to keep the best individual which is a potential solution including best design variables for the optimization problem. For this aim, the selection of the best individual is based on the avoiding the worst solution and becoming closer the best individual in the current population. To learn more detailed knowledge about the Jaya algorithm the paper presented by Rao (2016) can be investigated. In this figure, “ X ” represent any potential solution which is called an individual in the current population. “ X_{best} ” and “ X_{worst} ” are the best and the worst individual in the population. To measure the quality of the individuals the objective function is used.

6. Numerical example

In this section single pylon and two lateral spans cable-stayed bridge was selected as an example. The material and geometrical properties of the bridge were mentioned in the part of the description of the bridge. 3D FEM was created with using SAP2000 and linear static analysis performed under Combo-1 and Combo-2 as mentioned in section 3. For the optimization process the number of generation and population size were taken as 50 and 30, respectively. There are 14 design variables consist of cross-sectional area and strain of stay-cables. Due to the longitudinal and transverse

symmetry of cable configuration 28 stay-cables categorized as 7 groups. Thus the numbers of cross-sectional area of stay-cables were equal to seven. The other seven design variables were strain value for prestressing force. The design variables for cross-sectional area of stay-cables were discrete and the increment was taken 140 mm² because one of the 0.6 inch tendon cross-section area is 140 mm². The lower and upper bounds for discrete design variables were 5*140 mm² and 25*140 mm². The design variables for strain of stay-cables were continuous and the lower and upper bounds were taken into account -0.0001 and -0.007 according to the material properties of Grade 270 tendon, respectively. Two types of design constraints were taken into account: vertical displacement of the deck and stress of the stay-cables. For the case of Combo-1, the maximum vertical displacement at the point connecting stay-cables to the deck is 5 mm while this constraint was taken L/550 (L=lateral-span length) for all non-supporting point of deck for Combo-2. The allowable stay-cable stress was taken 1395 MPa ($0.75\sigma_u$) for both load combinations (AASHTO, 2012).

6.1 Numerical results

In this part firstly FEM results of the initial case of the bridge are summarized. The vertical displacement of the deck and horizontal displacement of pylon obtained from under the effect of the dead load of structural and non-structural elements of the bridge combined with prestressing load (Combo-1) are given Figs. 5 and 7, respectively. It can be noticed that under the effect of Combo-1 the maximum vertical deflection of the deck and the maximum horizontal deflections of the pylons are very close the zero. The tension stresses obtained from under the Combo-1 are given in Fig. 9. It is seen all stay-cable stresses under the allowable cable stress (1395 MPa). The maximum tension stress was occurred on A₃ cable about 749.52 MPa.

Combo-2 was created by adding the live load (4 kN/m²) to the loads of Combo-1. The vertical displacement of the deck and horizontal displacement pylon obtained from under the effects of Combo-2 are given Figs. 6 and 8,

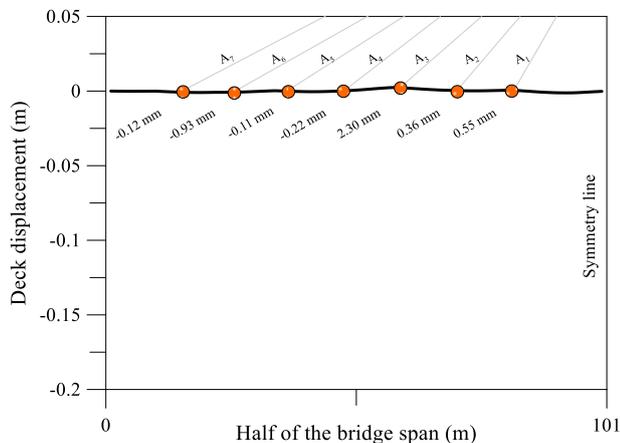


Fig. 5 Deck displacement under Combo-1 for initial bridge case

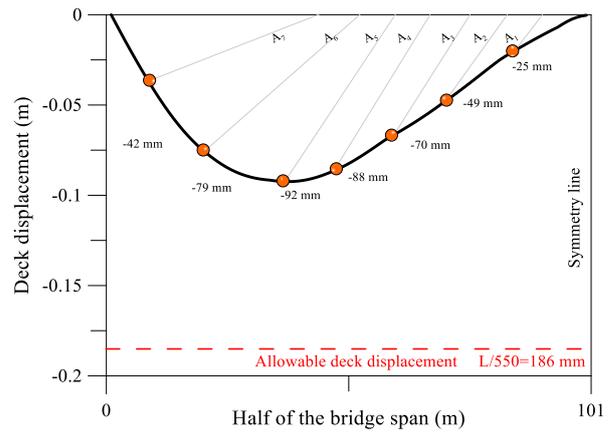


Fig. 6 Deck displacement under Combo-2 for initial bridge case

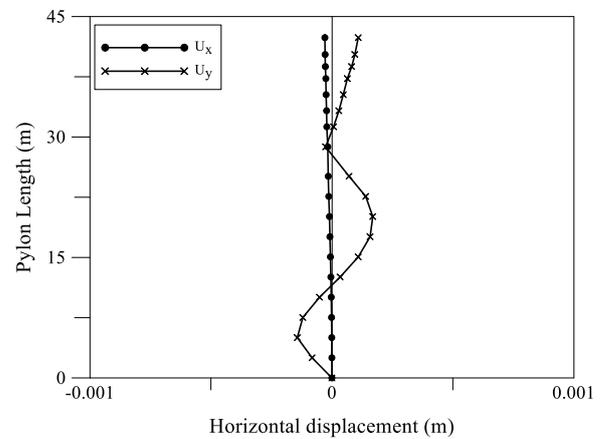


Fig. 7 Horizontal displacement of pylon under Combo-1 for initial bridge case

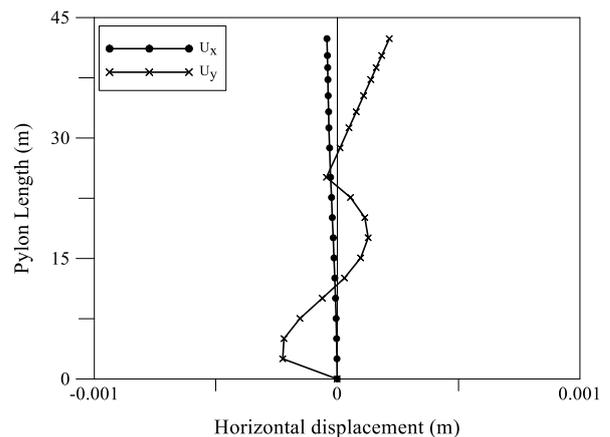


Fig. 8 Horizontal displacement of pylon under Combo-2 for initial bridge case

respectively. It is seen from these figures displacements are under the allowable limits. The tension stresses obtained from under the Combo-2 are given in Fig. 10. It is seen all stay-cable stresses under the allowable cable stress (1395 MPa). The maximum tension stress was occurred on A₃ cable about 901.12 MPa.

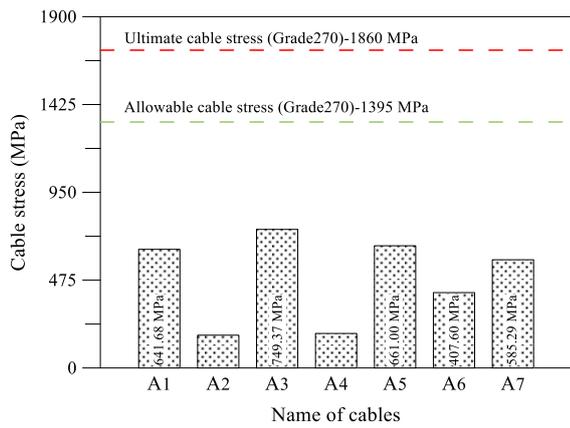


Fig. 9 Stay-cable stress under Combo-1 for initial bridge case

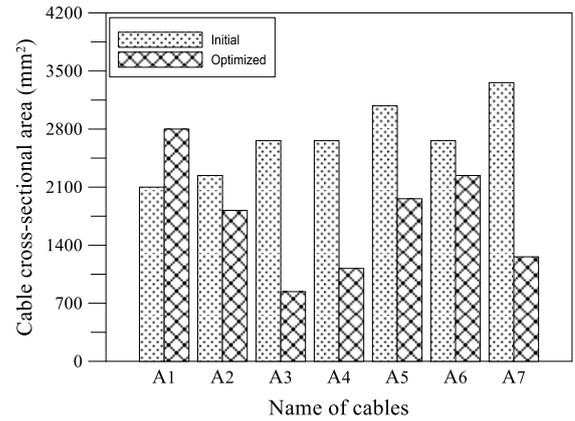


Fig. 12 Comparison of the initial and the optimal area of stay-cable

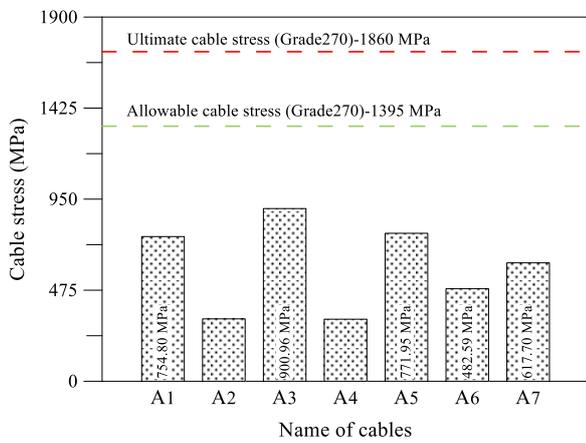


Fig. 10 Stay-cable stress under Combo-2 for initial bridge case

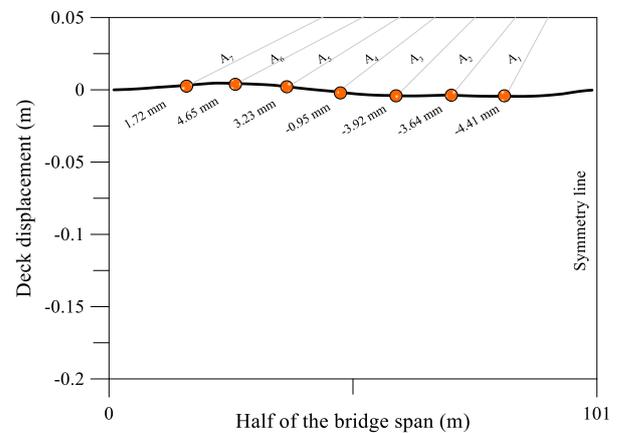


Fig. 13 Deck displacement under Combo-1 for optimized bridge stay-cable

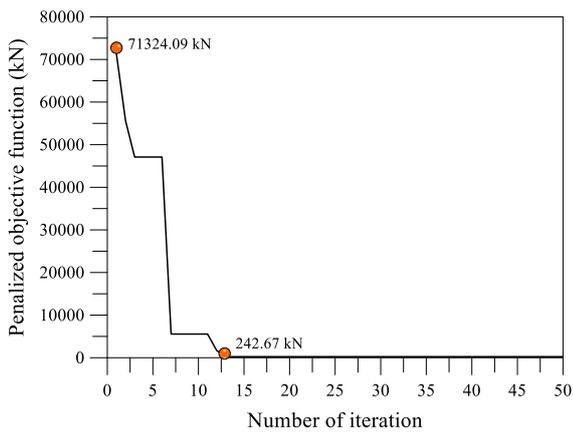


Fig. 11 Penalized objective function of the best solution

Penalized objective function of the best solution is given in Fig. 11. The process of optimization converged after 13 generation and the design variables were determined. A comparison between the cross-sectional area and strain of stay-cable of initial and optimized case of the bridge is presented in Fig. 12 and Table 3 together. According to the

Table 3 Comparison of design variables of cables

Cable no	Design variables (mm ²)		Strain of stay-cable	
	Initial sizes	Optimized sizes	Initial sizes	Optimized sizes
1	2100	2800	-0.0034	-0.00140
2	2240	1820	-0.0010	-0.00380
3	2660	840	-0.0039	-0.00490
4	2660	1120	-0.0010	-0.00570
5	3080	1960	-0.0034	-0.00400
6	2660	2240	-0.0021	-0.00470
7	3360	1260	-0.0030	-0.00460
Total weight (kN)	409.32	242.67		

results given in this table, the weights of stay-cables of initial case of the bridge were the 409.32 kN and after the optimization it obtained 242.67 kN. It is noticed that the weights of stay-cables of initial case of the bridge is reduced 40.7%.

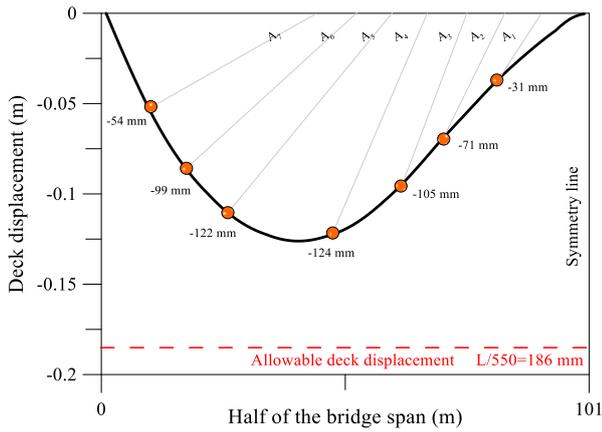


Fig. 14 Deck displacement under Combo-2 for optimized bridge stay-cable

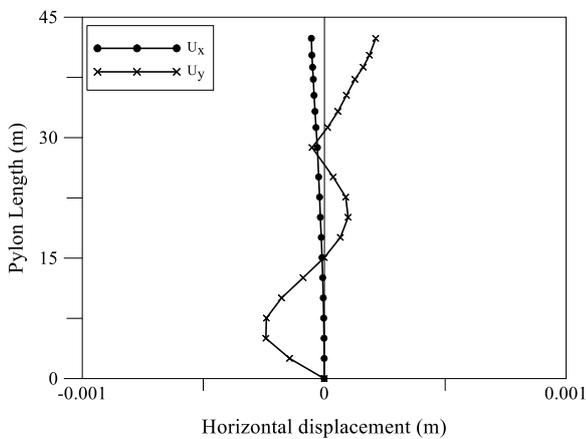


Fig. 15 Horizontal displacement of pylon under Combo-1 for optimized bridge

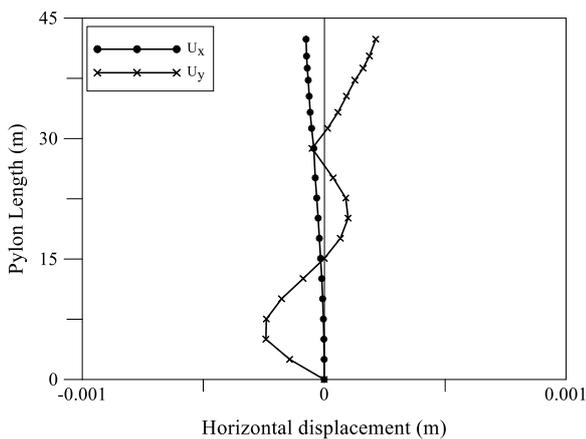


Fig. 16 Horizontal displacement of pylon under Combo-2 for optimized bridge

After the optimization, the vertical displacement of the deck and horizontal displacement of pylon for optimum solution occur due to Combo-1 are given in Figs. 13 and 15, respectively. It can be noticed that under the effect of Combo-1 the maximum vertical deflection of the deck and the maximum horizontal deflections of the pylons are very close the zero similar to the initial case of the bridge.

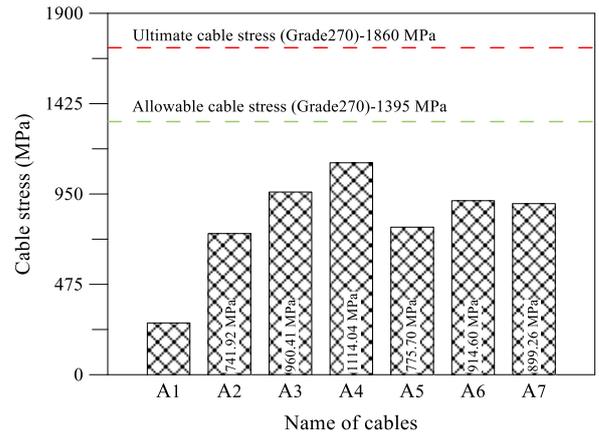


Fig. 17 Stay-cable stress under Combo-1 for optimized bridge

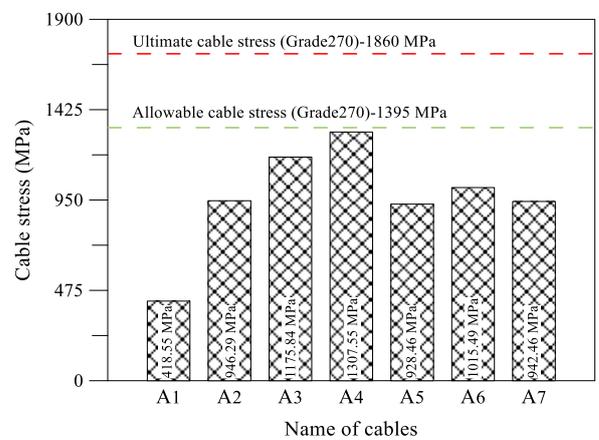


Fig. 18 Stay-cable stress under Combo-2 for optimized bridge

According to the optimum solution tensile stresses occurred under the effect of Combo-1 is given in Fig. 17. As seen in this figure tensile stresses in each stay-cable are not violates the stress constraints of stay-cables.

The vertical displacement of the deck and horizontal displacement of pylon for optimum solution occur due to Combo-2 are given in Fig. 14 and Fig. 16, respectively. It is seen from these figures displacements are under the allowable limits. It can be observed that the maximum deck displacement of initial and optimized case of the bridge under Combo-2 was occurred close to the connection zone of A₄, A₅ and A₆ stay-cables to the deck on account of this the cross-sectional area was determined larger by optimization process. According to the optimum solution tensile stresses occurred under the effect of Combo-2 is given in Fig. 18. As seen in this figure tensile stresses in each stay-cable are not violates the stress constraints.

7. Conclusions

Stay-cables ensure the allowable displacement and distribution of bending moments along the bridge deck with prestressing force and compared to the other structural

components the stay-cable is very expensive. Therefore the optimum design of the stay-cables and prestressing force are very important in achieving the performance expected from the cable-stayed bridges. This paper aims to obtain the stay-cables size and prestressing force optimization of cable-stayed bridge. For this purpose Manavgat Cable-Stayed Bridge was selected as an example. 3D FEM of the bridge was created with SAP2000. Analysis of the 3D FEM of the bridge was conducted under the different combined effect of self-weight of structural and non-structural element, prestressing force of stay-cable and live load. Stay-cable stress and deck were taken into account as constraints. To optimize this existing bridge a metaheuristic algorithm named Jaya was used in the optimization process. 3D FEM of the selected bridge was repeatedly analyzed by using OAPI properties of SAP2000. To carry out the optimization process the developed program which integrates the Jaya algorithm and the required codes for calling SAP2000 is coded in MATLAB. According to the obtained result from this study conclusions can be drawn as:

- To obtain strain value for prestressing force applied the stay-cables that will give zero vertical deck displacement by using trial-and-error procedure is time-consuming, expensive and unable to find optimum solution generally. To determine these strain value with the optimization technique is not difficult as trial and error procedure under certain assumptions.

- The maximum deck displacement of initial and optimized case of the single tower cable-stayed bridge under all load effects was occurred close between end support and center of the mid span therefore cross-sectional area of the stay-cables located this zone have larger area to meet design constraints.

- The total weight of stay-cables of initial case of the bridge is 409.32 kN and after the optimization it is obtained 242.67 kN. It is noticed that the optimization process provide to 40% reduction total weight stay-cables of initial case of the bridge.

- It is suggested that lateral load such as wind and earthquake motions will be taken into account in the design combination in new studies.

- It is possible to infer that applying an optimization procedure reduce the weight of stay-cables significantly and determine to proper strain for prestressing force applied the stay-cables of the bridges. It is also concluded that the Jaya algorithm is one of the other method can be affectively used in the optimization of cable-stayed bridges.

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