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Mixed mode I/II fracture criterion to anticipate behavior of the orthotropic materials

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Abstract. The new energy-based criterion, named Reinforcement Strain Energy Density (ReiSED), is proposed to investigate the fracture behavior of the cracked orthotropic materials in which the crack is embedded in the matrix along the fibers. ReiSED is an extension of the well-known minimum strain energy density criterion. The concept of the reinforced isotropic solid as an advantageous model is the basis of the proposed mixed-mode I/II criterion. This model introduces fibers as reinforcements of the isotropic matrix in orthotropic materials. The effects of fibers are qualified by defining reinforcement coefficients at tension and shear modes. These coefficients, called Reduced Stress (ReSt), provide the possibility of encompassing the fiber fraction in a fracture criterion for the first time. Comparing ReiSED fracture limit curve with experimental data proves the high efficiency of this criterion to predict the fracture behavior of orthotropic materials.

Keywords: minimum strain energy density criterion; reinforcement coefficients; reinforced matrix; fracture limit curve

1. Introduction

Composite materials are being used widely in engineering fields such as civil, aerospace, and marine industries due to their low weight and high strength as well as high flexibility at ascertaining physical and mechanical properties in required and favorable orientations (Altunisik *et al.* 2017). Studying the fracture of composite materials plays a significant role in the recent studies of fracture mechanics (Li *et al.* 2015). Some research has been conducted for improving fracture properties of composites, especially those that are employed as construction materials (Golewski 2017a, b, c). Also, based on the fracture behavior of composite materials, both macroscopic and microscopic approaches are employed to assess the fracture toughness (Golewski *et al.* 2016a, b, Golewski 2019) and performance (Golewski 2018) of composite structures. Other research has been conducted for ascertaining the fracture properties of bi-material bonded joints (Wang *et al.* 2018, Arouche *et al.* 2019), or for designating the failure mode of composite bolted joints via numerical and experimental investigation for various geometries (Shan *et al.* 2018, Zhou *et al.* 2019). Also, the empirical study and analysis of fracture behavior of asymmetric composite joints were conducted to establish a failure criterion for the particular examined joints (Shahverdi *et al.* 2016). Also, scholars strived to determine the effect of composite layup on the failure of the bonded joints owing to their noticeable application in the industry (Kupski *et al.* 2019, Baek *et al.* 2019). The growing use of composite materials in sensitive industries requires a momentous evaluation of the behavior of these materials under various loadings (Akbas 2019). As a result, for reliable and precise designing, it is necessary to benefit accurate criterion in order to anticipate the onset of fracture or failure of these materials as there are appropriate criteria to determine the fracture of isotropic ones (Toribio and Ayaso 2003). Following available criteria for anisotropic materials, fracture study of these materials can be classified in two empirical and theoretical methods (Fakoor *et al.* 2019). Due to the fracture complexity of anisotropic materials under mixed-mode I/II loadings, preliminary criteria were based on curve fitting of experimental data consisting of two or three experimental constants. Wu (1967) is known as the pioneer of this matter since he introduced the criterion via conducting tests on Balsa wood and Scotch ply. Although researchers like Leicester (1974), Williams and Birch (1976) declared that mode II is ineffective at mixed mode loadings, experimental surveys of Mall and Murphy (1983) proved the definitive interaction between K_I and K_{II} at the fracture onset and proposed two criteria. In addition, Chow and Woo (1979) declared the dependency of mode I and II stress intensity factors at mixed mode loadings. However, due to the lack of information about pure mode II toughness they failed to present a criterion. Since there are plenty of complexities in fracture of orthotropic materials, specifically in mixed mode I/II (Quade *et al.* 2019), the empirical criteria have high accuracy. The only downside of these criteria is that they cannot be applied to the general state of composite materials, because these criteria are practical for those with distinct degrees of anisotropy used in relevant tests. Therefore, the results just were applicable for materials with that specific degree of anisotropy. Moreover, to investigate the fracture behavior, the experimental tests are costly especially in mode II, which is an obstacle to utilize the

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empirical study of materials with different anisotropic degrees (Al-Fasih *et al.* 2018). Hence, the theoretical criteria were also studied simultaneously with the experimental survey of the fracture behavior of composite materials.

Primarily, to study orthotropic materials, Linear Elastic Fracture Mechanics (LEFM) theory was used so that the isotropic criteria such as Maximum Tangential Stress (MTS), Maximum Strain Energy Release Rate (SER) and Minimum Strain Energy Density (SED) were applied to orthotropic materials (Saouma *et al.* 1987, Carloni and Nobile 2005, Nobile and Carloni 2005). Employing stress state of orthotropic materials obtained by Sih *et al.* (1965), Jernkvist (2001a, b) applied energy-based isotropic criteria (SER and SED) to orthotropic materials. The size of his specimens allowed him to benefit LEFM. In addition, inspired by MTS, he presented Maximum Principle Stress (MPS) criterion for collinear distribution of a crack along the fibers. Fakoor and Rafiee (2013) utilized Jenkvist's assumptions and applied Maximum Shear Stress (MSS) to orthotropic materials. They replaced maximum normal stress by shear stress in the criterion of Buczek and Herakovich (1985) in wood.

The criteria as mentioned earlier are based on the LEFM theory in which the Fracture Process Zone (FPZ) at the crack tip is adequately small in comparison to the size of the body. Since orthotropic materials are quasi-brittle, creation of FPZ at the crack tip is inevitable (Vasic and Smith 2002, Muralidhara *et al*. 2010, Vasic *et al.* 2007). In consideration with dissipation energy as a consequence of micro-cracks formation in the vicinity of the main crack as well as creation and distribution of the fracture process zone, Anaraki and Fakoor (2010a) presented a criterion based on maximum strain energy release rate via regarding the effective elastic properties of the body containing micro-cracks around the main crack. One year later, they proposed the Strength-Based Criterion (SBC) in which micro-cracks in FPZ have the primary role as well (Anaraki and Fakoor 2011b). They determined resistance of plane that is the fracture toughness of aligned micro-cracks based on tension strength of micro-cracks and derived damage factor based on the strength of material along and across the fibers. Fakoor and Khansari (2016) measured properties of damage zone at the crack tip regarding the interference of micro-cracks in FPZ and propounded the Representative Circle Elements (RCL) criterion by micro-mechanical approach. Romanowicz and Seweryn (2008) employed a non-local stress approach for mixed-mode I/II fracture investigation of wood components. They utilized strain energy density for micro-cracks based on the derived formulation by Gambarotta and Lagomarsino (1993).

Delamination is another type of failure, which is considered in several studies. Kharazan *et al*. (2014) employed a finite element (FE) approach for the delamination modeling of laminated composite structures. Rizov (2017) studied mode II delamination of an end-loaded split functionally graded beam by theoretical approach and considering material non-linearity. The mechanical response of ELS was modeled analytically by using a power-law stress-strain relation. Fracture process zone (FPZ) was considered in their model by cohesive zone elements.

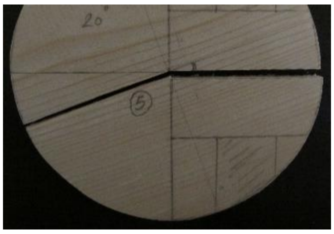
Van der Put (2007) proposed a new concept in fracture of highly orthotropic materials like wood. He derived his theory based on the failure of an elliptical crack in a 2D plane. He described that dissipated energy caused by the crack growth in orthotropic material has to rely on the strength of the isotropic matrix because orthotropic airy stress function cannot explain the fracture behavior of these kinds of materials. In addition, he derived Wu's fracture criterion to predict the wood failure based on the strength of matrix along and across the fibers without considering FPZ (Van der Put 2015). Considering Van der Put's theory, Anaraki and Fakoor (2010b) presented a mixed-mode fracture criterion based on Reinforcement Micro-Crack Systems (RMS) in which the resistance of the material in FPZ to micro-cracks' formation was the dominant parameter. Several experimental observations prove that crack propagation in orthotropic materials will be in the isotropic matrix, and the fiber effects can be modeled as reinforcement of isotropic matrix (Fakoor and Shokrollahi 2018, Farid and Fakoor 2019, Fakoor 2017). Based on this experimental observation and Van der Put's theory, they proposed energy-based mixed-mode fracture criteria for fracture investigation of composite materials (Fakoor and Farid 2019).

In this paper, the propounding concept of restricted isotropic solid, failure of highly orthotropic material is considered in the isotropic matrix. Accordingly, a new method is derived to demonstrate the effect of fibers on the matrix in orthotropic materials via reinforcement coefficients called ReSt. The principal and innovative characteristic of ReSt coefficients is encompassing fiber fractions in the relevant formula. Then the practical SED criterion is employed to anticipate the onset of in-plane crack growth. The crack is embedded along the fibers in the reinforced matrix under mixed-mode I/II loadings, and the plane strain conditions are applied. Also, crack growth direction is predicted by the SED criterion in the matrix that is the other novelty of this criterion in comparison to the aforementioned ones in which crack growth path is assumed along the fibers. The resulting criterion is validated by experimental data of wooden specimens gained by Jernkvist (2001a, b).

2. Isotropic solid model

The reinforced isotropic solid is based on empirical observation of the fracture of orthotropic materials. Fig. 1 shows that a crack with any arbitrary direction to fibers in orthotropic materials kinks along fibers and propagates in the isotropic matrix of composite materials.

In this model, the orthotropic materials are considered as reinforced materials in which the crack propagates through the isotropic solid reinforced by fibers (Farid and Fakoor 2019). The embedded fibers in the matrix of composite materials tolerate most of the subjected load, reinforcing the matrix. Furthermore, fibers are far stronger than the matrix itself; thus, a crack in a matrix is incapable of tearing them



(a) Crack orientation to fibers is 20

(b) Crack orientation to fibers is 40

(c) Crack orientation to fibers is 60

(d) Crack orientation to fibers is 80

Fig. 1 Empirical observation of crack propagation along fibers for different location of the crack to fibers

apart in case it faces the fibers. It is presumed that the fracture mechanism of these kinds of materials can be appropriately described through making relations between stress states of orthotropic materials and the isotropic matrix.

In this research, the relation between stress states of isotropic matrix and orthotropic lamina is defined by factors called “reinforcement factors”. The effect of reinforcement fibers is driven by a micromechanics approach (Fakoor and Farid 2019). The factors are obtained from comparing the load subjected to matrix and fibers at composite lamina, separately, which is called Reduced Stress (ReSt) method. The significant advantage of this method is the inclusion of the fiber fraction in the stress state, determining subjected load by fibers far more precisely.

2.1 Reduced Stress (ReSt) Micromechanical Model

This model has been introduced in our previous works (Farid and Fakoor 2019). For studying the effects of fibres on the matrix, the following Representative Volume Element (RVE) of the case study is chosen (Fig. 2).

The applied load is tolerated by fibers and also the matrix in composite materials; the following method facilitates the calculation of the exact amount of stress at the crack tip by three types of loadings, normal and shear stress, in-plane problems. Fig. 3 depicts the three possible independent in-plane loads subjected to the discrete RVE.

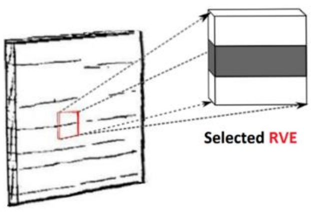
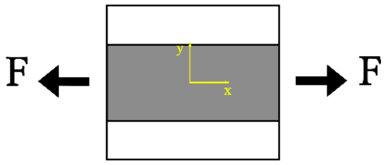
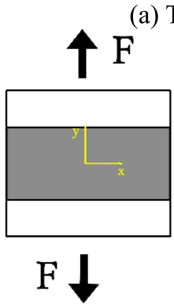


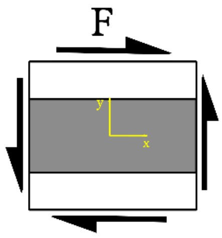
Fig. 2 Selected RVE in the present study



(a) Tension along fibers



(b) Tension across fibers



(c) Shear

Fig. 3 RVE under in-plane loadings

The first state elucidates that RVE is subjected to tension load along the fibers (see Fig. 3(a)). Satisfying the continuity conditions, the displacement of RVE along $\boldsymbol{x}$-direction should be consistent. Therefore, the matrix and fiber strains equal to the total RVE strain ($\varepsilon = \varepsilon_m = \varepsilon_f$). According to the constitutive equation and equality of strains, the stress at the matrix is related to the stress of RVE via $\frac{\sigma}{\sigma_m} = \frac{E_{xx}}{E_m}$. Then, the reinforcement factor along fibers,ξ_1, is defined as

$$\xi_1 = \frac{E_{xx}}{E_m} \tag{1}$$

The Eq. (1) illustrates the concept of reduced stress. In other words, the contributed stress of the matrix is reduced by ξ_1 factor provided that the composite body is subjected to a load along fibers.

The second case states (see Fig. 3(b)) the relation between the stress of the matrix and the composite body across fibers, along the $\boldsymbol{y}$-axis. Since the load on the matrix, fibers, and RVE are equal ($F = F_f = F_m$), the stress in the matrix, fibers, and RVE is the same across the fibers, along the $\boldsymbol{y}$ direction. Therefore

$$\xi_2 = 1 \tag{2}$$

$\xi_2 = 1$ expresses that fibers play no role in bearing the loads in this state since the stress of isotropic matrix equals the stress of the composite body.

The third state (Fig. 3c) presents the shear stress relation between the matrix and the composite. Based on the relations derived (Farid and Fakoor 2019), the dependency of the matrix shear stress to the shear stress of the composite body is defined via the following formula.

$$\tau = G_{12}\left(1 - V_f\right)\left(\frac{1}{G_m} + \frac{1}{G_f}\right)\tau_m \tag{3}$$

Eq. (3) displays the fiber fraction (V_f) plays a role in the amount of applied shear stress. As a result, the reinforced coefficient at shear loading, ξ_3, will be

$$\xi_3 = G_{12}\left(1 - V_f\right)\left(\frac{1}{G_m} + \frac{1}{G_f}\right) \tag{4}$$

Eq. (4) demonstrates the effect of fibers on shear stress subjected to the matrix. Assuming the $G_f = G_{12}$ and $G_m = G_{21}$ at orthotropic materials, ξ_3 factor is

$$\xi_3 = \left(1 - V_f\right)\left(1 + \frac{E_{11}(1 + \nu_{21})}{E_{22}(1 + \nu_{12})}\right) \tag{5}$$

As a result of these three states, in-plane stresses in orthotropic lamina is related to in-plane stresses of the isotropic matrix by ξ_i reinforcement factors in which

$$\sigma_{11}^{iso} = \frac{\sigma_{11}^{ortho}}{\xi_1}, \sigma_{22}^{iso} = \frac{\sigma_{22}^{ortho}}{\xi_2}, \sigma_{12}^{iso} = \frac{\sigma_{12}^{ortho}}{\xi_3} \tag{6}$$

Eq. (6) represents the relation between stress states of the composite materials as a whole body and the pertinent matrix as a part of it.

3. Theoretical background of problem

3.1 Hypotheses

In this paper, the behavior of a crack embedded along fibers in the orthotropic plane is studied (Fig. 4). The crack follows the plane strain conditions. It is also assumed that the crack is subjected to mixed-mode loadings. As Fig. 4 illustrates, unidirectional and straight fibers strengthen the lamina. It means the studied material is orthotropic, which has high strength and stiffness along fibers and low tensile strength across them or at the transverse direction. Accordingly, reinforced composites by unidirectional fibers are vulnerable to the crack embedded in the matrix between fibres. Therefore, these kinds of cracks tend to grow along the fibers. In this paper, it is assumed that the crack embedded in the matrix is along the fibers leading the stress state around the crack tip is intensely affected by singular stresses. Fig. 5 demonstrates the position of the crack along fibers in the matrix schematically.

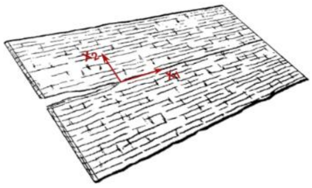


Fig. 4 The schematic figure of the studied crack in an orthotropic material

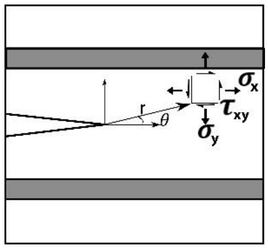


Fig. 5 The arbitrary stress state at the crack tip

3.2 Stress state in the vicinity of the crack tip

Experimental tests on wooden materials demonstrate that a crack along fibers in the matrix of orthotropic lamina starts to deflect and grow in a different direction. Fig. 6 displays this fact, and it is proof of the theory of Van der Put, who believed that the matrix failure causes the failure of the orthotropic solid. Therefore, referring to the concept of the constrained isotropic solid, the matrix constrained between fibers bears the less stress. It means the stress state at the crack tip will be the stress state at the isotropic matrix with reinforcement coefficients. Stress state at the crack tip in isotropic solid is the following formulas, which are obtained using Williams (1957) extends and considering the reinforcement factors. In these formulas, the functions, $f_{ij}(\theta)$ and $g_{ij}(\theta)$ are angular functions.

$$\begin{aligned}
\sigma_{11} &= \frac{(f_{11}(\theta)K_I + g_{11}(\theta)K_{II})}{\xi_1\sqrt{2\pi r}} \\
\sigma_{22} &= \frac{(f_{22}(\theta)K_I + g_{22}(\theta)K_{II})}{\xi_2\sqrt{2\pi r}} \\
\sigma_{12} &= \frac{(f_{12}(\theta)K_I + g_{12}(\theta)K_{II})}{\xi_3\sqrt{2\pi r}}
\end{aligned} \tag{7}$$

Fig. 6 The crack growth path in the matrix constrained between fibers

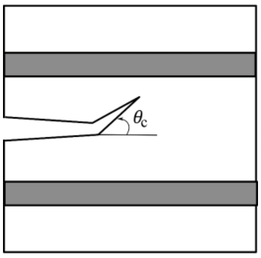


Fig. 7 The crack growth between fibers

3.3 Deriving the new fracture criterion

Considering the crack grows in the isotropic elastic homogenous continuum, the well-known and practical SED criterion is used to study the fracture behavior of this type of material under mixed-mode I/II loadings. Conforming to this criterion, if the strain energy density factor gets the critical minimum value at a specific distance from crack tip, the crack will initiate to extend at the critical path (Sih 1974). The path of crack growth, θ_c, is showed in Fig. 7 schematically and the SED criterion for this state of the crack under mixed mode loadings will be extended.

The strain energy density factor, S, is the fundamental quantity in the SED criterion and is calculated using the following equation (Sih 1974). In this equation, the effect of moisture and temperature changes is eliminated

$$S = r\frac{dW}{dV} = r\int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \tag{8}$$

Considering the problem hypothesis and conditions of Linear Elastic Fracture Mechanics, Eq. (8) is reduced to

$$S = r\frac{dW}{dV} = \frac{r}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} \sigma_{ij}\varepsilon_{ij} \tag{9}$$

σ_{ij} and ε_{ij} in Eq. (9) are stress and strain state of the problem, respectively. Regarding Eqs. (7)-(9), S restates as

$$\mathrm{S} = A_{11}K_I^2 + 2A_{12}K_IK_{II} \tag{10}$$

In which

$$A_{11} = \frac{1}{4\pi}\left(\frac{C_{11}}{\xi_1^2}{f_{11}}^2 + \frac{C_{22}}{\xi_2^2}{f_{22}}^2 + \frac{C_{66}}{\xi_3^2}{f_{12}}^2 + 2\frac{C_{12}}{\xi_1\xi_2}f_{11}f_{22}\right) \tag{11}$$

$$A_{12} = \frac{1}{4\pi}\left(\frac{C_{11}}{\xi_1^2}f_{11}g_{11} + \frac{C_{22}}{\xi_2^2}f_{22}g_{22} + \frac{C_{66}}{\xi_3^2}f_{12}g_{12} + \frac{C_{12}}{\xi_1\xi_2}(f_{11}g_{22} + f_{22}g_{11})\right) \tag{12}$$

$$A_{22} = \frac{1}{4\pi}\left(\frac{C_{11}}{\xi_1^2}{g_{11}}^2 + \frac{C_{22}}{\xi_2^2}{g_{22}}^2 + \frac{C_{66}}{\xi_3^2}{g_{12}}^2 + 2\frac{C_{12}}{\xi_1\xi_2}g_{11}g_{22}\right) \tag{13}$$

To mathematically derive the SED criterion, three conditions are applied. The first one is $S = \mathrm{S}_c$, that expresses the critical value of the strain energy density factor. The other ones that locate the crack growth are $\partial S/\partial\theta = 0$ and $\partial^2 S/\partial\theta^2 > 0$. S_c is a material constant so that it is obtained by pure mode I conditions in which $K_I = K_{I_c}$ and $K_{II} = 0$. For a crack along the fibers and under pure mode I loading, collinear crack growth occurs. It means $\theta_c = 0$ and

$$\mathrm{S}_c = \frac{K_{I_c}^2}{4\pi}\left(\frac{C_{11}}{\xi_1^2}{f_{11}}^2(0) + \frac{C_{22}}{\xi_2^2}{f_{22}}^2(0) + \frac{C_{66}}{\xi_3^2}{f_{12}}^2(0) + 2\frac{C_{12}}{\xi_1\xi_2}f_{11}(0)f_{22}(0)\right) \tag{14}$$

$f_{11}(0) = f_{22}(0) = 1$ and $f_{12}(0) = 0$ are concluded by using William (1957) expansion. Therefore, S_c equals

$$S_c = \frac{1}{4\pi}\left(\frac{C_{11}}{\xi_1^2} + \frac{C_{22}}{\xi_2^2} + 2\frac{C_{12}}{\xi_1\xi_2}\right) \tag{15}$$

For locating the crack growth, the second and third conditions are applied as follows

$$\dot{A}_{11}K_I^2 + 2\dot{A}_{12}K_IK_{II} + \dot{A}_{22}K_{II}^2 = 0 \tag{16}$$

$$\ddot{A}_{11}K_I^2 + 2\ddot{A}_{12}K_IK_{II} + \ddot{A}_{22}K_{II}^2 > 0 \tag{17}$$

In which $\dot{A}_{ij}$ and $\ddot{A}_{ij}$ are the first and second derivative of A_{ij}, respectively. As a result, the ReiSED criterion is derived as the following equation:

$$A_{11}(\theta)K_I^2 + 2A_{12}(\theta)K_IK_{II} + A_{22}(\theta)K_{II}^2 = \frac{1}{4\pi}\left(\frac{C_{11}}{\xi_1^2} + \frac{C_{22}}{\xi_2^2} + 2\frac{C_{12}}{\xi_1\xi_2}\right){K_{I_c}}^2 \tag{18}$$

Or

$$A_{11}(\theta)K_I^2 + 2A_{12}(\theta)K_IK_{II} + A_{22}(\theta)K_{II}^2 = \frac{1}{4\pi}\left(\frac{C_{11}}{\xi_1^2} + \frac{C_{22}}{\xi_2^2} + 2\frac{C_{12}}{\xi_1\xi_2}\right){K_{I_c}}^2 \tag{19}$$

In this criterion, the coefficient of K_{II}^2 is named as damage factor, $\rho = A_{22}(\theta)/A_{11}(0)$, which depends on the elastic properties of the material as well as the fiber fraction. Moreover, it represents that the fracture toughness of the material is varied at any orientation to the crack tip. Fig. 8 depicts the variation of ρ to the different orientations in Norway spruce wood. It is evident that for the other types of orthotropic materials, the amplitude of curve changes and the total behavior of ρ is the same. For pure mode I, $\theta_c = 0$, the damage factor is

$$\rho = \frac{C_{66}}{\xi_3^2\left(\frac{C_{11}}{\xi_1^2} + C_{22} + 2\frac{C_{12}}{\xi_1}\right)} \tag{20}$$

Damage factor of the SED criterion for orthotropic materials at $\theta_c = 0$ is

$$\rho = \frac{C_{66}}{C_{11} + C_{22} + 2C_{12}} \tag{21}$$

Regarding the principle role of damage factor on fracture behavior of materials and comparing Eq. (20) and Eq. (21), it reveals ξ_i has a dominant effect on ρ.

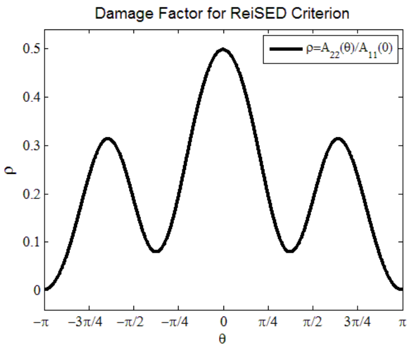
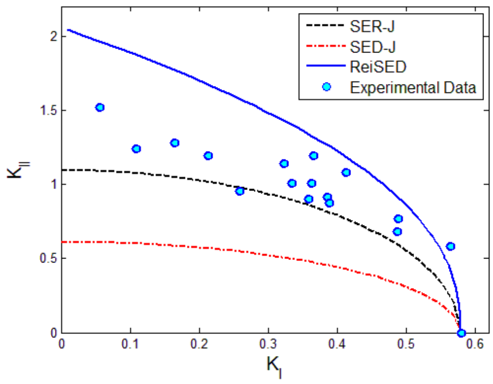
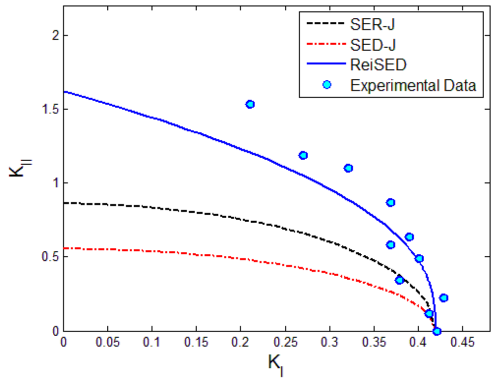


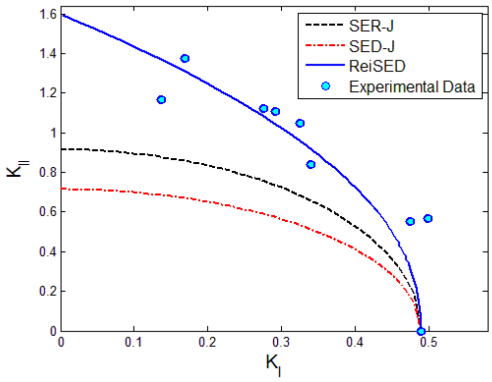
Fig. 8 Dependency of the damage factor to crack growth path in Norway Spruce sample



(a) Norway Spruce



(b) Red Spruce



(c) Scot Pine

Fig. 9 Fracture limit curve of ReiSED and the classic criteria including with experimental data

4. Result and discussion

4.1 Results

Fig. 9 depicts the fracture limit curves pertinent to ReiSED and the energy-based criteria of Jernkvist (2001a) (minimum strain energy density and maximum strain energy release rate hereafter referred to as SED-J and SER-J, respectively) in order to compare and evaluate the criterion by experimental data of three types of wood (Jernkvist 2001a). The properties of wooden materials are found in (Jernkvist 2001a).

4.2 Discussion

The SED criterion is of remarkable importance in industrial problems due to its significantly precise prediction of brittle isotropic materials. However, Fig. 9 illustrates that SED-J is the most conservative fracture limit curve. This can be for several reasons. First, in isotropic materials, a crack under mixed-mode loadings kinks, which needs more energy. In contrast, the collinear crack growth assumption states that the total energy are employed to tear the plane and to distribute the crack. Therefore, material failure happens at low toughness. Second, SED-J strongly depends on the shear stiffness through compliance elements, C_{12} and particularly C_{66}, which play the dominant role in ρ. Damage factor for SED-J is obtained as the following equation

$$\rho = \frac{C_{66}}{f_{11}(0)^2 C_{11} + C_{22} + 2f_{11}(0)C_{12}} \tag{22}$$

While in this criterion the collinear crack growth assumption neglects the shear resistance effects in material fracture. Thus, the total energy consumes to distribute the crack and overcome the tension resistance; nevertheless, under mixed-mode loadings, crack is inevitably affected by shear stress that cannot be omitted. Third, C_{12} element in ρ indicates the interaction of tension and shear stiffness, which contradicts the initial assumption of the collinear crack growth as well. Forth, according to the basic hypothesis of the SED criterion, $\theta_c = 0$ is not the theoretical, critical crack growth for a crack under mixed-mode loadings.

Although SER-J is an energy-based criterion, it fails to anticipate the fracture initiation correctly. It seems the assumptions are the main reason for its inefficiency. First off, it is assumed that crack growth occurs in the perfectly brittle material, and the available energy is applied to distribute the crack entirely. Accordingly, plenty of parameters that dissipate the energy are neglected, such as FPZ creation at the crack tip. Also, the collinear crack growth is the other reason of conservatism of this criterion (despite the principle concept of SER criterion) because conforming to the Griffith theory, crack distributes in a plane with the normal of equivalent critical stress. The equivalent critical stress is the tension stress that has the same effect of mixed-mode loadings in the solid. Indeed, at mixed-mode loading, the plane perpendicular to the equivalent critical loadings is not the plane along the main crack. Therefore, it is expected that SER-J is not a proper criterion. However, the ReiSED criterion, including ReSt coefficients, is appropriately compatible with the experimental data.

The distinct difference of this criterion is the fiber fraction in its damage factor. According to the damage factors of derived criterion, it is concluded that ξ_3 has a significant impact on the fracture behavior of materials under mixed-mode loadings. This criterion predicts that the more value of ξ_3 causes fibers have more effect on matrix failure by bearing shear loadings. It is definite that shear fracture that leads fiber breakages at different orientations to the main crack with a rough surface (Jernkvist 2001b) dissipates more energy than crack growth in the matrix via tension stress. The more value of ξ_3 means the most parts of energy of the body is applied to tear the planes apart by shear leads requiring more energy for material failure because the shear breakage of fibers needs more energy than the shear failure of isotropic matrix. ReSt coefficients balance the required energy for shear fracture and the other factors in fracture via fiber fraction. Therefore, these coefficients cause the ReiSED criterion encompasses the effects of shear and tension stress and their interactions in fracture under mixed-mode loadings and predicts the fracture behavior of the orthotropic materials precisely.

5. Conclusions

According to the reinforced isotropic matrix composite materials, the new model is applied to investigate the fracture of anisotropic materials. In this model, fibers act as a reinforcement matrix, which reduced the stress exerted on the matrix. The tension and shear reinforcement effects of fibers are calculated by reinforcement coefficients called ReSt. These coefficients depend on elastic properties and the fiber fraction of the material. Based on the observation, that ReiSED criterion finely fits the experimental data of the well-known conducted tests by the scholars, the following conclusions are:

1. The postulate, that the crack kinks in the matrix and distributes along the path predicted by the ReiSED criterion is proved. It is concluded that the crack growth onset occurs at the particular orientation predicted by fracture criteria. It grows along the fiber, that is observed in the tests conducted by the Fakoor *et al.* (2018).
2. Fiber reinforcement coefficients at tension along the fiber and shear loadings, ReSt coefficients, which depend on elastic properties and fiber fraction of the composite body, play a critical role in anticipating the fracture behavior of these materials.
3. According to the damage factor presented at the text, ξ_3 coefficient, which is indicative of shear effects of the reinforced isotropic matrix, has a noticeable impact on the mixed-mode fracture. It is shown where the shear loading has a dominant influence on the fracture behavior of these materials; the ReiSED criterion precisely predicts the behavior.
4. Also, based on the damage factor presented at the text, it is concluded that the fracture toughness of the crack tip, is various at the different orientation to the main crack in the matrix and depends on the ReSt coefficients (elastic properties and fiber fraction) as well.

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